Performance Evaluation of Spatial Modulation Patterns in Compressive Sensing Terahertz Imaging

Adolphe Ndagijimana*, Miguel Heredia Conde†, and Iñigo Ederra Urzainqui*
*Antennas Group, Public University of Navarre, Pamplona, Spain
†Center for Sensor Systems (ZESS), University of Siegen, Siegen, Germany
adolphe.ndagijimana@unavarra.es, heredia@zess.uni-siegen.de, and inigo.ederra@unavarra.es

***Abstract*—Terahertz (THz) imaging traditionally uses pixel-to-pixel mechanical raster scanning, which is slow and limits the resolution and potential applications. Compressive Sensing Terahertz (CS-THz) imaging has the potential to solve these challenges by reducing the number of required measurements. However, there is an existing research gap between the current CS-THz implementations, which often use random binary masks without further consideration, and specialized Compressive Sensing works, which focus on RIP and coherence reduction but ignore the physics underlying THz wave propagation such as the effect of diffraction on masks patterns. This paper discusses and evaluates the use of low-coherence sensing matrices as mask patterns for CS-THz imaging. Although not previously used in THz imaging, Best Antipodal Spherical Code masks show the best image reconstruction performance among the considered alternatives. We also demonstrate the feasibility of phase-only modulation.**

***Index Terms*—THz imaging, Optical Switch, Compressive Sensing, Spatial Light Modulation**

I. Introduction

THz imaging is a promising technology with potential applications in security [1], food industry/agriculture [2], etc. Traditionally, THz imaging uses mechanical pixel-by-pixel raster scanning in many applications due to a lack of adequate, cost-effective array detectors. The mechanical pixel-by-pixel raster scanning is usually slow. Hence there is a trade-off between the resolutions and the speed of the imaging system.

Compressive sensing (CS) allows sampling signals at a lower rate than dictated by the Nyquist theorem. The most common implementation of CS in the THz imaging system is the single-pixel camera, which involves sampling with different spatial modulation patterns. Previous implementations involve using a pre-patterned metal board, blocking Terahertz radiation [3]; this still used mechanical/manual switching, which is slow and limited to binary amplitude modulation. Photoexcitation of semiconductor substrates allows non-mechanical pattern switching with the projection of visible patterns on a proper semiconductor [4], [5].

Electromagnetic waves are subject to diffraction when hitting an obstacle or propagating through an aperture of the size in the range or less than its wavelength. These effects are negligible in the optical domain but not in THz as the wavelength and pixel pitch are of the same order of magnitude [6]. Hence, due to the coherence and diffraction effects, the resulting wavefront interacting with the scene exhibits a spatial modulation that may largely differ from the one intended with the commanded mask, affecting the quality of the imaging system, achievable resolution, and the minimum number of measurements required, among others [7].

These challenges make the study of sensing schemes and mask implementation crucial for achieving a good signal-to-noise ratio in the CS-THz imaging system. This research discusses the realization of well-known sensing matrices in two sensing scheme designs and presents quality image reconstruction using CS. We introduce using Best Antipodal Spherical Code (BASC) and Low-Density Parity-Check (LDPC) codes sensing matrices in THz imaging, which outperform the alternatives used in prior works. For the random Gaussian sensing matrix, The proposed phase-only setup improves the existing amplitude modulation setup.

II. Background review and related work

A. THz Spatial Light Modulation

THz spatial light modulation is achieved by translating visible-light spatial modulation into the modulation of a THz beam when incident to an appropriate semiconductor. Photoexcitation of the semiconductor substrates below the bandgap wavelength increases excess carrier density, modifying the refraction index and coefficient of extinction [4].

Different factors affect the THz modulation. The comparison and study of essential factors for intrinsic Si, Ge, and GaAs are presented in [5].

THz spatial light modulation using a photo-injected zone plate allows reflective THz spatial light modulation by placing a THz wave reflector behind. This configuration allows both phase-only modulation and amplitude modulation [4].

B. Compressive sensing

CS involves sampling and reconstructing signals at a lower rate than the Shannon-Nyquist sampling rate. It models signal acquisition as a linear system, given a signal $x \in \mathbb{C}^n$, and sensing matrix $A \in \mathbb{C}^{m \times n}$ to yield $y \in \mathbb{C}^m$. CS allows sampling at $m << n$.

Successful signal reconstruction in CS relies on the sparsity of x and the isometric behavior of the sensing matrices for differences of s-sparse signals [8]. The restrictive isometric principle is hard to prove as computing δ_{2s} is NP hard [9]. A more probabilistic way is expressed in terms of coherence.

When expressed in a suitably basis, many natural signals are sparse or approximate to a sparse representation. This opens up

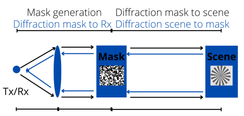


Fig. 1: Reflective CS THz imaging setup.

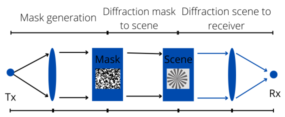


Fig. 2: Refractive CS THz imaging setup.

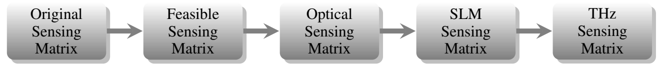


Fig. 3: Differently from optical CS imaging, in CS THz imaging, the commanded masks undergo a series of physics-related transforms before interacting with the scene.

multiple applications, given a proper sensing scheme ensuring low coherence. [9] presents a summary of the different relevant sensing matrices.

C. THz imaging with compressive sensing

Using the single-pixel idea, [3] demonstrated that it is possible to perform THz imaging using fewer measurements than those suggested by the Nyquist rate using spatial modulation of THz radiation with random binary masks obtained through metallic PCB prints and a single detector. However, the use of metallic print still required a mechanical switch. Optical switch-based THz Spatial light Modulator(SLM) allows spatial light modulation and eliminates the need for mechanical scans [10], [11]. This modulation technique allows high-speed signal acquisition and image reconstruction using CS [12], [13].

CS THz imaging still has multiple challenges, such as the diffraction effect, which leads to a difference between the optically projected masks and the commanded CS imaging sensing masks. In [14] with extra precautions for limiting the diffraction effect; Bernouilli, Hadamard, DCT, and random Gaussian masks were investigated. However, the projection of these masks modifies their geometry.

Provided that most THz detectors are intensity-only measurements, reconstructing the scene from phase-blind measurements is the fundamental inverse problem in THz imaging, described with (1), which is different from classic CS. This project uses the Phase Retrieval via Wirtinger Flow described in [15], one of the most widely-used methods for solving the problem at hand and yielding state-of-art results.

$$y_i = |\langle \vec{a_i}, vec(x) \rangle|^2 \tag{1}$$

III. THz IMAGING SENSING MODELS

A well-designed CS-THz imaging system sensing scheme is essential for obtaining high-resolution quality images. Proper design allows uncoherent measurements yielding good signal reconstruction. In this work, we discuss a refractive design, described in Fig. 2, and a reflective design, shown in Fig. 1.

For the refractive design, the transmitter, T_x, radiates THz waves, which propagate to the lens and produce an approximately collimated beam that propagates toward the SLM, modeled as field E simulated using the equations in [16]. The SLM modulates the THz radiation yielding $M^{\mathrm{THz}} = M^{\mathrm{SLM}} \circ E$ with $\circ$ denoting element-wise multiplication. These masks diffract toward the scene according to linear model D_{MS}, interact with the scene, and then diffract toward the receiver through a condensing lens with D_{SR}. The diffraction is modeled using [17]. Each propagation with intensity-only measurement is described in (2), formulated as (1).

$$y_i = |\langle \overline{diag(D_{\mathrm{MS}} vec(M_i^{\mathrm{THz}})} D_{\mathrm{SR}}^H \vec{1}, vec(x) \rangle|^2 \tag{2}$$

For the reflective design, after interacting with the scene, the radiation diffracts to the SLM with D_{SM}. The SLM modulates the diffracted radiation, which further diffracts toward the receiver for collection. Each measurement is then described by (3), formulated as (1).

$$y_i = |\langle D_{\mathrm{MR}}^H \overline{diag(M^{\mathrm{SLM}})} D_{\mathrm{SM}}^H \overline{diag(D_{\mathrm{MS}} vec(M^{\mathrm{THz}}))} \vec{1}, vec(x) \rangle|^2 \tag{3}$$

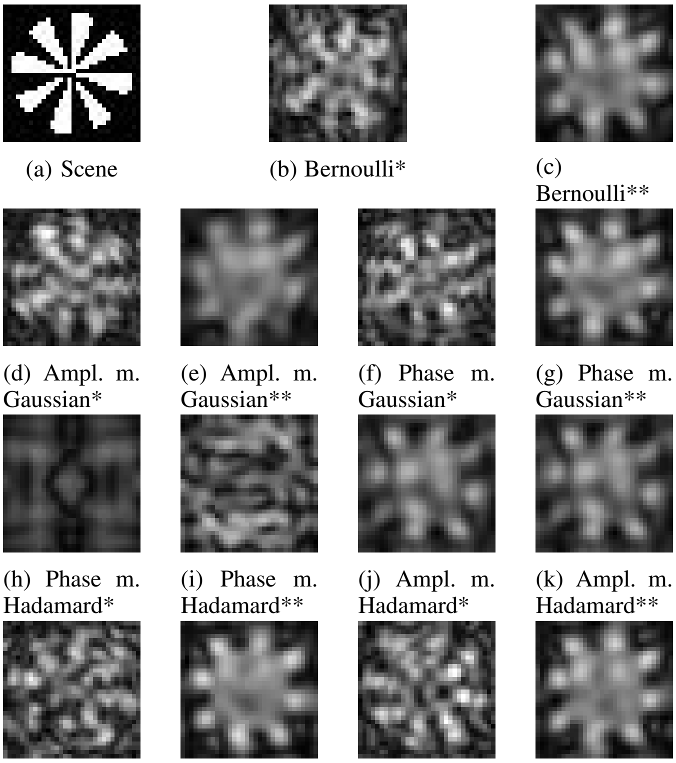
Fig. 3 shows how a standard sensing matrix is transformed during THz imaging to give rise to the THz sensing matrix, $A \left[\vec{a_i}^H \right]_{i=1}^{m}$, where we used $\vec{a_i}^H$ to denote the transposed vectors of $\vec{a_i}$ from (2) and (3) depending on the design. The hardware implementation of the SLM is limited to 256 discrete modulation steps, the feasible matrix, which is also equivalent to the optical sensing matrix in our simulation. The optical masks allows the THz modulation to yield the SLM masks, then undergo diffraction and give rise to the sensing matrix. Unlike previous studies, we also investigate phase-only modulation, which is possible with the technique in [4].

IV. RESULTS AND DISCUSSIONS

The coherence between the columns of a sensing matrix is an essential factor for obtaining high-quality image reconstruction from a few measurements. Most well-known CS matrices are designed to minimize the maximum coherence. Due to THz nature, the resulting THz sensing matrix is different from the original standard sensing matrices. Thus, the coherence between the THz sensing matrix columns differs from the original one.

We simulate a THz system of $0.35\,\mathrm{THz}$ with a resolution of 32×32 with 1×1mm resolution step for our target. We use a Siemens star as the binary target, meaning either reflect or not, as shown in Fig. 4a. We investigated the performance of well-known sensing matrices; Bernoulli, Gaussian, Hadamard, BASC, and LDPC codes. Where possible, we implemented phase-only and amplitude modulation. Below are the results.

All sensing matrices are affected by diffraction, affecting the coherence, especially between matrix columns corresponding to neighboring pixels. As the diffraction increases, the maximum coherence increases, medium diffraction sometimes dramatically boosts correlation between neighboring pixels, as a consequence of the mixing of neighboring pixels, the overall accuracy will also be lower. The Table I and Fig. 4 summarize the image reconstruction quality for $m = \frac{n}{8}$,this being the inflexion point of ratio $\frac{m}{n}$ vs. performance metrics. However, one crucial fact for very high diffraction is that coherence for



(a) Scene (b) Bernoulli* (c) Bernoulli** (d) Ampl. m. Gaussian* (e) Ampl. m. Gaussian** (f) Phase m. Gaussian* (g) Phase m. Gaussian** (h) Phase m. Hadamard* (i) Phase m. Hadamard** (j) Ampl. m. Hadamard* (k) Ampl. m. Hadamard** (l) LDPC* (m) LDPC** (n) BSAC* (o) BSAC**

Fig. 4: CS THz imaging results with $m = \frac{n}{8}$ for masks of different nature; with * for the refractive design and ** for the reflective design; and ampl. for the amplitude modulation matrix and m. for modulated.

TABLE I: Performance of different sensing matrices at $m = \frac{n}{8}$

Sensing matrix	Refractive design			Reflective design		
	RMSE	SSIM	PSNR	RMSE	SSIM	PSNR
Hadamard	0.44*	0.07*	6.94*	0.47	0.01	6.46
Ampl. mod. Hadamard	0.38*	0.17*	8.35*	0.42	0.11	7.52
Bernoulli	0.36	0.21	8. 78*	0.36	0.24*	8.74
Ampl. mod. Gaussian	0.38	0.14	8.34	0.35*	0.26*	9.03*
Phase mod. Gaussian	0.36*	0.21	8.82*	0.37	0.22*	8.60
BASC	**0.35**	**0.24**	**8.97**	**0.35***	**0.29***	**9.05***
LDPC codes	0.36	0.22	8.81	0.36	0.26*	8.84*

Ampl. stands for amplitude and mod. for modulated.
Bold describes the best overall performance for a given metric.
* indicates the best performance for a given sensing matrix.

any subset of columns is generally high, making the inversion of the problem ill-posed.

For *Randon Bernoulli sensing matrix* with $p = 0.5$, the reconstructed image is distorted but recognizable as shown in Fig. 4b and 4c. As the probability p of allowing transmission increases, the coherence increases, resulting in more distorted pixels. It seems thus recommendable to use lower p; this affects the intensity detected at the sensor as most THz radiation would be blocked — creating a trade-off between p and THz transmitter power.

For *Random Gaussian sensing matrix* implementation, shown in Fig. 4d, 4e, 4f, and 4g, phase-only implementation is also possible. In the refractive design, the phase-modulation method performs better than amplitude-modulation and is also practically as good as for the Bernoulli sensing matrix.

Most previous implementations of *Hadamard sensing matrix* in single-pixel THz imaging involve converting into amplitude modulation by using 0 instead of -1, which changes the coherence of the sensing matrix. Phase-only modulation offers proper implementation, but since we are using intensity-only measurements, the performance expected from low-coherence behavior was not obtained, as shown in Fig. 4i. On the other hand, the intensity modulation presented better performance but not as good as the previously presented sensing matrices, as shown in Fig. 4k.

LDPC codes sensing matrix [18] also showed a slight improvement over the previous sensing matrices, as seen in Fig. 4l, 4m, and Table I.

BASC sensing matrix [19] construction yields minimal coherence between code words, but they had to be adapted the feasible hardware implementation. They were the best performing matrices with observed improvement as shown in Table I and Fig. 4.

Phase modulation in some cases is better than *amplitude modulation*, as can be seen in the performance of the random Gaussian sensing matrix. A significant advantage of phase-only modulation is less THz power requirement due to no beam blocking at the modulation.

For amplitude-based sensing matrix, the *reflective sensing system design* outperforms the *refractive sensing system design*, since the mask is reinforced when reflected, as shown on Table I and Fig. 4; and but it requires much more power.

V. Conclusion and Outlook

Using an intensity-only measurement setup, we demonstrated that appropriate selection of the spatial modulation patterns contributes to the quality of CS THz imaging due to their initial coherence and the diffraction effects, which can further increase the latter. Using a Siemens star as a target and Wirtinger Flow as a reconstruction method, a set of low-coherence sensing matrices has been evaluated for their ability to produce an acceptable THz image with only one eighth number of pixels as the number of measurements. Phase-only modulation has been demonstrated and, as it is non-blocking, requires less THz illumination power. BASC-based sensing matrices exhibit the best performance as their design minimizes inter-column coherence. Simulation shows results that are backed by both electromagnetic and CS theory. The next step is the hardware implementation of the system to validate the performance of the different alternatives experimentally.

Acknowledgment

This project has received funding from the European Union's Horizon 2020 research and innovation program under the Marie Skłodowska-Curie grant agreement No 860370.

REFERENCES

[1] S. Augustin and H.-W. Hübers, "Phase-sensitive passive terahertz imaging at 5-m stand-off distance," *IEEE Transactions on Terahertz Science and Technology*, vol. 4, no. 4, pp. 418–424, 2014.

[2] D. Etayo, J. C. Iriarte, I. Palacios, I. Ederra, and R. Gonzalo, "Active thz imaging system to measure water content evolution in leaves," in *Millimetre Wave and Terahertz Sensors and Technology IV*, vol. 8188. SPIE, 2011, pp. 81–86.

[3] W. L. Chan, K. Charan, D. Takhar, K. F. Kelly, R. G. Baraniuk, and D. M. Mittleman, "A single-pixel terahertz imaging system based on compressed sensing," *Applied Physics Letters*, vol. 93, no. 12, p. 121105, 2008.

[4] T. F. Gallacher, R. Sondena, D. A. Robertson, and G. M. Smith, "Optical modulation of millimeter-wave beams using a semiconductor substrate," *IEEE transactions on microwave theory and techniques*, vol. 60, no. 7, pp. 2301–2309, 2012.

[5] A. Kannegulla, M. I. B. Shams, L. Liu, and L.-J. Cheng, "Photo-induced spatial modulation of thz waves: opportunities and limitations," *Optics express*, vol. 23, no. 25, pp. 32 098–32 112, 2015.

[6] M. Reiche and P. Jung, "Deepinit phase retrieval," *arXiv preprint arXiv:2007.08214*, 2020.

[7] M. Burger, J. Föcke, L. Nickel, P. Jung, and S. Augustin, "Reconstruction methods in thz single-pixel imaging," in *Compressed Sensing and Its Applications*. Springer, 2019, pp. 263–290.

[8] E. J. Candès and M. B. Wakin, "An introduction to compressive sampling," *IEEE signal processing magazine*, vol. 25, no. 2, pp. 21–30, 2008.

[9] M. H. Conde, "Fundementals of compressive sensing," in *Compressive Sensing for the Photonic Mixer Device*. Springer, 2017, pp. 89–204.

[10] S. Augustin, J. Hieronymus, P. Jung, and H.-W. Hübers, "Compressed sensing in a fully non-mechanical 350 ghz imaging setting," *Journal of Infrared, Millimeter, and Terahertz Waves*, vol. 36, no. 5, pp. 496–512, 2015.

[11] J. Hieronymus, S. Augustin, and H.-W. Hübers, "Characterization of a thz slm and its application for improved high resolution thz imaging," in *2015 40th International Conference on Infrared, Millimeter, and Terahertz waves (IRMMW-THz)*. IEEE, 2015, pp. 1–2.

[12] S. Augustin, S. Frohmann, P. Jung, and H.-W. Hübers, "An optically controllable 0.35 thz single-pixel camera for millimeter resolution imaging," in *2017 42nd International Conference on Infrared, Millimeter, and Terahertz Waves (IRMMW-THz)*. IEEE, 2017, pp. 1–2.

[13] S. Augustin, P. Jung, S. Frohmann, and H.-W. Hübers, "Optically initiated spatial modulation of thz radiation at far-field distances using a compressed sensing protocol," in *2019 44th International Conference on Infrared, Millimeter, and Terahertz Waves (IRMMW-THz)*. IEEE, 2019, pp. 1–2.

[14] S. Augustin, S. Frohmann, P. Jung, and H.-W. Hübers, "Mask responses for single-pixel terahertz imaging," *Scientific reports*, vol. 8, no. 1, pp. 1–7, 2018.

[15] E. J. Candes, X. Li, and M. Soltanolkotabi, "Phase retrieval via wirtinger flow: Theory and algorithms," *IEEE Transactions on Information Theory*, vol. 61, no. 4, pp. 1985–2007, 2015.

[16] P. F. Goldsmith *et al.*, *Quasioptical systems*. Chapman & Hall New York, NY, USA, 1998.

[17] V. Katkovnik, A. Migukin, and J. Astola, "Backward discrete wave field propagation modeling as an inverse problem: toward perfect reconstruction of wave field distributions," *Applied optics*, vol. 48, no. 18, pp. 3407–3423, 2009.

[18] A. G. Dimakis, R. Smarandache, and P. O. Vontobel, "Ldpc codes for compressed sensing," *IEEE Transactions on Information Theory*, vol. 58, no. 5, pp. 3093–3114, 2012.

[19] H. Zörlein, D. E. Lazich, and M. Bossert, "On the noise-resilience of omp with basc-based low coherence sensing matrices," in *Proc. 10th Int. Conf. Sampling Theory Appl.(SampTA)*, 2013, pp. 468–471.