Journal of Taibah University for Science

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/tusc20

On designing an assorted control charting approach to monitor process dispersion: an application to hard-bake process

Nasir Abbas, Usman Saeed, Muhammad Riaz & Saddam Akber Abbasi

To cite this article: Nasir Abbas, Usman Saeed, Muhammad Riaz & Saddam Akber Abbasi (2020) On designing an assorted control charting approach to monitor process dispersion: an application to hard-bake process, Journal of Taibah University for Science, 14:1, 65-76, DOI: 10.1080/16583655.2019.1703481

To link to this article: https://doi.org/10.1080/16583655.2019.1703481

Published online: 19 Dec 2019.

JOURNAL OF TAIBAH UNIVERSITY FOR SCIENCE
2020, VOL. 14, NO. 1, 65–76
https://doi.org/10.1080/16583655.2019.1703481

RESEARCH ARTICLE

OPEN ACCESS | Check for updates

On designing an assorted control charting approach to monitor process dispersion: an application to hard-bake process

Nasir Abbas [a], Usman Saeed [a], Muhammad Riaz [a] and Saddam Akber Abbasi [b]

[a]Department of Mathematics & Statistics, King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia; [b]Department of Mathematics, Statistics and Physics, Qatar University, Doha, Qatar

ABSTRACT

The monitoring of process variability is an important feature to get optimal output from a process. Control charts are vital tools used for efficient process monitoring. The commonly used types of charts include Shewhart, cumulative sum and exponentially weighted moving average (EWMA) control charts. This study focuses on dispersion control charts using some efficient transformation for small and medium instabilities. We intend to propose an assorted method to monitor a range of disturbances in process dispersion, using the well-known max approach. We have used several measures to evaluate the suggested assorted control chart. Based on these measures, we have compared the proposed assorted method with many existing charts. The study proposal outperforms the existing counterparts in detecting various amounts of shifts in process dispersion. Finally, a real-life application of the proposed chart is demonstrated to monitor the flow width measurements in a hard-bake process.

ARTICLE HISTORY
Received 5 September 2019
Revised 5 December 2019
Accepted 7 December 2019

KEYWORDS
Average run length; control chart; cumulative sum; dispersion monitoring; extra quadratic loss; exponentially weighted moving average

1. Introduction

A series of activities that takes some inputs and results into some output is termed as a process. Each process has its own dynamics that lead to a targeted output. For instance, a manufacturing process differs from a mechanical process in terms of procedures and inputs/outputs. The stability of any process is one of the major concerns for the continuity of the process. The process stabilization is generally required immediately with the beginning of mass production in any process. Otherwise, there may be a lot of waste output from the process that leads to process deterioration. The list of such processes may include business process, health process, banking process, among others. One such process is the hard-bake process. The hard-bake is a common method used to stabilize the printed features for providing optimum performance at etches for the photoresist pattern. In this process, one potential variable of interest is flow width measurements of wafers (X) that needs timely attention for effective performance of the process. We will propose an efficient monitoring mechanism of such a process in this study.

In a standard statistical process control (SPC) environment, we focus on removing the assignable cause(s) after the detection of any out of control (OoC) signals. There are many sources of variability in the processes, and SPC is a very handy tool to differentiate between natural and unnatural variations. The quality of a process is determined by different parameters such as location, shape and dispersion. The dispersion parameter is of prime importance as the stability of other parameters (like location) depends on dispersion. Generally, dispersion charts are used for two main reasons (i) if the variation in the process increases, there is a possibility that more defective units will be produced (ii) if the variation in the process decreases then more units will be near the target value and hence process capability will also increase. These changes can be quickly detected by the dispersion charts. These charts are important while interpreting the results of a location chart because they assume that standard deviation remains constant.

The Shewhart range (R) chart and the Shewhart standard deviation (S) chart are used to monitor the process variability of small subgroup sizes [1] introduced the cumulative sum (CUSUM) chart to monitor process variability among different subgroups. Many authors have evaluated the performance of CUSUM and exponentially weighted moving average (EWMA) control charts that were based on the subgroup standard deviation (cf. [2–9] and [10]). One-sided EWMA control chart based on the natural log was suggested by [11] to monitor subgroup variance [12] have proposed a CUSUM chart based on the logarithmic transformation of the subgroup variance [13] proposed a MaxMin EWMA chart to monitor process variability [14] have discussed and compared several control charts to monitor variation in the process [15] introduced a maximum (MAX) control chart to monitor the centre and spread of the variable simultaneously [16] proposed a MAX-EWMA control chart that can be used to monitor the process

CONTACT Nasir Abbas nasirabbas55@yahoo.com

location and dispersion simultaneously. The chart was based on plotting the Maximum (MAX) of the standardized location and dispersion statistics against a single control limit [17] extended the idea of MAX statistic to a CUSUM control chart and showed that their proposed method works better than the MAX-EWMA [18] proposed a new two-sided S^2 chart based on logarithmic transformation for monitoring variation in the process. A new CUSUM-S^2 to monitor the process variation was proposed by [19,20] introduced some useful control charts for location based on different sampling schemes [21] used different methods to increase the sensitivity of the mixed EWMA-CUSUM control charts for the location parameter.

Inspired by these improvements and modifications, this study proposes a new assorted method that can be used to identify small, intermediate and large shifts (in process dispersion) simultaneously. The proposal will serve as a single charting structure, instead of having multiple designs, and hence it will provide ease in the implementation for practitioners.

From here on, the paper is structured as the classical charts for dispersion and some of their modifications are presented in section 2; measures used to evaluate and compare the performance of different charts are given in section 3; the proposed assorted method and its construction is discussed in section 4; section 5 provides the performance of the proposed assorted method along with its comparison with the counterparts; section 6 is about how the proposed methods are applied in a real situation; section 7 gives the concluding remarks.

2. The control charts for process dispersion

Shewhart, CUSUM and EWMA are the 3 classical approaches to monitor the process dispersion. Below is a brief outline of these and some other modified charts. Assume X being a random variable following a normal distribution with mean (μ_0) and given a standard deviation (σ_0) i.e. $X_{ij} \sim N(\mu_0, (\delta\sigma_0^2))$ where $i = 1, 2, 3, \ldots\ldots$ and $j = 1, 2, \ldots\ldots, n$. where n is the size of the i^{th} sample. The disturbed/dislocated dispersion in a shifted process is denoted by σ_1 and is described as $\sigma_1 = \delta\sigma_0$.

2.1. The Shewhart R chart

The Shewhart R chart (cf. [22]) is used to monitor the process variability for small group sizes (sample size less than or equal to 10). Let $R_1, R_2, \ldots\ldots, R_m$ be the sample ranges of m samples.

Define $R = W\sigma$, where σ is the notation used for the standard deviation of the process and the standard error of R is given as

$$\sigma_R = d_3\sigma,$$

where d_3 is the standard error of the statistic W. The general upper control limit (UCL) of the R chart is

$$UCL = \bar{R} + Ld_3\frac{\bar{R}}{d_2}$$

Similarly, the 3-sigma UCL of the R chart is

$$UCL = \bar{R} + 3d_3\frac{\bar{R}}{d_2}$$

The process declares an OoC if $R_i > UCL$. where $i = 1, 2, 3, \ldots.$.

2.2. The Shewhart S chart

The Shewhart S chart (cf. [22]) is used to monitor the standard deviation (σ) in the process. Assume that at disposition there are m preliminary samples each of size n, and let S_i be the standard deviation of the ith sample. Then the average of m standard deviation is defined as

$$\bar{S} = \frac{1}{m}\sum_{i=i}^{m} S_i$$

The general and 3-sigma UCLs of the Shewhart S chart is

$$UCL = \bar{S} + L\frac{\bar{S}}{c_4}\sqrt{1 - c_4{}^2}, \quad UCL = \bar{S} + 3\frac{\bar{S}}{c_4}\sqrt{1 - c_4{}^2}$$

If S_i fall outside the UCL, then the process is considered as OoC.

2.3. The EWMA lnS^2 control chart

For the monitoring of process variance, [11] applied the EWMA scheme to the normal approximation of natural logarithmic $\left(\frac{S^2}{\sigma_0{}^2}\right)$ where $\sigma_0{}^2$ is the IC process variance. To enhance the efficiency for monitoring the process variability, they re-adjust the EWMA statistic to 0 if it is less than 0. The re-adjustment of smaller EWMA statistics to 0 may improve the EWMA statistic inertia problem and increase its detection ability. They used the following EWMA statistic

$$EWMA_i = \max\{(1-\lambda)EWMA_{i-1} + \lambda ln(S_i^2), ln(\sigma_0^2)\}$$

where $EWMA_0 = \ln(\sigma_0^2)$, λ is the smoothing constant, $\sigma_0^2 = 1$ and S_i^2 is the sample variance. The upper control limit of the EWMA statistic is

$$UCL = L\sigma_{EWMA}$$

where L is the charting constant and

$$\sigma_{EWMA} = \sqrt{\frac{\lambda}{2-\lambda}\left[\frac{2}{n-1} + \frac{2}{(n-1)^2} + \frac{4}{3(n-1)^3} - \frac{16}{15(n-1)^5}\right]}$$

We have OoC signals if $EWMA_i$ is greater than UCL.

2.4. The CUSUM InS^2 control chart

[12] proposed one-sided the CUSUM $\ln S^2$ control chart to monitor process variance. The CUSUM statistic used in their study is given by

$$C_i = \max\{0, lnS_i^2 - k + C_{i-1}\}, \quad i = 1, 2, \ldots.$$

where $C_0 = u$ for $0 \leq u < h$ and S_i^2 is the sample variance. The OoC signal is issued as soon as $C_i > h$.

2.5. The χ-CUSUM control chart

[23] proposed the CUSUM control chart based on a chi-square transformation for the monitoring of process variability. They proved that $\left(\frac{\chi_n^2}{n}\right)^{\frac{1}{3}}$ is approximately normal distribution with mean $1 - 2/(9n)$ and variance $2/(9n)$. Furthermore, if the observations are independentally and identically distributed $N(\mu, \sigma)$, then

$$\chi_i = \frac{\left(\frac{S_i^2}{\sigma_0^2}\right)^{\frac{1}{3}} - \left(1 - \frac{2}{9(n-1)}\right)}{\sqrt{\frac{2}{9(n-1)}}},$$

will an approximately standard normal distribution when $\sigma = \sigma_0$. Based on the said transformation, a CUSUM statistic by [14] is given as

$$C_i^+ = \max\{0, \chi_i - k + C_{i-1}^+\}$$

where $C_0^+ = 0$ and k is the reference value. The control limit of this statistic if h.

2.6. The P_σ CUSUM control chart

The inverse normal transformation was applied by [14] to design a P_σ CUSUM control chart. The following statistic is used in this study

$$C_i^+ = \max\{0, V_i - k + C_{i-1}^+\}$$

where

$$V_i = \phi^{-1}\left\{F_{\chi^2_{n-1}}\left(\frac{(n-1)S_i^2}{\sigma_0^2}\right)\right\},$$

$\phi^{-1}(.)$ is the inverse cumulative distribution function (CDF) of the normal distribution, $F_{\chi^2_{n-1}}(.)$ is the CDF of chi-square distribution with $n - 1$ degrees of freedom.

2.7. The CUSUM R control chart

[1] proposed a CUSUM chart, based on the subgroup range, to monitor the process variability. The plotting statistic used in this study is $S_r = \sum_{i=1}^{r}(x_i - k)$. The quantity k is called the reference value and h is the control limit for the chart.

2.8. The CUSUM S control chart

[2] proposed a CUSUM S control chart to monitor the process dispersion. The statistic used in this study is given as follows:

$$C_i = \max\{0, S_i - k + C_{i-1}\}, \quad i = 1, 2, \ldots.$$

where S_i is the sample standard deviation, $C_0 = 0$ and k is the reference value. Immediate corrective action is taken if $C_i > h$, where h is the decision interval.

3. Performance evaluation

This section gives a brief framework of the measures used for the evaluation of proposed and other control charts under discussion. Let X be a random variable that follows a normal distribution i.e. $X_{ij} \sim N(\mu_0, (\delta\sigma_0^2))$, $i = 1, 2, \ldots\ldots..$ and $j = 1, 2, \ldots., n$. Here it should be observed: $\delta = 1$ corresponds to a situation where the process is in control (IC); $\delta \neq 1$ corresponds to an OoC situation.

From the above notations, we can define the shift as $\delta = \frac{\sigma_1}{\sigma_0}$ where σ_0 and σ_1 are IC and OoC standard deviations, respectively, and n is the sample size.

Using these terms, some performance measures are discussed hereafter.

Run Length (RL): It is the number of observed subgroups before a signal is received by the chart. ($1 \leq RL < \infty$). The value of RL is desired to be large under IC situation. On the contrary, when the process is OoC, we want the value of RL to be small.

Average run length (*ARL*): A very well-known measure to determine the effectiveness of a chart is named *ARL*. It is the average number of observed subgroups before a signal is received by the chart. In addition, *ARL* is classified into two types, the IC *ARL* that is denoted by ARL_0 and the OoC *ARL* that is denoted by ARL_1. ARL_0 of a chart requires to be maximized because it indicates the average number of subgroups until a false alarm is received. On the contrary ARL_1 needs to be minimized as it tells us about how quickly a chart detects the OoC situation.

Standard deviation run length (*SDRL*): The consistency of RL is studied by SDRL. Mathematically, $RL = \sqrt{E(RL^2) - (ARL)^2}$.

Extra quadratic loss (*EQL*): It is the weighted average *ARL* of a chart in a given interval of shifts from $\delta_{\min}$ to $\delta_{\max}$ (see [24] for more details). Mathematically , *EQL* is defined as

$$EQL = \frac{1}{\delta_{\max} - \delta_{\min}} \int_{\delta_{\min}}^{\delta_{\max}} \delta^2 ARL(\delta) d\delta.$$

Sequential extra quadratic loss (*SEQL*): *SEQL* relates to a certain shift (say δ_i) of the *EQL*, mathematically described as

$$SEQL_i = \frac{1}{\delta_i - \delta_{\min}} \int_{\delta_{\min}}^{\delta_i} \delta^2 ARL(\delta) d\delta,$$

where $i = 2, 3, \ldots\ldots, \delta_{\max}$ (cf. [25]).

Relative average run length (*RARL*): It is the weighted average *ARL* of a chart relative to the benchmark chart in a given interval of shifts from $\delta_{\min}$ to $\delta_{\max}$. It measures the efficiency of a chart in terms of its *ARL* benchmark for each value of shift (cf. [24]). Mathematically, *RARL* is defined as

$$RARL = \frac{1}{\delta_{max} - \delta_{min}} \int_{\delta_{min}}^{\delta_{max}} \frac{ARL(\delta)}{ARL_{benchmark}(\delta)} d\delta,$$

where $ARL(\delta)$ is the *ARL* of the chart under study and $ARL_{benchmark}(\delta)$ is the *ARL* of benchmark chart for a given value of δ. A chart having a minimum *EQL* is generally set to be the benchmark. This implies that *RARL* value for the benchmark chart will always be 1 and *RARL* will be greater than 1 for other charts under study.

Sequential relative average run length (*SRARL*): *SRARL* is the *RARL* of chart up to a certain value of δ (say δ_i), mathematically described as

$$RARL = \frac{1}{\delta_i - \delta_{\min}} \int_{\delta_{\min}}^{\delta_i} \frac{ARL(\delta)}{ARL_{benchmark}(\delta)} d\delta,$$

where $i = 2, 3, \ldots\ldots, \delta_{max}$ (cf. [25]).

4. The design structure of the one-sided $S^2 - Assorted_{k,\lambda}$ chart

The design structure of the one-sided $S^2 - Assorted_{k,\lambda}$ chart is discussed in this section to monitor process variations where the goal is to target a range of shifts (that includes small, medium and large). The proposed $S^2 - Assorted_{k,\lambda}$ is designed for upward detection in the process variability.

The sample variance is defined as

$$S_i^2 = \frac{\sum_{i=1}^{n} (X_i - \bar{X}_i)^2}{n-1},$$

We define the following statistic that may be used for the monitoring of process variance

$$V_i = \phi^{-1}\left\{F_{\chi^2_{n-1}}\left(\frac{(n-1)S_i^2}{\sigma_0^2}\right)\right\} \sim N(0,1)$$

For large shifts in the process dispersion, we define a Shewhart's-type statistic U_{1i} as

$$U_{1i} = \frac{V_i}{C_s} \quad (1)$$

where c_s is the control limit coefficient for the *Shewhart* control chart.

For moderate shifts in the process dispersion, we define a CUSUM type statistic U_{2i} as

$$U_{2i}^+ = CUSUM_i^+ / h_c \quad (2)$$

where $CUSUM_i^+ = max[0, V_i - k + CUSUM_{i-1}^+]$ is the CUSUM statistic and h_c is its control limit.

Similarly, for small shifts in the process dispersion, we define a EWMA-type statistic U_{3i} as

$$U_{3i} = \frac{EWMA_i}{L_e\sqrt{\frac{\lambda}{2-\lambda}[1-(1-\lambda)^{2i}]}} \quad (3)$$

where $EWMA_i = \lambda V_i + (1-\lambda)EWMA_{i-1}$ is the EWMA statistic and L_e is its control limit. The value sensitivity parameter λ lies between 0 and 1.

Finally, using the max approach, we define the plotting statistic of the proposed chart as

$$U_i = \max(U_{1i}, U_{2i}^+, U_{3i}) \quad (4)$$

U_i in Equation (4) will always have a positive value. Accordingly, the control limit is set as

$$UCL = 1. \quad (5)$$

For any subgroup if U_i exceeds 1, it implies an OoC signal from the proposed assorted chart.

Justification for UCL = 1: The justification for selecting 1 as UCL is outlined below:

$U_i = \max(U_{1i}, U_{2i}^+, U_{3i})$ (cf. Equation (4)), so $U_i > 1$ indicates the following:

(i) either $U_{1i} > 1$ (cf. Equation (1)) $\Rightarrow$ the Shewhart statistic V_i has exceeded its control limits that are denoted by c_s;
(ii) and/or $U_{2i}^+ > 1$ (cf. Equation (2)) $\Rightarrow$ the CUSUM statistic $CUSUM_i$ has exceeded its control limits that are denoted by h_c;
(iii) and/or $U_{3i} > 1$ (cf. Equation (3)),$\Rightarrow$ the EWMA statistic $EWMA_i$ has exceeded its control limits that are denoted by L_e;

The design parameters $S^2 - Assorted$ are k and λ i.e. these two quantities identify how sensitive the proposed chart is for a different amount of shifts. The amount of shift and the corresponding optimal design parameters are portrayed in Table 1.

Table 1. Ranges of sensitivity parameters for different categories of shift for the $\boldsymbol{S^2} - \boldsymbol{Assorted}$ chart.

	Category of shift		
Sensitivity parameter	Small	Medium	Large
Λ	0.05 to 0.15	0.16 to 0.25	0.4 to 1
k	0.1 to 0.25	0.26 to 0.5	More than 0.5

After choosing the design parameters, the next task is to set the three limit coefficients (h_c, L_e, and c_s) so that the overall *ARL* of the chart is fixed at the desired level. For the said purpose we have implemented the following criteria:

Objective: The objective is to minimize *EQL*.

Constraints: The constraints are overall ARL_0 is fixed at the desired level and *ARL*s of three individual charts (i.e. Shewhart, CUSUM and EWMA) are equal. Mathematically, $ARL_0 =$ such that $ARL_{Shewhart} = ARL_{EWMA} = ARL_{CUSUM}$.

For an IC situation, we need to adjust the limits coefficients (h_c, L_e and c_s) of $S^2 - Assorted_{k,\lambda}$ chart under the said criteria to fix ARL_0 at 200. For the said purpose, we have chosen 15 combinations of the design parameters k and λ covering a wide range of shifts. For a fixed $ARL_0 = 200$ the three limit coefficients are selected in such a way that equal contribution is coming from all the three charts i.e. the individual ARL_0 values for three charts are exactly the same. Fifteen combinations of the design parameters and the corresponding control limit coefficients are given in Table 2, where the optimal choice of parameters is indicated by bold values and the overall ARL_0 is fixed at 200.

Special Cases: Following charts are a special case of the proposed $S^2 - Assorted_{k,\lambda}$:

- *Shewhart's* (S^2) *chart, when* h_c and L_e approach ∞;
- *CUSUM* (S^2) *chart, when* c_s and L_e approach ∞;
- *EWMA* (S^2) *chart, when* c_s and h_c approach ∞.
- *Combined Shewhart's CUSUM* (S^2) *chart, when* L_e approaches ∞.
- *Combined Shewhart's EWMA* (S^2) *chart, when* h_c approaches ∞.

It is to be mentioned that the current study is designed under normality; however, one may extend it for other distributional environments, such as Weibull, Burr, Power, etc. (cf. [26,27] and [28])

Table 2. Charting constant at $ARL_0 = 200$.

			$ARL_0 = 200$		
Case	k	λ	h_c	L_e	c_s
1	0.1	0.25	11.3000	2.7000	2.8230
2		0.40	11.3000	2.7900	2.8300
3		0.55	11.3000	2.8000	2.8300
4	0.25	0.25	6.9500	2.7000	2.8300
5		0.40	6.9500	2.7900	2.8300
6		0.55	6.9500	2.8020	2.8300
7	0.5	0.05	4.2490	2.2150	2.8350
8		0.4	4.2470	2.7900	2.8300
9		0.55	4.2100	2.7950	2.8200
10	**1**	**0.05**	**2.2298**	**2.2100**	**2.8295**
11		0.15	2.2260	2.5700	2.8100
12		0.55	2.2160	2.7400	2.7600
13	1.5	0.05	1.3700	2.3500	2.7000
14		0.15	1.3900	2.5400	2.8000
15		0.25	1.3900	2.6000	2.8100

5. Performance evaluations

This section discusses the efficiency assessments and comparisons of the $S^2 - Assorted_{k,\lambda}$ chart and some existing charts. The existing charts comprise *Shewhart R*, *Shewhart S*, *EWMA* $\ln S^2$, *CUSUM* $\ln S^2$, *CUSUM R*, χ *CUSUM*, P_σ *CUSUM*, and *CUSUM S* charts. A number of performance measures including *ARL*, *EQL*, *SEQL*, *RARL*, and *SRARL* have been used. To evaluate these measures, we addressed several OoC situations in an attempt to evaluate these measures by considering variable shifts (small, moderate and large) ranging from 1 to 3.

The algorithm for the computation of these measures is given as follows:

(i) A random sample of size n is generated from a normal distribution with mean μ and variance σ^2;
(ii) set the control limit coefficients using implementation criteria given in Section 4;
(iii) calculate the plotting statistic U_i using Equation (4);
(iv) based on the choices of λ and k, execute RL operational measure using steps (i)–(iii), (cf. Tables 1 and 2);
(v) calculate the *ARL* and *SDRL* by repeating step (vi) and generating a distribution of RLs;

Furthermore, [25] discussed the procedure to calculate *SEQL* and *SRARL*.

5.1. Performance evaluation of the proposed $S^2 - Assorted_{k,\lambda}$ chart

This section provides the assessment of the proposed $S^2 - Assorted_{k,\lambda}$ chart in terms of measures like *ARL* and *EQL*. Many (k, λ) pairs are studied at different levels of δ_1. The results are given in Table 3 at $ARL_0 = 200$. We have presented the *ARL* graphs in Figure 1 and Table 3. The findings support the following:

- The case 10 is the optimal choice because it has a minimum ARL_1 for varying shifts (cf. Figure 1(d)) and $EQL = 20.18$ (cf. Table 3). The charting constants of this case are ($h_c = 2.2298, L_e = 2.2100, c_s = 2.8295$) with sensitivity parameters $k = 1.00$ and $\lambda = 0.05$ at $ARL_0 = 200$.
- Four different types of charts are portrayed in Figure 1: Figure 1(a) shows a contrast of *ARL* values at $ARL_0 = 200$ with $\lambda = 0.40$ and varying k ($k = 0.1, k = 0.25$ and $k = 0.5$) for shift (δ) ranges from 1.05 to 1.5. Figure 1(b) shows a contrast of *ARL* values at $ARL_0 = 200$ with the fixed value $k = 1.00$ and varying $\lambda(\lambda = 0.05, \lambda = 0.15$ and $\lambda = 0.55)$ for shift (δ) ranges from 1.05 to 1.5. Different k and λ are used for various amounts of shifts and vice versa (cf. Figure 1(c,d)). The results depicted that the $S^2 - Assorted_{1.00,0.05}$ has a minimum ARL_1.

Table 3. ARL and EQL of the S^2 − ***Assorted*** chart for Case 1 to Case 15.

Shift	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1.01	154.87	158.59	158.34	163.28	161.69	163.52	159.05	177.45	170.15	**154.82**	163.32	176.85	160.78	171.29	168.52
1.02	126.47	129.92	129.80	133.59	132.53	135.68	128.38	148.82	144.22	**125.48**	136.22	153.76	134.06	140.80	143.79
1.03	102.98	106.54	107.77	111.60	110.63	114.43	106.60	123.77	122.23	**101.06**	113.73	134.89	110.78	117.45	119.06
1.04	88.86	89.94	93.05	95.42	93.76	95.72	89.05	109.53	103.41	**87.49**	97.20	114.46	92.76	101.07	104.67
1.05	74.01	77.01	77.13	79.35	81.89	80.29	74.06	91.45	90.83	**74.01**	81.80	98.48	76.28	84.04	87.44
1.10	39.77	40.54	40.74	40.63	41.21	41.24	38.32	47.76	47.02	**37.88**	41.96	52.81	38.18	41.59	45.39
1.15	25.22	25.88	26.58	24.65	24.72	25.44	23.80	27.75	27.45	**23.15**	25.32	32.01	23.90	25.16	26.92
1.20	17.59	18.36	19.01	17.06	17.49	17.66	16.61	18.31	18.09	**16.27**	17.33	21.38	16.70	16.92	17.80
1.25	13.25	13.74	14.22	12.70	12.93	13.06	12.49	13.24	13.44	**12.04**	12.39	14.82	12.55	12.25	12.88
1.30	10.36	10.76	11.34	9.85	10.21	10.38	9.99	10.08	10.27	**9.24**	9.54	11.03	10.04	9.77	10.07
1.35	8.31	8.72	9.30	8.14	8.22	8.51	8.21	8.15	8.03	**7.75**	7.82	8.68	8.35	7.83	8.03
1.40	6.93	7.28	7.67	6.81	6.82	7.08	6.98	6.69	6.72	**6.38**	6.55	6.99	7.04	6.59	6.58
1.45	5.88	6.18	6.42	5.80	5.89	6.06	5.94	5.79	5.83	**5.35**	5.67	5.80	6.06	5.66	5.59
1.50	5.13	5.30	5.49	5.10	5.13	5.27	5.26	5.02	4.94	**4.80**	4.94	5.09	5.34	4.98	4.89
1.55	4.51	4.69	4.79	4.49	4.50	4.65	4.63	4.43	4.42	**4.25**	4.36	4.42	4.67	4.36	4.32
1.60	4.01	4.08	4.21	4.06	4.05	4.13	4.17	3.97	3.96	**3.85**	3.92	3.96	4.22	4.01	3.88
1.65	3.69	3.69	3.84	3.63	3.66	3.72	3.81	3.62	3.53	**3.44**	3.56	3.52	3.76	3.61	3.53
1.70	3.36	3.36	3.43	3.35	3.31	3.36	3.49	3.31	3.26	**3.11**	3.24	3.20	3.44	3.31	3.22
1.75	3.06	3.10	3.15	3.08	3.04	3.08	3.17	3.00	3.00	**2.90**	2.97	2.94	3.14	3.05	2.97
1.80	2.87	2.87	2.88	2.88	2.80	2.87	2.96	2.84	2.79	**2.64**	2.73	2.73	2.98	2.81	2.79
1.85	2.69	2.68	2.63	2.67	2.62	2.65	2.78	2.64	2.61	**2.53**	2.58	2.54	2.73	2.61	2.57
1.90	2.50	2.45	2.56	2.52	2.50	2.50	2.58	2.46	2.46	**2.32**	2.41	2.37	2.54	2.48	2.44
1.95	2.40	2.34	2.37	2.38	2.33	2.34	2.47	2.33	2.29	**2.22**	2.32	2.24	2.40	2.32	2.28
2.00	2.24	2.22	2.27	2.23	2.18	2.21	2.31	2.19	2.21	**2.10**	2.19	2.12	2.26	2.20	2.17
EQL	21.27	21.83	22.30	21.57	21.71	22.00	21.17	23.19	22.90	**20.18**	21.58	24.64	21.42	21.85	22.44

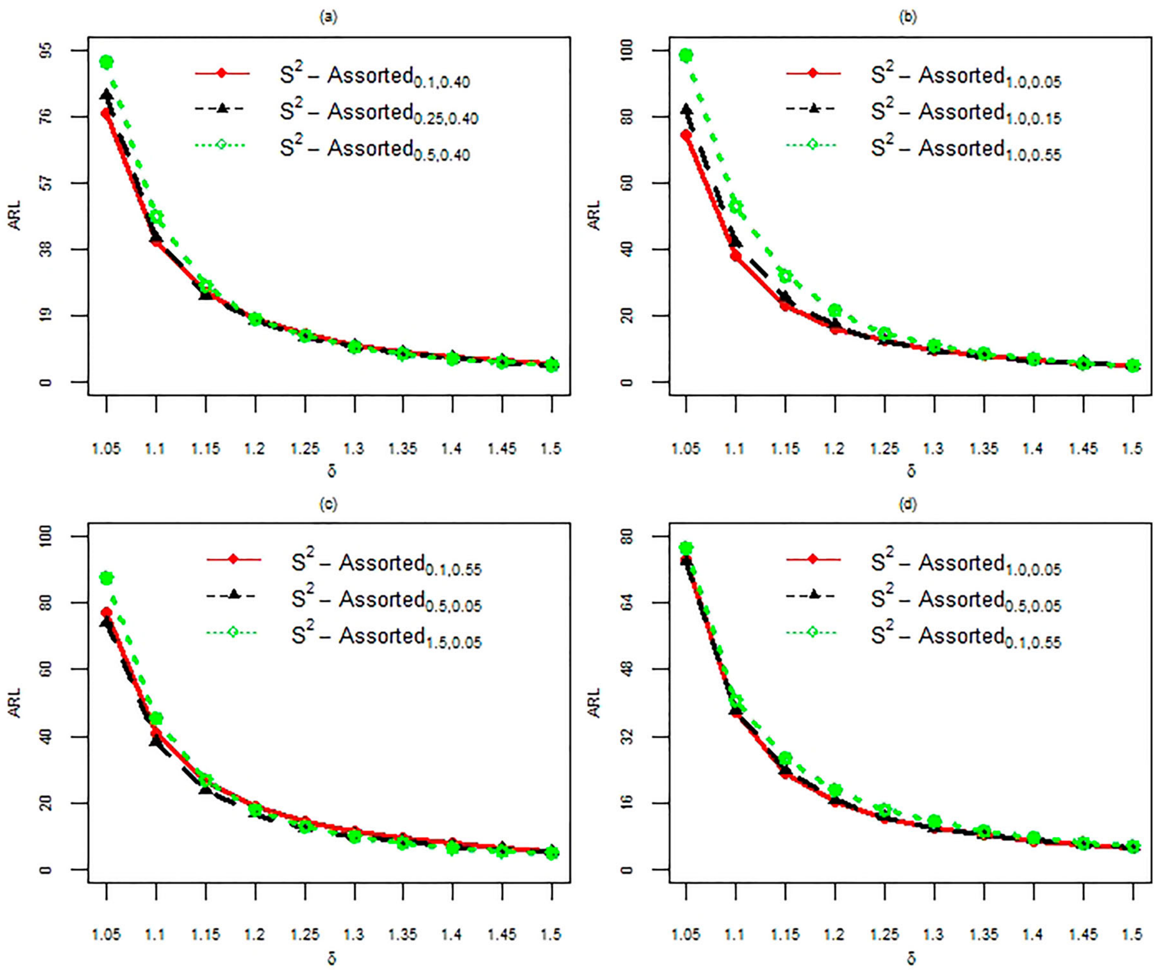

Figure 1. ARL comparison of the S^2 − $\boldsymbol{Assorted}_{k,\lambda}$ chart for: (a) varying values of $\boldsymbol{k}$ and fixed λ at $\boldsymbol{ARL}_0 = 200$; (b) varying values of λ and fixed $\boldsymbol{k}$ at $\boldsymbol{ARL}_0 = 200$; (c) varying values of λ and $\boldsymbol{k}$ at $\boldsymbol{ARL}_0 = 200$; (d) varying values of $\boldsymbol{k}$ and λ at $\boldsymbol{ARL}_0 = 200$.

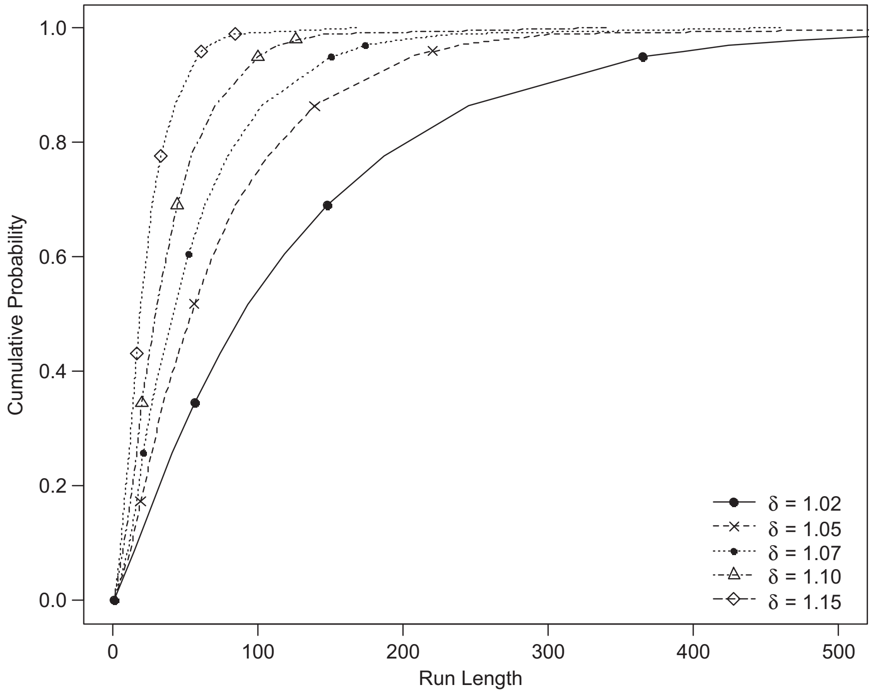

Figure 2. Run length curves of the S^2 − ***Assorted*** $_{k,\lambda}$ chart for: (a) varying values of δ for Case 10.

- To get further insight, Figure 2 presents run-length curves for Case 10 (optimal choice) at different levels of shift (δ). Figure 2 shows that for the proposed S^2 − *Assorted* chart, the probability for shorter run lengths increases as the shift level gets higher. This helps in quicker detection with an increase in the magnitude of shifts.
- The sensitivity of the proposed $S^2 - Assorted_{k,\lambda}$ chart improves with a reduction in λ at a particular selection of k and it is valid for all k attributes.
- The sensitivity of the proposed $S^2 - Assorted_{k,\lambda}$ chart improves with a reduction in k at a particular selection of λ and it is valid for all values of λ (cf. Table 3).

5.2. Comparative analysis

The efficiency assessments and contrasts of the $S^2 - Assorted_{k,\lambda}$ with contending charts including *Shewhart R*, *Shewhart S*, *EWMA* lnS^2, *CUSUM* lnS^2, *CUSUM R*, χ *CUSUM*, P_σ *CUSUM*, and *CUSUM S* are discussed in this section. The comparative assessment is based on two techniques: firstly, based on individual measures (such as *ARL*); secondly, based on overall measures (such as *SEQL*, *EQL*, *RARL* and *SRARL*). In addition to assessing these measures, we have discussed distinct OoC scenarios by considering the different amounts of shift (δ) (cf. Table 4)

Table 4. Performance comparison based on ARL, EQL and RARL of the S^2 − ***Assorted*** and other competing charts.

		δ_1					
Chart		1.10	1.20	1.30	1.40	1.50	2.00
S^2 − *Assorted*	ARL	**37.87**	**16.27**	**9.74**	**6.89**	**5.09**	**2.21**
	SEQL	**122.91**	**78.77**	**59.16**	**48.11**	**40.99**	**25.56**
	SRARL	1	1	1	1	1	1
Shewhart-R	ARL	68.75	30.72	16.55	10.20	6.96	2.40
	SEQL	141.68	102.69	80.50	66.37	56.66	34.65
	SRARL	1.40	1.63	1.68	1.66	1.61	1.42
Shewhart-S	ARL	65.10	28.30	15.10	9.20	6.30	2.40
	SEQL	139.43	99.59	77.44	63.52	54.04	32.96
	SRARL	1.36	1.54	1.57	1.54	1.49	1.32
EWMA lnS^2	ARL	43.00	18.10	11.00	7.60	6.00	3.20
	SEQL	126.01	82.53	62.46	51.03	43.66	28.40
	SRARL	1.06	1.09	1.10	1.11	1.11	1.21
CUSUM lnS^2	ARL	42.94	18.07	10.75	7.63	5.98	3.18
	SEQL	125.94	82.46	62.34	50.89	43.55	28.32
	SRARL	1.07	1.09	1.09	1.10	1.11	1.21
CUSUM R	ARL	40.40	17.60	10.82	7.81	6.13	3.13
	SEQL	125.34	81.22	61.42	50.27	43.12	28.14
	SRARL	1.03	1.05	1.06	1.08	1.09	1.20
CUSUM S	ARL	38.80	16.85	10.36	7.50	5.85	3.01
	SEQL	123.77	79.69	60.08	49.09	42.06	27.33
	SRARL	1.01	1.02	1.03	1.04	1.05	1.16
χ CUSUM	ARL	41.04	17.17	10.23	7.26	5.66	2.90
	SEQL	125.17	81.18	61.12	49.78	42.52	27.34
	SRARL	1.04	1.06	1.05	1.05	1.06	1.13
P_σ CUSUM	ARL	41.04	17.15	10.21	7.24	5.65	2.98
	SEQL	125.37	81.27	61.17	49.81	42.54	27.42
	SRARL	1.04	1.06	1.05	1.05	1.06	1.14

- The $S^2 - Assorted_{1.00,0.05}$ chart has minimum *EQL* value (i.e. 25.56), so it is considered as a benchmark chart. The *EQL′s* of other competing charts including *Shewhart R*, *Shewhart S*, *EWMA* lnS^2, *CUSUM* lnS^2, *CUSUM R*, *CUSUM S*, χ*CUSUM* and *the* P_σ *CUSUM* charts are 34.65, 32.96, 28.40, 28.32, 28.14, 27.33, 27.34 and 27.42, respectively (cf. Table 4).
- Because the $S^2 - Assorted_{1.00,0.05}$ is conceived as a benchmark chart, so its *RARL* is equal to 1. All contending charts have RARLs (1.42, 1.32, 1.21, 1.21, 1.20,

1.16, 1.13 and 1.14) (cf. Table 4) greater than 1, which shows the superiority of the proposed charts.

- As we have seen that the proposed $S^2-Assorted_{1.00,0.05}$ chart has lowest $EQL(25.56)$. To check the sensitivity of the $S^2-Assorted_{1.00,0.05}$ chart and competing charts such as *Shewhart R*, *Shewhart S*, *theEWMA* lnS^2, *theCUSUM* lnS^2, *the CUSUM R*, *the CUSUM S*, *the*χ*CUSUM* and *the* P_σ *CUSUM* on each amount of shift, we should determine Sequential Extra Quadratic Loss (*SEQL*). The *SEQL′s* demonstrates that the efficiency of the proposed $S^2-Assorted_{1.00,0.05}$ chart is superior to others competing for charts for a varying amount of shifts (cf. Table 4). For example, at $\delta=1.30$ the SEQL values of the $S^2-Assorted_{1,0.05}$, *Shewhart R*, *Shewhart S*, *the EWMA* lnS^2, *the CUSUM* lnS^2, *the CUSUM R*, *the CUSUM S*, *the*χ*CUSUM* and *the*P_σ *CUSUM* are 9.74, 16.55, 15.10, 11.00, 10.75, 10.82, 10.36, 10.23 and 10.21, respectively. The results advocate that the detection ability of the $S^2-Assorted_{1.00,0.05}$ chart based on *ARL*, *SEQL*, *EQL*, *SRARL* and *RARL* is superior to all aforementioned charts discussed in this research.

6. An application

This section gives a real-life implementation of the proposed chart where the manufacturing of semiconductors is going on. An outline of the process related to photolithography is given and finally the application of proposed assorted and some classical methods is explained.

6.1. Hard-Bake process

The hard-bake is a well-known procedure used to stabilize the printed topographies for providing optimum performance at etches for the photoresist pattern. The temperature of hard-bake differs but always remains less than 200 C^0 depending on the resistance. The removal of the solvent is ensured by the final bake step. This improves the bonding in wet plating processes so that the resistance is increased against plasma etches. The objective is to minimize the delay between hard-bake and plating so that the rehydration of the substrate can be prevented. A repetition of the hard-bake process before etch is optional if the delay after bake is more than 1 h. Sometimes in manufacturing processes, the hard-bake substrates are stored in a dry box because letting the material move from hard-bake to etch is less practical (cf. [29]).

An image of a spin coating photoresist process is shown in Figure 3, where five quality characteristics of interest namely resist dispenser, photoresist, resist flies, wafer (width flow measurements) and vacuum chuck is labelled (cf. [30]).

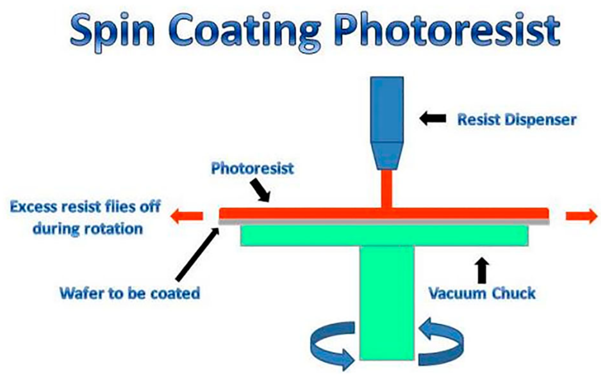

Figure 3. A pictorial display of a spin coating photoresist.

6.2. An application of quality control

In this section, we use a dataset extracted from semiconductor manufacturing of a hard-bake process to monitor flow width measurements of wafers. In an application, it was observed that the flow width measurement of wafers is the main variable of interest in this study (cf. [22], as shown in Figure 3. The dataset consists of 25 samples of size 5 each. The application of the proposed chart on the said dataset is outlined in the following steps:

First, we constructed the following control charts of hard-bake measurements process data with their respective settings (such that $ARL_0=200$) as listed below:

The proposed $S^2-Assorted_{1.00,0.05}$ chart with charting constant ($h_c=2.2298$, $L_e=2.2100$ and $c_s=2.8295$) and $UCL=1$;
The S^2 *Shewhart* chart with control limit coefficient ($K=3.84$) and upper control limits ($UCL=0.07277$);
The S^2 *CUSUM* chart with $k=1.0$, control limit coefficient ($h=1.88$) and ($UCL=1.88$);
The S^2 *EWMA* chart with $\lambda=0.05$ and $L=1.81$ and $UCL=0.2898$.

Figure 4 portrays the graphical display of the charts under discussion on hard-bake measurements data. None of the charts shows any false alarms for the first 25 in-control samples.

6.3. Application through data perturbation

To address various possible factors of OoC situations, we manipulated the dataset by data perturbation (cf. [31] and [32]). We used a range of distortions (1.1σ, 1.5σ and 2σ) to perturb the data and applied the proposed $S^2-Assorted_{1.00,0.05}$ and S^2 *Shewhart*, S^2 *CUSUM* charts and S^2 *EWMA* charts. Figures 5–7 and Table 5 demonstrate the graphic and tabular depiction and detection abilities of the resulting charts.

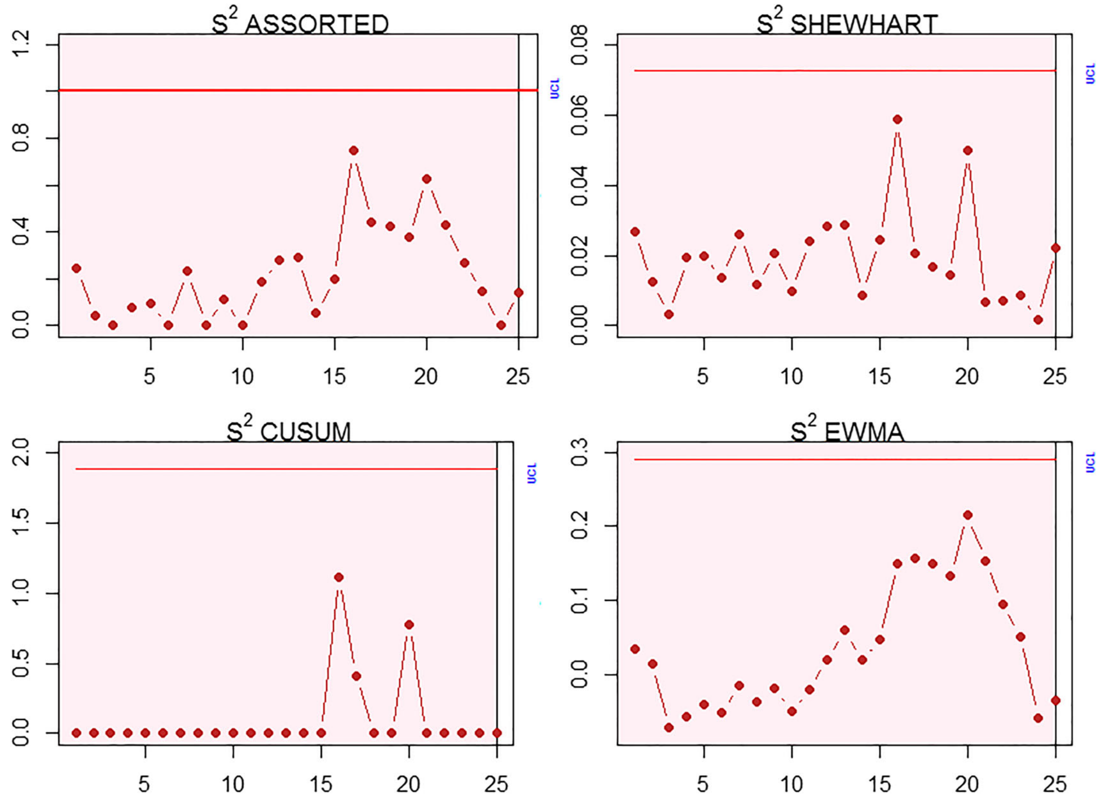

Figure 4. Graphical representation of the $\boldsymbol{S}^2$Assorted, $\boldsymbol{S}^2$ Shewhart, $\boldsymbol{S}^2$ CUSUM and $\boldsymbol{S}^2$ EWMA charts.

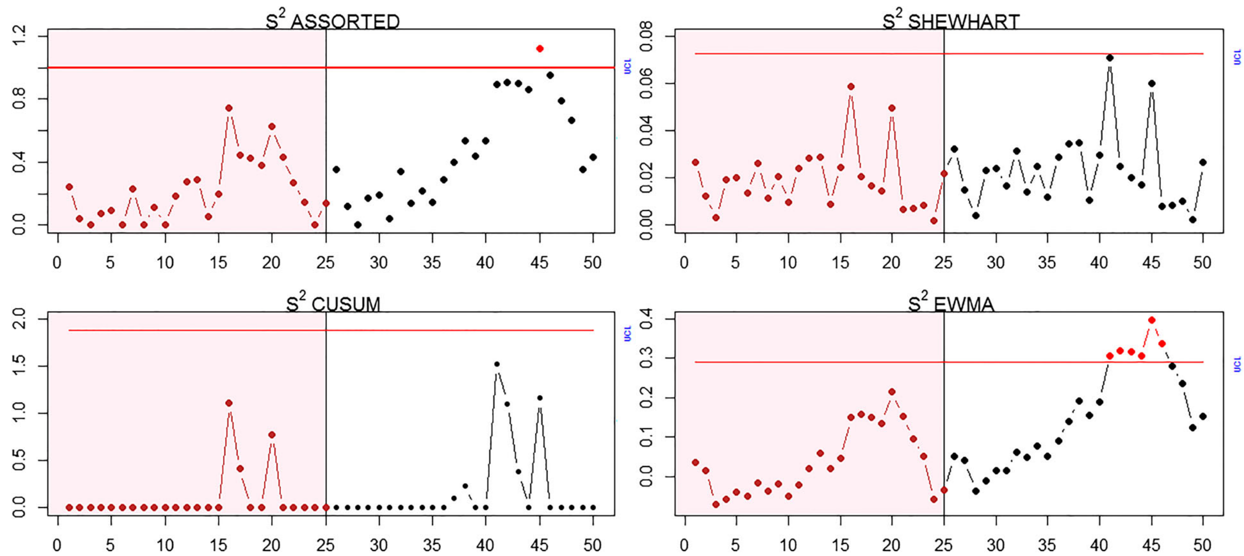

Figure 5. Graphical comparison at shift $= 1.1\boldsymbol{\sigma}$.

The results of our finding are outlined as follows:

- The $S^2 - Assorted_{1.00,0.05}$ chart detected a single OoC signal for (1.1σ) shift at sample number 45 (cf. Figure 5).
- The S^2 *Shewhart* and S^2 *CUSUM* charts did not detect any OoC signal for (1.1σ) shift (cf. Figure 5).
- The S^2 *EWMA* chart captured 5 OoC signals for (1.1σ) shift at sample numbers 40–46 (cf. Figure 5).
- The $S^2 - Assorted_{1.00,0.05}$ chart detected 19 OoC points for (1.5σ) shift at sample numbers 31–50 (cf. Figure 6).
- The S^2 *Shewhart* chart detected two OoC signal for (1.5σ) shift at sample number 41 and 45 (cf. Figure 6).
- The S^2 *CUSUM* chart detected 17 OoC points for (1.5σ) shift at sample numbers 37, 39 and 41–50 (cf. Figure 6).
- The S^2 *EWMA* chart captured 19 OoC points for (1.5σ) shift at sample numbers 31–50 (cf. Figure 6).

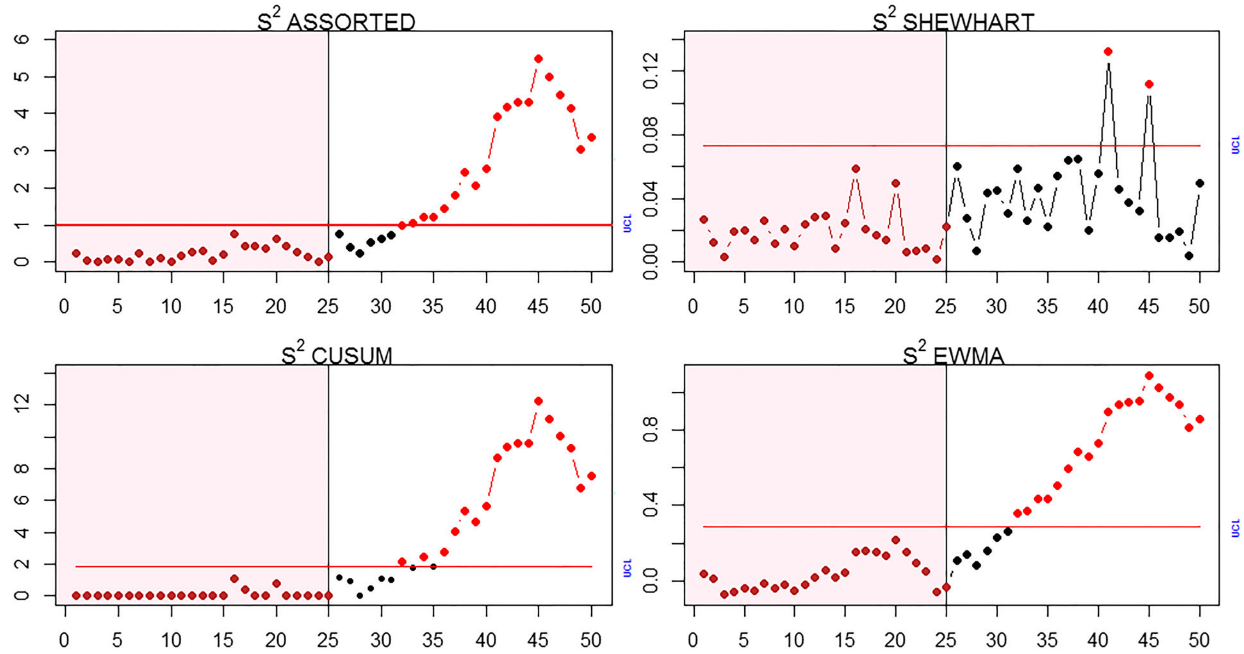

Figure 6. Graphical comparison at shift = 1.5σ.

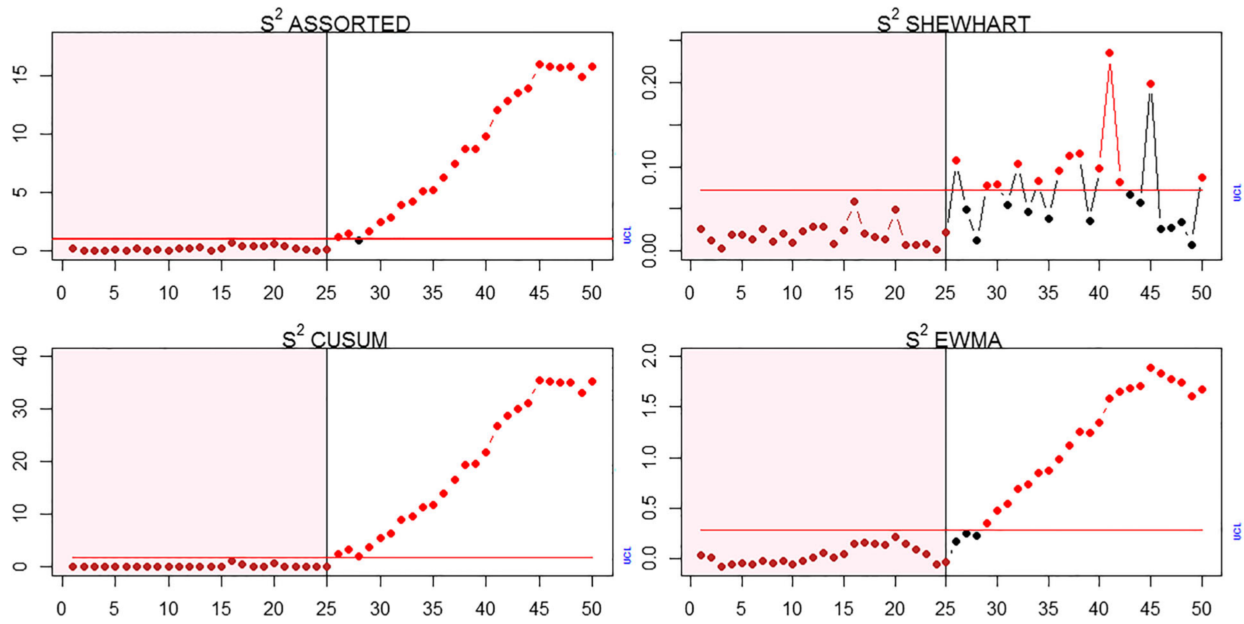

Figure 7. Graphical comparison at shift = 2σ.

Table 5. Detection summary through data perturbation.

Control chart	OOC detection	No. of signals	False alarms	Shift
S^2 Assorted	1	1	0	1.1σ
S^2 Shewhart	0	0	0	1.1σ
S^2 CUSUM	0	0	0	1.1σ
S^2 EWMA	5	6	0	1.1σ
S^2 Assorted	19	19	0	1.5σ
S^2 Shewhart	2	2	0	1.5σ
S^2 CUSUM	17	17	0	1.5σ
S^2 EWMA	19	19	0	1.5σ
S^2 Assorted	24	24	0	2σ
S^2 Shewhart	13	13	0	2σ
S^2 CUSUM	25	25	0	2σ
S^2 EWMA	22	22	0	2σ

- The $S^2 - Assorted_{1.00,0.0.05}$ chart detected 24 OoC points for (2σ) shift (cf. Figure 7).
- The S^2 *Shewhart* chart detected 13 OoC signal for (2σ) shift (cf. Figure 7).
- The S^2 *CUSUM* chart efficiently detected 25 OoC points for (2σ) shift (cf. Figure 7).
- The S^2 *EWMA* chart detected 22 OoC points for (2σ) shift (cf. Figure 7).

The above analyses indicate that the proposed chart is very effective in detecting a range of shifts in the process dispersion.

The possible explanation for these OoC signals might be the inclusion of special cause(s) in the hard-bake process such as substrate preparation, photoresist coating, edge bead removal, exposure, post exposure bake, developing the image and hard bake. These special cause variations can be possibly the result of external variables including temperature, gas flow and chemical composition. Fixing the issues with these variables in a timely manner is very important because it may damage the whole process and as a result waste of time and cost etc.

7. Summary and conclusions

Control charts have a central position among all the tools included in the SPC tool kit. These charts are classified based on the size of the shift they target.The *Shewhart R* and the *Shewhart S* are the fundamental control charts for detecting large shifts, while the *CUSUM* lnS^2 and the *EWMA* lnS^2 charts are used for moderate and smaller shift in variation. In this study, we proposed an assorted ($S^2 - Assorted_{k,\lambda}$) charting mechanism to detect smaller to larger shifts in process variability as a single chart. The efficiency of the $S^2 - Assorted_{k,\lambda}$ chart is assessed by using the well-known measures such as *ARL*, *EQL*, *SEQL*, and *RARL* and compared with *Shewhart R*, *Shewhart S*, *EWMA* lnS^2, *CUSUM* lnS^2, *CUSUM R*, χ*CUSUM*, P_σ *CUSUM* and *CUSUM S* charts.

A thorough result assessment advocated that the $S^2 - Assorted_{k,\lambda}$ chart is effective for all kinds of shifts. The performance of the $S^2 - Assorted_{k,\lambda}$ control chart at $k = 1.0$ and $\lambda = 0.05$ is best in terms of different run length properties. Because, the $S^2 - Assorted_{1.00,0.05}$ chart has lowest *EQL*, it is therefore considered a benchmark chart. The *RARL* of contending charts are more than 1 which indicates that the performance of the $S^2 - Assorted_{1.00,0.05}$ chart is best among the *Shewhart R*, *Shewhart S*, *EWMA* lnS^2, *CUSUM* lnS^2, *CUSUM R*, χ*CUSUM*, P_σ *CUSUM* and *CUSUM S* charts. Furthermore, *SEQL* is calculated to investigate the performance of the aforementioned charts at different amounts of shifts and it also supports the proposed chart. The scope of the current study may be extended in other directions as well such as profiles monitoring, non-parametric design structures and multivariate control charts.

Acknowledgements

Authors Nasir Abbas, Usman Saeed and Muhammad Riaz are thankful to King Fahd University of Petroleum and Minerals for providing excellent research facilities.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

The work was partially sponsored by Deanship of Scientific Research (DSR), King Fahd University of Petroleum and Minerals, Dhahran 31261, through project # IN171007.

ORCID

Nasir Abbas http://orcid.org/0000-0002-8622-5467
Usman Saeed http://orcid.org/0000-0003-3629-8518
Muhammad Riaz http://orcid.org/0000-0002-7599-6928
Saddam Akber Abbasi http://orcid.org/0000-0003-1843-8863

References

[1] Page ES. Controlling the standard deviation by cusums and warning lines. Technometrics. 1963;5(3):307–315. doi:10.1080/00401706.1963.10490100.
[2] Tuprah K, Ncube M. A comparison of dispersion quality control charts. Seq Anal. 1987;6(2):155–163. doi:10.1080/07474948708836122.
[3] Ng CH, Case KE. Development and evaluation of control charts using exponentially weighted moving Averages. J Qual Technol. 1989;21(4):242–250. doi:10.1080/00224065.1989.11979182.
[4] Abbasi SA, Riaz M, Miller A, et al. EWMA dispersion control charts for normal and non-normal processes. Qual Reliab Eng Int. 2015;31(8):1691–1704. doi:10.1002/qre.1702.
[5] Riaz M. On Enhanced Interquartile range charting for process dispersion. Qual Reliab Eng Int. 2015;31(3):389–398. doi:10.1002/qre.1598.
[6] Abujiya MR, Lee MH, Riaz M. New EWMA S2 control charts for monitoring process dispersion. Scientia Iranica. 2017;24(1):378–389. doi:10.24200/sci.2017.4041.
[7] Lyu J, Chen M. Automated visual inspection expert system for multivariate statistical process control chart. Expert Syst Appl. 2009;36(3, Part 1):5113–5118. doi:10.1016/j.eswa.2008.06.047.
[8] Pérez-González CJ, Colebrook M, Roda-García JL, et al. Developing a data analytics platform to support decision making in emergency and security management. Expert Syst Appl. 2019;120:167–184. doi:10.1016/j.eswa.2018.11.023.
[9] Yang S-F, Lin J-S, Cheng SW. A new nonparametric EWMA Sign control chart. Expert Syst Appl. 2011;38(5):6239–6243. doi:10.1016/j.eswa.2010.11.044.
[10] Sanusi RA, Riaz M, Abbas N, et al. Using FIR to improve CUSUM charts for monitoring process dispersion. Qual Reliab Eng Int. 2017;33(5):1045–1056. doi:10.1002/qre.2090.
[11] Crowder SV, Hamilton MD. An EWMA for monitoring a process standard deviation. J Qual Technol. 1992;24(1):12–21. doi:10.1080/00224065.1992.11979369.
[12] Chang TC, Gan FF. A cumulative Sum control chart for monitoring process variance. J Qual Technol. 1995;27(2):109–119. doi:10.1080/00224065.1995.11979574.
[13] Amin RW, Wolff H, Besenfelder W, et al. EWMA control charts for the smallest and largest observations. J Qual Technol. 1999;31(2):189–206. doi:10.1080/00224065.1999.11979914.
[14] Acosta-Mejia CA, Pignatiello JJ, Venkateshwara Rao B. A comparison of control charting procedures for monitoring process dispersion. IIE Trans. 1999;31(6):569–579. doi:10.1080/07408179908969859.
[15] Chen G, Cheng SW. Max chart: Combining X-bar chart and S chart. Stat Sin. 1998;8(1):263–271.

[16] Chen G, Cheng SW, Xie H. Monitoring process mean and variability with One EWMA chart. J Qual Technol. 2001;33(2):223–233. doi:10.1080/00224065.2001.11980069.

[17] Cheng SW, Thaga K. (2010). *The Max-CUSUM Chart BT - Frontiers in Statistical Quality Control 9* (H.-J. Lenz, P.-T. Wilrich, W. Schmid, editors.). doi:10.1007/978-3-7908-2380-6_6

[18] Castagliola P. A New S2-EWMA control chart for monitoring the process variance. Qual Reliab Eng Int. 2005;21(8): 781–794. doi:10.1002/qre.686.

[19] Castagliola P. A new CUSUM-S2 control chart for monitoring the process variance. J Qual Maint Eng. 2009;15(4): 344–357. doi:10.1108/13552510910997724.

[20] Mehmood R, Riaz M, Does RJMM. Control charts for location based on different sampling schemes. J Appl Stat. 2013;40(3):483–494. doi:10.1080/02664763.2012.740624.

[21] Ajadi JO, Riaz M, Al-Ghamdi K. On increasing the sensitivity of mixed EWMA–CUSUM control charts for location parameter. J Appl Stat. 2016;43(7):1262–1278. doi:10.1080/02664763.2015.1094453.

[22] Montgomery DC. Introduction to statistical quality control. 8th ed. New Jersey: John Wiley; 2019, August.

[23] Wilson EB, Hilferty MM. The distribution of chi-square. Proc Natl Acad Sci U S A. 1931;17(12):684–688. doi:10.1073/pnas.17.12.684.

[24] Wu Z, Jiao J, Yang M, et al. An enhanced adaptive CUSUM control chart. IIE Trans. 2009;41(7):642–653. doi:10.1080/07408170802712582.

[25] Abbas N, Saeed U, Riaz M. Assorted control charts: an efficient statistical approach to monitor pH values in ecotoxicology lab. J Chemom. 2019;33(6):e3129), doi:10.1002/cem.3129.

[26] Al Mutairi AO, Volodin A. Special cases in order statistics for the alternative parametrization of the generalized power function distribution. J Taibah Univ Sci. 2018;12(3):285–289. doi:10.1080/16583655.2018.1465726.

[27] Basheer AM. Alpha power inverse Weibull distribution with reliability application. J Taibah Univ Sci. 2019; 13(1):423–432. doi:10.1080/16583655.2019.1588488.

[28] Haq MAu, Elgarhy M, Hashmi S. The generalized odd Burr III family of distributions: properties, applications and characterizations. J Taibah Univ Sci. 2019;13(1):961–971. doi:10.1080/16583655.2019.1666785.

[29] Lithography Process Overview. 2019, December. http://www.imicromaterials.com/technical/lithography-process-overview.

[30] Fundamentals of Photolithography. 2019, October. https://www.uvu.edu/physics/nanotech/docs/fundamentals_of_photolithography.pdf.

[31] Liu K, Kargupta H, Ryan J. Random projection-based multiplicative data perturbation for privacy preserving distributed data mining. IEEE Trans Knowl Data Eng. 2006;18(1):92–106. doi:10.1109/TKDE.2006.14.

[32] Kargupta H, Datta S, Wang Q, et al. Random-data perturbation techniques and privacy-preserving data mining. Knowl Inf Syst. 2005;7(4):387–414. doi:10.1007/s10115-004-0173-6.