The Neurobehavioral Basis of the Parallel Individuation (PI) and Approximation Number System (ANS)

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Abstract

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Research on numerical cognition proposes that there are two systems for the perception of numerical quantity, a small-number system (1~3) invoking parallel individuation, or "subitizing", and a large-number system (4+) that is based on Weberian magnitude estimation (Hyde, 2011). Many numerical cognitive neuroscientists have focused on studying how the magnitude of numerosities (small vs. large numbers) and numerical distance (close vs. far differences between numbers) are influential factors when processing numbers and change detection. However, is there a difference when numerosities are increasing or decreasing? The effects of direction on numerical change processing are lesser known.

This 128-channel EEG study investigated the neurobehavioral basis of differentiation between small vs. large-number perception and effects of change directionality. During EEG data collection, participants were sequentially presented with stimulus arrays of 1 to 6 dots, with parameters like size and location controlled for, to minimize varying non-numerical visual cues during habituation. Participants were instructed to press a key whenever they detect a change in the number of dots presented.

The current study adapts a dot-stimuli numerical change study design from Hyde and Spelke (2009, 2012). In their EEG study, the researchers examined event-related-potential (ERP) differences during the processing of small (1, 2, 3) and large (8, 16, 24) numbers. For this study,

we chose to examine a narrower numerical range from 1~6, so that small (1, 2, 3) vs. large (4, 5, 6) contrasts were along a numerical continuum. In contrast to Hyde and Spelke (2009, 2012), where participants passively-viewed the sequential presentation of dot arrays, this study employed an active change detection paradigm, where participants' reaction time (RT) and accuracy in detecting change in the number of dots were recorded.

We investigated the effects of Direction and Size in numerical change detection, where Direction is operationally defined as Decreasing and Increasing change in numeric set size, while Size is divided into Small-to-Small, Large-to-Large and Crossovers. Numerical change conditions were categorized into six groups: "Increasing Small-to-Small" (e.g., 1-to-2, 2-to-3), "Decreasing Small-to-Small" (e.g., 2-to-1, 3-to-2), "Increasing Large-Large" (e.g., 4-to-6, 5-to-6), "Decreasing Large-Large" (e.g., 5-to-4, 6-to-5), Increasing Small-to-Large" (e.g., 2-to-4, 3-to-5, 3-to-6) and "Decreasing Large-to-Small" (e.g., 4-to-2, 5-to-2, 6-to-3), where the last two groups are operationally defined as Crossovers. There was also a "No Change" condition, where the number of dots remain the same for up to five presentations. ERP analyses were conducted for the N1 component (125-200 ms) over the left and right occipital-temporal-parietal (POT) junction and for the P3b component (435-535 ms) over the midline parietal area (Pz).

During the No Change condition, results show that the N1 amplitude was modulated by the cardinal values of the habituated numbers 1~6. Within this continuous range, we found N1 amplitudes commensurate with cardinal values in the small range (1, 2, 3), but not in the large range (4, 5, 6), suggesting that numbers in the subitizing range are individuated as objects in working memory.

Meanwhile, in the Change condition, there was a significant main effect of Direction on N1 peak latency, where the Increasing condition showed earlier peaks. In the Decreasing Small-to-Small condition, N1 amplitudes were the lowest (even lower than N1 peaks for No Change conditions), while the other five Change conditions all produced higher N1 negativities than No Change conditions. These results imply that when the number of dots get small enough to parallel individuate, instead of encoding items into visual short-term memory, the brain is "off-loading" items from our perceptual load.

Intriguingly, although the Decreasing Small-to-Small condition had the lowest N1 negativities, it produced the highest P3b positivity. Distinctions in P3b waveforms reflect a clear categorical break between small vs. large numbers, where easier/small number change conditions have higher amplitudes than harder, large number conditions, suggesting more difficulty with updating the context in the latter. However, in contrast to the earlier N1, there was no main effect of Direction on P3b peak latency, but there was an interaction effect of Direction by Size.

Interestingly, there was also a similar interaction effect of Direction by Size for reaction times, with similar trends showing that Decreasing conditions produced shorter reaction times for the Large-to-Large and Crossover conditions, yet this pattern was reversed in the Small-to-Small condition. This lends more support to the implication of the "off-loading" phenomenon when processing decreases of numerosities in the small range (1~3). Meanwhile, when it comes to context-updating at later stages, and a behavioral response is required for this change detection task, the Large-to-Large condition prove to be the most difficult, as there was lower accuracy, longer reaction times, later and lower P3b peaks.

N1 and P3b amplitudes are complementary to each other, with the early N1 being more sensitive to Direction, and the later P3b being more sensitive to Size. This suggests that the posterior parietal cortex might encode Direction first, followed by Size. This study proposes a model that is an adaptation to the P3b context-updating model (Donchin, 1981), where the early, sensory N1 interplays with the later, cognitive P3b. These findings suggest a neurobehavioral basis for the differentiation of small vs. large number perception at early stages of processing that is sensitive to encoding vs. off-loading objects from perceptual load and visual short-term memory, as well as a later stage that involve higher-order cognitive processing on the magnitude of set size that is employed in numerical change detection tasks.

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Chapter 1: Introduction

1.1. Neurobehavioral Basis of Numerical Cognition

The ability to process numerical information is crucial to navigate the human world. An understanding of numerical quantity is informative to decisions regarding basic survival, for example, to enable one to choose the habitat with the most plentiful supply of food and mates, whilst minimizing the risk of incoming predators. Research shows that human adults, infants and animals have the ability to process non-symbolic numerical quantities as represented by comparing related sets of objects (Dehaene, 2009; Feigenson et al., 2004; Nieder & Dehaene, 2009).

1.2. Non-verbal numerical mechanisms

Two basic non-verbal mechanisms for numerosity processing have been identified: The Object File System (OFS; or Object Tracking System, OTS) and the Approximate Number System (ANS) (Feigenson et al., 2004; Piazza, 2010; Piazza & Izard, 2009).

In numerical cognition, the OFS/OTS allows individuals to "precisely keep track of small numbers of individual objects and for representing information about their continuous quantitative properties" (Feigenson et al., 2004, p. 310). It is considered a pre-verbal domain-general mechanism to track the spatio-temporal characteristics of a limited number of items (approximately 3-4), which are assigned with a visual index. One numerical mechanism of the OFS/OTS is "subitizing", which allows humans to rapidly and accurately determine the numerosity of small sets without using a counting routine (Kaufman et al., 1949; Mandler & Shebo, 1982; Pylyshyn, 2001; Trick & Pylyshyn, 1994). The crucial signature of the OFS/OTS is its limited storage capacity (i.e., 3-4 elements), which mimics that of visual short-term memory

(VSTM). For this reason, some authors have suggested the OFS/OTS is intimately linked to VSTM (Knops et al., 2014; Piazza, 2010).

Peter Gordon (1994) proposed that small number exact enumeration was the outcome of a process whereby working memory systems can represent up to three items at the same time, which he termed “parallel individuation”. Characterizations have linked parallel individuation to the idea of object files, which were introduced by Kahneman, Treisman and Gibbs (Kahneman et al., 1992) as a format for object representation that listed properties such as shape, color and so on. Parallel individuation forms a distinct representation for each object, and these representations process numerical content by retaining information about numerical identity, where mentally stored items can be compared on a one-to-one basis with visible objects in the scene to detect numerical matches or mismatches (Hyde, 2011).

In the Approximate Number System (ANS), each numerosity is represented as a Gaussian curve of activation on a metaphorical mental number line. There are two main competitive mathematical models that formally describe the Approximate Number System: In the linear model (Gallistel & Gelman, 1992, 2000; Whalen et al., 1999), the curves of activation are linearly spaced with an increasing standard deviation (i.e., scalar variability) as a function of numerical magnitude. Conversely, in the logarithmic model (Dehaene, 2009), the standard deviation of the Gaussian curves is constant, whereas the distance between numerical magnitude is logarithmically compressed. Despite the differences in their formulation, both models explain behavioral results and make similar predictions.

Comparing the difference between two numbers involves magnitude estimation, a perceptual judgment of relative quantity that follows Weber’s Law. Weber’s Law of psychophysics describes that the just-noticeable-difference between two stimuli increases proportionally with magnitude or intensity (Dehaene et al., 2003; Piazza, 2010). In other words, if the distance between two numbers is far apart enough, we are more sensitive in detecting the difference in the two values.

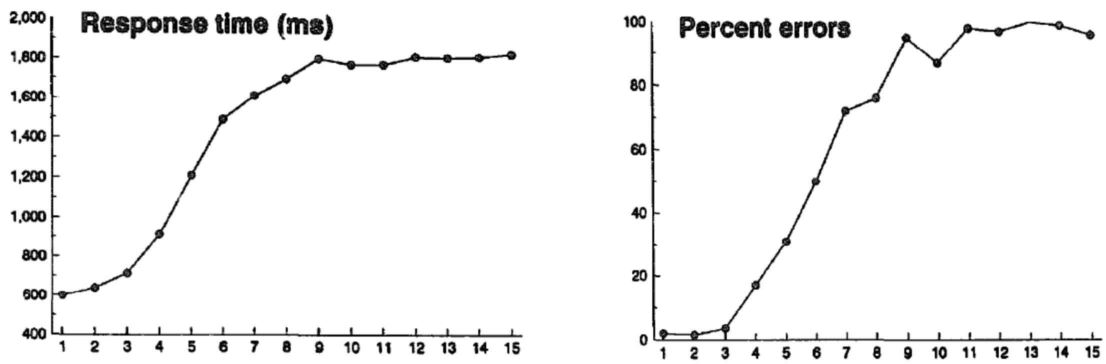
The characteristic signature of the ANS is the ratio-dependent effect, which states that the discriminability between two numerical sets crucially depends on their numerical ratio, thereby obeying Weber's law. The numerical ratio is derived from the value of the smaller number divided by the larger number. The more the numerical ratio approaches "1", the harder the discrimination is. Conversely, when the numerical ratio approaches "0", identifying the larger between two numerical sets becomes an easy task. For instance, to compare 1 to 4, the numerical ratio is 0.25, which makes it easier to discriminate than to compare 3 to 4, with a numerical ratio of 0.75.

1.3. Non- symbolic Numerical Processing in Adults

1.3.1. Subitizing

Different experimental paradigms have been implemented to assess subitizing in adults. Typically, individuals are asked to enumerate the items in a visually presented set. When few elements are displayed (1-3), we engage in subitizing, where the accuracy is high and reaction times are fast (Kaufman et al., 1949; Trick & Pylyshyn, 1994). Subitizing seems to rely heavily on attentional and visual short term memory capacity, which allows participants to track the spatiotemporal characteristics of a few items (Burr et al., 2010; Piazza, 2010; Piazza et al., 2011). When the number of items increases beyond 3, individuals can rely on estimation or counting, depending on the time at their disposal. When the items are presented for a short time, individuals can only estimate the presented numerosity. In this case, reaction times increase with numerosity to a point, and then reach a plateau (i.e., reaction times remain constant despite the increase in displayed numerosity), whereas the accuracy is markedly lower compared to subitizing (Dehaene, 1992; see *Figure 1*). On the other hand, with unlimited time, individuals can count the items, thereby increasing response time as a function of numerosity, while accuracy remains high.

Figure 1: Performance in timed quantification of numerical visual displays (stimulus duration: 200ms)



Note: Response time (left graph) and percentage of errors (right graph) are plotted as a function of the actual numerosity of the display. *Note*. Adapted from *Mandler & Shebo, 1982*, in "Varieties of numerical abilities," by S. Dehaene, 1992, *Cognition, 44.* Copyright 1992 by Elsevier Academic Press.

Several indexes may be used to obtain a reliable measure of subitizing and counting skills, such as the RTs slopes separately for subitizing and counting range, or the estimation of the subitizing range as the discontinuity point in RT slopes (Reeve et al., 2012; Schleifer & Landerl, 2011). Sella and colleagues (2013) adopted a match-to-sample task in which a sample set ranging from 1 to 9 was briefly presented, followed by a target set whose numerosity differed for one element (-1 or +1). Participants had to determine whether the two sets had the same or different numerosity. The accuracy in comparing numerical sets around the Object Tracking System capacity (i.e., 3 vs. 4, 4 vs. 5) represented a valid and pure assessment of the subitizing limit (Sella et al., 2013).

1.3.2. Numerical Estimation

Numerical estimation is thought to rely on a non-verbal system for the representation of numerical magnitude in the Approximate Number System (ANS) (Feigenson et al., 2004; Piazza, 2010). Currently it is widely accepted that subitizing and estimation are selectively connected to the OFS and the ANS, respectively (Burr et al., 2010; Hyde, 2011; Trick & Pylyshyn, 1994). Although it is conceivable that estimation can operate both below and above the subitizing range, only numerosities beyond subitizing range show the classic variability signature obeying Weber's law (Revkin et al., 2008), where the noticeable difference between two numerical stimulus is detectable as the magnitude between them widens. On the other hand, the Weber signature can emerge also for small numerosities if attentional resources are diverted by means of a concurrent task (Burr et al., 2010; Piazza et al., 2011). That is, dual task conditions appear to disrupt the functioning of the OFS and therefore impairs the ability to subitize.

Mental representations of numbers can be processed with an analogical magnitude code, for the estimation of quantity and magnitude that is associated with a number. An intuitive sense of magnitude has been shown to have phylogenetic and ontogenetic continuity (Dehaene, 2009). Specifically, the ability to distinguish between nonsymbolic numerical magnitudes is thought to rely on an ANS (Feigenson et al. 2004; Nieder & Dehaene, 2009).

Two basic properties of the ANS are as follows: (1) The ANS is used to process magnitudes greater than three or four items– that is, above the range of the exact number processing system (Demeyere et al., 2014; Le Corre & Carey, 2007). (2) Accuracy and reaction times on numerical discrimination tasks are sensitive to the numerical ratio or the distance of the numerical quantities being compared. As an example, the ratio of the numbers 4 and 6 is 0.67 and the distance of the numbers 4 and 6 is 2. Performance on numerical discrimination tasks (e.g., judging which of two numbers is numerically larger) decreases as the ratio between the two quantities being compared increases.

Similarly, performance on numerical discrimination tasks decreases as the distance between the two numbers being compared decreases (i.e., the numerical distance effect). Typically, participants are faster and more accurate at discriminating between the numbers 4 and 6 compared to discriminating between the numbers 5 and 6 (Halberda & Feigenson, 2008; Moyer & Landauer, 1967). In view of these effects, it has been argued that numerical quantities are analog and approximate rather than exact, since distance and ratio effects would not be present if each number was represented fully independently of adjacent numbers. Consistent with this interpretation, it is assumed that close numbers (such as 5 and 6) share overlapping representations and this overlap increases with the relative size of the numbers (their numerical ratio).

1.3.3. Numerical Acuity: In Non-symbolic Number Comparison

To investigate the ability to discriminate between numerical quantities, various studies have presented stimuli that are usually comprised of dots (or other geometrical shapes), which can be presented side-by-side, sequentially, or intermixed using different colors for the two sets (for a comparison of different presentation modalities, see (Agrillo et al., 2015; Price et al., 2012).

The ability to compare non-symbolic quantities, also known as number acuity, is usually assessed by calculating the Weber fraction –*w*– (Halberda et al., 2008, 2012). The calculation of the Weber fraction can vary according to the underlying model for the Approximate Number System. Nevertheless, the Weber fraction can be considered as the constant standard deviation ("noise") within a Gaussian curve; this corresponds to an internal representation of each numerosity on the mental number line (Halberda et al., 2008; Pomè et al., 2021). Therefore, the smaller the standard deviation (i.e., Weber fraction), the more precise the numerical representation. To put it concretely, an individual with a Weber fraction of 0.14 can reliably individuate the larger between two numerical sets, when one of the two numerosities is at least 14% larger than the other one. Then, for example, the individual can reliably discriminate 7 vs. 8 and 14 vs. 16.

The ANS's imprecision of its representations grows with the target numerosity, such that the ability to nonverbally discriminate two quantities depends on their ratio (Moyer & Landauer, 1967). This ratio dependence is observed when adults estimate numbers of items (Halberda et al., 2012; Whalen et al., 1999), judge the more numerous of two arrays (Agrillo et al., 2015; Barth et al., 2003), and estimate the results of arithmetic events (Pica et al., 2004). Because of the inexactness of ANS representations, two quantities cannot be distinguished when the distance between them is too small. According to Halberda et al. (2008), the finest numerical ratio that adults can consistently discriminate has been identified as 7:8 (ratio: 0.88). This limit can also be described as a Weber fraction that measures the smallest numerical change to a stimulus that can be reliably detected. The Weber fraction is equal to the difference between the two numbers divided by the smaller number; for example, to compare 7 to 8, when we apply the formula: (8 - 7)/7 = 0.14. In a study comparing French adults and the Munduruku, an Amazonian group that has number words up to five, when they are asked to indicate the more numerous of two simultaneously presented arrays containing 20–80 dots, French adults' Weber fraction is 0.12 and Amazonian adults' Weber fraction is 0.17. Thus, on average these adults could discriminate ratios differing by about 7:8 (Pica et al., 2004).

1.4. Neuroscience of Numerical Cognition

1.4.1. Large Number Representation in the Brain

The ANS, as outlined earlier, is a frequently posited non-verbal system for numerical quantification for numbers larger than 3-4 (Piazza 2010; Hyde 2011; Hyde & Spelke, 2011). In contrast to a discrete representation of numerical values, the ANS is suggested to facilitate a degree of quantification that is analogous to estimating. Neuroimaging studies seeking to identify the neural bases for the ANS have typically used passive fixation, numerosity comparison, or approximate calculation using dot arrays.

The ratio-dependent effect as a characteristic signature of the ANS, where the discriminability between two sets depends on their numerical ratio (Weber's law). Work with non-human primates has demonstrated neural tuning curves that are consistent with this formulation (see Nieder, 2005, for a review). Single unit recording highlights the existence of neurons that demonstrate a 'preferred' numerosity in the prefrontal and posterior parietal cortices, where the preferred numerosity was "1" for the IPS neuron, and "4" for the PFC neuron (Nieder & Dehaene, 2009; Nieder & Miller, 2003). Importantly, for each cell's preferred numerosity, a progressive reduction in activity was found for numerosities as a function of numerical distance. Thus, a neuron that was maximally active for the number 3, for example, was less responsive to the numbers 2 and 4, with the overall lowest response reflecting the array with the smallest cardinal value. The neurons consecutively arranged overlapping tuning curves retained an inherent order of cardinalities, allowing for numerosities to exist in relation to one another, thus reflecting meaningful quantity information.

Later work by Nieder and colleagues demonstrated that, whilst similar properties were exhibited by neurons in the prefrontal and posterior parietal cortex, those in the posterior parietal cortex were responsive to numerosity sooner than those in the prefrontal cortex (Nieder & Miller, 2004). The authors conclude that quantity is initially extracted by neurons within the posterior parietal cortex, then fed forward and expanded in the prefrontal cortex to support online executive processes.

Piazza et al. (2006) sought to confirm whether the bilateral Intraparietal Sulcus (IPS) encoded numerical quantity, and that activation in this region also followed Weber's law. Using a passive fMRI task, the authors demonstrated that the horizontal IPS was responsive to non-symbolic numerosity. By extracting the level of activity from the peak voxel in each subject's data set, the authors were able to generate curves that were comparable to psychophysical curves extracted from the same sample (Piazza et al., 2006). Moreover, both behavioral and brain data were in line with Weber's law, providing strong evidence for an Approximate Number System (Piazza et al., 2006).

Many EEG studies have used a nonsymbolic number discrimination task to study ERPs from brain regions that support the ANS. Investigations have revealed parietal neural activation during nonsymbolic number discrimination tasks (Ansari et al., 2006; Cantlon et al., 2009; Holloway et al., 2010).

1.4.2. Small Number Representation in the Brain

As outlined previously, the OFS/OTS is a mechanism by which objects are represented as distinct entities and individuated in parallel. With a signature capacity limited to around 3 or 4 items, the OFS/OTS is linked to visual short-term memory. Small numbers are therefore easier to visually discriminate with the aid of subitization, where we rapidly and precisely enumerate small quantity of elements within an array (Mazza & Caramazza, 2015). Whilst the occurrence of subitizing is not in itself disputed, the degree to which the ANS might support estimation of small sets (i.e. sets that are within the subitizing range), or in what circumstances the OFS/OTS might take over, and whether these systems are dissociable, has been widely debated.

Early neuroimaging studies that sought to contrast small versus larger set sizes were indicative of a single system, where regions that support enumeration of small arrays were shown to be activated similarly than when enumerating larger arrays (Piazza et al. 2002). This led some researchers to suggest that the ANS operates over the entire range of numbers: a "one system view" (see Hyde, 2011, who reviewed evidence for the "one system view").

Recent work, however, converges to suggest that set size does indeed affect encoding, although the neural systems are still not well established. For example, ERP data demonstrate a distinction between small and larger numerical quantities that would be outside of the subitizing range (Hyde & Spelke, 2009). Evidence for reduced response latencies for arrays within the subitizing range has been demonstrated in the posterior temporal cortex (Vuokko et al., 2013).

1.4.3. Subitizing vs. Estimation in the Brain

Processes of visual quantification within and outside the subitizing range have also been distinguished on a neuro-functional level. While quantification outside the subitizing range was found to engage superior parietal (Demeyere et al., 2012; He et al., 2014; Vetter et al., 2011; Vuokko et al., 2013) and frontal (Vuokko et al., 2013) brain regions, neural correlates of subitizing were associated with posterior temporo-parietal (Demeyere et al., 2014; He et al., 2014; Vetter et al., 2011; Vuokko et al., 2013) and occipito-parietal areas (Demeyere et al., 2012).

Working on the basis that absence of the ratio-dependent effect is indicative of OTS activation, Agrillo et al. (2015) demonstrated that it was the presence (or absence) of task-irrelevant stimuli in the visual field that determined ratio-dependence in small numerosities. By comparing a number of classic experimental formats for dot comparison tasks, the authors demonstrated that when the sum total of dots was greater than the subitizing amount, the ratio-dependent effect was observed (Agrillo et al., 2015). This supports the proponents of a “two system view”, which suggest that attentional load modulates the engagement of the OTS and the ANS, where items presented outside of attentional limits cannot be represented as individual units (Hyde, 2011).

1.4.4. Neuroanatomy of Visual Numerical Processing

There is evidential consistency that numerical processing and magnitude comparison are linked to anatomical correlates described by previous research, where the posterior parietal cortex (PPC) – particularly the right intraparietal sulcus (IPS) – has been broadly implicated in non-symbolic numerical processing (see reviews by Cantlon et al., 2009; Cohen Kadosh et al., 2008)

In an EEG study, Hyde and Spelke (2012) source-localized the effect of the P3, a cognitive event-related-potential during the visual processing of 1~3 dots, where they revealed activity in the right temporal-parietal junction (RTPJ). This activation towards the right parietal regions is related to fMRI findings that found similar activation when directly contrasting small versus large number processing in a number comparison task (Ansari et al., 2007). Ansari and colleagues (2007) suggested that this activation was related to heightened stimulus-driven attention for small number comparisons compared to large comparisons.

Other studies of right TPJ function suggest that this brain region enables the “bottom-up” reorientation of attention to novel stimuli, or “odd-ball” stimuli like those presented in our study (see Corbetta & Shulman, 2002, for a review). According to Hyde and Spelke (2012), based on the localization and previous work on the functional profile of this region, the P3 modulation may reflect the reorientation of attention upon the overt detection of a number change in the small number range, a process that is not seen for large numbers.

In an fMRI study by Culham et al. (1998), in a visual task where participants have to track multiple “bouncing balls” on the screen (presentation can go up to 9 “balls), the researchers found that as they increased the number of objects needed to be tracked, there is an increase in brain activity over parietal-occipital regions linked to visuopatial attention, including the superior parietal lobule, the intraparietal sulcus, and the lateral parietal occipital- junction (Culham et al., 1998).

Chapter 2: EEG in Numerical Cognition

Many numerical processing studies in the brain have used functional MRI (fMRI), a method that is known for poor temporal resolution, as compared to EEG (Hyde & Spelke, 2012; Luck, 2005). Although fMRI scans can produce neuroanatomically-precise activation maps during the function of numerical processing with high spatial resolution, there is poor detail on the temporal order of activation for different brain regions. In other words, fMRI activation maps can explain *where* activity is happening when we process cues of numerosity, but this method does not fully explain *how* and *when* numerical processes interact with related brain areas that aid us to abstract, compare, and operate on numerosities.

As described in the previous section, when processing numerosities in rapidly changing visual streams, human brains can instantaneously engage in automatic perceptual and cognitive mechanisms that allows for accurate enumeration and efficient numerical approximation. Such mechanisms often need less than 1 second to activate in the brain. However, as fMRI relies on waiting for oxygenated blood to gradually flow to brain areas with higher activity, this neuroimaging method is limited to a low temporal resolution of several seconds by the sluggish nature of the hemodynamic response (Luck, 2005).

Event-related potentials (ERPs) derived from EEG experiments are particularly well suited for studying number processing given their precise temporal resolution of 1 millisecond that allows for neural and cognitive mechanisms to be precisely tracked millisecond by millisecond (Hyde & Spelke, 2012; Luck, 2005).

Meanwhile, though fMRI has superior spatial resolution in the millimeter range, most cognitive neuroscientists still view the ERP technique as an important complement to hemodynamic measures (Luck, 2005). ERP studies on number processing have been reconciling the literature on the divide between neurocognitive processing of small vs. large numbers (Fornaciai & Park, 2017; Hyde & Spelke, 2009, 2012; Libertus et al., 2007). By investigating the spatiotemporal process of number representation, we may come to better understand how information from numerosities are derived from sensory stimulation and perceptual load in the brain.

2.1. N100: Early, Sensory ERP

The N100, also known as the N1, is an early-occurring ERP that is the first negative polarity observed after the onset of a stimulus (Luck, 2005). This sensory, perceptual ERP waveform is often recorded after 100ms, but not before 200ms (Luck & Kappenman, 2012). The peaks of the N1 ERP are visual cortical responses that reflect bottom-up information such as stimulus features (Crowley & Colrain, 2004; Vaughan, Arezzo, & Picton, 1988), as well as top-down influences such as selective attention (Golob, Johnson, & Starr, 2002; Hillyard, Hink, Schwent, & Picton, 1973) and short-term memory load (Conley, Michalewski, & Starr, 1999; Golob & Starr, 2000). According to Yurgil and Golob (2013), attention to infrequent targets presented amid frequent standard stimuli typically enhances N100 amplitudes.

Hyde and Spelke (2009; 2012) have shown that small and large number processing dissociates in pattern and in time: small numbers modulate an early visuospatial attentional component (N1) by the cardinal value of the display: irrespective of the previous numerical context, where the N1 amplitude is largest for three objects, smaller for two objects, and smallest for one object.

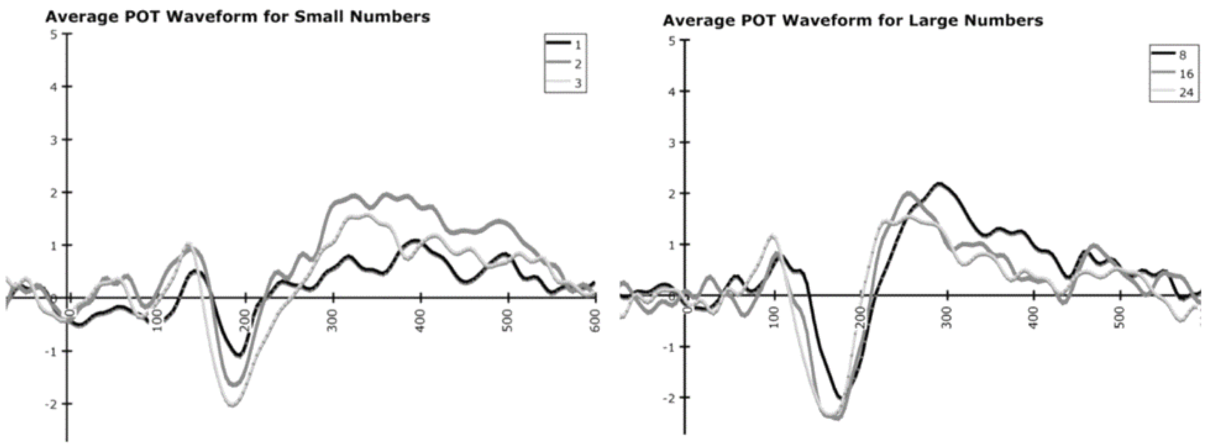
Of note worth discussing is the nomenclature of the targeted ERPs in the current study, specifically “N1” versus “N170”. Research has shown that the N170 is mainly generated in the fusiform gyrus (Ghuman et al., 2014; Luck, 2005), and this ERP is elicited during facial processing, perceptual expertise and linguistic processing (Maurer et al., 2005; Luck & Kappenman, 2012).

In their numerical EEG study where participants saw “2” or “3” dots, Hyde and Spelke (2012) ran source localization identified neural generators of the observed first negative-going wave at 145-183 ms. Greater current density in response to “3” dots than “2” was observed over the right extrastriate visual areas (i.e. middle occipital and fusiform areas), left inferior parietal areas, as well as the “left middle temporal, right superior temporal, and right anterior temporal regions” (Hyde & Spelke, 2012, p. 2183). The authors imply that the combination of neural activity from these posterior regions originated the posterior N1 response targeted in their study. More research is needed to distinguish N1 from N170, as their time windows overlap.

2.1.1. N1 Response to Cardinality

Hyde and Spelke (2009) found that the amplitude of N1 ERP responses scaled to cardinal value in the small-number group (1, 2, 3), but not in the large number group (8, 16, 24). In other words, N1 amplitude was largest for “3” and smallest for “1”, but there was no such scaling for the large numbers that they tested (see Figure 2). In Hyde and Spelke (2009), N1 peak latency differences were observed between small and large numbers, where N1 peaked nearly 20 ms earlier for large numbers (~155 ms) compared to small numbers (~173 ms).

Figure 2: Average waveform over POT sites measured in the small number range (Left plot) and in the Large number range (Right plot) in Hyde and Spelke (2009).



Note: Peak N1 amplitudes in response to adapted displays of each cardinal value, where N1 showed higher negativities for larger numerosities. Adapted from "All Numbers Are Not Equal: An Electrophysiological Investigation of Small and Large Number Representations," by D. Hyde and E. Spelke, 2009, *Journal of Cognitive Neuroscience, 21*, p. 1048, Copyright 2009 by MIT Press.

Given the very large differences between the small (1, 2, 3) and large number groups (8, 16, 24) from Hyde and Spelke (2009; 2012), more research is needed to determine if there is a similar difference in scaling of N1 response to numerical quantities that continue from 3, 4, 5 and onwards.

2.1.2. N1 Response to Numerical Change

According to Luck et al. (2000), the visual N1 component reflects the operation of a discrimination process within the focus of attention. Vogel and Luck (2003) found that N1 amplitude reflects the difficulty of target discrimination, suggesting that perceptual load modulates attention at an early processing stage, but following perceptual discrimination

The parietal cortex of the brain serves a vital role in everyday perception, fMRI research shows that the intraparietal sulcus is involved in the integration of relevant feature- and spacebased cues to optimize the deployment of attention in visual discrimination (Egner et al., 2008). According to Tan et al. (2015), a parietal-localized N1 component reflects the discrimination process to the change target, where visual stimuli that involve high perceptual loads require a higher level of discrimination processing (Lavie, 2010; Lavie & Tsal, 1994).

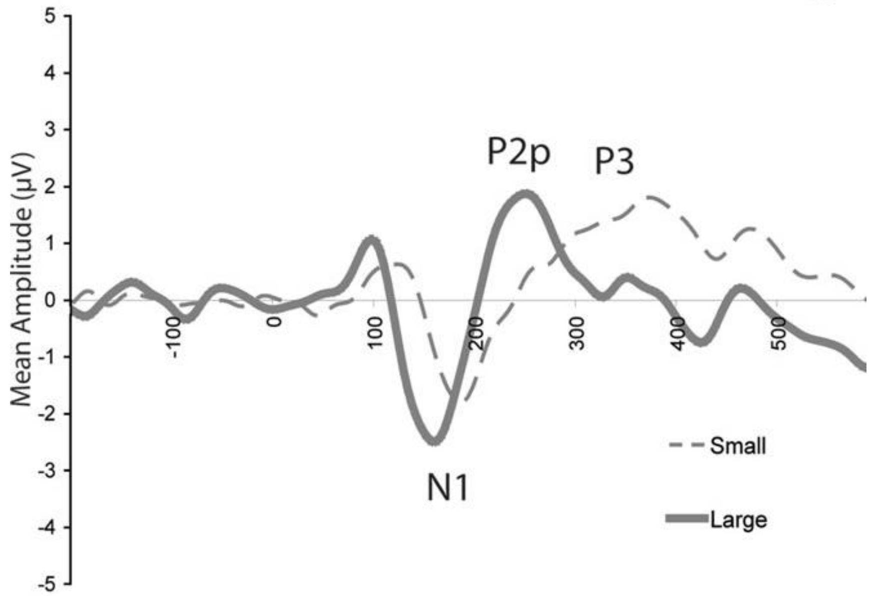
Temple and Posner (1998) have found ERP N1 negativities associated with change in numerical distance over the posterior Parietal-Occipital-Temporal (POT) region, where participants saw presentations of dots in amounts of 1, 4, 6, or 9, and they were asked to respond as rapidly as possible to indicate whether the stimulus was larger or smaller than “5” (the “control” number in the study). The researchers found that dots with further numerical distance from “5” produced higher N1 negativities, while closer numerical distances produced lower N1 amplitudes (Temple & Posner, 1998). These findings of higher N1 signals while processing larger numerical distances imply that more neural activity is generated over brain regions of numerical cognition as numbers get harder to tell apart.

2.2. Mid-latency ERPs and Numerical Change

2.2.1. P2p vs. P3 ERP

Previous ERP research also showed how small and large numbers evoke distinct mid-latency components over posterior scalp sites between 200 and 400 msec that were modulated in contrasting ways. In several ERP studies, higher P2p ERPs are produced when processing change in large numbers, such as 8, 16, 24 in Hyde and Spelke (2012), as well as Fornaciai and Park (2017), who examined the P2p response in arrays of 100~400 dots.

Figure 3: Grand average waveform in response to Small and Large numbers over the posterior scalp by Hyde and Spelke (2012).



Note: Small numbers (1, 2, 3) produced a P3 ERP, while Large numbers (8, 16, 24) produced a P2p ERP. Adapted from "Spatiotemporal dynamics of processing nonsymbolic number," by D. Hyde and E. Spelke, 2012, *Human Brain Mapping, 33*, p. 2192. Copyright 2012 by PubMed Central.

Although Hyde and Spelke (2012) found that large numbers (8, 16, 24) evoked a P2p that peaked around 280 msec over widespread left and right posterior sites, they discovered that small numbers (1, 2, 3) evoked a P3 that peaked around 334 msec over more left posterior and central sites. This shows that there is a spatiotemporal distinction between the P2p for large numbers and a later peaking P3 for small numbers over the posterior regions of the brain (see **Figure 3** for the ERPs produced when participants passively viewed a sequential presentation of dots, where P3 is elicited by small numerosities within the subitizing range by Hyde and Spelke, 2012).

Hence, I will conduct an analysis of ERP responses associated with the change (oddball) conditions, with a focus on amplitude and latency of responses from later positive waveforms. I will also examine reaction time and accuracy in these change conditions, and relate it to ERP differences in conditions of number type (small vs. large), and change direction (increasing or decreasing) between number pairs.

While the aforementioned components have been repeatedly shown to be more positive in response to target trials compared to non-target trials, multiple investigations have shown them to be modulated by distinct factors, indicating that these components reflect different cognitive processes (Brown et al., 2015; Kiat et al., 2018). Therefore, we will be considering various approaches to analyze our final later ERP component, taking into consideration peak latency and region of origin.

2.3. P3: Higher-order Cognitive ERP

2.3.1. P3a vs. P3b Component

The P300, also known as the P3, is a positive, late-occuring (often recorded around 300ms after stimulus onset) ERP waveform that is related to higher-order cognitive functions (Kiat et al., 2018; Polich, 2007; Sutton et al., 1965). Within the P300 family, there are two different components often studied (Polich, 2011):

a) **P3a**: Often measured over frontal regions, peaking between 250–280 ms post-stimulus, it is usually evoked by a distractor, and it originates from stimulus-driven, frontal attention mechanisms during task processing.

b) **P3b**: Often measured over parietal regions, peaking between 250-500 ms post-stimulus (note the longer time window than P3a), it is associated with attention and subsequent memory processing, and it originates from temporal–parietal activity.

Although these two different ERP components have been repeatedly shown to have more positive amplitudes in response to target trials compared to non-target trials, multiple investigations have shown them to be modulated by distinct factors, indicating that these components reflect different cognitive processes (Brown et al., 2015; Kiat et al., 2018).

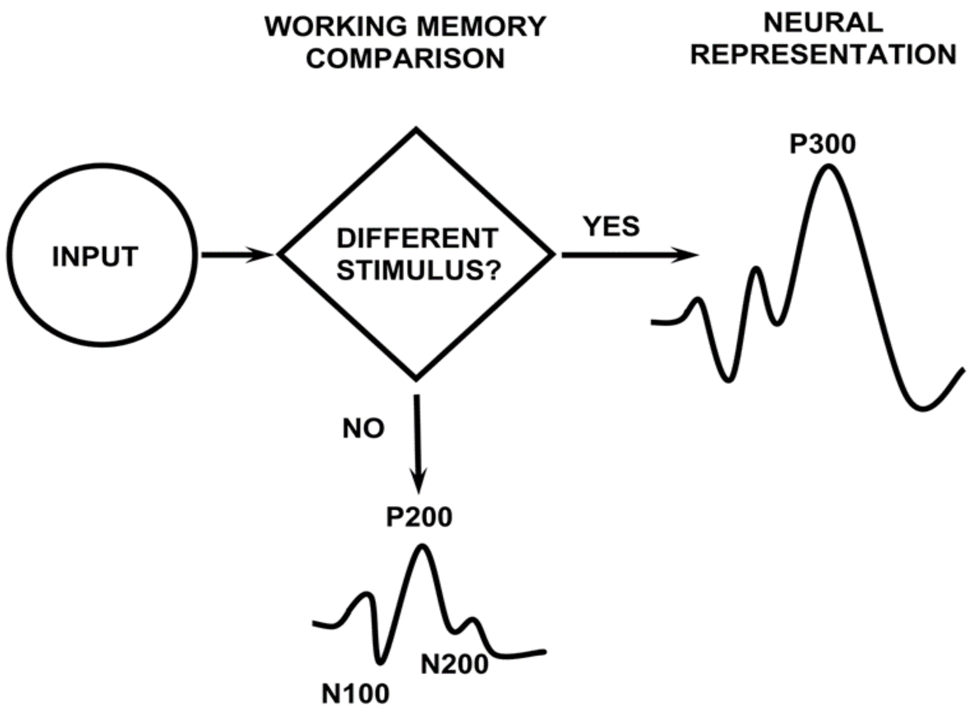
P3a differs from P3b in the following ways: (i) its scalp distribution (frontal vs. parietal) (ii) it often peaks 60–80 ms earlier than the P3b (Courchesne et al., 1975). While the P3a is elicited by highly deviant or task-irrelevant distracters, such as infrequently presented loud noises during an arithmetic task, the P3b is associated with task-relevant stimuli (Simons et al., 2001; Spencer et al., 2001). Therefore, we will be considering various approaches to analyze our final later ERP component, taking into consideration peak latency and region of origin.

In the case of simple go-No-go oddball paradigms with no distractors, the P3b is widely studied, as it is shown that this ERP reflects attentional resource allocation with respect to top-down goals stored in short-term memory (Donchin & Coles, 1988; Polich, 2011). As the current study employs a numerical change detection task with straightforward go-No-go instructions, this dissertation will focus on the P3b ERP. Of note, according to Luck (2005), when ERP researchers refer to the P300 component, they "almost always mean the P3b component" (p. 42).

2.3.2. The P300 Context-Updating Theory

Based on the Context-Updating Theory first outlined by Donchin, (1981), Figure 4 below is a schematic illustration of a theoretical account of the oddball task by Polich (2007). This theory posits that the P3 indexes brain activities underlying revision of the mental representation induced by incoming stimuli (Donchin, 1981). After initial sensory input, an attention-driven comparison evaluates the representation of the previous event in working memory. If no stimulus attribute change is detected, the current mental model or *schema* of the stimulus context is maintained, and only sensory potentials are evoked. When a new stimulus attribute is detected, on the other hand, the "updating" of the neural stimulus representation in working memory occurs and P3 is produced.

Figure 4: Schematic illustration by Polich (2007) of the P300 Context-Updating Model by Donchin (1981).



Note: Stimuli enter the system and a memory comparison process is engaged that ascertains whether the current stimulus is the same as the previous stimulus or not (Polich, 2007). If the incoming stimulus is the same, the neural model of the stimulus environment is unchanged, and only sensory potentials are evoked (N100, P200, N200). If the incoming stimulus is not the same and the subject allocates attentional resources to the target, the neural representation of the stimulus environment is updated, such that a P300 potential (P3b) is elicited in addition to the sensory potentials (Polich, 2007). Adapted from "Neuropsychology of P300" (p. 162) by J. Polich, 2012, in S. J. Luck & E. S. Kappenman (Eds.), ***The Oxford Handbook of Event-Related Potential Components.*** **Copyright 2011 by Oxford University Press.**

2.3.3. P3b Amplitude and Resource Allocation

Polich (2012) found that as primary task difficulty increases, the target stimulus P300 amplitude from an oddball task decreases. When difficult tasks require more processing resources, the P300 peak produces lower mean amplitudes (Kramer et al., 1985). Polich (2011) also noted that increases in working memory load reduce P3 amplitude in a manner that suggests that fewer attentional resources are engaged because of increased task demands to process these items (Gomer et al., 1976; Kok, 2001; Wijers et al., 1989).

2.3.4. P3b Latency and Reaction Time

P3b latency is thought to index classification speed, which is proportional to the time required to detect and process a target item (Kutas et al., 1977; Magliero et al., 1984). Additionally, P300 timing is sensitive to both stimulus- and response-related variables when responding is fast (cf. Ilan & Polich, 1999; Verleger et al., 2005); this conclusion suggests that P300 may originate from the neural events that link stimulus perception and event response (Verleger et al., 2005).

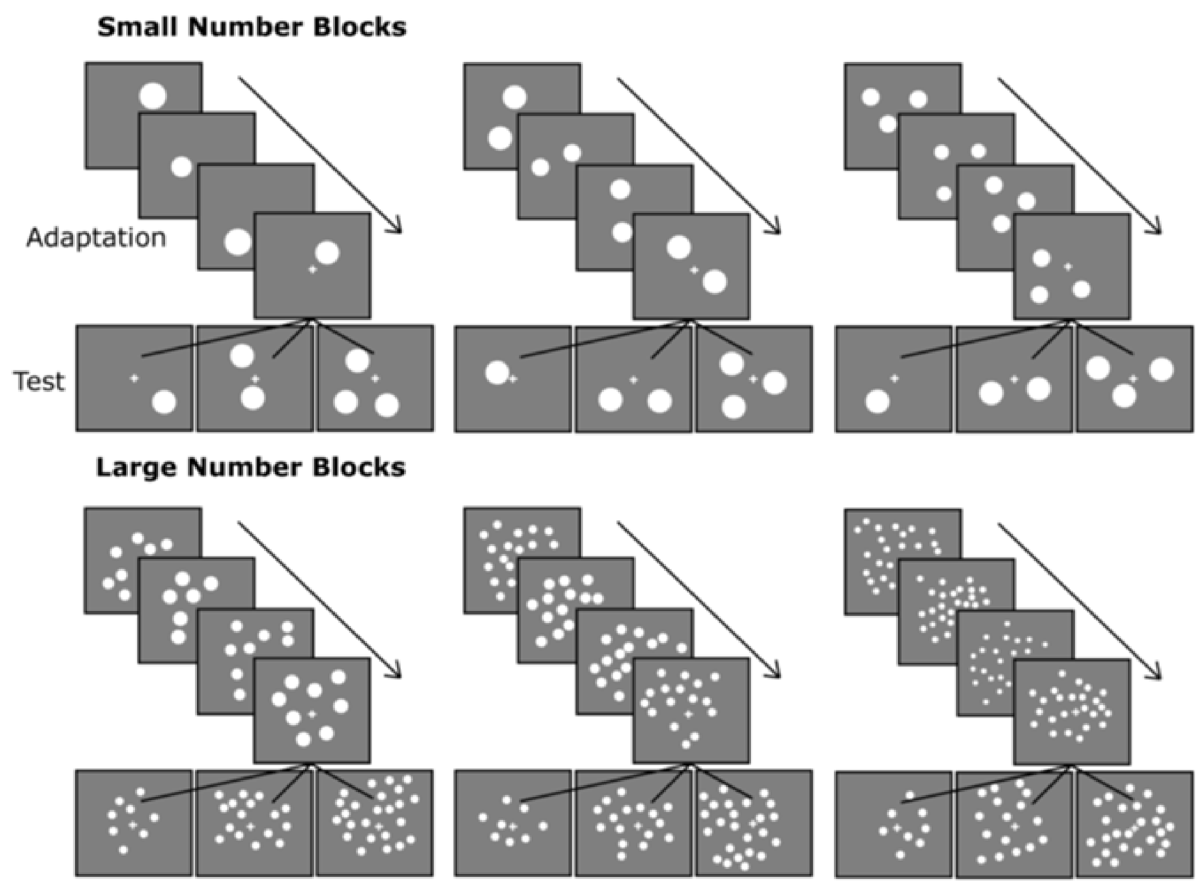
All in all, ERP studies on number processing suggest that differences observed between small and large number representation result from early differences in attentional selection, and these differences determine whether an array of objects will be represented by the approximate numerical magnitude system or through parallel individuation. In other words, it seems to be the case that the two systems are not specialized for small and large numbers per se, rather early attentional selection and its corresponding limits determine whether objects will be represented as distinct individuals or approximate numerical magnitudes.

Chapter 3: Study Rationale

3.1. ERPs from Small vs. Large Numbers in a Narrower Continuum (1~6)

The Hyde and Spelke experiments (2009; 2012) examined ERP differences during a passive observation of small and large numerical set sizes that were perceptually distinct from each other, where there is a large gap between the small (1, 2, 3) and large (8, 16, 24) number ranges used in their study (see Figure 5).

Figure 5: Schematic description of adaptation and test number pairs presented to participants in EEG study by Hyde and Spelke (2009).



Note: Adapted from "All Numbers Are Not Equal: An Electrophysiological Investigation of Small and Large Number Representations," by D. Hyde and E. Spelke, 2009, *Journal of Cognitive Neuroscience, 21,* p. 1041, Copyright 2009 by MIT Press.

The current study aims to examine the Small vs. Large distinction within a narrower and continuous range of 1 to 6, to see if there was a clear categorical boundary between small and large number response. One of the study's goals was to analyze ERP responses after the participants have adapted/habituated to the individual numerical quantities in the "No Change" condition, where the same number of dots are presented for three to five slides. At the post-adaptation time point, peak amplitude and latency of the N1 response (associated with early sensory perception) will be analyzed to see how these differ in response to different numerosities of 1~6. In particular, we aim to investigate whether there was a distinct N1 response pattern when changes crossed over between small (1~3) and large (4~6) set sizes as compared to N1 responses within the same set size changes (i.e. Small-to-Small and Large-to-Large).

3.2. Direction: Increasing vs. Decreasing Numerosities

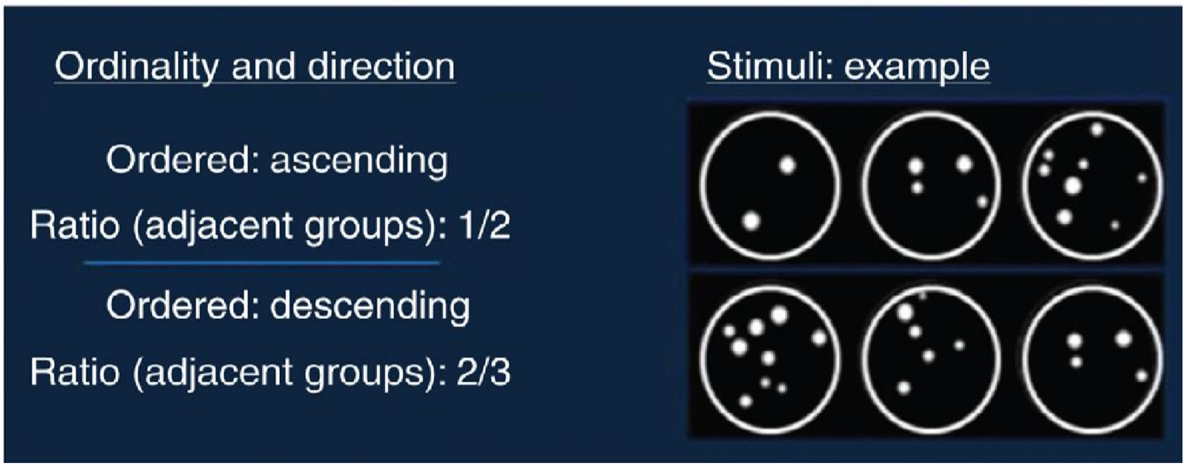
In the field of numerical change processing studies, magnitude effects of numeric size and numerical distance are well-known, but there is growing evidence for another factor that affects discriminability: *change direction*, by which a relative increase in quantity is identified more rapidly and with greater accuracy than a relative decrease in quantity (Kaan, 2005; Paulsen & Neville, 2010). Unlike the numerical range used in the present study (i.e., 1~6), both of these studies used much larger numerical magnitudes in their change processing studies. Paulsen and Neville (2010) examined a numerical range of 10~60 dots, while the study from Kaan (2005) used spelled-out English number-words that ranged from “twelve” ~ “seventy-two”.

Change direction effects have been found in symbolic number judgments and non-symbolic numerosity judgments. Kaan (2005) had participants compare sequentially presented spelled-out number-word stimuli (e.g., “thirty - sixty”) and found that participants responded more quickly and accurately when the second numerical stimulus was larger relative to when the second stimulus was smaller than the first stimulus.

In the case of using sequentially-presented dot stimuli in numerical studies, Paulsen and Neville (2010) found a behavioral interaction between magnitude and numerical distance of dot stimuli that led to the examination of change direction. In their study, participants were required to judge whether the second stimulus (S2) of a pair of sequentially presented dot–array stimuli contained the same or a different number of dots than the first stimulus (S1) of the pair. A direction effect was observed, in that accuracy was greater and reaction time was faster when the second numerosity was larger in magnitude than the first (e.g., 16:24), compared to when it was smaller (e.g., 24:16). S1:S2 pairs that decreased in magnitude also elicited a greater negativity around 400 ms compared to pairs that increased in magnitude.

On the other hand, in an ERP study by Rubinsten et al. (2013) that examined the effects of direction (ascending vs. descending quantity), the researchers asked participants to decide whether the three circles include dots that are presented in an ordered or nonordered fashion, based on quantity (see Figure 6 for an example of the authors' stimuli).

Figure 6: Direction-variable numerical stimuli by Rubinsten, 2016 (each stimulus is represented by a black rectangle that includes three simultaneously presented circles with dots).



Note: The task is to decide whether the three circles include dots that are presented in an ordered or nonordered fashion (in the current example, based on quantity). Adapted from *"Ordinal instinct: A neurocognitive perspective and methodological issues"* (p. 277) by O. Rubinsten, 2016, in A. Henik (Ed.), *Continuous issues in numerical cognition: How many or how much. (pp. 271–288). Copyright 2016 by Elsevier Academic Press.*

In contrary to the findings where there seem to be a behavioral advantage for detecting change for increasing numerosities in an ascending, ordinal manner (Kaan, 2005; Paulsen & Neville, 2010), Rubinsten et al. (2013) found faster reaction times for descending sequences. Rubinsten et al. (2013) also found higher P3 amplitudes for descending sequences were associated with higher activity over the right parietotemporal area, but less positive P3 amplitudes over the left parietotemporal area.

As there have been rather few ERP studies investigating the brain's neural mechanisms of processing direction effects of numerical change, (Kaan, 2005; Paulsen et al., 2010; Rubinsten et al., 2013), there is no well-established ERP marker for this process yet. However, in the context of discrimination, a great number of studies have found that the N1, N2, P2p and P3, vary as a function of the difficulty of the discrimination process in different ways (e.g. size of numeric sets, numerical distance), in that the amplitudes of the N1 and N2 are positively correlated with the difficulty level of the discrimination, while the amplitude of the P3 negatively varies with the increase of the difficulty.

Chapter 4: Research Questions, Hypotheses and Predictions

4.1. Cardinal Values in No-Change Trials

We seek to investigate if there is a discontinuity between systems of small number processing and large number processing within the continuous range of 1 to 3 and 4 to 6, respectively, similar to results found with more numerically separated values used in previous research (Hyde & Spelke, 2012). Our hypotheses and predictions relate to trials in which cardinal values are presented without a change and therefore no response is expected. In other words, there are no behavioral data to consider and only hypotheses relating to ERP data will be considered. In previous studies by Hyde & Spelke (2009; 2012) they found effects of small vs, large cardinality in the Parietal-Occipital-Temporal (POT) region whereby values within the small number range (1-3) were scaled to N1 ERP amplitudes. On the other hand, their study found no clear scaling of numerosity to amplitude for the larger numerical values (8-16-24) that they tested.

4.1.1. Research Question 1:

Our first research question relates to whether this electrophysiological difference will be found within a continuous range of numerical values, specifically when participants are presented with numerosities 1~6 for the present experiment.

4.1.2. Hypothesis 1:

Over the POT area, the sensory visual N1 component is related to spatial attention and is read off individual items in Working Memory within the smaller number range (1~3). On the other hand, numerical estimation is not read off of exact individuation of numerical values in the larger number range (4~6).

4.1.3. Prediction 1:

In evaluating responses to no-change trials, we predict that there will be scalar N1 ERP amplitude differences within the range of small cardinal values ($1 < 2 < 3$), whereas no such scalar ordering of ERP numbers is predicted within the continuous, larger range of 4 to 6.

4.2. Behavioral Effects of Size and Direction in Change Trials

For trials in which there was a change in numerical value of the stimulus, participants were asked to respond by pressing a key for the study's change detection task. We are interested in performance variables such as *reaction time* and *accuracy* for these change conditions, as well as whether these behavioral responses are affected by the *set size* and the *directionality* of the change (increasing vs. decreasing). In terms of the *directionality* factor, previous research has found that there is a performance advantage in the increasing set size conditions over the decreasing set size conditions, but these studies looked at number comparisons with much larger numerical set sizes of 10 to 70 (Paulsen et al., 2010), or used spelled-out number words again in the larger value ranges (Kaan, 2005).

4.2.1 Research Question 2:

Our second research question relates to the *size* of numeric sets on change detection behavioral measures like accuracy and reaction time, and whether the increased set size advantage is found within the smaller set size in the present experiment. Set size changes are grouped into three categories: Small-to-Small (e.g., 1→2, 2→3), Large-to-Large (e.g., 4→5, 5→6, 4→6) and Crossover conditions, which can be Small-to-Large (e.g., 2→4, 3→6), or Large-to-Small (e.g., 4→2, 6→3).

4.2.2 Hypothesis 2:

Numerical change is harder to detect among large numbers, since large number estimation does not access exact numerical quantities available in processes involving parallel individuation and working memory.

4.2.3 Prediction 2:

We predict faster reaction times and higher accuracy when change detection is easier, particularly while processing smaller numbers in the subitizing range (1~3), as compared to large number processing (4~6). We also predict these advantages for crossover conditions to be midway between Small and Large conditions in behavioral measures of processing efficiency (reaction times and accuracy). Our research questions relate to how set size is related to accuracy and reaction time, and whether the increased set size advantage is found within the smaller set size in the present experiment.

4.3. Direction and Size Effects of Numerical Change

Within the 1~6 numerical range, we aimed to investigate if there is an advantage for trials that involve *Increasing* quantities over *Decreasing* quantities. The present study asks whether change detection in smaller numerical ranges reflect the same or different kinds of processes in which directionality could have an effect.

It bears to note that the aforementioned ERP research on how the direction of numerical change can influence neurobehavioral outcomes are all using different experimental paradigms from the current study. For instance, Rubinsten et al. (2013) concurrently presented three sets of numerosities to participants to compare and they are to respond if their numerosities are arranged in an ordinal manner.

The numerosities of 1~6 that are examined in the present study are outside the range to those of the previously mentioned numerical processing neurobehavioral studies that also investigated the effects of change direction. Kaan (2005) showed participants a sequence of two spelled-out English number words in a range between “twelve” and “seventy-two”, where they are asked to indicate whether the second number word signifies a quantity smaller or larger than the first number word. Though Paulsen and Neville (2010) also used dot stimuli similar to the current study, their tested range is much larger (10~60). Additionally, their participants are asked to respond when they see the same number of dots, whereas our study’s participants responded when they see a change in the number of dots.

Due to the differences in experimental paradigms and numerical ranges, findings from these studies on change direction have been hard to reconcile. Hence, we have few *a priori* predictions about the behavioral and ERP effects of increasing vs. decreasing magnitudes in the change conditions. Therefore, we leave the question open as to whether directionality will lead to increases or decrease in task difficulty. However, working backwards, we can assume that reaction time and accuracy differences between increasing and decreasing quantities will provide evidence for processing differences as will differences in the ERP deflections associated with increasing vs. decreasing quantity.

For directionality effects, we are primarily guided by previous research on large number changes that show advantages for increasing over decreasing changes in magnitude (Kaan, 2005; Paulsen et al., 2010). While the mechanism for such asymmetry has multiple explanations, there are no *a priori* mechanistic hypotheses at this point for the current experiment. However, we can identify two possible hypotheses regarding the possible role of directionality of change in the smaller number range.

4.3.1. Research Question 3:

Are there behavioral differences in the Small vs. Large vs. Crossover changes, where directionality of numerical change interacts with set size?

4.3.2. Hypothesis 3 (UEH of Direction):

Based on the Uniform Effects Hypothesis (UEH) of *Direction*, responses are uniform across all quantities. Since previous research has shown better performance for increasing magnitude in large number changes (e.g. 10~70), such advantages could exist across the board and within the 1~6 number range of the current experiment.

4.3.3. Prediction 3 (UEH of Direction):

Across the board for the Small, Large and Crossover conditions, reaction time is faster and accuracy is higher for increasing over decreasing magnitude changes.

4.3.4. Hypothesis 4 (IEH of Direction and Size):

Based on the Interaction Effects Hypothesis (IEH) of *Direction* and *Size*, directionality effects are different in the small number (1~3) and large number (4~6) range reflecting different effects due to changes in the working memory/parallel individuation range compared to numerical estimation.

4.3.5. Prediction 4 (IEH of Direction and Size):

Performance in detecting changes within the larger numerical range (4~6) is predicted to show similar patterns with previous research, in which higher accuracy and faster reaction times for increasing over decreasing numerical changes. This effect might be higher in larger numbers than smaller numbers in the subitizing range. Changes in the small number range (1~3) should show a difference effect, either no directionality effects or better accuracy and reaction times in the decreasing set size range.

Note that the above two hypotheses represent two possible and contradictory mechanisms with distinct sets of predictions, neither of which are committed to in any *a priori* manner, but do represent two "possible worlds" with regards to directionality effects.

4.4. N1 Response to Numerical Change

In this study, while participants are behaviorally responding to numerical change on the screen with a key press, their EEG data (in response to the stimuli) is simultaneously being recorded. This gives our study an avenue to study ERPs in response to numerical change. The ERPs of interest to us are the N1 and the P3b.

4.4.1. Research Question 5:

For the N1 component measured over the lateralized POT areas, are there amplitude and latency differences between small and large numbers, and in terms of change direction (increasing or decreasing)?

4.4.2. Hypothesis 5:

During change conditions, the N1 component over the POT area is related to visuospatial attention and of visual short-term memory, where heavier perceptual loads are linked to higher N1 negativities.

4.4.3. Prediction 5:

As more objects are encoded in of visual short-term memory, the N1 amplitude over the POT area is predicted to get higher. As Hyde and Spelke (2009, 2012) found that N1 responses showed earlier peaks for larger numbers, we also predict that smaller numbers will have longer latencies.

4.5. P3b Response to Numerical Change

4.5.1. Research Question 6:

For the P3b component measured over the mid-parietal (Pz) area, are there amplitude and latency differences between small and large numbers, and in terms of change direction (increasing or decreasing)?

4.5.2. Hypothesis 6:

In accordance with the P3b context-updating theory (Polich, 2007), when measuring numerical change processes over the Pz area, the P3b component is scaled to the ease of change detection, by how much easier it is to update the context from its previous one.

4.5.3. Prediction 6:

When the numerical change gets easier to detect (smaller sizes), the P3b is predicted to show higher amplitudes, reflecting greater ease in updating the context. Polich (2012) describes that the peak latency of the P3b component can be interpreted as indexing stimulus evaluation time, where it peaks earlier for easier tasks and more difficult tasks showed longer latencies. Thus, we predict higher P3b amplitudes and shorter P3b latencies during easier, small number processing.

Chapter 5: Methods and Study Design

For this study, a set of numerical stimuli with the number of dots ranging from 1~6 was adapted from Hyde and Spelke (2009), where they tested small numbers (1, 2, 3) and much larger numbers (8, 16, 24).

The design for the current study is outlined in *Figure 7* below. If we compare our design with the paradigm used in Hyde and Spelke (2009, *see Figure 5 in Chapter 3*), we see that our large number set is much smaller (4, 5, 6), Also, unlike Hyde and Spelke (2009), the change conditions occur not only within the small and large sets, but also between sets in the "Crossover" conditions.

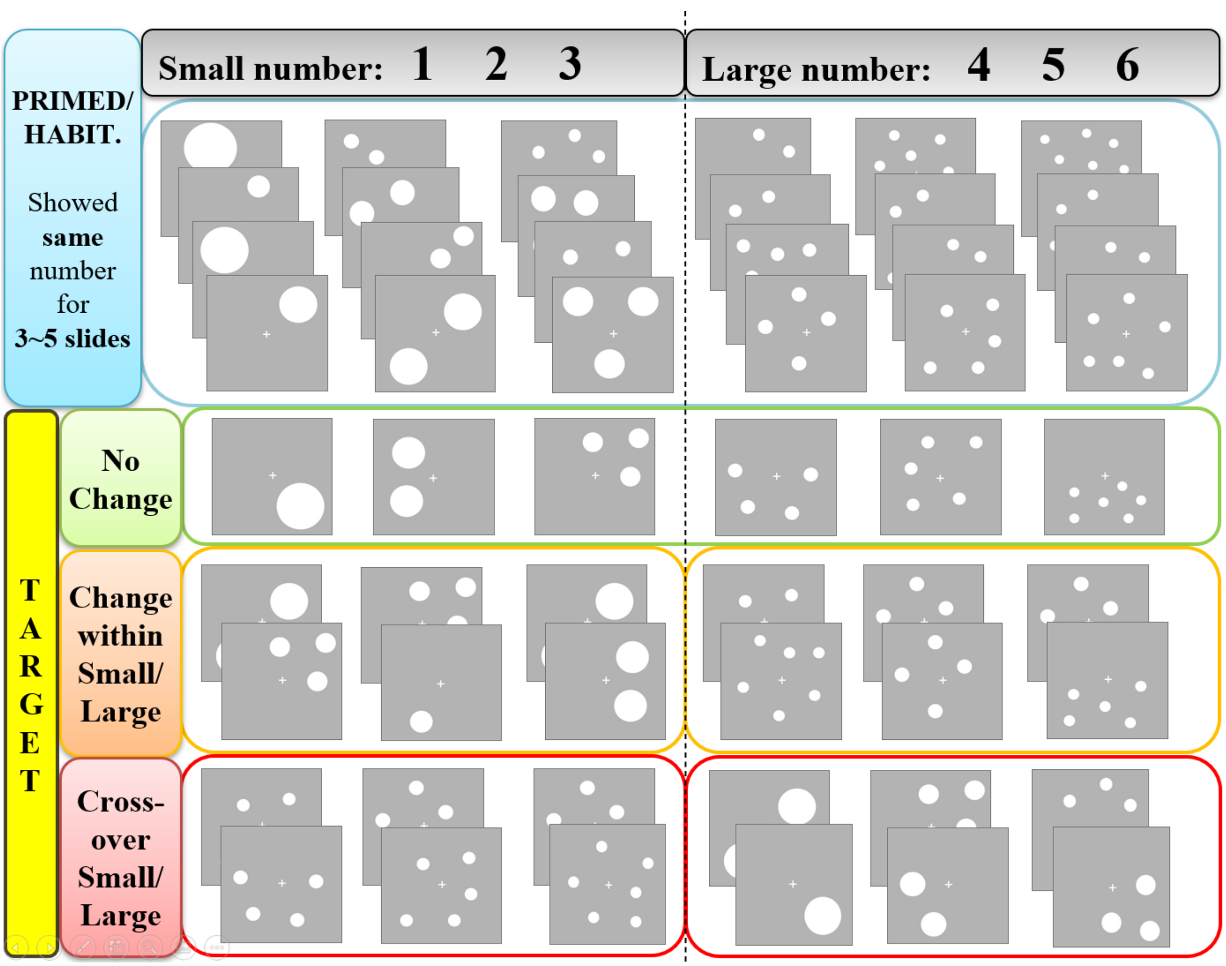


Figure 7. Current study design (adapted and revised from Hyde and Spelke, 2009)

5.1. Population and Sample

This dissertation study (IRB protocol # 22-288) involves a retrospective analysis of EEG data that was collected under IRB protocol # 13-068 from 17 neurotypical adult volunteers at Teachers College, Columbia University. Please see Appendix A for IRB approval letter and consent forms associated with the current study.

Because of COVID-19 restrictions, no additional data were collected for the present study as had been originally proposed. Two participants were excluded from the final analysis, restricting the sample to n= 15 (4 male, 11 female). These exclusions were due to several factors, including excessive noise in raw EEG recordings, artifact exceeding acceptable limits during ERP processing, or technological failures. Mean participant age was 27.31 years old.

Two participants were excluded from the final analysis, restricting the sample to n= 15 (4 male, 11 female) due to several factors, including excessive noise in raw EEG recordings, artifact exceeding acceptable limits during ERP processing, or technological failures. Mean participant age was 27.31 years old**.** All participants were right-handed (based on self-reporting), and all were proficient in the English language to give informed consent, as TESOL results are required for admission into Teachers College. Demographic data from each participant can be found in the Appendix B (Table 1A).

The following were considered ***exclusionary*** criteria for participants:

1. 18 years old and younger
2. Visual impairment without ability to compensate via corrective lenses
3. Neurological disorders; history of seizure disorders; history of traumatic brain injury.

5.2. Stimuli

Adapting stimuli from Hyde and Spelke (2009; 2012), this study's stimuli consisted of square images (650 × 650 pixels) of 1, 2, 3, 4, 5 or 6 white dots on a gray background, constructed as to control for continuous parameters other than the number. Using MATLAB and Adobe Illustrator, S.B. Kim (a former doctoral student of the lab) adapted design protocols from previous studies to create our stimuli (Hyde & Spelke, 2009; Piazza et al., 2004). Specifically, the design protocol equated "the intensive parameters (individual dot size and inter-dot spacing) of the arrays across the target stimuli and varied the extensive parameters (total area occupied and total luminance) of the target arrays randomly so that these variables were equated, on average, across adaptation stimuli with the constraint that the values for the extensive parameters were drawn randomly from fixed distributions that spanned the range of values used for target stimuli" (Hyde & Spelke, 2009, p. 8).

To minimize the influence of non-numerical visual cues, size and location of the dots needed to be pseudorandomized within each trial, as this prevents a linear covariation between numerosity and total area (Cutini et al., 2014). This design resulted in target stimuli that were equally similar in regards to the continuous parameters other than number, because these values had already been presented equally often in the "primed" habituation stimuli. This method of controlling for intensive and extensive parameters has been employed in several recent neuroimaging studies of numerical cognition (Hyde & Spelke, 2009; Izard et al., 2008; Piazza et al., 2004).

In EEG studies, it is important to keep blinks and eye movement artifacts to a minimum to ensure cleaner electrophysiological data. Not using high contrast images is one particular method of reducing these artifacts. To decrease the level of eye strain for the participants while they go through our study, the contrast ratio of our stimuli's gray background against white dots (40 cd/m2) has a contrast ratio of 2.1:1 (~50%). To prevent excessive eye movement artifacts that will contaminate our data, we encouraged our participants to have their eyes faced towards the center of the monitor, aided by a small white fixation cross in the center, which will be present during both the displays and the inter-stimulus intervals. They were also encouraged to close their eyes during breaks.

A total of 270 test trials were divided into five experimental blocks. Before the start of the actual experiment, each participant goes through a practice block with 10 trials. For the actual five experimental blocks, there were a total of 60 test trials in each block. In each block, stimulus consist of number pairs with no change (NC, n = 12), Small-to-Small (SS, n = 12), Large-to-Large (LL, n = 12), Small-to-Large (SL, n = 12) and Large-to-Small (LS, n = 12) conditions. Each unique number pair was tested 120 times throughout the whole study.

Each trial consisted of a habituation phase where the same number of dots is presented one after another for three to five slides (as the "primed" number), and this is then followed by the "target", where the number of dots will change. If the participant detects a numerical change in the "target", they are instructed to press a key. Throughout the study, the oddball "target" appears at ~10% of the time. Stimuli were presented for 250 ms each, with interstimulus intervals at 750-1250 ms. This stimulus jitter is applied to reduce overlap in the ERP response to successive images (Luck, 2005) and to reduce as much as possible the repetitive nature of the stimuli. Please see Figure 8 for an example of a trial, where "3" is the primed number showed on three separate dot arrays, and "5" is the target number.

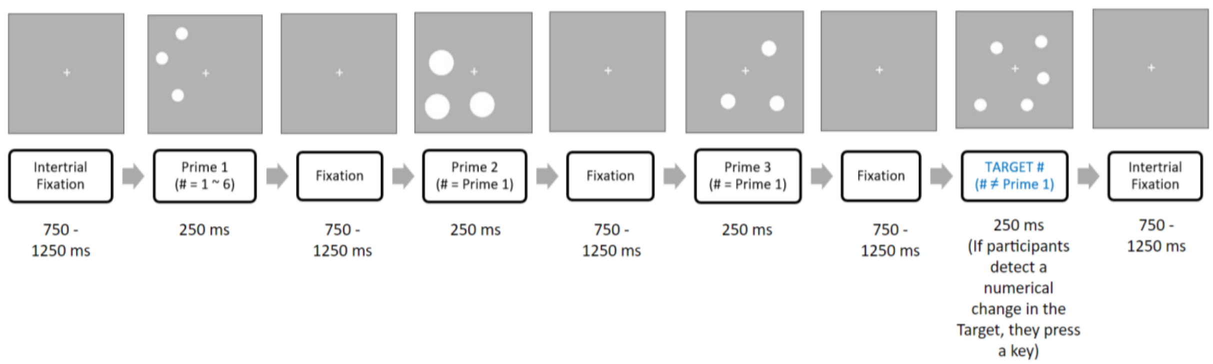


Figure 8: Example of a trial where "3" is the primed number showed on three separate dot arrays, and "5" is the target number.

Using random-number-generated protocols from EPrime, trials were presented randomly within blocks, while items from each condition were balanced with each trial and block. Breaks were provided between each of the five experimental blocks. The total duration of the experimental portion of the study was approximately 40 minutes-1 hour.

5.3. Study Design

In our study, ***small*** numbers are defined as ***1, 2, 3***, and ***large*** numbers are defined as ***4, 5, 6***. As part of the habituation, participants first saw the same numerical value (1 ~ 6) presented on three to five trials. Varying the number of habituation trials to between three and five ensured that the onset of the "target" stimuli would be unpredictable to the participants. After presenting the same quantity of dots to promote habituation/adaptation in the brain to the "primed" number, the "target" stimulus that appears next could be categorized into one of the six types of "change" conditions (as outlined below, with each unique number pair as examples):

1. Increasing Small-to-Small (**iSS**): 1→2, 1→3, 2→3
2. Decreasing Small-to-Small (**dSS**): 2→1, 3→1, 3→2
3. Increasing Large-to-Large (**iLL**): 4→5, 4→6, 5→6

4. Decreasing Large-to-Large (**dLL**): 5→4, 6→4, 6→5

5. Increasing Small-to-Large (**iSL**): 1→4, 2→4, 2→5, 3→4, 3→5, 3→6

6. Decreasing Large-to-Small (**dLS**): 4→1, 4→2, 4→3, 5→2, 5→3, 6→3

7. No change (as a control): 1→1, 2→2, 3→3, 4→4, 5→5, 6→6

Please see *Figure 9* for the study's design elements and how specific number pairs are grouped into the "change" conditions.

Target # / Primed #	1	2	3	4	5	6
1	Same #	inc.SS	inc.SS	inc.SL	N/A	N/A
2	dec.SS	Same #	inc.SS	inc.SL	inc.SL	N/A
3	dec.SS	dec.SS	Same #	inc.SL	inc.SL	inc.SL
4	dec.LS	dec.LS	dec.LS	Same #	inc.LL	inc.LL
5	N/A	dec.LS	dec.LS	dec.LL	Same #	inc.LL
6	N/A	N/A	dec.LS	dec.LL	dec.LL	Same #

<table>
<tr><td rowspan="3">Pairs of numerical change (e.g., "1→2" = the Primed # is "1", followed by the Target # as "2")</td><td rowspan="2">No change (Same number)</td><td colspan="2">Small → Small (SS)</td><td colspan="2">Large → Large (LL)</td><td colspan="2">Cross-over Small-to-Large (SL) & Large-to-Small (LS)</td></tr>
<tr><td>inc.</td><td>dec.</td><td>inc.</td><td>dec.</td><td>increase</td><td>decrease</td></tr>
<tr><td>1→1; 2→2; 3→3; 4→4; 5→5; 6→6</td><td>1→2; 1→3; 2→3</td><td>2→1; 3→1, 3→2</td><td>4→5; 4→6; 5→6</td><td>5→4; 6→4; 6→5</td><td>1→4; 2→4; 2→5; 3→4; 3→5; 3→6;</td><td>4→1; 4→2; 4→3; 5→2; 5→3; 6→3</td></tr>
</table>

Figure 9: Design elements of current study

We chose number pairs that have a difference of one, two, three, and not more. This is so that the differences were not too perceptually distinct. For instance, if a number pair has a difference of 4, such as a change of 5→1, it would be too perceptually distinct. The same goes for if a number pair has a difference of 5, such as a change pair of 1→6. Therefore, we have set the maximum difference between a pair of numbers to 3. Furthermore, within the small values (1 ~ 3), it is not possible to have a difference greater than 2. So, we wanted to keep the differences between each number pair small enough to keep the distances constant.

The Hyde and Spelke (2009) experiment examined differences that occurred between small (1, 2, 3) and large (8, 16, 24) sets. However, the differences between these numbers are perceptually distinct with a large gap between the small and large number ranges. Furthermore, in Hyde and Spelke (2009), the changes remained within the small or large number sets. No changes crossed between small and large number values.

The current study aims to look for categorical small-large differences within a narrower range (1~ 6) than Hyde and Spelke (2009; 2012). We also investigated changes that crossover between small (1~3) and large number (4~6) values.

5.4. Behavioral Data Collection

During the experiment, each participant was seated in a comfortable chair placed inside an electrically-shielded, sound attenuated EEG suite, at a distance of 70 cm from a 22" LCD Dell monitor. Participants are instructed to pay attention to the displays of dot patterns, and to press a key whenever they detect a change in the number of dots. The response was embedded to keep participants engaged, and to interpolate behavioral findings to ERP data.

The previous studies by Hyde and Spelke (2009; 2012) were passive-viewing EEG experiments, meaning that their participants were not required to respond as they viewed the dot arrays, while their electrophysiological data was simultaneously being recorded. Prior to the present study, in our previous attempts to replicate the experiments by Hyde and Spelke (2009; 2012) to examine numerical ERP effects, participant boredom was a serious concern, as many of them fell asleep, were inattentive, or complained of being unpleasantly bored. Instead of passive viewing of thousands of dot patterns, our study asked participants to detect changes in the numerical value, and to press a key when such changes occurred.

To keep the participants further engaged, we gave them a score update during breaks between each of the 5 testing blocks. At the end of the experiment, each participant was given a score of how many correct responses they got, and they were given a reward of a lottery scratch off card for every 50 items they got correct. Behavioral data (accuracy and reaction time) was collected via EPrime. Variables of numerical change are directionality: Increasing vs. Decreasing, and size (Small-Small, Large-Large, Crossovers).

For this study, a set of numerical stimuli with the number of dots ranging from small (1~3) and large (4~6) cardinalities. This stimulus presentation paradigm was adapted from Hyde and Spelke (2009), where participants viewed an ongoing stream of different numerosities, presented rapidly and sequentially. During the No Change conditions, participants saw the same number of dots repeatedly (albeit in different locations and sizes in each array), and the N1 response to the cardinalities of 1~6 was recorded and analyzed.

Chapter 6: Data Processing and Analysis

6.1. Pre-/Post-Processing

First, continuous raw EEG data were digitally filtered offline using a 0.3 high-pass filter and a 30 Hz low-pass filter (FIR Passband Gain: 99.0 % [-0.1 dB], Stopband Gain: 1.0 % [-40.0 dB], Rolloff: 2.00 Hz). The data were segmented into epochs of 500 ms that included 100ms prior to stimulus onset and 400ms following stimulus presentation. The segmentation protocol also incorporated an offset that reflects a necessary millisecond correction due to an expected delay between the timestamp (time reported by experimental control module) and the actual time the stimulus was presented onscreen to the participant (Electrical Geodesics, 2015). The offset value was acquired by running timing tests prior to each run using a Cedrus Stim Tracker.

The segmented data were then subjected to automatic artifact detection and bad channel replacement protocols to remove eyeblinks and physiological artifacts (e.g., electrocardiogram, electromyogram, electrooculogram). Electrode channels that exceeded 200 microvolts (μV) were replaced using spherical spline interpolation from data acquired at surrounding sensors. Trials were discarded from analysis if they contained eye blinks (EOG >140 μV), or if more than 40% of the channels were bad. Following the automatic artifact rejection protocol, trial segments were manually reviewed and marked as bad if necessary.

Baseline correction was then carried out with respect to a 100ms portion of each epoch preceding stimulus presentation. This portion of the total epoch reflects random activity not associated with stimulus processing, which can introduce significant variance to the data, making group differences more difficult to observe. Baseline correction minimized such confounds by averaging the amplitude at all points across the pre-stimulus segment and then subtracting that value from the samples in the post-stimulus segment (Luck, 2014).

Data were then re-referenced from the vertex electrode (applied during recording) to the average of all electrodes. As a final step, all trials for each participant were averaged to generate the ERP waveforms within individuals and within conditions, so that event-related brain activity most relevant to the stimulus presentation could be observed and further analyzed (Luck, 2014). The pre-/post-processing protocol was completed for all recorded data.

6.2. ERP Data Analysis Protocol

Post-processed averaged ERP data files for each participant and condition were exported from NetStation for statistical analysis. Post-processed data files were read into an R database (R Core Team, 2016) and measures of amplitude and latency obtained. R scripts developed in-house specifically for this experiment were used to obtain peak latency and adaptive mean amplitude measures for each component. Peak latency measures were calculated by identifying the maximum positive and negative voltage deflection within a pre-selected time window. These values were then used to calculate the adaptive mean amplitude, which selected five samples or a 10 ms window on either side of the identified peak latency and averaged the sampled amplitude values.

For this study, the recording sites selected for statistical analysis focused on scalp locations in the parietal-occipital-temporal (POT) area. The N1 and P3b components were represented by electrode montages (please refer to next section in 6.3 for exact location of these montages). Individual files were grand-averaged together in MATLAB to visualize the produced N1 and P3b ERP waveforms elicited by the No Change and Change Conditions by numerical change direction (Decreasing vs. Increasing) and size of numeric sets (Small, Large, Crossovers).

Measures for each target component (N1 and P3b) were calculated based on expected scalp topography represented by specified electrodes and time windows from prior studies (Coch et al., 2005; Luck, 2014; Woodman, 2010) and adapted for a high-density recording net.

6.3. Regions of Interest (Montaging)

Three regions of interest (ROI)/montages were selected based on previous empirical evidence of their clear contribution in producing ERPs related to numerical change. These include the N1 (Hyde & Spelke, 2012; Libertus et al., 2007; Temple & Posner, 1998), and the P3b components (Ilan & Polich, 1999; Polich, 2011; Rubinsten et al., 2013). For the early N1 ERP to the No Change conditions and to the Change conditions, we extracted electrocortical information from montages encompassing the right and left parietal-occipital-temporal (POT) junctions (i.e. secondary visual cortex). For the P3b response to Change conditions, ERP responses were recorded from montaged electrodes over the Pz area, which is above the posterior-parietal region (over the midline of the scalp). Please see *Figure 10* for specific electrodes used for examined montages:

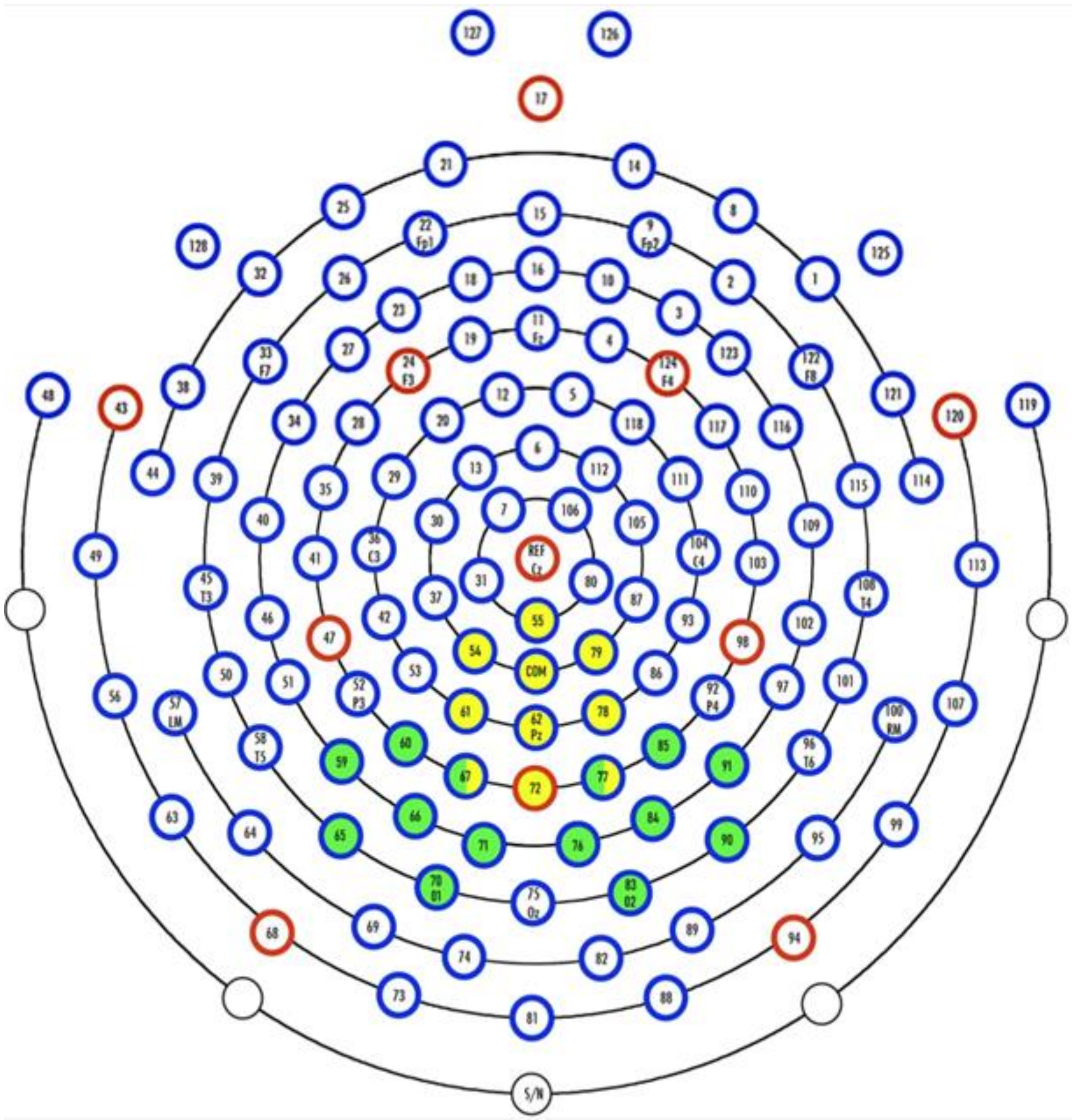


Figure 10: Map of electrode groupings used for averaging and analysis: Left and Right POT area (green, N1 component) and Pz area (yellow, P3b component).

6.2.1. N1 over Left POT (Montage 1): Located over the POT area in the left hemisphere and consisted of electrodes 66, 65, 59, 60, 67, 71, 70.

6.2.2. N1 over Right POT (Montage 2): Located over the POT area in the right hemisphere and consisted of electrodes 84, 76, 77, 85, 91, 90, 83

6.2.3. **P3b (Montage 3):** Located over the central location of the posterior parietal areas above the scalp's midline (Pz area) and consisted of electrodes 62, 78, 77, 72, 67, 61, 54, 55, 79.

6.4. Statistical Analysis

R Studio and SPSS were used to conduct statistical analyses. During the habituation phase where the number of dots remained the same, the effects of cardinal value were statistically assessed by comparing the mean amplitude and mean latency for the N1 ERP over the POT area. This study employed a Linear Mixed Model Analysis to investigate N1 amplitudes over bilateral posterior parietal sites. The Mixed Model Analysis (Cnaan et al., 1997) can efficiently take into account between-participant and between-block variability as random effects, while testing our manipulations of different cardinal values on N1 production.

For ERPs produced during trials when there is a change in the number of dots, the means for peak amplitude and latency for the N1 (125-200ms, over the POT area) and the P3b (435-535ms, over the Pz area) were analyzed. During numerical change trials, behavioral measures were recorded and analyzed as means for reaction time and mean accuracy.

Assumptions of homogeneity and normality were investigated prior to conducting between-group analyses (Levene's statistic and Shapiro-Wilk test). Each dependent variable (Amplitudes for N1 and P3b, Latencies for N1 and P3b, reaction time and accuracy) will be analyzed by using a 2 × 3 within-subjects repeated measures analysis of variance (ANOVA) with the factors of Direction (Increasing vs. Decreasing change) and Size (Small-to-Small, Large-to-Large, Crossovers). The ANOVA provided *p*-values for differences between group means for each of the components within the change conditions by Direction and Size, as well as to analyze any significant interaction effects of Direction by Size. When appropriate, the Greenhouse-Geisser method for the violation of the sphericity assumption was applied.

Further analyses were conducted by means of pairwise comparisons (*t*-tests). As we had established *a priori* hypotheses for the main effect for Size, where smaller numbers will show superior behavioral performance, as well as ERP differences of amplitude and latency between small and large numbers, one-sided significance tests will be used for the pairwise comparisons of Size. On the other hand, as there are no well-established hypotheses for the main effect of Direction, two-sided significance tests will be used for the pairwise comparisons of Direction.

To correct for multiple comparisons, we followed the False Discovery Rate (FDR) method (Benjamini & Hochberg, 1995). For the pairwise *t*-tests, only FDR-corrected *p*-values are reported. The FDR correction has been shown to be an effective practice for neuroimaging data where multiple-testing across related spatial and temporal datapoints is a common problem (Benjamini & Yekutieli, 2001), and where more conservative methods, like the Bonferroni correction controlling for the Type I error rate, do not offer a good solution (Genovese et al., 2002). Results were deemed significant when the false discovery rate among the rejected tests was estimated to be lower than 5%.

The relationships between the obtained latency and adaptive mean amplitude from the ERP measures and the behavioral measures were explored using two-sided Pearson's correlation analyses. In the next chapter, the results obtained using these parameters and methods are reported.

Chapter 7: Results

For this study, a set of numerical stimuli with the number of dots ranging from small (1~3) and large (4~6) cardinalities. This stimulus presentation paradigm was adapted from Hyde and Spelke (2009), where participants viewed an ongoing stream of different numerosities, presented rapidly and sequentially. During the No Change conditions, participants saw the same number of dots repeatedly (albeit in different locations and sizes in each array), and the N1 response to the cardinalities of 1~6 was recorded and analyzed.

7.1. N1 ERP to Cardinalities

7.1.1. N1 from Left vs. Right POT

To evaluate bilateral differences over the POT areas in the brain, we ran a repeated measures ANOVA to compare measured N1 responses to cardinality from the left and right POT. There was a significant effect of laterality [$F(1,14) = 5.93$, $p < 0.05$, $\eta^2_g = 0.06$]. To investigate whether the right or left POT had higher N1 amplitudes, we ran a linear mixed model fit by restricted maximum likelihood (with subject as a random factor). We found that the left POT has a mean amplitude of -3.72 µV (S.E. = 0.4), and the right POT has a higher mean amplitude of -4.47 µV (S.E. = 0.26) (see Table 1).

Table 1:
Linear mixed model of N1 mean amplitude to Cardinality over the Left vs. Right POT area

	Left vs. Right POT
(Intercept)	-3.72^{***}
	(0.40)
Right POT N1	-0.75^{**}
	(0.26)

	Left vs. Right POT
AIC	752.03
BIC	764.81
Log Likelihood	-372.02
Num. obs.	180
Num. groups: partic	15
Var: partic (Intercept)	1.84
Var: Residual	3.08
$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$	

Post hoc *t*-tests using Satterthwaite's method revealed that significantly higher N1 amplitudes were observed over the right POT compared to the left POT [$t(14) = 2.868$, $p < 0.005$]. For a visual comparison of the plotted waveforms from the bilateral POT areas, please see *Figure 1A* in Appendix C.

Though there are higher amplitudes measured over the right POT, the left POT exhibited an almost identical scaling of N1 waveforms in response to the six numerosities. These similar trends can observed in the profile plot for estimated marginal (E.M.) means of N1 amplitudes to the six different cardinal values (see *Figure 11* below), where blue signifies the left POT, and red signifies the right POT.

As effects were significantly stronger in the right POT compared to the left POT, yet both provide parallel reactivity functions, further analyses for this region are restricted to data from the POT over the right hemisphere.

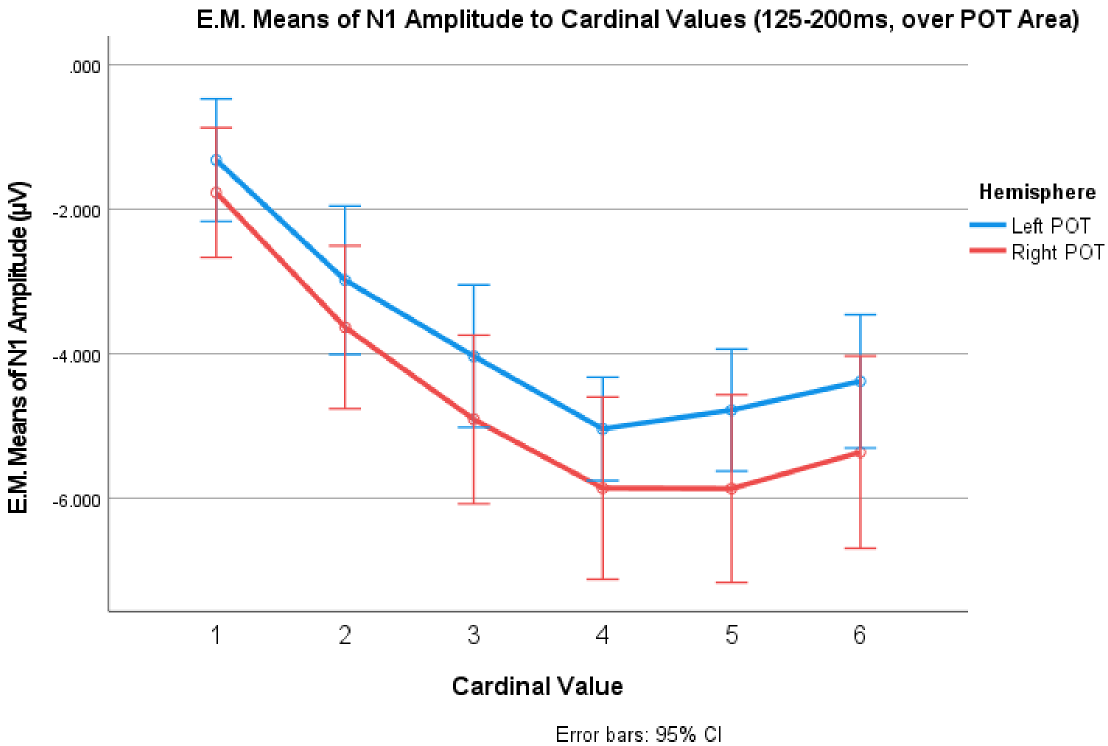


Figure 11: Bilaterally-similar effects of N1 amplitude to cardinal values over the left (blue) and right (red) POT area during a time window of 125-200 ms.

7.1.2. N1 Amplitude to Cardinalities of 1~6

In all of the following grand-averaged plots for the N1 waveforms, graph points that are higher on the Y-axis indicate weaker effects, in that the negative-going signal was less deflected in that polarity. Therefore, strength of effects for the N1 ERP is to be read in this inverted manner.

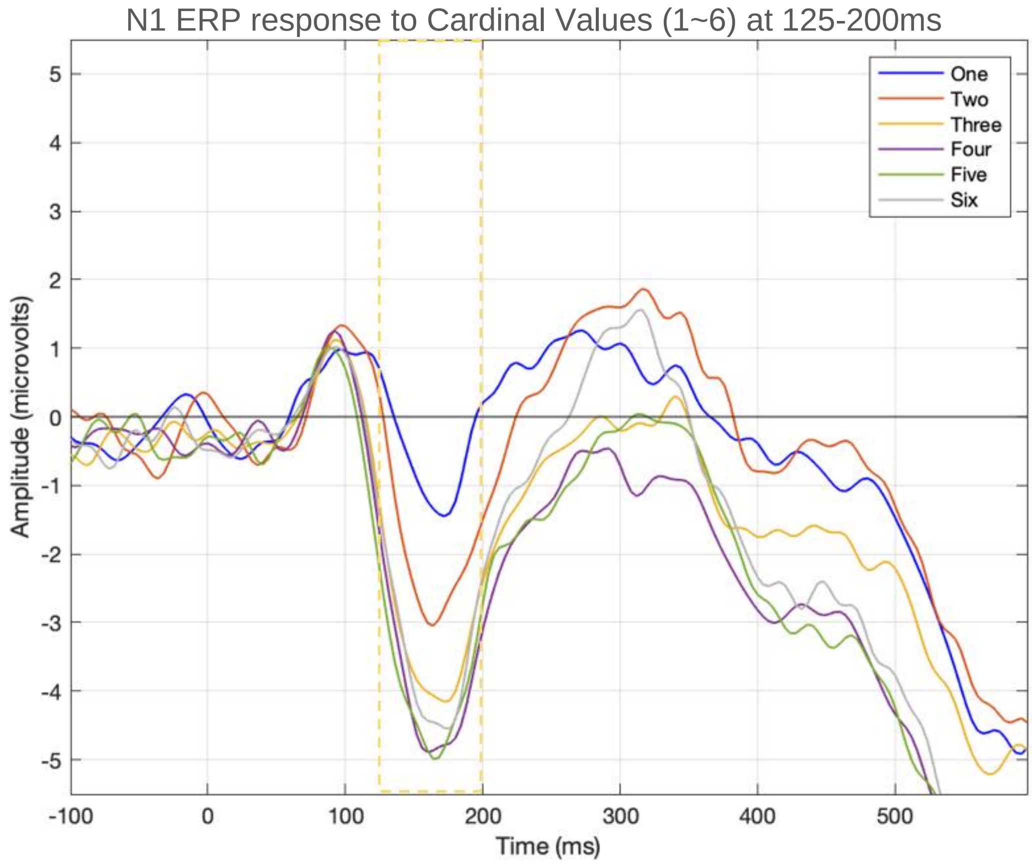


Figure 12: Grand-averaged waveform of N1 amplitudes over the POT area as an electrophysiological response to each of the tested cardinal values (1~6).

Figure 12 is a plot of grand-averaged N1 waveforms over the right POT area, where measured N1 amplitudes increase as the cardinal condition increases in numerical value for the range 1 to 3. This might imply that as more items are encoded in working memory, the N1 amplitude produced higher negativities commensurate with the increased processing load. However, we do not see scalar effects within the large number range of 4 to 6, suggesting that such working memory effects are not in play.

Descriptive statistics show that the mean N1 amplitude increases from “1” to “2”, and to “3”. From “3” to “4”, the mean N1 amplitude only slightly increases, followed by a gradual plateau of N1 amplitudes at “5” and “6” (see *Table 4A* in Appendix B).

When assessing the POT N1 amplitudes towards the six cardinal values, Mauchly's test suggests that the sphericity assumption was violated, χ^2 (14) = 31.370, $p < 0.01$.[1] Therefore, a repeated-measures ANOVA with a Greenhouse-Geisser correction[2] was conducted, showing that the N1 amplitude means for the six cardinal values revealed that the mean amplitudes to each cardinal value is significant from each other, where for the Right POT: $F(2.50, 35.02) = 34.08$, $p < 0.000$, $\eta^2_g = 0.34$ and for the left POT: $F(2.61, 36.61) = 38.01$, $p < 0.000$, $\eta^2_g = 0.42$.

Post-hoc pairwise *t*-tests with Bonferroni adjustments for the six cardinalities were conducted (see Table 4A in Appendix B). Pairwise comparisons showed that the N1 mean amplitude for the cardinal value “1” was significantly lower than the mean amplitudes generated for the other cardinal values (2~6). Meanwhile, the N1 mean amplitude for “2” was significantly different than 3~6, while “3” was significantly different than “1”, “2”, and “4”. For cardinal values “4” onwards and up, there were no significant differences in N1 mean amplitude among the larger cardinal values (>3). These results justify our decision to designate cardinal values 1~3 as small numbers, and 4~6 as large numbers in our experimental design. See *Figure 13* (below) for a plot of mean N1 amplitudes for each of the six cardinal values.

[1] According to Field (2013), if Mauchly's test yielded a *p*-value less than 0.05, then we should refer to the Greenhouse-Geisser epsilon (GG ε) to see if it is less than 0.75.
[2] GG ε for the left and right POT amplitudes were both less than 0.75.

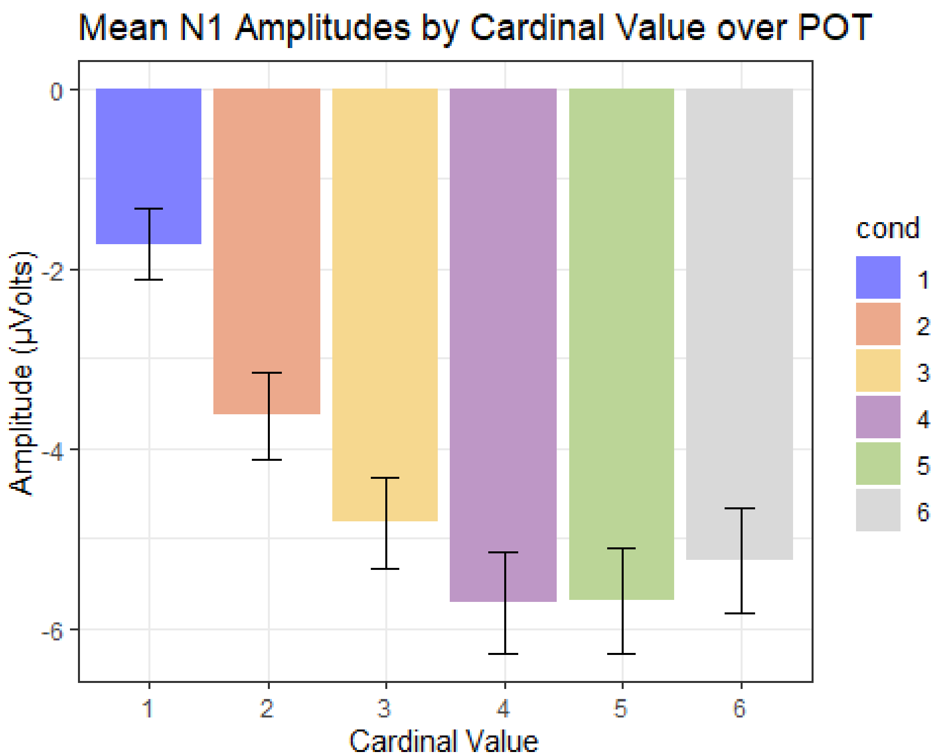


Figure 13: Mean N1 amplitudes plotted by 6 cardinal values over POT (125 – 200 ms)

7.1.3. N1 Latency to Cardinality

Repeated measures ANOVA revealed that there were no significant main effects of laterality (Left vs. Right POT) on N1 peak latency to the cardinalities of 1~6 [$F(1, 13) = 0.261$, $p = 0.618$, $\eta^2_g = 0.02$]. There were also no main effects of cardinal condition [$F(5, 65) = 1.427$, $p = 0.226$, $\eta^2_g = 0.099$]. Finally, there were no interaction effects of laterality and cardinality [$F(5, 65) = 0.881$, $p = 0.423$, $\eta^2_g = 0.063$].

Figure 14 (below) is the plot for estimated marginal (E.M.) means of N1 latencies to the six different cardinal values, where blue signifies the left POT, and red signifies the right POT. There were no differences between both hemispheres for N1 latency.

It is important to confirm that our data do not show a positive relationship between N1 latency and cardinal value. Should N1 peak latency increases as cardinal value increases, this might indicate that there is a presence of a serial process in numerical processing. This finding is similar to Hyde and Spelke (2009), as they also did not find differences in N1 latencies for their small numerosities (1, 2, 3) as well.

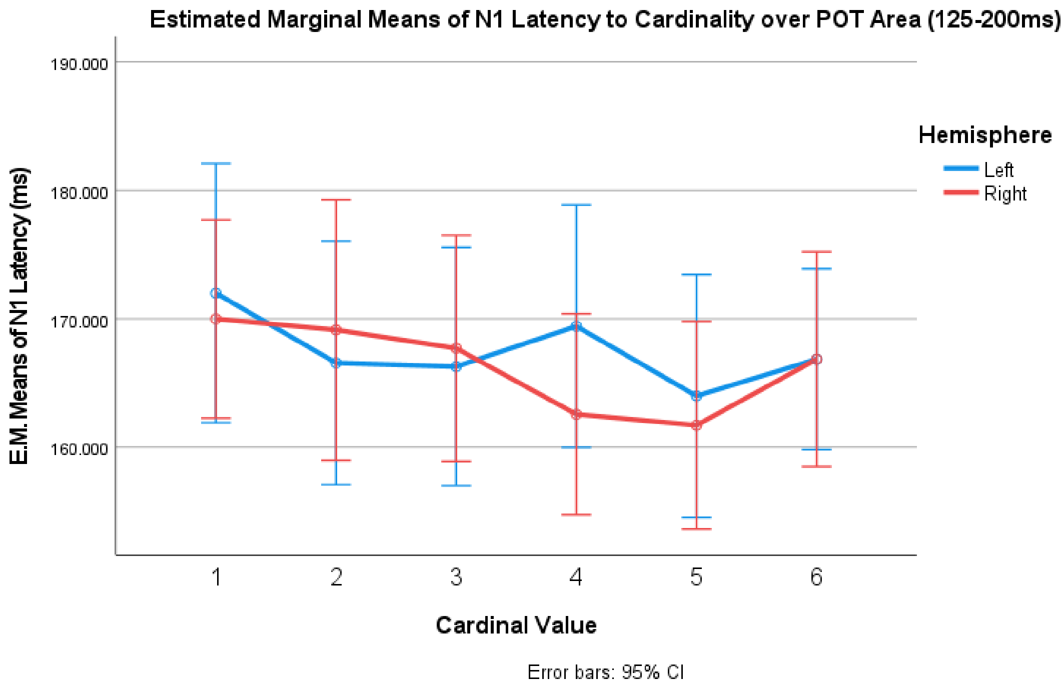


Figure 14: Estimated marginal means of N1 latency (milliseconds) to 6 cardinal values

In summary of the N1 response to cardinality, the present data strongly support the predictions of Hypothesis 1, in that cardinal values will show scalar properties for N1 amplitudes in the small number cardinalities (1~3), but not in the large number cardinalities (4~6), which are close and continuous with the small number sets. Just like Hyde and Spelke (2009), we did not find significant effects of N1 latency for cardinality set size.

7.2. Effects of Direction, Size, and Direction by Size

This study used a 2 × 3 within-subjects experimental design with the factors of change direction (Decrease vs. Increase) and set size of numerical change (SS: Small-to-Small, LL: Large-to-Large, SL and LS: Crossovers). This study investigated six dependent variables to numerical change direction, including reaction time, accuracy, N1 amplitude and latency over the right POT, as well as the P3b amplitude and latency over the Pz area.

7.2.1. Reaction Time: Sig. for Direction, Size, and Direction by Size

Reaction time results for the Change conditions by Direction and Size can be seen in the boxplot below (*Figure 15*). The data for reaction time was normally-distributed and showed homogenous variance[3].

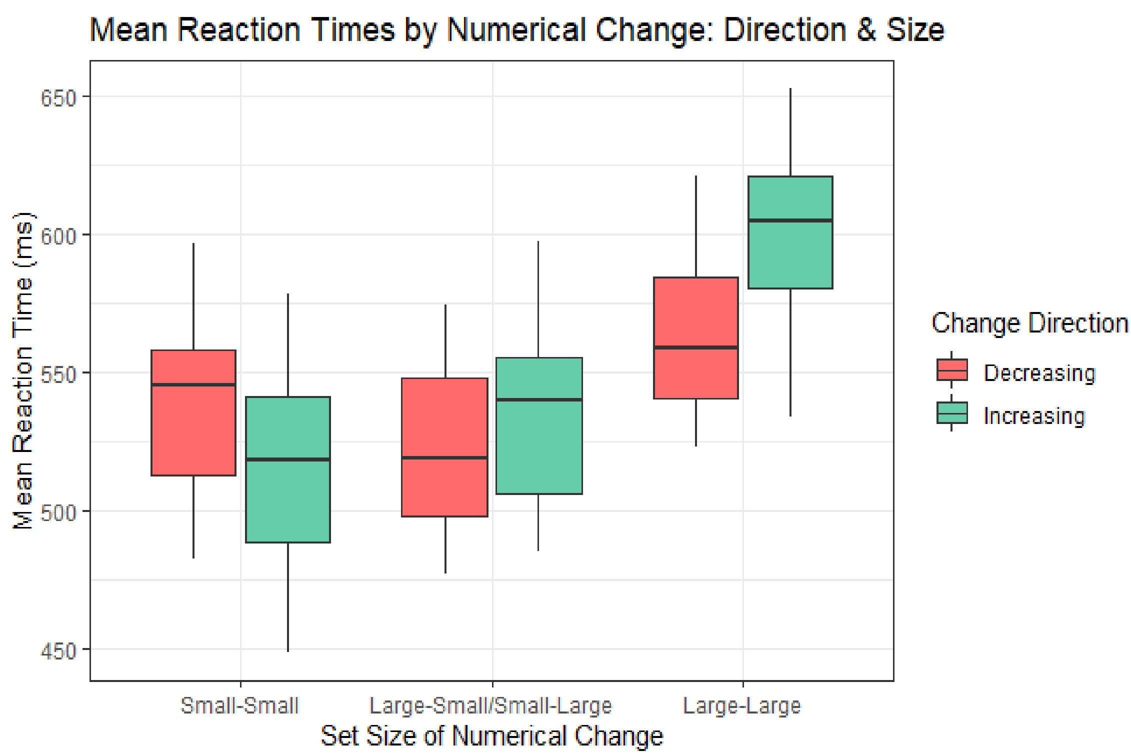


Figure 15: Mean reaction times (milliseconds) by Numerical Change (Direction and Size)

[3] As assessed by the Shapiro-Wilk normality test ($p = 0.489$) and the Levene's test of equal variance ($p = 0.771$).

Mean reaction times to different change conditions from shortest to longest are as follows: Decreasing-Large-Small, Increasing-Small-Small, Increasing-Small-Large, Decreasing-Small-Small, Decreasing-Large-Large, with the longest mean reaction time observed for the Increasing-Large-Large condition. Please see *Table 5A* in Appendix B for descriptive statistics of Reaction Time by Numerical Change Condition.

Repeated measures ANOVA was conducted to analyze Reaction Times by numerical change variables (Direction and Size), with subject as a random factor[4]. Analyses showed that there was a significant main effect of Direction [$F(1, 14) = 7.863$, $p < 0.05$, $\eta^2_p = 0.360$], and a higher significant main effect of Size [$F(1.909, 26.730) = 55.319$, $p < 0.001$, $\eta^2_p = 0.798$]. There was also a significant interaction effect of Direction by Size on reaction time [$F(2, 28) = 20.039$, $p < 0.001$, $\eta^2_p = 0.589$]. We conclude that the means of reaction time are not all equal among the six different change conditions, and followed this up with post-hoc pairwise *t*-tests on mean reaction time (see *Table 6A* in Appendix B for detailed output).

As we had established *a priori* hypotheses for the main effect for Size, where smaller numbers will show shorter reaction times, one-sided significance tests will be used for the pairwise comparisons of Size. On the other hand, as there are no well-established hypotheses for the main effect of Direction, two-sided significance tests will be used for the pairwise comparisons of Direction.

For Direction, two-tailed *t*-tests revealed that there are significant differences of mean reaction time among all three tested groups of Size. For the Small sets, "Decreasing" conditions produced longer reaction times than "Increasing" [$M = 21.412$, $t(14) = 3.204$, $p < 0.001$]. For the Crossover sets, "Decreasing" conditions produced shorter reaction times than "Increasing" [$M =$

[4] For the main effect of Size, sphericity was not met, as indicated by Mauchly's test, $\chi^2(2) = 0.371$, $p < 0.001$. Therefore, a Greenhouse-Geisser correction was conducted on the ANOVA for reaction time by Size.

-12.826, $t(14) = -2.297$, $p < 0.05$]. For Large sets, "Decreasing" conditions produced shorter reaction times than "Increasing", [$M = -38.319$, $t(14) = -5.350$, $p < 0.0001$].

For Size, one-tailed *t*-tests showed that there are significant differences of mean reaction time among all six combinations of set sizes. In the "Increasing" conditions, Small sets produced shorter reaction times than Crossover sets [$M = -15.937$, $t(14) = -2.281$, $p = 0.023$] and Large sets [$M = -37.369$, $t(14) = -4.958$, $p = 0.001$], while Crossover sets produced shorter reaction times than Large sets [$M = -75.688$, $t(14) = -10.094$, $p < 0.0001$]. In the "Decreasing" conditions, Small sets produced longer reaction times than Crossover sets [$M = 18.301$, $t(14) = 2.679$, $p = 0.013$], but shorter reaction times than the Large sets [$M = -15.957$, $t(14) = -2.171$, $p = 0.024$]. Crossover sets produced shorter reaction times than Large sets [$M = -59.751$, $t(14) = -9.251$, $p < 0.0001$].

These trends can be observed in the plot below (*Figure 16*) for estimated marginal means of reaction time to numerical change by the three set sizes and change directionality, where red signifies the mean reaction time for the "Decreasing" condition, and green signifies the mean reaction times for the "Increasing" condition.

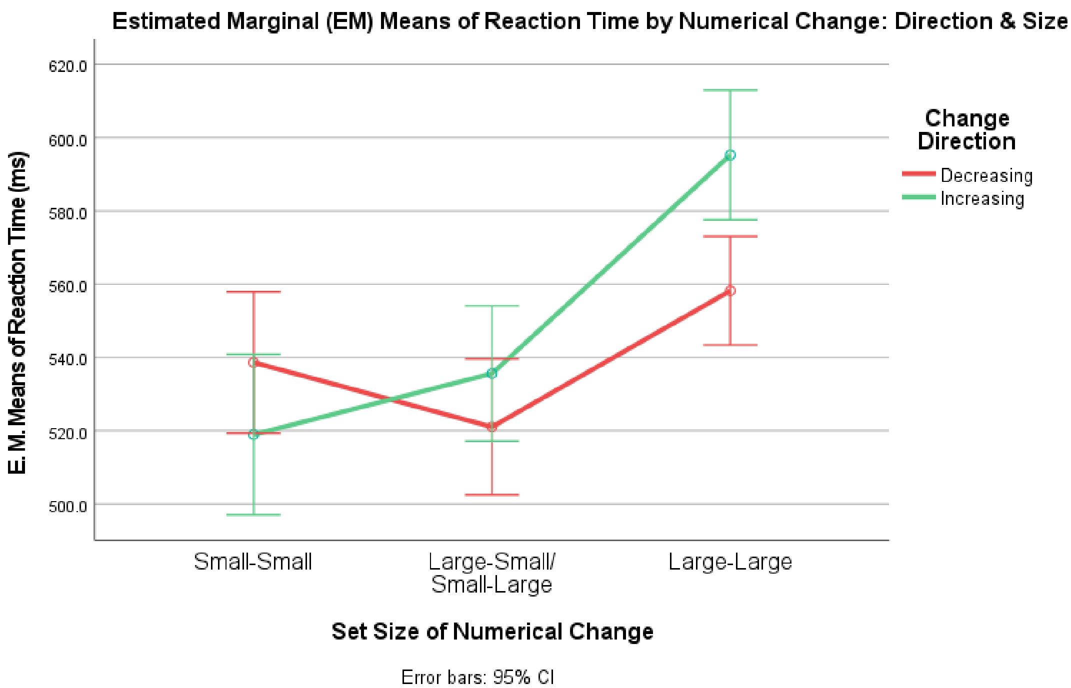


Figure 16: Estimated marginal means of reaction time (milliseconds) by Numerical Change (Direction and Size)

7.2.2. Accuracy: Sig. for Direction, Size, and Direction by Size

Mean accuracy to different change conditions from lowest to highest are as follows: Increasing-Large-Large, Decreasing-Large-Large, Increasing-Small-Large, Increasing-Small-Small, Decreasing-Small-Small, with the highest mean accuracy observed for the Decreasing-Large-Small condition. Please see *Table 7A* in Appendix B for descriptive statistics of Accuracy by Numerical Change condition. Accuracy data [5] for the Change conditions by Direction and Size are plotted in *Figure 17*.

[5] Levene's test of equal variance showed homogenous variance for accuracy data ($p = 0.144$). Normality tests on accuracy data by the six Change conditions revealed that dLS (Decrease Large-Small) was the only Change condition with accuracy data that was not normally-distributed ($W = 0.819$, $p < 0.01$), while the other five Change conditions (dSS, iSS, dLL, iLL, iSL, iSS) yielded normally-distributed accuracy data.

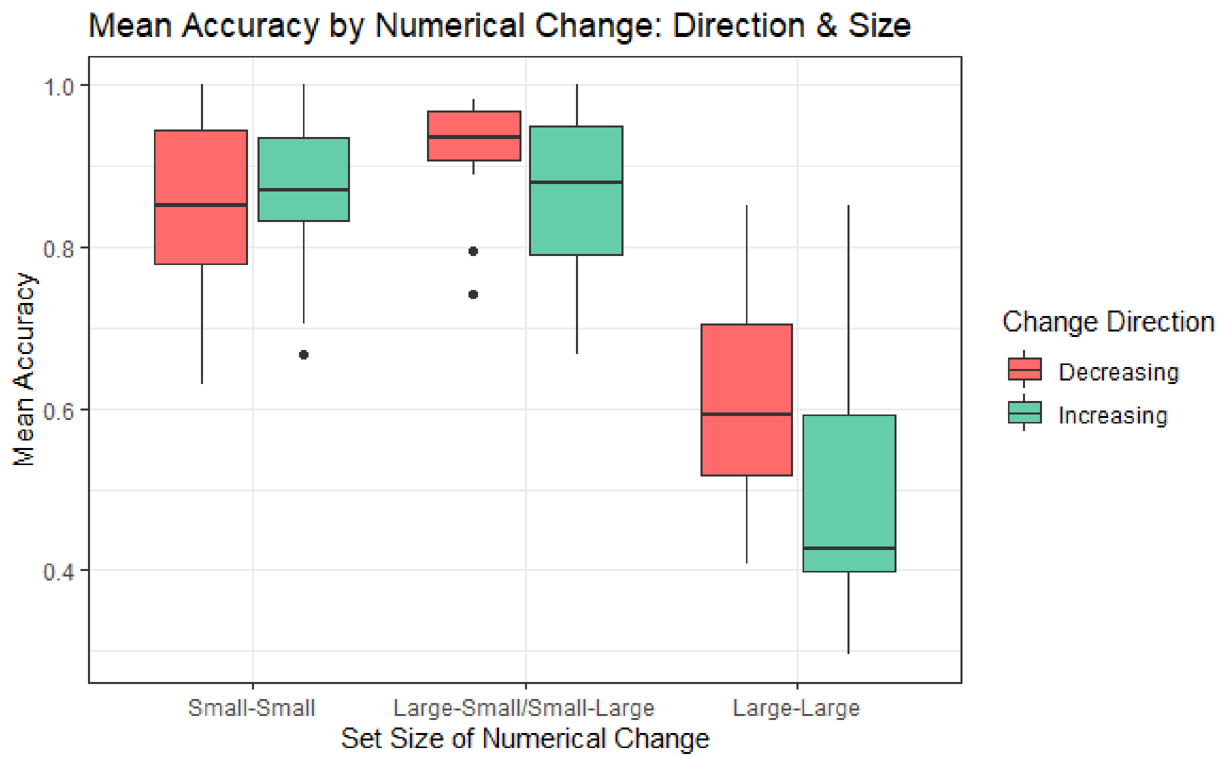


Figure 17: Mean accuracy plotted by Numerical Change (Direction and Size)

We ran a repeated measures ANOVA to analyze mean accuracy by numerical change variables (Direction and Size), with subject as a random factor. The analysis[6] on accuracy by Size showed that the three set sizes had significantly different mean accuracies [$F(1.202,16.822) = 96.914$, $p < 0.001$, $\eta^2_p = 0.874$]. There was a moderately significant main effect of Direction [$F(1, 14) = 8.085$, $p < 0.05$, $\eta^2_p = 0.366$], and a weaker interaction effect of Direction and Size on mean accuracy [$F(2, 28) = 4.553$, $p < 0.05$, $\eta^2_p = 0.245$].

In terms of Size, lower accuracies are observed for change in the Large-Large condition. In terms of Direction, higher accuracies are observed for Decreasing change, than Increasing change. We conclude that the means of accuracy are not all equal among the six different change

[6] For the main effect of Size, sphericity was not met, as indicated by Mauchly's test, $\chi^2(2) = 11.498$, $p < 0.005$. Therefore, a repeated-measures ANOVA with a Greenhouse-Geisser correction was conducted on accuracy by Size.

conditions, and follow this up with post-hoc pairwise *t*-tests on mean accuracy (see *Table 8A* in Appendix B for detailed output).

As we had established *a priori* hypotheses for the main effect for Size, where smaller numbers will show higher accuracy, one-sided significance tests will be used for the pairwise comparisons of Size. On the other hand, as there are no well-established hypotheses for the main effect of Direction, two-sided significance tests will be used for the pairwise comparisons of Direction.

For the effect of Direction, two-tailed *t*-tests revealed that only one out of the three set sizes had significant differences in accuracy, where the "Decreasing" condition produced higher accuracy rates only in the Large sets over the "Increasing" condition, M =0.101, $t(14)$ =5.25, $p <$ 0.0001.

For the effect of Size, one-tailed *t*-tests showed that there are significant differences on accuracy among five out of six paired combinations we analyzed. There were no differences when comparing the Small-to-Small and Small-to-Large set sizes in the "Increasing" condition. Meanwhile, in the "Decreasing" condition, the Small-to-Small sets produced lower accuracy rates than the Crossover (Large-to-Small) sets [M = -0.042, $t(14)$ = -1.919, p = 0.046], but this was only marginally significant.

In the "Decreasing" condition, Small sets produced higher accuracy rates than Large-to-Large sets [M = 0.262, $t(14)$ = 8.626, p < 0.001], while the Crossover (Large-to-Small) sets produced higher accuracy rates than Large sets [M =0.353, $t(14)$ = 11.290, p <0.0001. In the "Increasing" condition, Small sets produced higher accuracy rates than Large sets [M = 0.252, $t(14)$ = 7.930, p < 0.0001], while the Crossover (Small-to-Large) sets produced higher accuracy rates than Large sets [M =0.351, $t(14)$ = 10.453, p < 0.0001].

Both mean accuracies for the Large sets (in both directions) are significantly lower than all the other change conditions. These trends can be observed in the plot below (*Figure 18*) for estimated marginal means of accuracy to numerical change by the three different set sizes and change directionality, where red signifies the mean accuracy for the "Decreasing" condition, and green signifies the mean accuracy for "Increasing" condition.

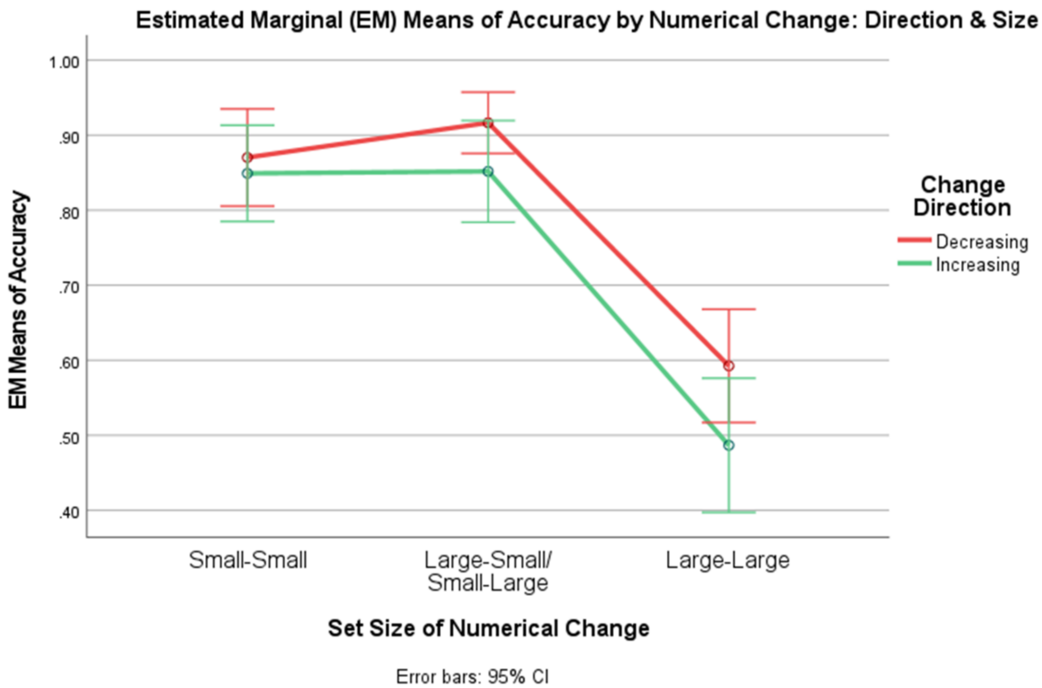


Figure 18: Estimated marginal means of accuracy by Numerical Change (Direction and Size)

7.3. N1 ERP over Right POT

There were bilaterally-similar patterns of ERP waveforms over the left and right POT. However, as supported by previous research (Ansari et al., 2007; Hyde & Spelke, 2012), there were higher N1 amplitudes over the right POT. Please see *Figure 2A* in Appendix C for a visual comparison of the plotted waveforms from the left and right POT. For grand-averaged N1 waveforms plotted for the six change conditions and the no-change condition (where the presented cardinal values remain the same), see *Figure 19* below.

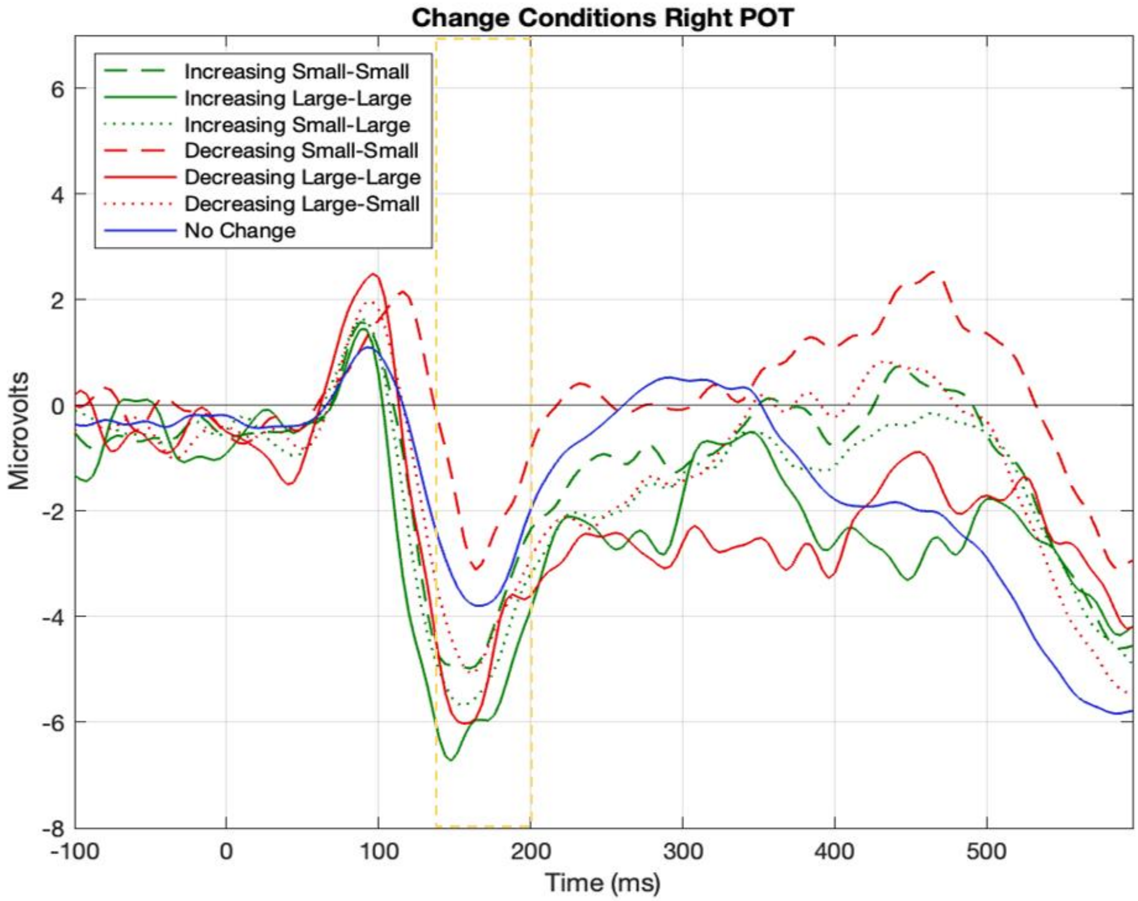


Figure 19: N1 ERP waveforms for different numerical Change conditions over the right POT area during the 125-200ms window.
Note: Compared to all the other Change conditions, "Decrease Small-Small" (red dashed line) produced the lowest N1 amplitude, even lower than the "No Change" condition (blue solid line).

7.3.1. N1 Amplitudes: Sig. for Direction, Size, and Direction by Size

Over the right POT area, the lowest N1 amplitude was measured in the "Decrease Small-Small" condition, compared to "No Change", while the rest of the change conditions showed higher amplitudes. These results indicate that at 125-200ms, the POT is "off-loading" objects from visual short-term memory with decreasing small numbers (in the subitizing range), but not for large numbers [7].

Figure 20 shows the mean N1 amplitudes plotted by numerical change variables (Direction and Size). Mean N1 amplitudes to different change conditions from lowest to highest are as follows: Decreasing-Small-Small, Decreasing-Large-Small, Increasing-Small-Small, Increasing-Large-Large, Decreasing-Large-Large, with the highest mean N1 amplitudes observed for the Increasing-Small-Large condition. Please see *Table 9A* in Appendix B for descriptive statistics of mean N1 amplitudes by Numerical Change condition.

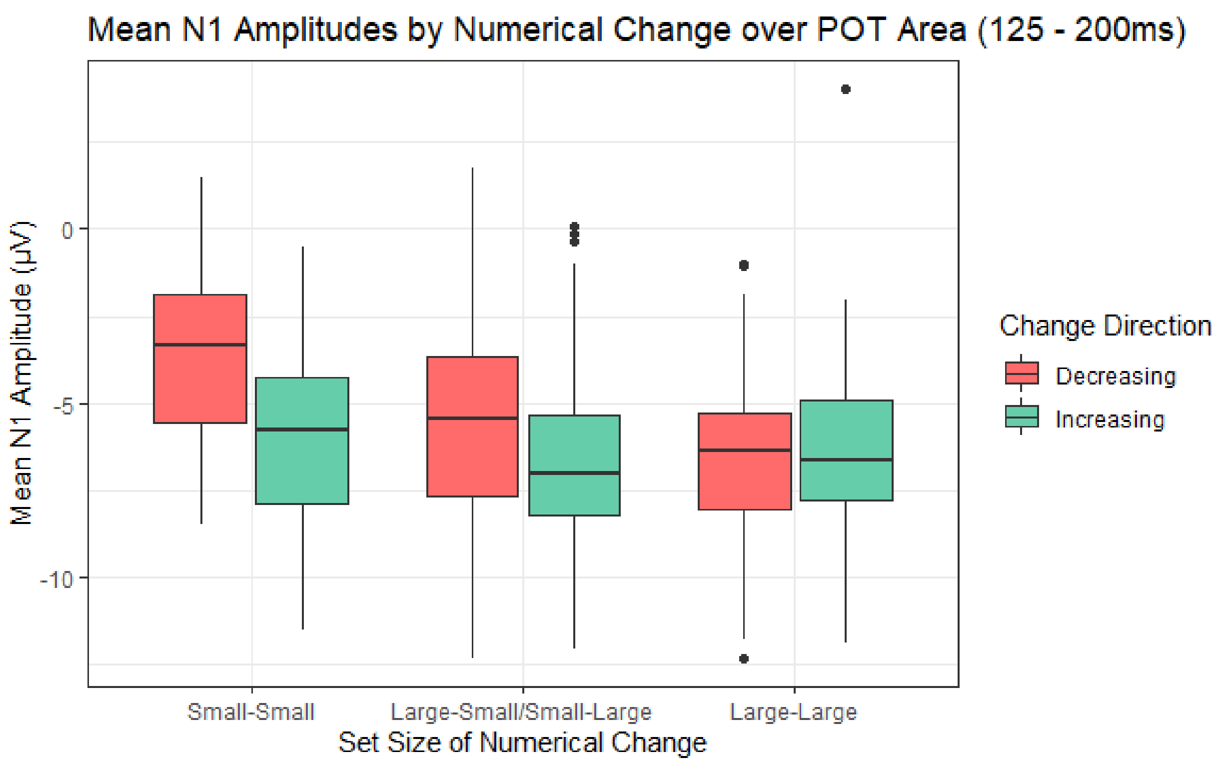


Figure 20: Mean N1 amplitudes plotted by Numerical Change (Direction and Size)

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[7] Levene's test showed that N1 amplitude data to Change conditions had homogenous variance (p = 0.092) Normality tests conducted on N1 amplitude data by the six Change conditions revealed that only the Large-Large numerical change conditions did not have normally-distributed N1, while N1 amplitudes for the other four Change conditions (dSS, iSS, dLS, iSL) were normally-distributed

Repeated measures ANOVA on mean N1 amplitudes by numerical change variables (Direction and Size), with subject as a random factor, revealed that there was a strong significant main effect of Direction on N1 amplitudes [$F(1, 14) = 51.98$, $p < 0.001$, $\eta^2_p = 0.788$], as well as a significant main effect of Size [$F(2, 28) = 26.599$, $p < 0.001$, $\eta^2_p = 0.655$]. There was also a significant interaction effect of Direction and Size[8] on mean N1 amplitudes over the POT area [$F(1.314, 18.397) = 15.79$, $p < 0.001$, $\eta^2_p = 0.53$]. We conclude that the means of N1 amplitudes are not all equal among the six different change conditions, and followed this up with post-hoc pairwise *t*-tests on mean N1 amplitude (see *Table 10A* in Appendix B for detailed output).

As we had established *a priori* hypotheses for the main effect for Size, where we predict N1 amplitudes to have higher negativities as numbers get larger, one-sided significance tests were used for the pairwise comparisons of Size. On the other hand, as there are no well-established hypotheses for the main effect of Direction, two-sided significance tests were used for the pairwise comparisons of Direction.

For the effects of Direction, two-tailed *t*-tests revealed that there are significant differences of mean N1 amplitudes among two out of the three set sizes: In Small sets, the "Decreasing" conditions showed lower N1 amplitudes (i.e., less negative deflection) than "Increasing" [$M = 2.324$, $t(14) = 7.222$, $p < 0.0001$]. In Crossover sets, "Decreasing" conditions showed higher N1 amplitudes than "Increasing" [$M = 1.194$, $t(14) = 4.36$, $p < 0.0001$]. Meanwhile, there were no significant effects of Direction on N1 amplitudes in the Large sets.

For the effects of Size, one-tailed *t*-tests showed that there are significant differences of mean N1 amplitudes among four out of six pairwise comparisons. In the "Decreasing" condition,

[8] For the interaction effect of Direction by Size on N1 amplitudes, sphericity was unmet, as indicated by Mauchly's test, $\chi^2(2) = 9.596$, $p < 0.01$. Therefore, a repeated-measures ANOVA with a Greenhouse-Geisser correction was conducted on mean N1 amplitudes by change direction and set size.

Small sets produced higher N1 amplitudes than Crossover sets [M = 1.859, $t(14) = 9.956$, $p <$ 0.001] and higher N1 amplitudes than Large sets [M = 2.881, $t(14) = 7.665$, $p < 0.001$]. In the "Increasing" condition, Small sets produced higher N1 amplitudes than Crossover sets [M = 0.729, $t(14) = 2.727$, $p < 0.05$] and higher N1 amplitudes than Large sets [M =0.557, $t(14)$ = 2.067, $p < 0.05$]. There were no significant differences between the Large sets and Crossover sets in the Decreasing condition, as well as between the Large sets and Crossover sets in the Increasing direction.

These trends can be observed in the plot below (*Figure 21*) for estimated marginal (E.M.) means of N1 amplitudes to numerical change by the three set sizes and change directionality, where red signifies the mean N1 amplitudes for the "Decreasing" condition, and green signifies N1 amplitudes for the "Increasing" condition.

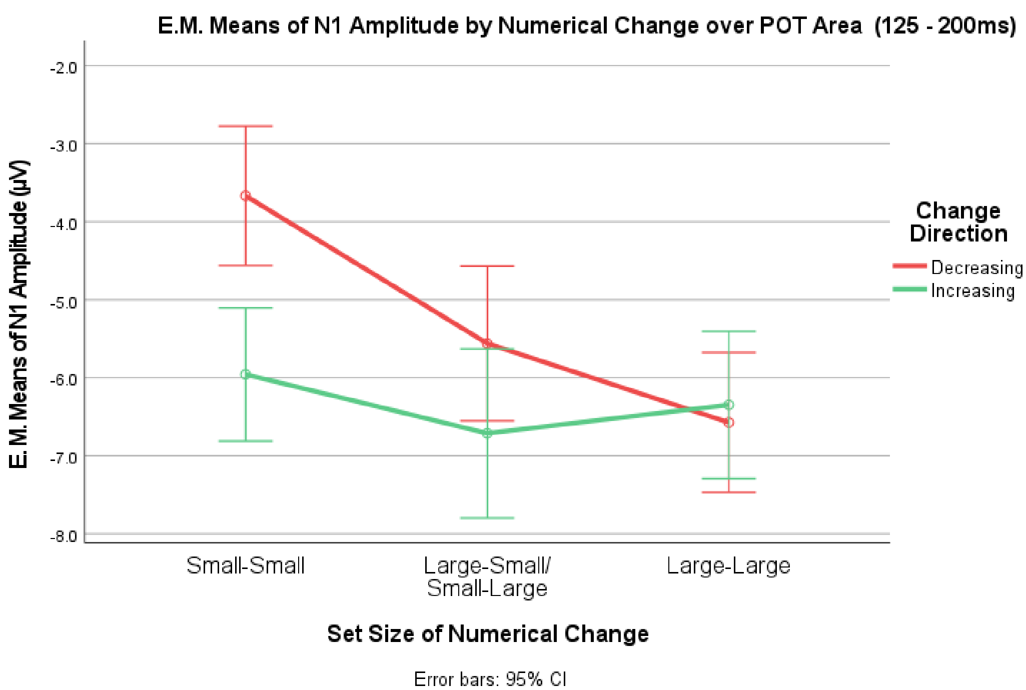

Figure 21: Estimated marginal means of N1 amplitude by Numerical Change (Direction and Size)

7.3.2. N1 Latency: Sig. for Direction (Not Size)

N1 latency[9] to Change conditions (Direction and Size) showed homogenous variance with a relatively-normal distribution. See *Figure 22* for mean N1 latencies plotted by numerical change variables (Direction and Size).

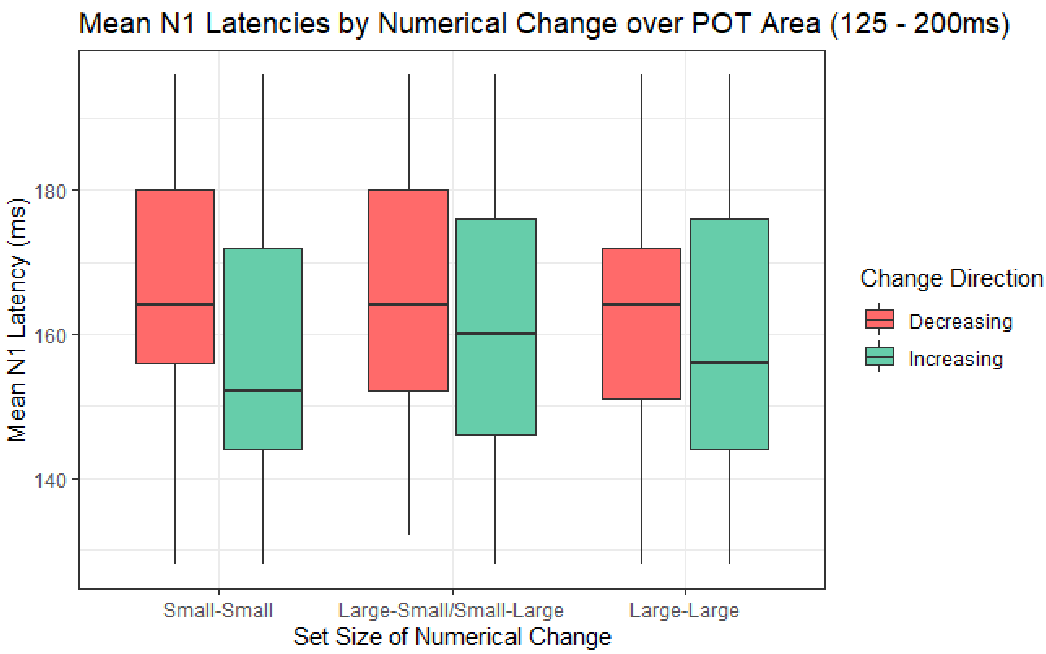

Figure 22: Mean N1 latencies plotted by Numerical Change (Direction and Size)

Mean N1 latencies to different change conditions from earliest to latest are as follows: Increasing-Small-Small, Increasing-Small-Large, Increasing-Large-Large, Decreasing-Large-Large, Decreasing-Large-Small, with the latest mean N1 latencies are observed for the Decreasing-Small-Small condition. Please see *Table 11A* in Appendix B for descriptive statistics of mean N1 latencies by Numerical Change condition.

[9] Levene's test showed that the N1 latency data to Change conditions (Direction and Size) had homogenous variance (p = 0.313). Shapiro-Wilk's tests of normality on the N1 latency data by the six Change conditions revealed that five out of the six Change conditions (dSS, iSS, dLL, iLL and iSL) was normally-distributed, while only the Decreasing-Large-Small conditions had N1 latency data that followed a marginally-normal distribution of: W = 0.851, p = 0.06.

Repeated measures ANOVA was conducted on mean N1 latencies by numerical change variables (Direction and Size), with subject as a random factor. There was a significant main effect of Direction on mean N1 latencies [$F(1, 14) = 13.1$, $p < 0.005$, $\eta^2_p = 0.483$], but there were no main effects of Size [$F(2, 28) = 0.472$, $p = 0.629$, $\eta^2_p = 0.033$]. There was also no interaction effect of Direction and Size on mean N1 latencies [$F(2, 28) = 2.836$, $p = 0.076$, $\eta^2_p = 0.168$]. Follow-up pairwise *t*-tests were conducted on the main effect of Direction, as it was a significant influence on peak N1 latencies.

As there are no well-established hypotheses for the main effect of Direction, two-sided significance tests will be used for the pairwise comparisons of Direction on mean N1 peak latencies. Paired-samples (by Direction) two-tailed *t*-tests revealed that there are significant differences of mean N1 latencies among two out of three numeric set sizes (see *Table 12A* in Appendix B for detailed output). For the Small sets, "Decreasing" conditions produced later N1 mean latencies than "Increasing" [$M = 9.6$, $t(14) = 3.246$, $p < 0.05$]. For the Crossover sets, "Decreasing" conditions also produced later N1 mean latencies than "Increasing", $M = 4.956$, $t(14) = 3.35$, $p < 0.05$. Only the Large numeric sets did not show differences between either directions on N1 peak latencies.

These trends can be observed in the plot below (*Figure 23*) for estimated marginal (E.M.) means of N1 latencies to numerical change by the three set sizes and change directionality, where red signifies the mean N1 latencies for the "Decreasing" condition", and green signifies the mean N1 latencies for the "Increasing" condition.

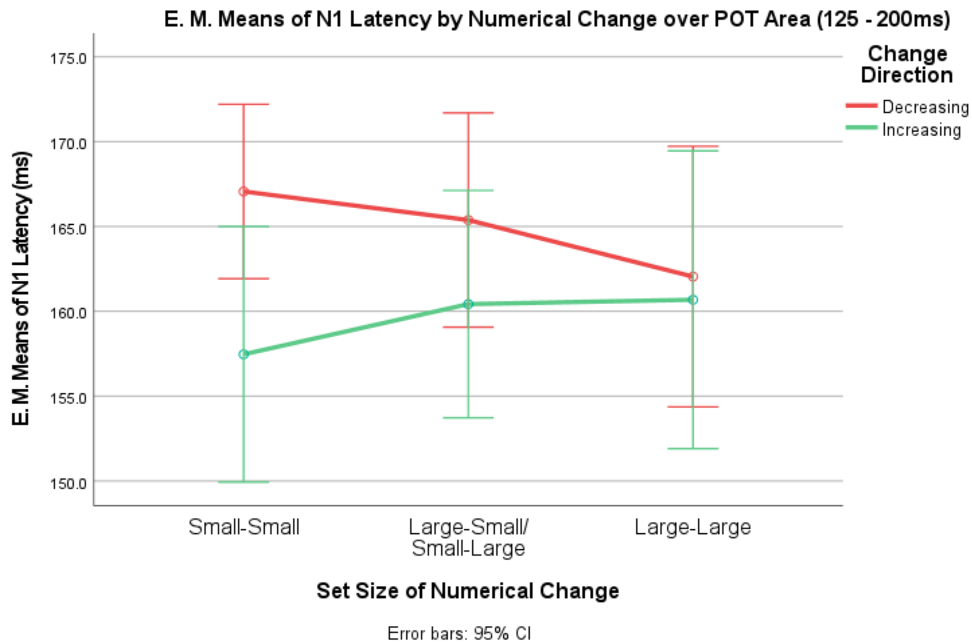

Figure 23: Estimated marginal means of N1 latency (milliseconds) by Numerical Change (Direction and Size)

7.4. P3b ERP over Mid-Parietal (Pz) Area

The current study investigated the P3b ERP as a biomarker that signals higher-cognitive processes involved in context-updating. For grand-averaged P3b waveforms plotted for the six change conditions and the no-change condition (where the presented cardinalities remain the same), please see *Figure 24.*

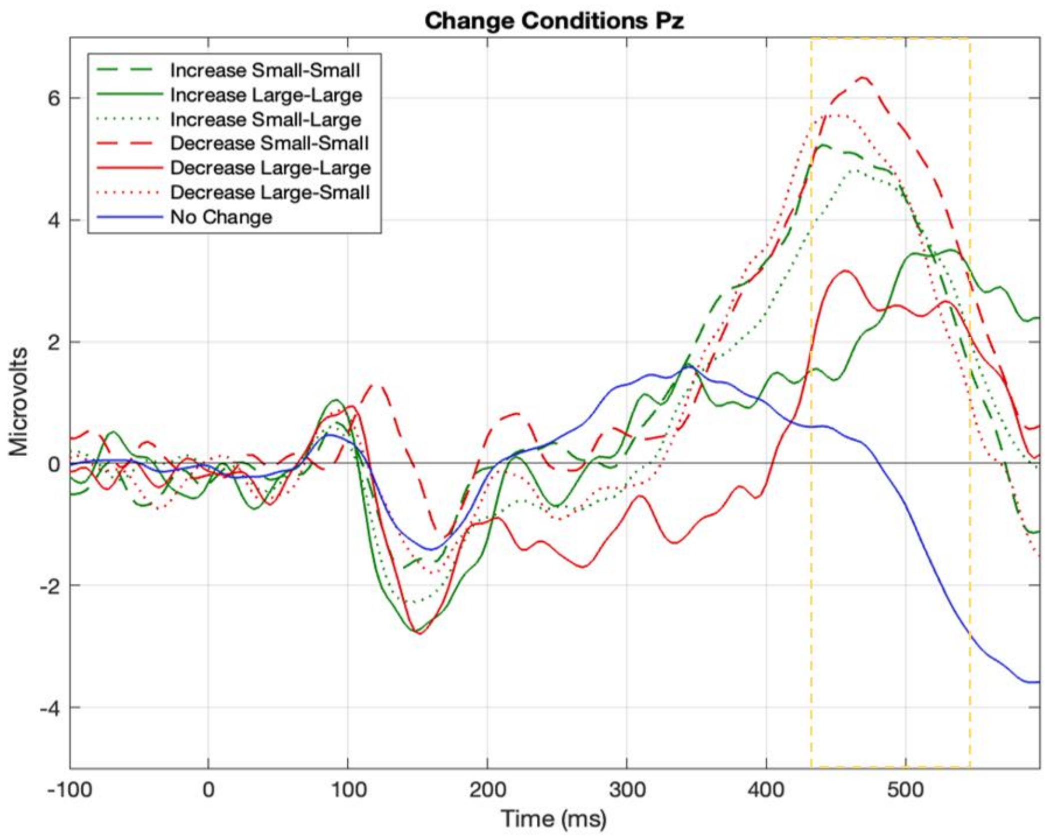

Figure 24: Grand-averaged P3b waveforms for seven different numerical change conditions over the mid-Parietal (Pz) area.
Note: **Conditions of No Change (blue solid line) produced the earliest and lowest peak at ~330ms post-stimulus, whereas Small sets (dashed lines) and Crossover sets (dotted lines) produced higher peaks at ~435ms, while the Large sets (red and green solid lines) produced lower peaks at ~500ms.**

7.4.1. P3b Amplitude: Sig. for Size (Not Direction)

P3b amplitude[10] to Change conditions (Direction and Size) showed homogenous variance was normal distributed. See *Figure 25* for mean P3b amplitudes plotted by numerical change variables (Direction and Size).

[10] Levene's test of equal variance showed that our P3b mean amplitude data to Change conditions (Direction and Size) had homogenous variance (p = 0.853). P3b mean amplitude data to the 6 Change conditions were normally-distributed, as assessed by the Shapiro-Wilk tests of normality.

Over the Pz area at 435-535 ms, “Decrease Small-Small” produced the highest P3b amplitude, while Increase and Decrease Large-Large have lower P3b amplitudes. As the number stays the same in No Change, there is a weak P3 signal instead. Change in the Decreasing Small-to-Small condition (1~3) had the highest P3b amplitude, while change in the Large-Large condition (with 4~6) had the lowest amplitude with no differences based on change direction.

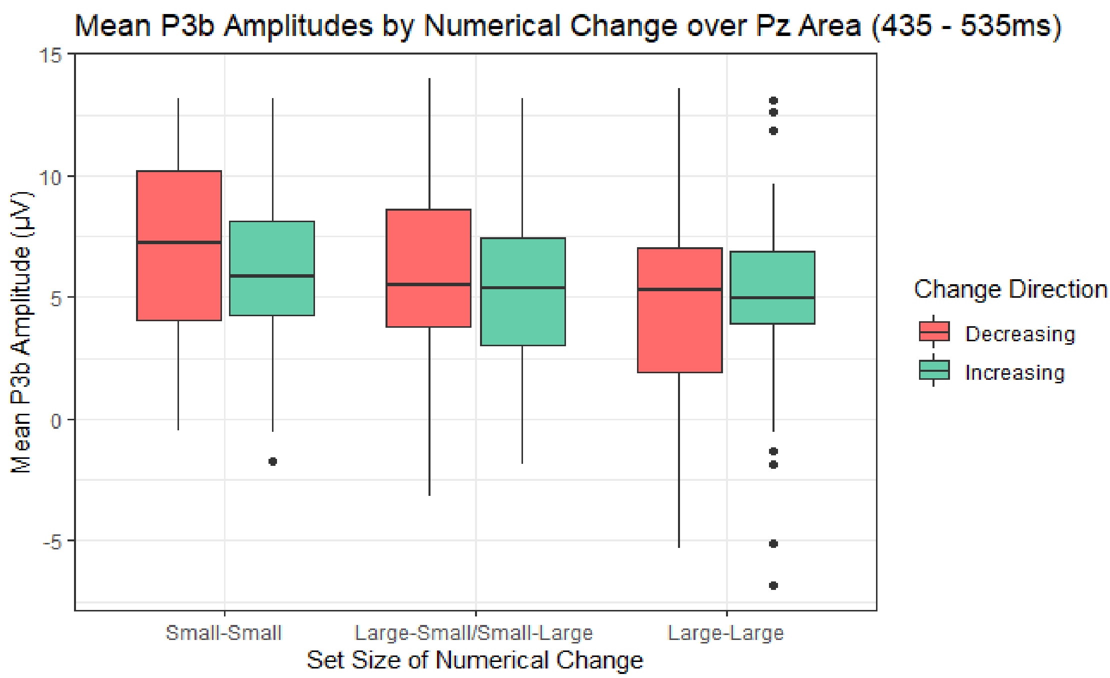

Figure 25: Mean P3b amplitudes plotted by Numerical Change (Direction and Size)

Mean P3b amplitudes to different change conditions from lowest to highest are as follows: Decreasing-Large-Large, Increasing-Large-Large, Increasing Small-Large, Increasing-Small-Small, Decreasing-Large-Small, with the highest mean P3b amplitudes observed for the Decreasing-Small-Small condition. Please see *Table 13A* in the Appendix B for descriptive statistics of mean P3b amplitudes by Numerical Change condition.

A repeated measures ANOVA was conducted to analyze mean P3b amplitudes by numerical change variables (Direction and Size), with subject as a random factor. There was a significant main effect of Size on mean P3b amplitudes [$F(2, 28) = 8.869$, $p = 0.001$, $\eta^2_p = 0.388$], but there were no main effects of Direction [$F(1, 14) = 1.661$, $p = 0.218$, $\eta^2_p = 0.106$]. Also, there was no interaction effect of Direction and Size on mean P3b amplitudes [$F(2, 28) = 1.545$, $p = 0.231$, $\eta^2_p = 0.099$].

As Direction did not significantly influence peak P3b mean amplitudes, post-hoc tests for the main effects of Direction were not conducted for this variable. As Size was the only significant effect on P3b amplitudes, we ran post-hoc pairwise *t*-tests to investigate which set sizes are contributing to this significant difference of mean P3b amplitudes among the Small vs. Large sets in the "Decreasing" condition (see *Table 14A* in Appendix B for detailed output).

As we had established *a priori* hypotheses for the main effect for Size, where we predict P3b amplitudes to be higher for tasks that are easier to update its context, especially for sets with smaller numbers, one-sided significance tests will be used for the pairwise comparisons of Size. Pairwise one-sided *t*-tests showed that there are significant differences of mean P3b amplitude among two pairs out of six combinations. In the "Decreasing" condition, Small sets showed larger P3b amplitudes than the Large sets [$M = 2.588$, $t(14) = 3.886$, $p < 0.01$], while the Crossover (Large-to-Small) sets showed larger P3b amplitudes than the Large sets [$M = 1.877$, $t(14) = 2.53$, $p < 0.05$].

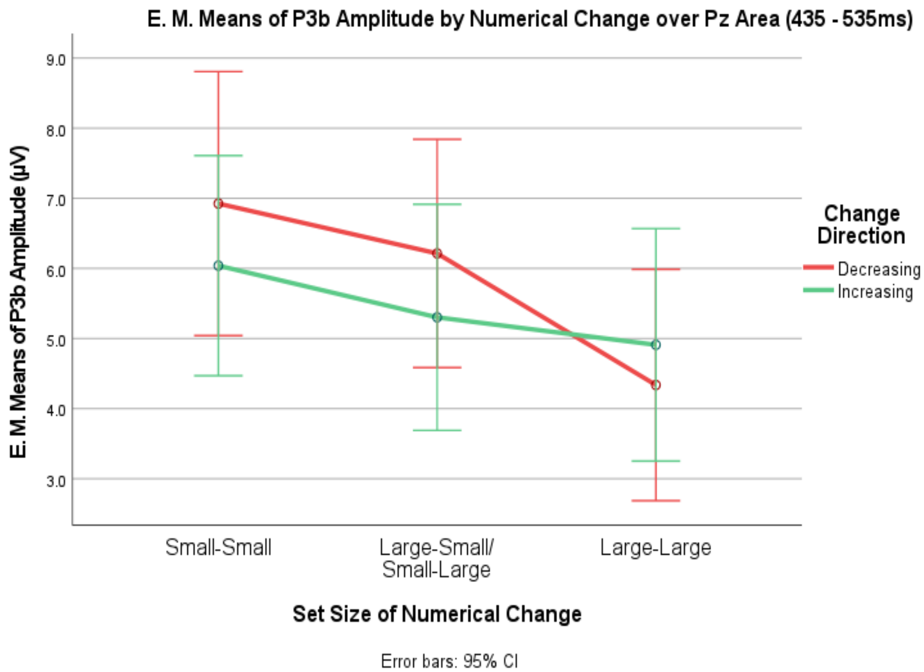

Figure 26: Estimated marginal means of P3b amplitudes by Numerical Change (Direction and Size)

These trends can be observed from the plot in *Figure 26* for estimated marginal (E.M.) means of P3b amplitudes to numerical change by the three set sizes and by both directions, where red signifies the P3b amplitudes for the "Decreasing" condition", and green signifies the P3b amplitudes for the "Increasing" condition.

7.4.2. P3b Latency: Sig. for Size, and Direction by Size

P3b latency[11] to Change conditions (Direction and Size) showed homogenous variance with a normal distribution. See *Figure 27* for mean P3b latencies plotted by numerical change variables (Direction and Size).

[11] Levene's test of equal variance showed that our P3b mean latency data to Change conditions (Direction and Size) had homogenous variance (p = 0.07). P3b mean latency data to the 6 Change conditions were normally-distributed, as assessed by the Shapiro-Wilk tests of normality

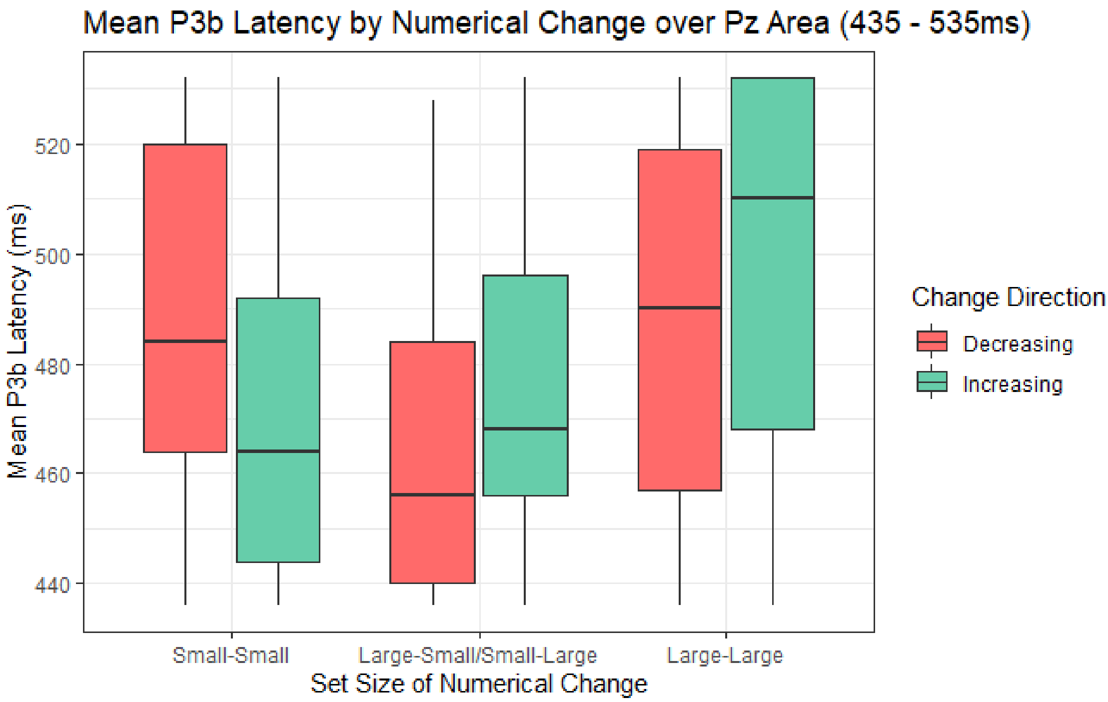

Figure 27: Mean P3b latencies plotted by Numerical Change (Direction and Size)

P3b latency was shortest for “dLS”, followed by “dLL” and “dSS”, but for Increasing conditions, P3b latency was shortest for “iSS”, followed by “iSL”, with “iLL” having the longest P3b latencies. Please see *Table 15A* in Appendix B for descriptive statistics of mean P3b amplitudes by Numerical Change condition.

A repeated measures ANOVA was conducted to analyze mean P3b latencies by numerical change variables (Direction and Size), with subject as a random factor. There was a significant main effect of Size on P3b mean latencies [$F(2, 28) = 14.095$, $p < 0.001$, $\eta^2_p = 0.502$], but there were no main effects of Direction [$F(1, 14) = 0.304$, $p = 0.59$, $\eta^2_p = 0.021$]. There was a significant interaction effect of Direction and Size on mean P3b latencies [$F(2, 28) = 5.931$, $p = 0.007$, $\eta^2_p = 0.298$]. We conclude that the means of P3b latencies are not all equal among the six different change conditions by Size, and followed this up with post-hoc pairwise *t*-tests (see

Table 16A in Appendix B for detailed output). As Direction did not significantly influence peak P3b mean latencies, post-hoc tests were not conducted for this variable.

As we had established *a priori* hypotheses for the main effect for Size, where we predict P3b latencies to be longer for more difficult tasks that are harder to update its context, especially for sets with larger numbers, one-sided significance tests will be used for the pairwise comparisons of Size. Pairwise one-sided *t*-tests showed that there are significant differences of mean P3b latencies among four out of six pairwise comparisons. In the "Decreasing" condition, Crossover (Large-to-Small) sets showed earlier P3b latencies than Small sets [M =23.956, $t(14)$ =3.794, $p < 0.001$], as well as Large sets showing longer P3b peak latencies [M =-26.8, $t(14)$ = -3.216 $p < 0.001$]. In the "Increasing" condition, Large sets showed later P3b latencies than Small sets [M =-15.333, $t(14)$ =-2.906, $p < 0.01$], as well as Crossover (Small-to-Large) sets [M =-22.649, $t(14)$ =-3.736, $p < 0.001$]. There were no significant differences between the Large sets and Crossover sets in the Decreasing condition, as well as between the Small sets and Crossover sets in the Increasing direction.

These trends can be observed in the plot below (*Figure 28*) for the estimated marginal (E.M.) means of P3b latencies to numerical change by the three set sizes and by both directions, where red signifies the P3b latencies for the "Decreasing" condition", and green signifies the P3b latencies for the "Increasing" condition.

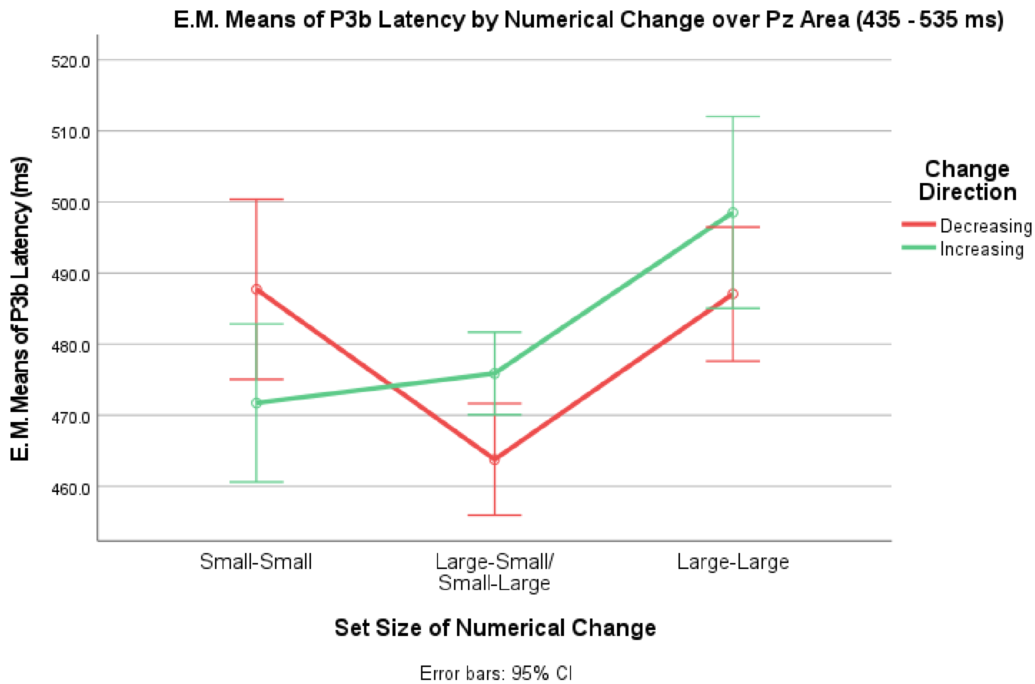

Figure 28: Estimated marginal means of P3b latencies by Numerical Change (Direction and Size)

7.5. Correlations of Brain and Behavior

To analyze the link between these variables (Reaction time/RT, Accuracy, N1 Amplitude and Latency, P3b Amplitude and Latency), two-way Pearson's correlation analyses were conducted with an FDR correction. Please see *Table 2* for the correlation findings.

Table 2: Correlations for Key Study Variables (Significant results are in bold)

Two-tailed Correlations and Confidence Intervals (C.I.s)						
					95% C.I.s (2-tailed) ¶	
Variable A	**Variable B**	**Pearson Correlation**	***p*, Sig. (2-tailed)**	**FDR, *p* (2-tailed)**	**Lower**	**Upper**
RT	**Accuracy**	**-0.645**	<0.001	**<0.001**	**-0.750**	**-0.502**
P3b Lat.	**RT**	**0.376**	<0.001	**0.002**	**0.182**	**0.540**
P3b Lat.	**Accuracy**	**-0.307**	0.003	**0.015**	**-0.482**	**-0.105**
P3b Amp.	**Accuracy**	**0.259**	0.014	**0.042**	**0.053**	**0.441**
P3b Amp.	**N1 Amp.**	**0.263**	0.012	**0.045**	**0.058**	**0.445**
N1 Amp.	N1 Lat.	0.221	0.037	0.093	0.013	0.408
P3b Amp.	P3b Lat.	-0.201	0.057	0.122	-0.391	0.007
P3b Amp.	N1 Lat.	0.190	0.073	0.137	-0.019	0.381
N1 Lat.	Accuracy	0.154	0.147	0.245	-0.056	0.349
P3b Amp.	RT	-0.145	0.172	0.258	-0.341	0.064
N1 Lat.	RT	-0.138	0.194	0.265	-0.335	0.072
N1 Amp.	Accuracy	0.094	0.380	0.475	-0.116	0.295
P3b Lat.	N1 Amp.	0.077	0.471	0.543	-0.133	0.279
P3b Lat.	N1 Lat.	0.020	0.850	0.893	-0.188	0.226
N1 Amp.	RT	0.014	0.893	0.893	-0.193	0.221

¶ Estimation is based on Fisher's r-to-z transformation with bias adjustment

Significant correlations are as follows: Higher accuracies are strongly positively correlated with shorter reaction times (r = -0.645, $p < 0.001$). Earlier P3b latencies are strongly positively correlated with shorter reaction times (r = 0.376, $p < 0.001$). Earlier P3b latencies are negatively correlated with lower accuracy (r = -0.307, $p < 0.05$). Higher N1 amplitudes are correlated with higher P3b amplitudes ($r = 0.263$, $p < 0.05$). Higher P3b amplitudes are correlated with higher accuracy ($r = 0.259$, $p < 0.05$).

7.6. Summary

This study examined the effects of Direction and Size on the dependent variables of Reaction Time, Accuracy, POT's N1 Amplitude, POT's N1 Latency, Pz's P3b Amplitude, Pz's P3b Latency to numerical change. For an overall summary of whether the main effects of Direction and Size are significant for these six dependent variables, and whether there is a significant interaction effect between Direction and Size for the listed six dependent variables, please refer to Table 3 below.

Table 3: Summary of ANOVA results of change variables for Direction, Size and Direction-by-Size for dependent variables of Reaction Time, Accuracy, POT's N1 Amplitude, POT's N1 Latency, Pz's P3b Amplitude, Pz's P3b Latency to numerical change. All p-values were FDR corrected. Significant results are bolded, with effect sizes (partial-eta-squared) in parentheses.

Variables	Direction	Size	Direction * Size
Reaction Time	**$p = 0.014$ (0.360)**	**$p < 0.001$ (0.798)**	**$p < 0.001$ (0.589)**
Accuracy	**$p = 0.013$ (0.366)**	**$p < 0.000$ (0.874)**	**$p = 0.019$ (0.245)**
POT N1 Amp.	**$p < 0.000$ (0.788)**	**$p < 0.000$ (0.655)**	**$p < 0.000$ (0.530)**
POT N1 Lat.	**$p = 0.003$ (0.483)**	$p = 0.629$ (0.033)	$p = 0.076$ (0.168)
Pz P3b Amp.	$p = 0.218$ (0.106)	**$p = 0.001$ (0.388)**	$p = 0.231$ (0.099)
Pz P3b Lat.	$p = 0.590$ (0.021)	**$p < 0.000$ (0.502)**	**$p = 0.007$ (0.298)**

All in all, there were significant interaction effects between the variables of Direction by Size for reaction time (RT), accuracy, as well as N1 amplitudes over the right POT area, and P3b latencies over the Pz area. Meanwhile, the only main effect on N1 latencies was driven by the Direction variable, where the Decreasing condition showed later N1 peaks. On the other hand, the only main effect on P3b amplitudes was the Size variable, where the Large-to-Large conditions showed the lowest P3b peaks.

Chapter 8: Discussion

The current EEG study used an active-change-detection paradigm in a set of numerical stimuli with arrays of dots ranging from small (1~3) and large (4~6) cardinalities. Our stimulus presentation paradigm was adapted from Hyde and Spelke (2009), where participants viewed an ongoing stream of dots in different numerosities, presented rapidly and sequentially. During the No Change conditions, participants saw the same number of dots repeatedly (albeit in different locations and sizes in each array), and the N1 response to the cardinalities of 1~6 was recorded and analyzed.

8.1. N1 ERP to Cardinalities in No-Change Trials

We investigated whether there is a discontinuity between systems of small number processing and large number processing within the continuous range of 1 to 3 and 4 to 6, respectively, similar to results found with more numerically separated values used in previous research (Hyde & Spelke, 2009, 2012). Previously, Hyde and Spelke (2009) found effects of small vs. large cardinalities in the Parietal-Occipital-Temporal (POT) region whereby values within the small number range (1, 2, 3) were scaled to N1 ERP amplitudes. Their study found no clear scaling of numerosity to amplitude for the larger numerical values (8, 16, 24).

Our first research question was as follows: Will there be differences in N1 amplitude and latency produced for smaller and larger numbers, specifically when participants are presented with a continuous range of the numerosities 1~6 for the present experiment? Our hypotheses and predictions were related to trials in which cardinal values were presented without a change.

Findings show that as cardinal value increases, more objects are encoded in early visual working memory, leading to higher N1 amplitudes. Findings revealed that the N1 amplitude for

“1” is different from “2”; “2” is different than “3”, and “3” is different than the later cardinal values. In the large number range (“4” and above), the amplitudes were not discernible from each other. Relative magnitudes of N1 deflections corresponded to ordered numerical magnitudes within the small-number range, but not within the large number range. This scaling of the N1 ERP to numerical magnitude replicates Hyde & Spelke (2012).
Scaling is clearer in our data, and the categorical break between “1” and “2”, followed by “3” and 4~6 is apparent.

8.1.1. N1 Amplitude for Cardinalities 1~6

We hypothesized that the sensory visual N1 component is related to spatial attention and is read off individual items in visual short-term memory (VSTM) within the smaller number range (1~3) over the POT area. We also hypothesized that numerical estimation, on the other hand, is not read off of exact individuation of numerical values in the larger number range (4~6). In evaluating responses to no-change trials, we found scalar N1 ERP amplitude differences within the range of small cardinal values ($1 < 2 < 3$), whereas no such scalar ordering of ERP numbers was for the continuous, larger range of 4 to 6. Our findings are supported by similar results from Hyde and Spelke (2009, 2012), where we found that N1 peak amplitudes are significantly different for 1, 2, 3 in the small, subitizable range, but not for our larger numerosities (4, 5, 6).

The current study found that as cardinal values increase in magnitude, higher N1 amplitudes over the POT are produced. This is supported by previous literature that described higher parietal cortical activity when there is an increase in the number of objects to be tracked in visual working memory (Culham et al., 1998, 2001a; Culham & Kanwisher, 2001).

From our findings, it can be implied that as the number of dots increase, more objects are encoded into working memory, which increases perceptual load, leading to higher N1 negativities with each increasing number for the cardinal values in the subitizable range. However, as a crucial signature of visual short-term memory is its limited storage capacity (i.e., 3-4 elements), the earlier scaling of N1 amplitudes is not seen for numerosities of 4 dots or more.

8.1.2. N1 Latency for Cardinalities 1~6

In trials with no change, where participants were habituated to each cardinal value by seeing the same number of dots, the current study did not find any significant differences in peak latency of the N1 ERP in response to cardinalities of 1 ~ 6.

It is important to note that our data do not show a positive relationship between N1 latency and cardinal value, and our findings are supported by similar results from Hyde and Spelke (2009), as they found no N1 latency differences for 1, 2, 3. Should N1 peak latency increases as cardinal value increases, it might indicate that there is a presence of a serial process in numerical processing. However, if an increase in cardinal value is accompanied by an N1 amplitude increase without latency differences, this may reflect the degree of perceptual load, or an increased load in working memory, as participants have to split their visuospatial attention (mental resources) to accommodate more objects in parallel (Gordon, 1994; Pylyshyn & Storm, 1988).

Neuroimaging research suggests multiple and distinct parietal processes are at work in the parallel individuation of a small number of objects (Xu, 2009), including a process of individuation that scales with the number of objects in the array. Therefore, the N1 effect observed over the bilateral POT areas in the range of 1~6 numerosities might reflect this parallel individuation process.

8.2. Effects of Size and Direction in Numerical Change

For trials in which there was a change in numerical value of the stimulus, participants were asked to respond by pressing a key. We were interested in performance variables such as reaction time and accuracy for these change conditions, as well as whether these behavioral responses were affected by the set size of numerical change (Small-to-Small vs. Large-to-Large vs. Crossovers) and the directionality of the change (Increasing vs. Decreasing). Participants of this study were only told to respond when they see a change in the number of dots. Therefore, there were not aware that Size of numeric sets and Direction of numerical change were variables that this current study is investigating.

Our research question was as follows: How is set size related to accuracy and reaction time, and is the increased set size advantage found within the smaller set size in the present experiment? Set size changes were categorized into: Small-to-Small (1~2, 1~3, 2~3), Large-to-Large (4~5, 5~6, 4~6) and Crossover conditions, which can be Small-to-Large (2~4, 3~6), or Large-to-Small (4~2, 6~3). We hypothesized that numerical change would be harder to detect among large numbers, since large number estimation does not access exact numerical quantities available in processes involving parallel individuation and working memory.

Within the 1~6 numerical range, we investigated whether there is an advantage for trials that involve Increasing quantities over Decreasing quantities. The present study questions whether change detection in smaller numerical ranges reflect the same or different kinds of processes in which directionality could have an effect.

In terms of directionality, previous studies have found that there is a performance advantage in the increasing set size conditions over the decreasing set size conditions, but these studies either looked at number comparisons with much larger numerical set sizes of 10 to 70, or they used a different experimental paradigm than our current study (Kaan, 2005; Paulsen et al., 2010).

Our third research question was as follows: Are there differences in the Small vs. Large vs. Crossover changes, where Direction of numerical change interacts with set size? For directionality effects, we were primarily guided by previous research on large number changes that show uniform advantages for increasing over decreasing changes in magnitude (Kaan, 2005; Paulsen et al., 2010). While the mechanism for such asymmetry has multiple explanations, there were no *a priori* mechanistic hypotheses for this study.

However, we identified two possible hypotheses regarding the possible role of directionality of change in the smaller number range: a) *Uniform Effects Hypothesis* (UEH), and b) *Interaction Effects Hypothesis* (IEH). UEH posits that directionality effects are uniform across all quantities. Since previous research from Kaan (2005) and Paulsen et al. (2010) has shown better performance for increasing magnitude in large number changes (e.g., 10~70), such advantages could exist across the board and within the 1~6 number range of the current experiment. In this case, across the board–for the Small, Large and Crossover conditions,– reaction time would be faster and accuracy would be higher for increasing over decreasing magnitude changes. On the other hand, IEH posits that directionality effects are different in the small number (1~3) and large number (4~6) range reflecting different effects due to changes in the working memory/parallel individuation range compared to numerical estimation. Performance in detecting changes within the larger numerical range (4~6) would thus show higher accuracy and faster reaction times for decreasing over increasing numerical changes if IEH were to be accepted.

This effect might be higher in larger numbers than smaller numbers in the subitizing range. Changes in the small number range (1~3) should show a difference effect, either no directionality effects or better accuracy and reaction times in the decreasing set size range.

8.2.1. Reaction Time: Direction by Size Effects

In regards to reaction time, the IEH was supported, as we found significant evidence of an interaction effect of Direction by Size. When paired by different Direction groups for the same sizes, we found that the “Decreasing” condition had shorter reaction times than “Increasing”, but only when set sizes of numerical involve larger numbers, which are the Crossovers (Large-to-Small) and Large-to-Large sets. This pattern is reversed when comparing mean reaction times for both change directions among Small-to-Small set sizes, where “Decreasing” change had significantly longer reaction times than “Increasing”.

8.2.2. Accuracy: Direction by Size Effects

With respect to accuracy, the IEH is supported, as we found significant evidence of an interaction effect of Direction and Size. When paired by different Direction groups for the same sizes, we found that the “Decreasing” conditions had higher accuracy than “Increasing”. Further analysis revealed that this significant result was driven by how Decreasing Large-to-Large had significantly higher accuracy than Increasing Large-to-Large. The other two sizes (Small-to-Small and Crossovers) did not show significant differences in mean accuracy in both change directions. However, when paired by different set sizes in the same Direction groups, we found significantly higher accuracy for numerical change sets with smaller numbers over larger numbers, with the exception of Increasing Small-to-Small vs. Small-to-Large (one out of the six pairs) that showed no significant differences in mean accuracy. These results are supported by well-established concepts, where as numerosity increases, the imprecision of the approximate number system systematically increases (Hyde, 2011).

Overall, we found shorter reaction times when change detection was easier, particularly while processing change when the number pairs involved smaller numbers in the subitizing range (1~3), as compared to large number processing (4~6). Similarly, we found higher accuracy when change detection is easier, particularly while processing change when the number pairs involve smaller numbers in the subitizing range (1~3), as compared to large number processing (4~6). We also found these advantages for crossover conditions to be midway between Small and Large conditions in behavioral measures of processing efficiency (Reaction time and Accuracy).

8.3. N1 and P3b ERPs of Numerical Change

In this study, while participants were behaviorally responding to numerical change on the screen with a key press, their ERP data (in response to the stimuli) is simultaneously being recorded. This gives our study an avenue to investigate the effect of change direction on amplitude and latency differences between small and large numbers. The ERPs of interest to us are the N1 and the P3b.

8.3.1. N1 Amplitudes: Direction by Size Effects

We hypothesized that, during change conditions, the N1 component over the POT area is related to visuospatial attention and visual short-term memory (VSTM), where heavier perceptual loads are linked to higher N1 negativities. We found that as more objects are encoded in VSTM, the N1 amplitude over the POT area is higher.

With respect to N1 amplitudes, the IEH was supported, as we found significant evidence of an interaction effect of Direction and Size. When paired by different Direction groups for the same set sizes, we found that the Decreasing condition had higher peak N1 amplitudes than Increasing. Further analysis revealed that this significant result was driven by the Small-to-Small and Crossover (Large-to-Small) set sizes, where Decreasing had significantly higher N1 amplitudes than Increasing. However, this pattern reversed in the Large-Large sets where Decreasing had marginally lower N1 amplitudes than Increasing (but this was not significant). This suggests that the N1 amplitude effect most likely reflects subprocesses within the system of object representation and attentive object tracking.

8.3.2. N1 Latencies: Direction (not Size) Effects

In contrast, we found that the UEH is supported in N1 latency, instead of the IEH, as only Direction was significant in influencing peak latencies of the N1 ERP, where Increasing change had shorter peak N1 latencies. Further analysis revealed that Direction did not have a significant effect on the N1 peak latencies for the Large-to-Large sets, but for the Small-to-Small and the Crossover (Small-to-Large) conditions, Increasing conditions had earlier N1 peak latencies. Dissimilar to Hyde and Spelke (2009, 2012) who found earlier latencies for N1 peaks as neural responses to changes in larger numbers (8, 16, 24) over small numbers (1, 2, 3), the current study, which tested a narrower, continuous range of numbers (1~6), did not find that larger numbers have earlier N1 peak latencies. These findings imply that the effect of change directionality has a larger effect on the N1 ERP, instead of the numeric set sizes.

8.3.3. P3b Amplitudes: Size (not Direction) Effects

We hypothesized that, in accordance with the P3b context-updating theory (Donchin, 1981; Polich, 2007), when measuring numerical change processes over the Pz area, the P3b component is scaled to the ease of change detection, by how much easier it is to update the context from its previous one. We found that when the numerical change gets easier to detect (smaller sizes), the P3b shows higher amplitudes, reflecting greater ease in updating the context.

With respect to the variable of P3b amplitudes, we found that the UEH is supported, instead of the IEH, as only Size was significant in influencing peak amplitudes of the P3b ERP. Analysis revealed that the Large-to-Large sets showed lower P3b amplitudes. Our finding of significant effect for Size on P3b amplitudes during numerical processing is supported by previous research from Libertus et al. (2007), where they found higher P300 amplitudes for small values as compared to large values.

8.3.4. P3b Latencies: Direction by Size Effects

Polich (2012) noted that the peak latency of the P3b component can be interpreted as indexing stimulus evaluation time, where it peaks earlier for easier tasks and more difficult tasks showed longer latencies. In accordance, we found shorter P3b latencies during easier, small number processing.

Regarding the P3b peak latency variable, we found that the IEH is supported, as we found significant evidence of an interaction effect of Direction and Size. When paired by different Direction groups for the same sizes, we found that the “Decreasing” condition showed shorter P3b peak latencies than “Increasing”, but only when set sizes of numerical involve larger numbers, which are the Crossovers (Large-to-Small) and Large-to-Large sets. This pattern is reversed when comparing mean P3b peak latencies for both change directions among Small-to-Small set sizes, where the “Decreasing” condition had significantly longer P3b peak latencies than “Increasing”.

Size was a more significant factor in influencing P3b latencies, but there were no main effects for Direction. This contrasts the earlier N1 latency results, where there the only main effect on it was Direction, where Increasing conditions produced earlier N1 peak latencies.

Yurgil and Golob (2013) found that high perceptual load elicited longer P3b latencies and smaller P3b amplitudes in an auditory oddball EEG study. Our findings suggest a neural basis for the differentiation of small vs. large number perception at early stages of processing, and a later stage that involves more complex numerical processing that is employed in our numerical change detection task. Our findings are also in accordance with Rubinsten et al. (2013), who found that direction of change in the descending order was associated with a late parietotemporal (300–600 ms) positivity.

8.4. Neurobehavioral Correlates in Processing Change Direction & Set Size

8.4.1. Stronger P3b and Behavioral Performance

Behavioral outcomes (accuracy and reaction time) were found to be strongly correlated with each other, as well as with amplitude and latency of P3b ERP signal. Our findings show that accuracy is positively correlated with P3b amplitude and strongly negatively correlated with P3b latency, while reaction time is strongly positively correlated with P3b latency.

8.4.2. Longer P3b Latency and Reaction Time

From visually comparing the plots of mean P3b latencies and mean reaction times by numerical change direction and size (see Figure 29), there are almost identical trends for the main effects of Direction, and the interaction effects of Direction by Size. This similarity is supported by our findings that P3b latency and reaction time have a strong positive correlation with each other. Our findings are supported by previous research that also found positive correlations between P3b latency and reaction times (Ilan & Polich, 1999; Kutas et al., 1977). This support the proposition that the latency of P300 corresponds to stimulus evaluation time (Polich, 2011).

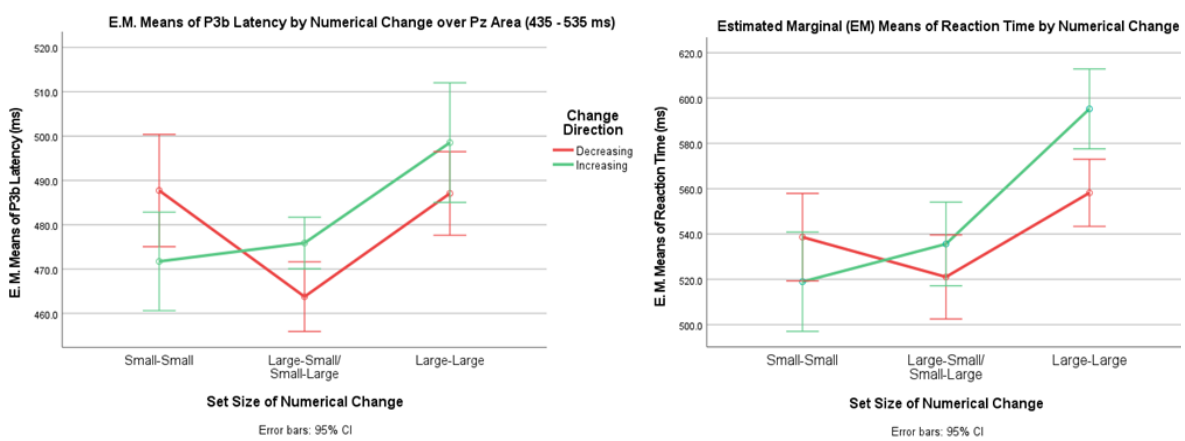

Figure 29: Similar trends for peak latency of the P3b ERP and reaction time, where for the Small-Small condition, Decreasing showed longer P3b latencies and longer reaction times. This effect of Direction was reversed for the other two Size conditions (Large-Large and Crossovers, which include Large-Small and Small-Large)

8.4.3. Links between N1 and P3b Amplitudes

By visually comparing the plots of mean amplitudes of the N1 and the P3b (see Figure 30), there are similar trends for the interaction effects of Direction by Size, especially in the Large-Large sets.

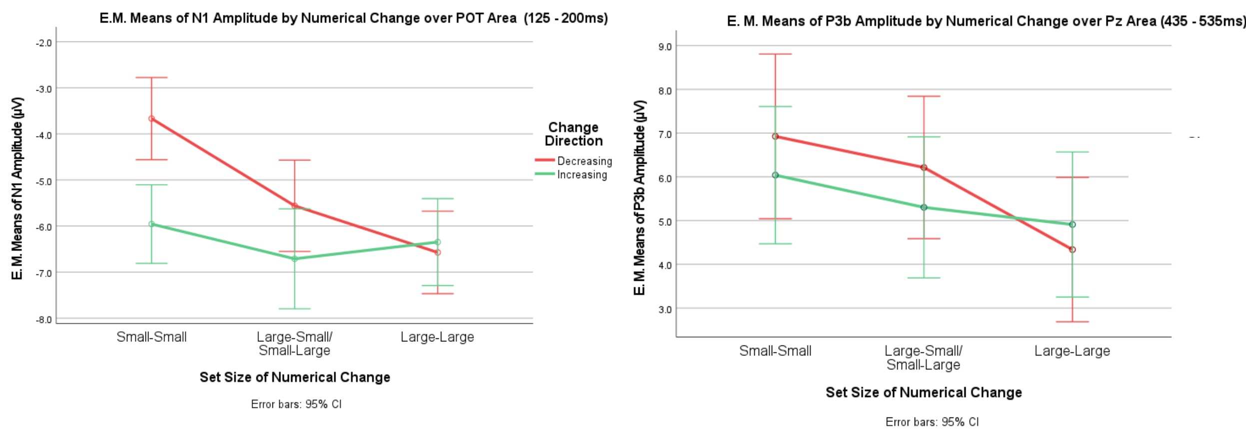

Figure 30: Similar trends for mean amplitudes of the N1 and the P3b ERP, where for the Large-Large condition, Decreasing showed lower N1 and P3b amplitudes. This effect of Direction was reversed for the other two Size conditions (Small-Small and Crossovers, which include Large-Small and Small-Large)

Our findings show that the amplitudes of the N1 and the P3b ERP have a moderate positive correlation. For the N1 waveforms to numerical change, the major effect was a shallower N1 deflection for the Decreasing Small-Small condition compared to all other conditions. In fact, this change condition showed a shallower deflection than even the no-change condition.

The off-loading phenomenon was observed from N1 amplitudes for the Decreasing Small-Small condition, where it produced the lowest N1 negativity (even lower than N1 peaks for No Change conditions), while the other five Change conditions all produced higher N1 negativities than No Change conditions. However, for the later P3b ERP, the Decreasing Small-

Small condition produced the highest amplitudes. This indicates that in our study, the less negative the N1 peak is, the more positive the P3b peak becomes.

One possibility is that this difference in the N1 deflection reflected an easier discrimination between the primes and targets for the Small-to-Small sets. If this were the case, then we would expect there to be a faster reaction time and greater accuracy for this change condition in the behavioral data. However, this was not the case. For reaction time, the Decreasing Small-to-Small sets was actually slower than the Increasing Small-to-Small sets. Also, there was no difference in accuracy between both directions in the Small-Small sets.

An alternative interpretation of the N1 outlier status of the Decreasing Small-to-Small condition is that only Small cardinalities are encoded within working memory and that there is a decrease in cognitive load when decreasing the number of items in visual working memory. In the case of the large number cardinal values, these are not encoded in working memory, and so are undifferentiated in the N1 for increasing and decreasing conditions. On the other hand, we do see an advantage for Decreasing over Increasing in the Large-to-Large conditions in the behavioral data, whereby Decreasing is faster and more accurate than Increasing.

8.5. Size Effects: Perceptual Load Theory

Based on the Perceptual Load Theory from the foundational work of Lavie and Tsal (1994) and as outlined by Lavie (2005, 2010), the efficiency of our early- and late-selection attentional processes is dependent on the processing demands of the current task, where the ability to pay attention to the task at hand inevitably deteriorates under conditions of high perceptual load on cognitive control processes such as working memory.

Theorists have associated early-selection attention to the perceptual processing stage (e.g., Treisman, 1969), while late-selection attention has been linked to the response selection stage (e.g., Deutsch & Deutsch, 1963). Perceptual load theory states that perception is a limited-capacity process (similar to early-selection views) and proceeds automatically until that capacity is filled (in line with late-selection views). When a task imposes high perceptual load, capacity is reached, resulting in performance consistent with early-selection attention (Murphy et al., 2016). Cognitive load, such as a high working memory requirement, can cause late-selection attention to fail (Lavie, 2005; Murphy et al., 2016).

Researchers who study numerical processing have increased perceptual loads by increasing the amount of objects that participants have to process, and found that increased reaction times were longer and accuracy was lower (Cartwright-Finch & Lavie, 2007; Luck et al., 2000; Murphy et al., 2016). In an fMRI study by Culham, Cavanagh and Kanwisher (2001), they found increased activity in parietal and frontal cortical areas when there is an increase in the number of objects that must be tracked. Behaviorally, as the size of numerosities increase, participants show longer reaction times and lower accuracy (Nan et al., 2006; Vetter et al., 2011; Vuokko et al., 2013).

ERP research by Rorden et al. (2008) suggest that increased perceptual task loads result in higher N1 responses to relevant information. In an ERP study where participants are asked to rapidly enumerate geometric objects on a screen, Nan et al. (2006) employed a mixed design where target numerosities are 1~6, to be mixed with distractors to total up to a maximum of 20 objects on the screen to discriminate and enumerate. Nan et al. (2006) found that as the target numerosity becomes larger, N1 amplitudes become progressively higher, but the strength of the N1 amplitude stops increasing after approximately 8 objects, indicating that this might be an intrinsic limit of our ability to efficiently engage in numerical processing after a certain threshold of perceptual and cognitive load. Within the same study, the authors also found that P3 amplitudes become lower as the number of objects to process increases.

8.6. Direction Effects on N1 vs. Size Effects on P3b

Research suggest that change detection should take place downstream from the encoding of numerical change at N1 (Hyde & Spelke, 2012). If early perceptual-level processing stages are influenced by the allocation of attentional resources, the N1 attention effects should be sensitive to manipulations of perceptual load. As more items are encoded in visual short-term memory (VSTM), measured N1 amplitudes increases as the condition increases in numerical magnitude. Meanwhile, as more items are off-loaded during VSTM, the measured N1 amplitude decreases.

In comparison, if N1 peak latency is more sensitive to Direction, and P3b peak latency is more sensitive to Size, this implies that the factor of Change Direction is encoded first (signaled by N1), followed by stronger effects for Set Size (as signaled by reaction time and P3b latency, where larger loads lead to longer reaction times and longer P3b latencies).

8.7. Proposed Adaptation to Perceptual Load and Context-Updating Theory in Working Memory Model of Numerical Change Processing

Taking all of these findings into consideration, we propose an adaptation to the P3b Context-Updating Model (Polich, 2007). Please see *Figure 31* for a schematic illustration of our proposed model.

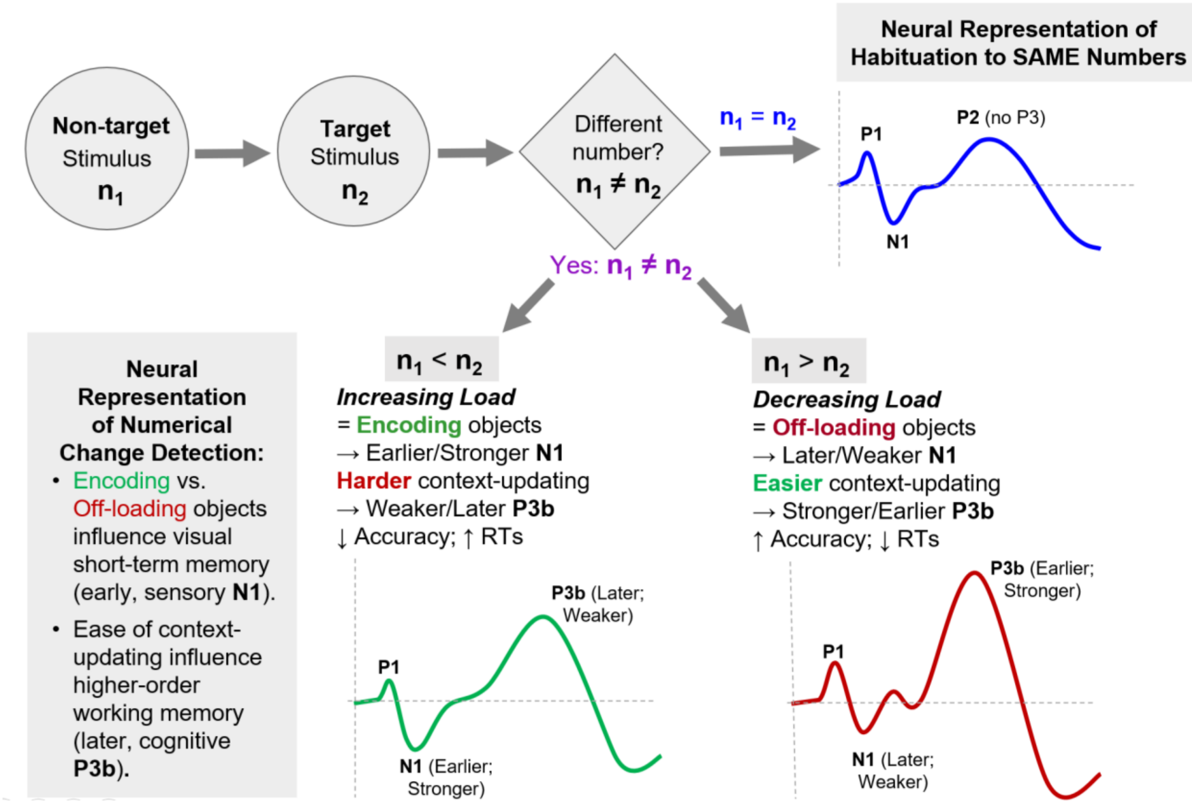

Figure 31: Visual short-term memory (VSTM) and Working Memory in Numerical Change: Model adapted from Polich (2007).

The context-updating theory of the P3b is related to updating one's working memory in change detection paradigms, where an incoming sensory input is evaluated as being the same or different from the previous context (Polich, 2007). If this input is different, it elicits an updating of a given neural representation which is reflected in a P3b deflection at ~400ms. We propose that at an earlier sensory stage (~125ms) in numerical context-updating, objects are encoded or off-loaded from visual short-term memory, which modulates the N1, before integrating this information at later cognitive stages.

Chapter 9: Conclusion

In contrast to Hyde & Spelke (2012), who examined distant small (1, 2, 3) vs. large (8, 16, 24) numbers, we examined a smaller numerical range (1~6), so that small (1, 2, 3) vs. large (4, 5, 6) contrasts were along a numerical continuum. Within this continuous range, we found N1 amplitudes commensurate with cardinal values in the small range (1, 2, 3), but not in the large range (4, 5, 6), where the process of encoding/off-loading objects in memory determines the amplitude strength, suggesting that numbers in the subitizing range are individuated in working memory.

Measured N1 amplitudes become increasingly higher in their negativity as more objects are encoded in working memory. This increase of the N1 signal might imply that as more items are encoded in working memory, more neural activity is generated over the parietal regions of numerical cognition. When change is happening in the Decreasing direction, the Small-to-Small sets show its unique outlier status where it showed the lowest N1 negativity at early stages, but it eventually shows the highest P3b positivity at later stages. Differences in P3b amplitude and latencies also reflect a clear categorical break between increasing vs. decreasing, and small vs. large numbers, where easier/small number change conditions have higher amplitudes than harder, large number conditions, suggesting more difficulty with updating the context in the latter.

Overall findings align with the context-updating model (Polich, 2007) where working memory representations differ between small and large numbers, as well as increasing and decreasing numerical change. We found that P3b and behavioral performance are correlated, with reaction time and P3b latency tightly linked. With N1 being more sensitive to Direction, and P3b being more sensitive to Size, this suggests that the posterior parietal cortex might encode Direction first, followed by Size. Based on the present findings, it is possible that early spatial attention is deployed differentially based on the number of objects in the set. This explanation fits nicely with the idea that a limited number of items can be individuated or tagged as what has been termed “object files” (Sears & Pylyshyn, 2000; Scholl & Pylyshyn, 1999). From the object files perspective, the small number system is not a “number system” per se, but a system for keeping track and encoding features of individuals in parallel (Feigenson & Carey, 2003).

Further, our results provide evidence that there are two distinct system of representations spanning across a range of 1~6 when processing numerical change while considering the effects of change directionality and set size. These two systems are not specialized for small and large numbers per se, rather early attentional selection and its corresponding limits determine whether objects will be represented as distinct individuals or approximate numerical magnitudes.

By understanding the processing stream and the neural generators involved in representing number at different stages of the processing stream, we may also gain insight into a currently debated issue within the literature on number processing: whether small and large numbers are truly represented by the same cognitive system or by distinct cognitive systems. We may also gain an understanding of the role that more general-purpose cognitive/brain systems play in producing, limiting, and altering representations of numbers.

What brain regions underlie these dissociations between small and large number systems? Do individuation and numerical approximation engage the same brain area in different ways or do different brain regions serve to represent distinct individuals and approximate numerical magnitudes? A complete picture of the timing, pattern, and localization of these effects can provide a better basis not only for linking these components to identifiable brain processes but also for understanding the dissociations between processing of individuals and of numerical magnitudes.

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Appendix A

IRB Approval Letter (Current study)

TEACHERS COLLEGE
COLUMBIA UNIVERSITY

Teachers College IRB *Exempt Study Approval*

To: Jean Ee Tang
From: Curt Naser, TC IRB Administrator
Subject: IRB Approval: 22-288 Protocol
Date: 05/06/2022

Thank you for submitting your study entitled, "*Reanalyses of ERP data in numerical cognitive processing*;" the IRB has determined that your study is **Exempt** from committee review (Category **4, 10**) on 05/06/2022.

Please keep in mind that the IRB Committee must be contacted if there are any changes to your research protocol. The number assigned to your protocol is **22-288**. Feel free to contact the IRB Office by using the "Messages" option in the electronic Mentor IRB system if you have any questions about this protocol.

Please note that your Consent form bears an official IRB authorization stamp and is attached to this email. Copies of this form with the IRB stamp must be used for your research work. Further, all research recruitment materials must include the study's IRB-approved protocol number.

As the PI of record for this protocol, you are required to:

- Use current, up-to-date IRB approved documents
- Ensure all study staff and their CITI certifications are on record with the IRB
- Notify the IRB of any changes or modifications to your study procedures
- Alert the IRB of any adverse events

You are also required to respond if the IRB communicates with you directly about any aspect of your protocol. Failure to adhere to your responsibilities as a study PI can result in action by the IRB up to and including suspension of your approval and cessation of your research.

You can retrieve a PDF copy of this approval letter from Mentor IRB.

Best wishes for your research work.

Sincerely,
Curt Naser, Ph.D.
TC IRB Administrator
curtn@axiom-mentor.com

Informed Consent (page 1)

INFORMED CONSENT FORM

Neurocognition of Language Laboratory / Language and Cognition Laboratory
Biobehavioral Sciences Department
Teachers College, Columbia University

Purpose of research:
The purpose of this research is gain information regarding brain activation during speech and language tasks.

Procedure:
The procedure involves coming up to the Neurocognition of Language / Language and Cognition Labs in the Biobehavioral Sciences Department for up to two hours. Your head size will be measured and you will have a net placed on your head that contains sensors within small sponges that sit directly on the scalp. The sponges are first soaked in a weak salt solution (potassium chloride) which helps pick up small electrical signals. The minute signals generated by brain activity are recorded through the sensors. You will listen to sounds through earphones, look at pictures or movies, or read words or sentences on a computer screen while your brain activity is recorded. You might be asked to evaluate and answer questions about the sounds, pictures, movies, words or sentences by pushing response buttons. The details of the experimental procedure will be explained to you by the experimenter.

Confidentiality:
All information collected in the study is confidential. Your name and other personal information will not be disclosed at any time. Your data are only identified by subject number. A copy of this form will be kept by the experimenter in a locked file cabinet which only the principal investigator can access. The original will be given to you to keep. The data you provide will be grouped with data others provide for reporting and presentation.

Risks:
As with all physiological recording, there is a minimal risk of electric shock. This is minimized by using a special isolated amplifier, and ensuring that you are never connected to ground. There is a risk of skin irritation, minimized by careful choice of electrolyte, which is a simple salt solution. There is also a small risk of skin infection, minimized by careful and complete disinfection of electrodes. The sensor net will be wet when applied, and this may be slightly uncomfortable at first. However, towels are provided so as to minimize discomfort and to protect your clothing. The experimental tasks can be repetitive, and you may find them somewhat boring and/or difficult to complete. However, you can take breaks during the experiment and continue only when you feel ready. Should you feel uncomfortable or concerned with the net application or the procedures used, feel absolutely free to discuss them with the experimenter. You may stop participating at any time with no penalty whatsoever.

Benefits, Freedom to Withdraw:
You should understand that the experiment is not designed to help you personally, but that the investigator hopes to gain more information concerning human speech and language, and the efficacy of certain research methods for this purpose. You will not be paid for your participation. You may withdraw from the study at any time without penalty.

- I state that I am over 18 years of age and in good physical health.
- I have read and understand the information presented above, and have had an opportunity to discuss this information with the experimenter.
- I wish to participate in a program of research conducted by the Neurocognition of Language / Language and Cognition Laboratory in the Biobehavioral Sciences Department at Teachers College, Columbia University.

Printed Name:______________________________

Signature:______________________________

Date:________________

Teachers College, Columbia University
Institutional Review Board
Protocol Number: 13-068
Consent Form Approved Until: 08/08/2020

Informed Consent (page 2)

Teachers College, Columbia University

PARTICIPANT'S RIGHTS

Principal Investigator: Prof. Karen Froud, Prof. Peter Gordon

Research Title: Studies of linguistic processing in adult humans using behavioral and electrophysiological methods

- I have read and discussed the Research Description with the researcher. I have had the opportunity to ask questions about the purposes and procedures regarding this study.
- My participation in research is voluntary. I may refuse to participate or withdraw from participation at any time without jeopardy to future medical care, employment, student status or other entitlements.
- The researcher may withdraw me from the research at his/her professional discretion.
- If, during the course of the study, significant new information that has been developed becomes available which may relate to my willingness to continue to participate, the investigator will provide this information to me.
- Any information derived from the research project that personally identifies me will not be voluntarily released or disclosed without my separate consent, except as specifically required by law.
- If at any time I have any questions regarding the research or my participation, I can contact the principal investigators, Prof Karen Froud / Prof. Peter Gordon, who will answer my questions. The investigators' phone number is (212) 678 8158.
- If at any time I have comments, or concerns regarding the conduct of the research or questions about my rights as a research subject, I should contact the Teachers College, Columbia University Institutional Review Board /IRB. The phone number for the IRB is (212) 678-4105. Or, I can write to the IRB at Teachers College, Columbia University, 525 W. 120th Street, New York, NY, 10027, Box 151.
- I will receive a copy of the Research Description and this Participant's Rights document.
- Video recording is part of this research. Check one:

  I ( ) consent to be video recorded.

  I ( ) do NOT consent to being video recorded.

  The video recordings will be viewed only by the principal investigator and members of the research team for the purposes of monitoring and artifact detection only.
- My signature means that I agree to participate in this study.

Signature: ____ Date:____________

Print Name: ______________________________

Investigator's Verification of Explanation

I certify that I have carefully explained the purpose and nature of this research to ______________________________ (participant's name) in age-appropriate language. He/She has had the opportunity to discuss it with me in detail. I have answered all his/her questions and he/she provided the affirmative agreement (i.e. assent) to participate in this research.

Investigator's Signature: _________________________________ Date: _______________

Teachers College, Columbia University
Institutional Review Board
Protocol Number: 13-068
Consent Form Approved Until: 08/08/2020

Recruitment Flyer

BRAIN AND NUMERICAL ESTIMATION
RESEARCH OPPORTUNITY

Participants are invited for a research study that will look at brain electrical activity using electroencephalography (EEG)

Title: The Neural Basis of Parallel Individuation and Numerical Estimation

Description: The Language and Cognition Lab at Teachers College Columbia University is seeking **typically developing, healthy adult volunteers (ages 18-65)** to participate in an EEG study on number processing. EEG is a non-invasive procedure that uses a sensor cap to record brain activity. Participants can expect to be in the lab for approximately **90 minutes**, where they will have the cap placed on their head and sit in front of a computer viewing a series of images while researchers collect data.

To Enroll or Get additional Info:

Contact Lab Manager Erin Reddick Kirby

emr2187@tc.edu | (978) 886-0414

IRB Protocol Number: 13-068

Principal Investigator: Dr. Peter Gordon

Lab Location: 525 W. 120th Street, Thorndike Hall Room 1153

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Appendix B

Table 1A
Demographic data for participants included in dataset

Participant	Gender	Handedness	Age	Accuracy
2	Female	Right	23	65.06
3	Female	Right	26	63.72
4	Female	Right	23	51.56
5	Female	Right	24	56.06
6	Male	Right	43	67.18
7	Female	Right	25	67
8	Female	Right	28	55.34
9	Female	Right	33	50.14
10	Female	Right	29	58.46
11	Male	Right	25	66.44
12	Male	Right	24	58.48
13	Female	Right	29	59.8
14	Female	Right	26	67.92
15	Male	Right	29	54.06
17	Female	Right	26	72.32

Table 2A
Categorization of number pairs (Primed-Target) into Direction of change, Set Size, absolute difference, distance, and ratio between the Primed number and the Target number

Primed	Target	Direction	Set Size	Abs. Diff.	Distance	Ratio
1	**3**	Increasing	Small-to-Small	2	Far	0.33
1	**2**	Increasing	Small-to-Small	1	Med	0.5
2	**3**	Increasing	Small-to-Small	1	Close	0.67
1	**1**	Null	No Change	0	N/A	1
2	**2**	Null	No Change	0	N/A	1
3	**3**	Null	No Change	0	N/A	1
2	**1**	Decreasing	Small-to-Small	1	Med	0.5
3	**2**	Decreasing	Small-to-Small	1	C	0.67
3	**1**	Decreasing	Small-to-Small	2	F	0.33
Primed	**Target**	**Direction**	**Set Size**	**Abs. Diff.**	**Distance**	**Ratio**
4	**6**	Increasing	Large-to-Large	2	Close	0.67
5	**4**	Decreasing	Large-to-Large	1	Close	0.8
6	**5**	Decreasing	Large-to-Large	1	Close	0.83
4	**4**	Null	No Change	0	N/A	1

5	**5**	Null	No Change	0	N/A	1
6	**6**	Null	No Change	0	N/A	1
4	**5**	Increasing	Large-to-Large	1	Close	0.8
5	**6**	Increasing	Large-to-Large	1	Close	0.83
6	**4**	Decreasing	Large-to-Large	2	Close	0.67
Primed	**Target**	**Direction**	**Set Size**	**Abs. Diff.**	**Distance**	**Ratio**
3	**4**	Increasing	Small-to-Large	1	Close	0.75
4	**3**	Decreasing	Large-to-Small	1	Close	0.75
2	**4**	Increasing	Small-to-Large	2	Med	0.5
3	**5**	Increasing	Small-to-Large	2	Med	0.6
4	**2**	Decreasing	Large-to-Small	2	Med	0.5
5	**3**	Decreasing	Large-to-Small	2	Med	0.6
1	**4**	Increasing	Small-to-Large	3	Far	0.25
2	**5**	Increasing	Small-to-Large	3	Med	0.4
3	**6**	Increasing	Small-to-Large	3	Med	0.5
4	**1**	Decreasing	Large-to-Small	3	Far	0.25
5	**2**	Decreasing	Large-to-Small	3	Med	0.4
6	**3**	Decreasing	Large-to-Small	3	Med	0.5

Table 3A

Means and S.D. of N1 Amplitude to Six Cardinal Values for Right vs. Left POT

	N1 Amplitude of Right POT		N1 Amplitude of Left POT	
Cardinality	**Right (Mean)**	**Right (S.D.)**	**Left (Mean)**	**Left (S.D.)**
One	-1.771	1.555	-1.299	1.415
Two	-3.632	1.952	-3.043	1.731
Three	-4.906	2.019	-3.960	1.668
Four	-5.859	2.182	-4.994	1.207
Five	-5.865	2.252	-4.674	1.469
Six	-5.361	2.301	-4.365	1.537

Table 4A

Post hoc paired samples t-tests of Right POT area's N1 amplitude to the Six Cardinal Values

Cardinal									95% C.I. of the Diff.	
n_1		n_2	*t*	df	Sig. (1-tailed)	Mean	Std. Dev.	S.E. Mean	Lower B.	Upper B.
1	**-**	**2***	**4.527**	**13**	**0.000***	**1.861**	**1.538**	0.411	0.973	2.749
1	**-**	**3***	**6.481**	**13**	**0.000***	**3.135**	**1.810**	0.484	2.09	4.181
1	**-**	**4***	**7.212**	**13**	**0.000***	**4.088**	**2.121**	0.567	2.863	5.312
1	**-**	**5***	**7.361**	**13**	**0.000***	**4.094**	**2.081**	0.556	2.893	5.296
1	**-**	**6***	**6.182**	**13**	**0.000***	**3.59**	**2.173**	0.581	2.336	4.845
2	**-**	**1***	**-4.527**	**13**	**0.000***	**-1.861**	**1.538**	0.411	-2.749	-0.973
2	**-**	**3***	**3.946**	**13**	**0.001***	**1.274**	**1.208**	0.323	0.577	1.972
2	**-**	**4***	**6.634**	**13**	**0.000***	**2.227**	**1.256**	0.336	1.502	2.952
2	**-**	**5***	**8.970**	**13**	**0.000***	**2.233**	**0.932**	0.249	1.696	2.771
2	**-**	**6***	**5.275**	**13**	**0.000***	**1.729**	**1.227**	0.328	1.021	2.437
3	**-**	**1***	**-6.481**	**13**	**0.000***	**-3.135**	**1.810**	0.484	-4.181	-2.09
3	**-**	**2***	**-3.946**	**13**	**0.001***	**-1.274**	**1.208**	0.323	-1.972	-0.577
3	**-**	**4***	**2.824**	**13**	**0.007***	**0.952**	**1.262**	0.337	0.224	1.681
3	-	5	2.670	13	0.01	0.959	1.344	0.359	0.183	1.735
3	-	6	1.303	13	0.108	0.455	1.306	0.349	-0.299	1.209
4	**-**	**1***	**-7.212**	**13**	**0.000***	**-4.088**	**2.121**	0.567	-5.312	-2.863
4	**-**	**2***	**-6.634**	**13**	**0.000***	**-2.227**	**1.256**	0.336	-2.952	-1.502
4	**-**	**3**	**-2.824**	**13**	**0.007***	**-0.952**	**1.262**	0.337	-1.681	-0.224
4	-	5	0.026	13	0.490	0.007	0.937	0.25	-0.534	0.547
4	-	**6**	-1.693	13	0.057	-0.498	1.100	0.294	-1.133	0.138
5	**-**	**1***	**-7.361**	**13**	**0.000***	**-4.094**	**2.081**	0.556	-5.296	-2.893
5	**-**	**2***	**-8.970**	**13**	**0.000***	**-2.233**	**0.932**	0.249	-2.771	-1.696
5	-	3	-2.670	13	0.010	-0.959	1.344	0.359	-1.735	-0.183
5	-	4	-0.026	13	0.490	-0.007	0.937	0.25	-0.547	0.534
5	**-**	**6**	**-2.882**	**13**	**0.006***	**-0.504**	**0.655**	0.175	-0.882	-0.126
6	**-**	**1***	**-6.182**	**13**	**0.000***	**-3.59**	**2.173**	0.581	-4.845	-2.336
6	**-**	**2***	**-5.275**	**13**	**0.000***	**-1.729**	**1.227**	0.328	-2.437	-1.021
6	-	3	-1.303	13	0.108	-0.455	1.306	0.349	-1.209	0.299
6	-	4	1.693	13	0.057	0.498	1.100	0.294	-0.138	1.133
6	**-**	**5***	**2.882**	**13**	**0.006***	**0.504**	**0.655**	0.175	0.126	0.882

*. The mean difference is significant at the 0.0083 level, based on Bonferroni adjustments for multiple comparisons. Significant results are in bold.

Table 5A

Descriptive statistics of Reaction Time by Numerical Change Condition.

	Mean	Std. Deviation	N
dSS	541.259	33.75	15
dLS	522.958	31.77	15
dLL	557.216	25.00	15

iSS	519.847	36.73	15
iSL	535.784	30.83	15
iLL	595.535	29.48	15

Table 6A:
Paired samples t-tests for multiple comparisons of mean reaction to the change conditions by Direction and Size

Two-sided pairwise t-tests for multiple comparisons of mean reaction time to the change conditions by Direction

	Mean	S.D.	Std. Err. Mean	95% C.I. of the Diff.		*t*	df	Significance	
				Lower	Upper			*p*, 2-sided	FDR, 2-sided
dSS - iSS	21.412	25.882	6.683	7.079	35.745	3.204	14	0.006	0.009
dLS - iSL	-12.826	21.625	5.584	-24.801	-0.850	-2.297	14	0.038	0.038
dLL - iLL	-38.319	27.739	7.162	-53.680	-22.957	-5.350	14	<0.001	<0.001

One-sided pairwise t-tests for multiple comparisons of mean reaction time to the change conditions by Size

	Mean	S.D.	Std. Err. Mean	95% C.I. of the Diff.		*t*	df	Significance	
				Lower	Upper			*p*, 2-sided	FDR, 2-sided
dSS - dLS	18.301	26.459	6.832	3.649	32.954	2.679	14	0.018	0.027
dSS - dLL	-15.957	28.469	7.351	-31.723	-0.191	-2.171	14	0.048	0.048
iSS - iSL	-15.937	27.056	6.986	-30.920	-0.954	-2.281	14	0.039	0.047
iSS - iLL	0.252	0.123	0.032	0.184	0.320	7.930	14	<0.001	<0.001
dLS - dLL	0.353	0.121	0.031	0.286	0.420	11.290	14	<0.001	<0.001
iSL - iLL	0.351	0.130	0.034	0.279	0.423	10.453	14	<0.001	<0.001

Table 7A:

Descriptive statistics of Accuracy by Numerical Change Condition

	Mean	Std. Deviation	N
dSS	0.830	0.191	15
dLS	0.872	0.187	15
dLL	0.568	0.158	15
iSS	0.820	0.156	15
iSL	0.817	0.175	15
iLL	0.467	0.168	15

Table 8A:

Two-sided pairwise t-tests for multiple comparisons of mean accuracy to the change conditions by Direction

	Mean	S.D.	Std. Err. Mean	95% C.I. of the Diff.		*t*	df	Significance	
				Lower	Upper			*p*, 2-sided	FDR, 2-sided
dSS - iSS	0.010	0.104	0.027	-0.048	0.068	0.367	14	0.719	0.719
dLS - iSL	0.054	0.119	0.031	-0.012	0.120	1.764	14	0.100	0.150
dLL - iLL	0.101	0.075	0.019	0.060	0.143	5.250	14	<0.001	<0.001

One-sided pairwise t-tests for multiple comparisons of mean accuracy to the change conditions by Size

	Mean	S.D.	Std. Err. Mean	95% C.I. of the Diff.		*t*	df	Significance	
				Lower	Upper			*p*, 1-sided	FDR, 1-sided
dSS - dLS	-0.042	0.085	0.022	-0.089	0.005	-1.919	14	0.038	0.046
dSS - dLL	0.262	0.118	0.030	0.197	0.327	8.626	14	<0.001	<0.001
iSS - iSL	0.002	0.086	0.022	-0.045	0.050	0.111	14	0.457	0.457
iSS - iLL	0.252	0.123	0.032	0.184	0.320	7.930	14	<0.001	<0.001
dLS - dLL	0.353	0.121	0.031	0.286	0.420	11.290	14	<0.001	<0.001
iSL - iLL	0.351	0.130	0.034	0.279	0.423	10.453	14	<0.001	<0.001

Table 9A:
Descriptive statistics of mean N1 amplitudes by Numerical Change condition.

	Mean	Std. Deviation	N
dSS	-3.682	1.490	15
dLS	-5.541	1.656	15
dLL	-6.563	1.495	15
iSS	-6.001	1.435	15
iSL	-6.735	1.812	15
iLL	-6.389	1.583	15

Table 10A:
Two-sided pairwise t-tests for multiple comparisons of mean N1 Amplitude to the change conditions by Direction

	Mean	S.D.	Std. Err. Mean	95% C.I. of the Diff.		*t*	df	Significance	
				Lower	Upper			*p*, 2-sided	FDR, 2-sided
dSS - iSS	2.324	1.246	0.322	1.634	3.014	7.222	14	<0.001	<0.001
dLS - iSL	1.194	1.061	0.274	0.607	1.781	4.360	14	<0.001	<0.001
dLL - iLL	-0.174	1.170	0.302	-0.822	0.474	-0.576	14	0.574	0.574

One-sided pairwise t-tests for multiple comparisons of mean N1 Amplitude to the change conditions by Sizes

	Mean	S.D.	Std. Err. Mean	95% C.I. of the Diff.		*t*	df	Significance	
				Lower	Upper			*p*, 1-sided	FDR, 1-sided
dSS - dLS	1.859	0.723	0.187	1.458	2.259	9.956	14	<0.001	<0.001
dSS - dLL	2.881	1.456	0.376	2.075	3.687	7.665	14	<0.001	<0.001
iSS - iSL	0.729	1.036	0.267	0.156	1.303	2.727	14	0.008	0.016
iSS - iLL	0.557	1.044	0.270	-0.021	1.135	2.067	14	0.029	0.044
dLS - dLL	0.383	1.495	0.386	-0.445	1.211	0.992	14	0.169	0.169
iSL - iLL	-0.346	1.285	0.332	-1.058	0.366	-1.043	14	0.157	0.188

Table 11A:
Descriptive statistics of mean N1 latencies by Numerical Change condition.

	Mean	Std. Deviation	N
dSS	167.067	9.278	15
dLS	165.384	11.406	15
dLL	162.049	13.867	15
iSS	157.467	13.603	15
iSL	160.428	12.097	15
iLL	160.684	15.849	15

Table 12A:
Post-hoc paired samples t-tests for multiple comparisons of mean N1 latencies to the change conditions by Direction

Two-sided pairwise t-tests for multiple comparisons of mean N1 Latency to the change conditions by Direction

	Mean	S.D.	Std. Err. Mean	95% C.I. of the Diff.		*t*	df	Significance	
				Lower	Upper			*p*, 2-sided	FDR, 2-sided
dSS - iSS	9.600	11.455	2.958	3.256	15.944	3.246	14	0.006	0.009
dLS - iSL	4.956	5.729	1.479	1.783	8.128	3.350	14	0.005	0.015
dLL - iLL	1.364	10.631	2.745	-4.523	7.252	0.497	14	0.627	0.627

Table 13A:
Descriptive statistics of mean P3b amplitudes by Numerical Change condition.

	Mean	Std. Deviation	N
dSS	6.926	3.400	15
dLS	6.214	2.940	15
dLL	4.337	2.980	15
iSS	6.040	2.834	15
iSL	5.303	2.913	15
iLL	4.911	2.995	15

Table 14A:

One-sided pairwise t-tests for multiple comparisons of mean P3b amplitude to the change conditions by Size

	Mean	S.D.	Std. Err. Mean	95% C.I. of the Diff.		t	df	Significance	
				Lower	Upper			*p*, 1-sided	FDR, 1-sided
dSS - dLS	0.711	1.909	0.493	-0.346	1.769	1.444	14	0.085	0.128
dSS - dLL	2.588	2.58	0.666	1.16	4.017	3.886	14	0.001	0.006
iSS - iSL	0.737	1.772	0.457	-0.244	1.718	1.611	14	0.065	0.130
iSS - iLL	1.129	3.045	0.786	-0.557	2.815	1.437	14	0.086	0.103
dLS - dLL	1.877	2.873	0.742	0.286	3.468	2.530	14	0.012	0.036
iSL - iLL	0.392	2.811	0.726	-1.164	1.949	0.541	14	0.299	0.299

Table 15A:

Descriptive statistics of mean P3b latencies by Numerical Change condition.

	Mean	Std. Deviation	N
dSS	487.733	22.874	15
dLS	463.778	14.200	15
dLL	487.067	17.030	15
iSS	471.733	20.071	15
iSL	475.884	10.499	15
iLL	498.533	24.383	15

Table 16A:
One-sided pairwise t-tests for multiple comparisons of mean P3b latencies to the change conditions by Size

	Mean	S.D.	Std. Err. Mean	95% C.I. of the Diff.		t	df	Significance	
				Lower	Upper			p, 1-sided	FDR, 1-sided
dSS - dLS	23.956	24.457	6.315	10.412	37.499	3.794	14	<0.001	<0.001
dSS - dLL	0.667	29.006	7.489	-15.396	16.730	0.089	14	0.465	0.465
iSS - iSL	-4.151	18.981	4.901	-14.662	6.360	-0.847	14	0.206	0.247
iSS - iLL	-15.333	20.435	5.276	-26.650	-4.017	-2.906	14	0.006	0.009
dLS - dLL	-26.800	32.279	8.334	-44.675	-8.925	-3.216	14	0.003	0.006
iSL - iLL	-22.649	23.480	6.063	-35.652	-9.646	-3.736	14	0.001	0.003

Appendix C

Figure 1A:
Topographic plot of ERP waveforms towards Cardinalities of 1~6; Recorded from Left and Right POT (parietal-occipital-temporal) area vs. Left and Right DLPFC (dorsolateral prefrontal cortex)

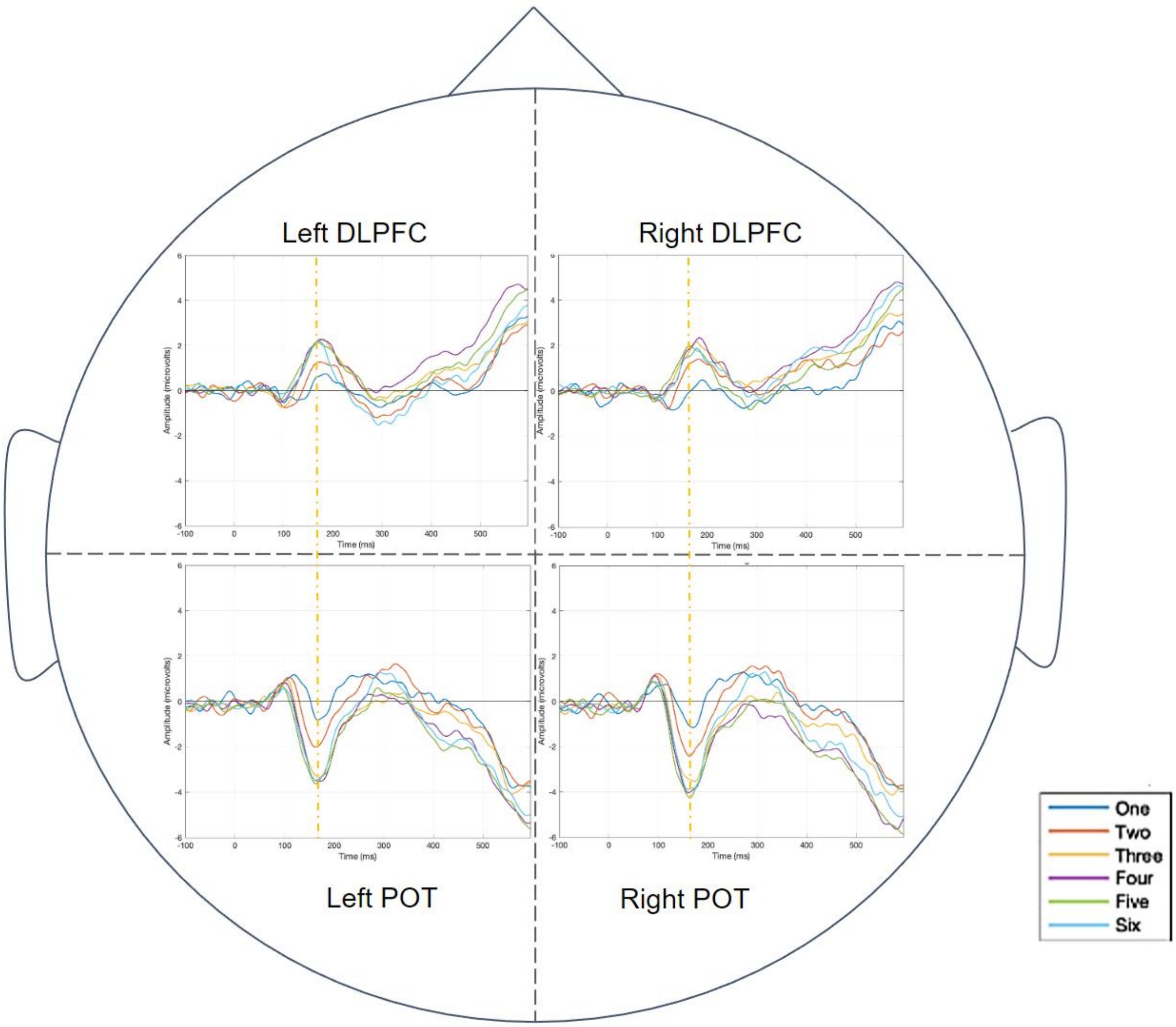

Figure 2A:
Topographic plot of ERP waveforms towards Change (Direction and Size); Recorded from Left and Right POT (parietal-occipital-temporal) area vs. Left and Right DLPFC (dorsolateral prefrontal cortex)