CONVERSION OF INFLUENCE COEFFICIENTS AMONG STATIC-COUPLE AND MULTIPLANE FORMATS ON TWO-PLANE BALANCING

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ABSTRACT

The influence coefficient (IC) method is typically used in balancing, especially on-site. For a two-bearing machine, there are basically two approaches to apply this method. The first one is to treat it as a multiplane balance problem involving a 2x2 matrix of complex ICs. In this approach, two direct ICs along with two cross-effect ICs are generated so that correction weights at one or two balance planes can be determined. The second one is to apply a static pare (in-phase) and/or couple pair (180 degrees out-of-phase) weights to reduce the vibration. The latter approach has been used extensively in the field, especially on steam turbine and generator rotors.

Dependent on vibration mode shapes and combinations as well as balance plane accessibility, sometimes applying static or couple pair weights can be a wise choice; other times weights at one or two end planes are needed. There are totally 4 possible sets of IC data due to weights at plane 1, plane 2, static pair, and couple pair. Influence coefficient data would typically be obtained by applying trial weights followed by trial weight runs. It is found, however, that these IC data can be converted easily without trial weight runs once any two of 4 sets are known. The above findings and conversion equations have been obtained analytically and verified by experimental results. This paper presents all available IC conversion equations together for the first time. Experimental verification is also provided by a rotor kit to demonstrate their accuracy.

Four real cases are presented to demonstrate their applications. The first case is to show the necessity of applying a couple pair weight by obtaining its IC converted from plane 1 and plane 2 ICs. The second case is to show a one-trial weight run method on symmetric

rotor. The third case is to show the necessity of applying individual weights each at plane 1 and plane 2 by obtaining their ICs converted from static and couple pair ICs. And the fourth case is to apply individual weights each at plane 1 and plane 2 after having one trial weight run only, with previously known static pair IC data.

INTRODUCTION

High consistent vibration in rotating machines is usually caused by mass unbalance. The life span of the machine could be reduced as a result of excessive stresses on the rotor, bearings and casing if vibration level is not reduced via balancing. The source of unbalance includes assembly variation and material non-homogeneity. This can happen on new machines or machines after rotor repair or overhaul. Though rotors are often low-speed or sometimes high-speed balanced by manufacturers or workshops before they are installed for service, unbalance may still occur afterwards due to various reasons such as deposits on or erosion and shifting of rotating parts as well as thermal effects. Hence, balancing is often required in the field and has been of great interest to rotor dynamic researchers as well as practicing engineers.

A rotor such as that on a steam turbine or generator is often supported by two bearings with two balance planes available at both ends. Vibrations are monitored by a pair of proximity probes at each bearing. This often requires two-plane balancing of the rotor. Everett (1987), and Foiles and Bently (1988) discussed two-plane balancing with amplitude or phase only, which would often require more trial weight runs in the field. The influence coefficient method is typically used in the field for trim balancing with two approaches. The first method is to treat it as a multiple-plane balancing problem involving 2x2 matrix of complex influence coefficients, as indicated by Thearle (1934). In this method, two direct influence coefficients along with two cross-effect influence coefficients are generated so that correction weights at two balance planes can be determined. The second method is to treat it as two single-plane balance problems using static (in-phase) and couple (180 degrees out-of-phase) components, respectively, as shown by Wowk (1995) and Eisenmann (1997).

Note that the static and couple components are referred to as in-phase and 180 degrees out-of-phase components, respectively. The static component is usually due to first and/or third modes that are very symmetric to the rotor mass center while the couple component is typically due the second mode that is nearly anti-symmetric. Static weight is defined as a weight vector at the first balance plane, always accompanied by the same weight vector at the second balance plane. And couple weight is defined as a weight vector at the first balance plane, always accompanied by the opposite or 180 degrees out-of-phase weight vector at the second balance plane. Therefore, the current “static” and “couple” vibration or weight vectors are really referred to as “in-phase” and “180 degrees out-of-phase”, respectively. “Static” and couple” terms have their origin from rigid rotor balancing and have been extended to flexible rotors for balancing the first, the second, and/or even the third modes. These terms are well defined in the standards of balancing and have been widely accepted and used in the field. Therefore, these terms are used throughout this paper so that practicing engineers can apply the solutions from the paper with the above definitions.

Yu (2009) indicated the relationship of influence coefficients between static-couple and multiplane methods on two-plane balancing. Thus, static or couple influence coefficients due to static or couple weights can be obtained directly without having to place static or couple trial weights if influence coefficients used in the multi-plane approach are known. From static and couple influence data as well as cross effects, influence data for the multi-plane method can be obtained directly as well without having to place any trial weights at either plane. Yu (2012) applied the relationship into two-plane balancing of symmetric rotors. Yu (2014 and 2015) used these conversion equations to deal with two-plane rotor balancing problems in the field effectively.

Sometimes one may have influence coefficients due to one end weight only, but not both ends by using the first method, plus those due to static or couple weight only, but not both by using the second method. It has been found by Yu (2020) that obtaining or knowing half of influence coefficients from the first method, plus half of those from the second method, can yield the other half of influence coefficients for both methods. In other words, all influence coefficients in different formats can be obtained without a further trial run once one has half of the influence coefficients from each of two methods.

The current paper will summarize all possible conversion equations, followed by experimental verification. Then three distinctive cases will be demonstrated to show their effectiveness in IC evaluation and balancing. Moreover, the corresponding spreadsheets that have been developed recently to easily convert ICs between these different expressions, will be presented here.

THEORY

Note that each influence coefficient is a complex number that contains not only sensitivity of synchronous 1X vibration amplitude from a particular probe versus balance weight but also 1X vibration phase lag relative to the weight. When influence data is concerned, a common reference of orientation should be used for both weight and vibration vectors. Since phase lag of the synchronous 1X vibration vector is always referenced to the probe against shaft rotation from vibration test equipment such as ADRE®, phase lag of the weight vector should also be referenced to the probe against shaft rotation in calculation. As a result, developed influence vectors such as static and couple influence coefficients would be meaningful by looking vibration phase lags relative to the weight vector.

Multiplane Balance Model

As shown in Figure 1, synchronous 1X vibration vectors are expressed as $\vec{A}_1$ and $\vec{A}_2$ measured by probes 1 and 2, respectively. Their orientations and are defined by phase lag relative to their probe orientation (Figure 1 shows the instance when Keyphasor® pulse occurs). Balance weights at weight planes 1 and 2 are expressed as $\vec{W}_1$ and $\vec{W}_2$ with their orientations and referenced to the probe orientation, respectively. Assuming the system is linear, changes in 1X vibration vectors due to weight placement can be given by

$$\begin{bmatrix} \vec{h}_{11} & \vec{h}_{12} \\ \vec{h}_{21} & \vec{h}_{22} \end{bmatrix} \begin{Bmatrix} \vec{W}_1^{(1)} \\ \vec{W}_2^{(1)} \end{Bmatrix} + \begin{Bmatrix} \vec{A}_1^{(0)} \\ \vec{A}_2^{(0)} \end{Bmatrix} = \begin{Bmatrix} \vec{A}_1^{(1)} \\ \vec{A}_2^{(1)} \end{Bmatrix} \tag{1}$$

where $\vec{h}_{11}$, $\vec{h}_{12}$, $\vec{h}_{21}$, and $\vec{h}_{22}$ form the 2x2 influence coefficient matrix. Superscripts "(0)" and "(1)" represent status without and with weights, respectively. Typically, the four ICs, through two trial runs, can be computed as follows:

$$\begin{bmatrix} \vec{h}_{11} & \vec{h}_{12} \\ \vec{h}_{21} & \vec{h}_{22} \end{bmatrix} = \begin{bmatrix} \vec{A}_1^{(1)} - \vec{A}_1^{(0)} & \vec{A}_1^{(2)} - \vec{A}_1^{(0)} \\ \vec{A}_2^{(1)} - \vec{A}_2^{(0)} & \vec{A}_2^{(2)} - \vec{A}_2^{(0)} \end{bmatrix} \begin{bmatrix} \vec{W}_1^{(1)} & \vec{W}_1^{(2)} \\ \vec{W}_2^{(1)} & \vec{W}_2^{(2)} \end{bmatrix}^{-1} \tag{2}$$

where superscript "(0)" represents status without weights, and superscript "(1)" and "(2)" denote status with the first and second sets of weights. Note that the two sets of weights must be chosen in a way that the weight matrix is not singular.

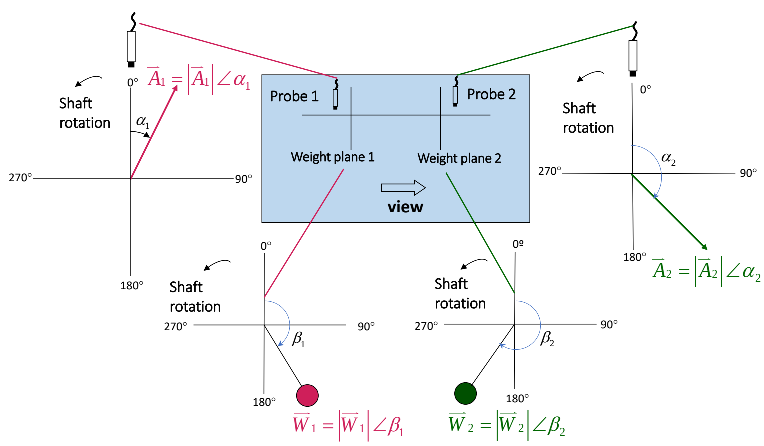


Figure 1 Diagram of vibration and weight vectors when Keyphasor® pulse occurs

Static-Couple Balance Model

In the static-couple method, as shown in Figure 2, vibration vectors at both ends of the shaft are expressed as combinations of static and couple components as follows:

$$\vec{S} = \frac{\vec{A}_1 + \vec{A}_2}{2} \tag{3a}$$

$$\vec{C} = \frac{\vec{A}_1 - \vec{A}_2}{2} \tag{3b}$$

where $\vec{S}$ and $\vec{C}$ are defined as static and couple components, respectively. Similarly, static weight $\vec{W}_S$ is defined as being in-phase each at two ends with the same amount while couple weight $\vec{W}_C$ is defined as the weight vector at plane 1 accompanied with the $-\vec{W}_C$, 180 degrees out-of-phase with the same amount. The static-couple balance method fits to both rigid and flexible rotors.

The following static-couple balance model was introduced by Yu (2009):

$$\begin{bmatrix} \vec{H}_{SS} & \vec{H}_{SC} \\ \vec{H}_{CS} & \vec{H}_{CC} \end{bmatrix} \begin{Bmatrix} \vec{W}_S^{(1)} \\ \vec{W}_C^{(1)} \end{Bmatrix} + \begin{Bmatrix} \vec{S}^{(0)} \\ \vec{C}^{(0)} \end{Bmatrix} = \begin{Bmatrix} \vec{S}^{(1)} \\ \vec{C}^{(1)} \end{Bmatrix} \tag{4}$$

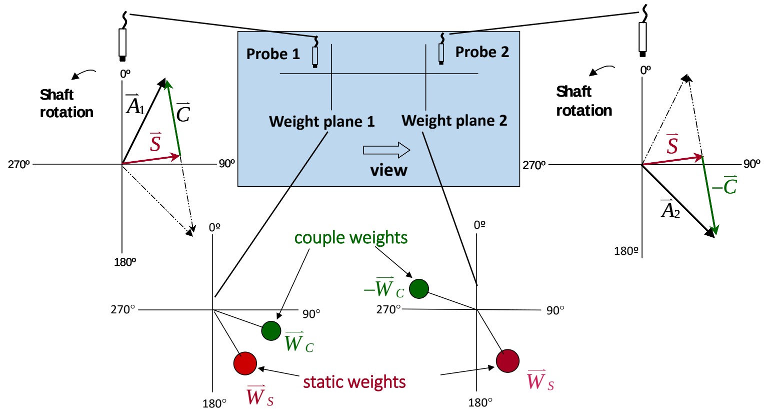


Figure 2 Diagram of static/couple vibration and weight vectors when Keyphasor pulse occurs

where superscripts "(0)" and "(1)" represent status without and with weights. Having vibration data before and after a static pair weight placement $\vec{W}_S$ (without a couple pair weight) yields:

$$\vec{H}_{SS} = \frac{\Delta\vec{S}_S}{\vec{W}_S} \tag{5}$$

and

$$\vec{H}_{CS} = \frac{\Delta\vec{C}_S}{\vec{W}_S} \tag{6}$$

where

$\Delta\vec{S}_S$ = static vibration component with static pair weight - static vibration component without static pair weight

$\Delta\vec{C}_S$ = couple vibration component with static pair weight - couple vibration component without static pair weight

Similarly, having vibration data before and after a couple weight placement $\vec{W}_C$ (without a static pair weight) yields:

$$\vec{H}_{CC} = \frac{\Delta\vec{C}_C}{\vec{W}_C} \tag{7}$$

and

$$\vec{H}_{SC} = \frac{\Delta\vec{S}_C}{\vec{W}_C} \tag{8}$$

where

$\Delta\vec{C}_C$ = couple vibration component with couple pair weight - couple vibration component without couple pair weight

$\Delta\vec{S}_C$ = static vibration component with couple pair weight - static vibration component without couple pair weight

Equations (5) and (7) have been widely used to compute the effect of static pair weight to the static component, and the effect of couple pair weight to the couple component, respectively. However, the cross-effect of static pair weight to the couple component or couple pair weight to the static component has not been used and has often been assumed to be zero. In a real rotor where asymmetry exists due to rotor structure or coupling effects, the cross-effect could be significant. Equations (6) and (8) include these cross effects. After both static and couple balancing without considering the cross-effects, residual unbalance response could still be high in some cases. However, if these four influence coefficients are obtained, both the static and couple vibration components can be effectively reduced by applying appropriate static and couple pair weights. Thus synchronous vibration levels at plane 1 ($\vec{A}_1 = \vec{S} + \vec{C}$) and plane 2 ($\vec{A}_2 = \vec{S} - \vec{C}$) will be reduced accordingly.

When the static or the couple component appears to be significantly large only, static or couple pair weight alone can be used. An optimized static or couple pair weight solution can be obtained to include the cross-effect.

Sometimes one needs to know individual probe IC due to static or couple pair weight. Static pair weight ICs to probes near planes 1 and 2 can be given by

$$\vec{h}_{1,S} = \frac{\Delta\vec{A}_{1,S}}{\vec{W}_S} \tag{9}$$

and

$$\vec{h}_{2,S} = \frac{\Delta\vec{A}_{2,S}}{\vec{W}_S} \tag{10}$$

where

$\Delta\vec{A}_{1,S} = \vec{A}_1$ with static pair weight - $\vec{A}_1$ without static pair weight

$\Delta\vec{A}_{2,S} = \vec{A}_2$ with static pair weight - $\vec{A}_2$ without static pair weight

Similarly, couple pair weight ICs to probes near planes 1 and 2 can be given by

$$\vec{h}_{1,C} = \frac{\Delta\vec{A}_{1,C}}{\vec{W}_C} \tag{11}$$

and

$$\vec{h}_{2,C} = \frac{\Delta\vec{A}_{2,C}}{\vec{W}_C} \tag{12}$$

where

$\Delta\vec{A}_{1,C} = \vec{A}_1$ with couple pair weight - $\vec{A}_1$ without couple pair weight

$\Delta\vec{A}_{2,C} = \vec{A}_2$ with couple pair weight $-$ $\vec{A}_2$ without couple pair weight

Note that $\Delta\vec{S}_S = \frac{\Delta\vec{A}_{1,S} + \Delta\vec{A}_{2,S}}{2}$ and $\Delta\vec{C}_S = \frac{\Delta\vec{A}_{1,S} - \Delta\vec{A}_{2,S}}{2}$, based on Equation (3). Then, using the above Equations from (5) to (12) can yield IC expressions of static and couple vibration components in terms of ICs of individual probes 1 and 2, all due to static/couple pair weight

$$\vec{H}_{SS} = \frac{1}{2}\left(\vec{h}_{1,S} + \vec{h}_{2,S}\right) \tag{13a}$$

$$\vec{H}_{CS} = \frac{1}{2}\left(\vec{h}_{1,S} - \vec{h}_{2,S}\right) \tag{13b}$$

$$\vec{H}_{CC} = \frac{1}{2}\left(\vec{h}_{1,C} - \vec{h}_{2,C}\right) \tag{13c}$$

$$\vec{H}_{SC} = \frac{1}{2}\left(\vec{h}_{1,C} + \vec{h}_{2,C}\right) \tag{13d}$$

and vice vasa

$$\vec{h}_{1,S} = \vec{H}_{SS} + \vec{H}_{CS} \tag{14a}$$

$$\vec{h}_{2,S} = \vec{H}_{SS} - \vec{H}_{CS} \tag{14b}$$

$$\vec{h}_{1,C} = \vec{H}_{SC} + \vec{H}_{CC} \tag{14c}$$

$$\vec{h}_{2,C} = \vec{H}_{SC} - \vec{H}_{CC} \tag{14d}$$

These complex number conversion calculations can be implemented on an excel spreadsheet.

Conversion Equations of ICs between Multiplane and Static-Couple Balance Models

In the field, the number of weights or amount of weights are limited at balance planes. In this case, even if a 2x2 IC matrix with multiplane balance model is available that may lead to placement of large amount of weights at two end planes, one would prefer to use less amount of static or couple pair weight only to reduce vibration to acceptable levels. Whether to use static or couple pair weight would depend on which vibration component is dominant and which weight placement is more efficient, and having sensitivity of static and couple ICs would help to make a better decision. Obtaining ICs for static-couple model, an engineer would be able to see how the rotor is running before, after, or close to the translational, pivotal, or other bending modes based on phase lag angle of static and couple ICs. The above-mentioned questions can be answered by conversion of ICs from multiplane to static-couple model.

On the other hand, an engineer would also need to know ICs expressed in terms of multiplane model from known static and couple ICs in some cases. Sometimes only one end balance plane can be used, due to unavailable empty holes or slot section for weight placement or difficult access on the other end balance plane. In thermal bow/rub situations, calculating additional equivalent unbalance caused by thermal bow using vibration excursion vectors compensated by the normal running condition vectors based on the multiplane balance model, would help to determine whether the thermal bow/rub location is close to balance plane 1 or 2. Using the 2x2 multiplane balance model, would also give the weight placement solution at planes 1 and 2 directly. All of these would require conversion of ICs from the static-couple to the multiplane balance model.

Yu (2009) established the conversion equations of ICs between these two balance models with detailed derivation step by step, which will not be repeated here again. Conversion equations from multiplane to static-couple balance model are given by

$$\vec{H}_{SS} = \frac{1}{2}\left(\vec{h}_{11} + \vec{h}_{22} + \vec{h}_{12} + \vec{h}_{21}\right) \tag{15a}$$

$$\vec{H}_{CS} = \frac{1}{2}\left(\vec{h}_{11} - \vec{h}_{22} + \vec{h}_{12} - \vec{h}_{21}\right) \tag{15b}$$

$$\vec{H}_{CC} = \frac{1}{2}\left(\vec{h}_{11} + \vec{h}_{22} - \vec{h}_{12} - \vec{h}_{21}\right) \tag{15c}$$

$$\vec{H}_{SC} = \frac{1}{2}\left(\vec{h}_{11} - \vec{h}_{22} - \vec{h}_{12} + \vec{h}_{21}\right) \tag{15d}$$

or alternatively, using Equation (14), can be expressed by

$$\vec{h}_{1,S} = \vec{h}_{11} + \vec{h}_{12} \tag{16a}$$

$$\vec{h}_{2,S} = \vec{h}_{21} + \vec{h}_{22} \tag{16b}$$

$$\vec{h}_{1,C} = \vec{h}_{11} - \vec{h}_{12} \tag{16c}$$

$$\vec{h}_{2,C} = \vec{h}_{21} - \vec{h}_{22} \tag{16d}$$

while those from static-couple to multiplane balance model can be expressed as

$$\vec{h}_{11} = \frac{1}{2}\left(\vec{H}_{SS} + \vec{H}_{CC} + \vec{H}_{SC} + \vec{H}_{CS}\right) \tag{17a}$$

$$\vec{h}_{21} = \frac{1}{2}\left(\vec{H}_{SS} - \vec{H}_{CC} + \vec{H}_{SC} - \vec{H}_{CS}\right) \tag{17b}$$

$$\vec{h}_{12} = \frac{1}{2}\left(\vec{H}_{SS} - \vec{H}_{CC} - \vec{H}_{SC} + \vec{H}_{CS}\right) \tag{17c}$$

$$\vec{h}_{22} = \frac{1}{2}\left(\vec{H}_{SS} + \vec{H}_{CC} - \vec{H}_{SC} - \vec{H}_{CS}\right) \tag{17d}$$

or alternatively, using Equation (14), can be expressed as

$$\vec{h}_{11} = \frac{1}{2}\left(\vec{h}_{1,S} + \vec{h}_{1,C}\right) \tag{18a}$$

$$\vec{h}_{21} = \frac{1}{2}\left(\vec{h}_{2,S} + \vec{h}_{2,C}\right) \tag{18b}$$

$$\vec{h}_{12} = \frac{1}{2}\left(\vec{h}_{1,S} - \vec{h}_{1,C}\right) \tag{18c}$$

$$\vec{h}_{22} = \frac{1}{2}\left(\vec{h}_{2,S} - \vec{h}_{2,C}\right) \tag{18d}$$

Equation (15) or (16) can be utilized as conversion equations of ICs from the multiplane to the static-couple balance model, while Equation (17) or (18) can be utilized as conversion equations of ICs from the static-couple to the multiplane balance model. These complex number conversion calculations can be built up on an excel spreadsheet for quick evaluation.

Conversion Equations of ICs between Multiplane and Static-Couple Balance Models in the Case of Symmetric Rotor

Many rotors such as double-flow low pressure turbine and generator rotors are actually symmetrical in terms of their geometry (i.e., each longitudinal side of the rotor is identical) as well as two identical supporting bearings. Their rated running speed is often between first and second critical speeds or between second and third critical speeds. Thus, synchronous vibration is typically composed of both static (in-phase) and couple (180 degrees out-of-phase) components, as measured at the two bearings supporting the rotor. To obtain both static and couple ICs, an engineer would typically use one trial run with static pair weight to obtain static IC, and another trial run with couple pair weight to obtain couple IC. There is often one more run performed with corrected static pair weight after a trial run with static pair weight, prior to performing a new trial run with couple pair weight. Yu (2012), however, found that one trial weight run can yield both static and couple ICs with high accuracy on symmetric rotors.

Equation (4) covers the general condition where the cross-effects $\vec{H}_{SC}$ and $\vec{H}_{CS}$ are included. For a symmetric rotor, however, these two cross-effect ICs can be assumed to be zero, i.e.,

$$\vec{H}_{CS} \approx \vec{H}_{SC} \approx 0 \tag{19}$$

due to its symmetric static and antisymmetric couple modes. Note that the symmetric static (in-phase) mode can be either the first mode or the third mode. Static pair weights placed at two ends would affect almost the static (in-phase) response only while couple pair weights placed at two ends would affect almost the couple (180 degrees out-of-phase) response only. Thus, the 2x2 ICs in Equation (17) can be expressed as

$$\begin{bmatrix} \vec{h}_{11} & \vec{h}_{12} \\ \vec{h}_{21} & \vec{h}_{22} \end{bmatrix} \approx \begin{bmatrix} \vec{H}_{SS}+\vec{H}_{CC} & \vec{H}_{SS}-\vec{H}_{CC} \\ \vec{H}_{SS}-\vec{H}_{CC} & \vec{H}_{SS}+\vec{H}_{CC} \end{bmatrix} \tag{20}$$

Note that in the case of symmetric rotor

$$\vec{h}_{11} \approx \vec{h}_{22} \tag{21a}$$

$$\vec{h}_{12} \approx \vec{h}_{21} \tag{21b}$$

and

$$\vec{h}_{1,S} \approx \vec{h}_{2,S} \tag{22a}$$

$$\vec{h}_{1,C} \approx -\vec{h}_{2,C} \tag{22b}$$

The corresponding static and couple ICs can be computed as

$$\vec{H}_{SS} \approx \frac{\vec{A}_1^{(1)}+\vec{A}_2^{(1)}-\left(\vec{A}_1^{(0)}+\vec{A}_2^{(0)}\right)}{\vec{W}_1^{(1)}+\vec{W}_2^{(1)}} \tag{23a}$$

$$\vec{H}_{CC} \approx \frac{\vec{A}_1^{(1)}-\vec{A}_2^{(1)}-\left(\vec{A}_1^{(0)}-\vec{A}_2^{(0)}\right)}{\vec{W}_1^{(1)}-\vec{W}_2^{(1)}} \tag{23b}$$

or alternatively in the following shorter form:

$$\vec{H}_{SS} \approx \frac{\vec{S}^{(1)}-\vec{S}^{(0)}}{\vec{W}_S^{(1)}} \tag{24a}$$

$$\vec{H}_{CC} \approx \frac{\vec{C}^{(1)}-\vec{C}^{(0)}}{\vec{W}_C^{(1)}} \tag{24b}$$

where $\vec{W}_S^{(1)} = \dfrac{\vec{W}_1^{(1)}+\vec{W}_2^{(1)}}{2}$ and $\vec{W}_C^{(1)} = \dfrac{\vec{W}_1^{(1)}-\vec{W}_2^{(1)}}{2}$.

Equation (23), or alternatively Equation (24), implies that placement of $\vec{W}_1$ an/or $\vec{W}_2$ can generate both static and couple ICs for a symmetric rotor where the cross-effects $\vec{H}_{SC}$ and $\vec{H}_{CS}$ are assumed to be zero. Even one weight at either balance plane1 or balance plane 2 would be sufficient to obtain both static and couple ICs. Assuming that balance wheel diameters are the same for two planes, for placement of weights at two planes, as long as their weight amounts are different, the two-plane weights can be placed at any orientation. If the two weight amounts are the same, both ICs are still obtainable as long as they are placed neither in-phase nor 180 degrees out-of-phase.

In practice, a trial weight at one plane of a symmetric rotor that tends to decrease both static and couple components can be placed based on vibration polar plots. After obtaining both static and couple ICs from Equation (23) or (24), one can determine to balance the static (in-phase) or the couple (out-of-phase) component only, or their combination. Most end-users are only interested in low vibration at rated speed. When the static $\vec{H}_{SS}$ is far lower than the couple $\vec{H}_{CC}$, sometimes it might be not feasible to place enough static weights to offset the static component. Then placement of couple weights only might be more efficient. No matter how the correction weights are placed, the true ICs can be obtained from the first trial weight run plus the correction weight run via the multiplane balance model. One may verify whether the initial static and couple ICs are close enough to the final computed true values, and also check to see if the cross-effects $\vec{H}_{SC}$ and $\vec{H}_{CS}$ are trivial or not. If needed, a final correction can be performed based on accurate ICs.

Conversion Equations of ICs from Combined Multiplane and Static-Couple Balance Models

The above conversion equations can be used to obtain ICs from multiplane to static-couple balance model, or from the latter to the former. Sometimes one may have ICs due to weight from one end only, but not both ends, plus those due to static or couple pair weight only but not both pairs. It was found by Yu (2020) that obtaining or knowing half of influence coefficients in the multiplane balance model, plus half of those in the static-couple balance model, can yield the other half of ICs in both models. In the other words, all ICs in different formats can be obtained without a further trial run once one has half of influence coefficients in both models. The detailed derivation steps will not be repeated here.

Knowing ICs from static pair weight $\vec{H}_{SS}$ and $\vec{H}_{CS}$, plus ICs from plane 1 weight $\vec{h}_{11}$ and $\vec{h}_{21}$, one can obtain

$$\vec{h}_{12} = \vec{H}_{SS} + \vec{H}_{CS} - \vec{h}_{11} \tag{25a}$$

$$\vec{h}_{22} = \vec{H}_{SS} - \vec{H}_{CS} - \vec{h}_{21} \tag{25b}$$

$$\vec{H}_{CC} = \vec{h}_{11} - \vec{h}_{21} - \vec{H}_{CS} \tag{25c}$$

$$\vec{H}_{SC} = \vec{h}_{11} + \vec{h}_{21} - \vec{H}_{SS} \tag{25d}$$

Knowing ICs from couple pair weight $\vec{H}_{CC}$ and $\vec{H}_{SC}$, plus ICs from plane 1 weight $\vec{h}_{11}$ and $\vec{h}_{21}$, one can obtain

$$\vec{h}_{12} = \vec{h}_{11} - \vec{H}_{CC} - \vec{H}_{SC} \tag{26a}$$

$$\vec{h}_{22} = \vec{h}_{21} + \vec{H}_{CC} - \vec{H}_{SC} \tag{26b}$$

$$\vec{H}_{SS} = \vec{h}_{11} + \vec{h}_{21} - \vec{H}_{SC} \tag{26c}$$

$$\vec{H}_{CS} = \vec{h}_{11} - \vec{h}_{21} - \vec{H}_{CC} \tag{26d}$$

Knowing ICs from static pair weight $\vec{H}_{SS}$ and $\vec{H}_{CS}$, plus ICs from plane 2 weight $\vec{h}_{12}$ and $\vec{h}_{22}$, one can obtain

$$\vec{h}_{11} = \vec{H}_{SS} + \vec{H}_{CS} - \vec{h}_{12} \tag{27a}$$

$$\vec{h}_{21} = \vec{H}_{SS} - \vec{H}_{CS} - \vec{h}_{22} \tag{27b}$$

$$\vec{H}_{CC} = \vec{H}_{CS} - \vec{h}_{12} + \vec{h}_{22} \tag{27c}$$

$$\vec{H}_{SC} = \vec{H}_{SS} - \vec{h}_{12} - \vec{h}_{22} \tag{27d}$$

Knowing ICs from couple pair weight $\vec{H}_{CC}$ and $\vec{H}_{SC}$, plus ICs from plane 2 weight $\vec{h}_{12}$ and $\vec{h}_{22}$, one can obtain

$$\vec{h}_{11} = \vec{h}_{12} + \vec{H}_{CC} + \vec{H}_{SC} \tag{28a}$$

$$\vec{h}_{21} = \vec{h}_{22} - \vec{H}_{CC} + \vec{H}_{SC} \tag{28b}$$

$$\vec{H}_{SS} = \vec{h}_{12} + \vec{h}_{22} + \vec{H}_{SC} \tag{28c}$$

$$\vec{H}_{CS} = \vec{h}_{12} - \vec{h}_{22} + \vec{H}_{CC} \tag{28d}$$

Equations (25) to (28) can be utilized as conversion equations of ICs when one only knows half of the ICs from the multiplane model, plus half of those from the static or couple pair ICs. Equation (25) can be used when ICs are available from plane 1 weight as well as from static pair weight. Equation (26) can be used when ICs are available from plane 1 weight as well as from couple pair weight. Equation (27) can be used when ICs are available from plane 2 weight as well as from static pair weight. Equation (28) can be used when ICs are available from plane 2 weight as well as from couple pair weight. These complex number conversion calculations can also be built up on an excel spreadsheet for quick evaluation.

EXPERIMENTAL VERIFICATION

To verify the above conversion equations of ICs between multiplane and static-couple balance models or from combined models, a real example same as that in Yu (2009) is used here. As shown in Figure 3, this is a Bently Nevada™ RK-4 Rotor Kit with a shaft diameter and length of 0.01m and 0.56m, respectively, supported by two brass bushing bearings and driven by a 75 W motor. Three 0.8-kg disks were attached to the shaft with one close to bearing #1 and two close to bearing #2, to create asymmetry mass distribution with respective to the two bearings. Therefore, cross effects would exist between static (or couple) pair weight and couple (or static) response, to simulate a general two-plane rotor balance problem (vibration modes not necessarily symmetrical or anti-symmetrical). The data acquisition and processing system consisted of two pairs of X-Y displacement proximity probes, one speed probe and one Keyphasor probe for speed and phase measurement. Two balance weight planes 1 and 2 are located adjacent to bearings #1 and #2 as well as their corresponding proximity probes. The shaft was rotating in counter-clockwise direction when viewed from the motor to the rotor kit.

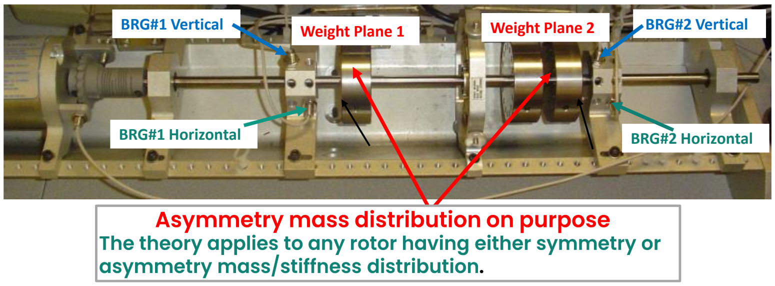


Figure 3 Rotor kit for IC conversion verification

The selected speed for balance calculation was set at 4800 rpm. Since higher amplitudes occurred in horizontal direction at 4800 rpm, IC calculations were carried out in terms of vibration readings measured by the two horizontal probes located 90-degree right to the top as shown in Figure 3. From an initial run without any balance weight placement, synchronous vibration vectors at bearings # 1 and #2 in horizontal direction were as follows:

$$\vec{A}_1^{(0)} = 5.962 \text{ mil pp } \angle 88°,\ \vec{A}_2^{(0)} = 3.742 \text{ mil pp } \angle 260°$$

With the following two 0.4-gram weights placed at planes 1 and 2 (see Figure 4):

$$\vec{W}_1^{(1)} = 0.4 \text{ gram } \angle 225°,\ \vec{W}_2^{(1)} = 0.4 \text{ gram } \angle 45°$$

the corresponding vibration vectors became

$$\vec{A}_1^{(1)} = 2.262 \text{ mils pp } \angle 269°,\ \vec{A}_2^{(1)} = 1.521 \text{ mil pp } \angle 118°$$

Placing the following two 0.8-gram weights at planes 1 and 2 (see Figure 4) after removing the above two 0.4-gram weights:

$$\vec{W}_1^{(2)} = 0.8 \text{ gram } \angle 135°,\ \vec{W}_2^{(2)} = 0.8 \text{ gram } \angle 135°$$

corresponded to the following vibration vectors:

$$\vec{A}_1^{(2)} = 6.157 \text{ mil pp } \angle 104°,\ \vec{A}_2^{(2)} = 5.572 \text{ mil pp } \angle 289°$$

Figure 4 shows polar plots and vibration vectors at around 4800 rpm for three different runs as well as two sets of weight placement.

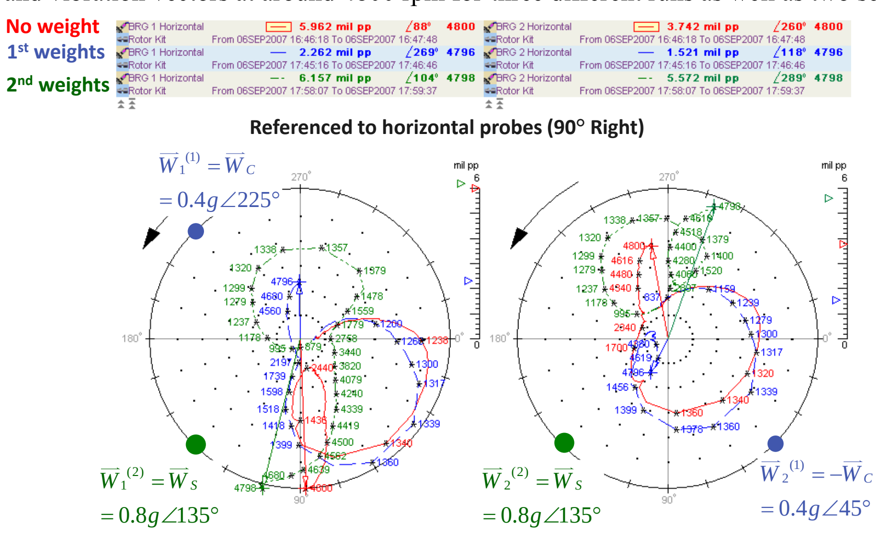


Figure 4 Polar plots and vibration vectors at 4800 rpm for initial run, and first and second trial runs with weight placements

ICs for the multiplane balance model at horizontal probes, as defined in Equation (1), is computed via Equation (2) as

$$\begin{bmatrix} \vec{h}_{11} & \vec{h}_{12} \\ \vec{h}_{21} & \vec{h}_{22} \end{bmatrix} = \begin{bmatrix} 11.3405\angle 43° & 9.2189\angle 223° \\ 7.8777\angle 218° & 4.8643\angle 58° \end{bmatrix} \text{mil pp/g}$$

The above two trial weight runs can also be viewed as couple pair weight placement (as the first trial weight run)

$$\vec{W}_C = 0.4 \text{ gram } \angle 225°$$

followed by static pair weight placement (as the second trial weight run)

$$\vec{W}_S = 0.8 \text{ gram } \angle 135°$$

Using Equation (3), static and couple vibration vectors for the initial run without weight placement, the first trial weight run with $\vec{W}_C$, and the second trial weight run with $\vec{W}_S$ are computed respectively as

$$\vec{S}^{(0)} = 1.158 \text{ mil pp } \angle 101°,\ \vec{C}^{(0)} = 4.841 \text{ mil pp } \angle 85°$$
$$\vec{S}_C^{(1)} = 0.594 \text{ mil pp } \angle 230°,\ \vec{C}_C^{(1)} = 1.834 \text{ mil pp } \angle 281°$$
$$\vec{S}_S^{(2)} = 0.388 \text{ mil pp } \angle 65°,\ \vec{C}_S^{(2)} = 5.859 \text{ mils pp } \angle 106°$$

Then ICs for the static-couple balance model as defined in Equation (4) can be calculated as

$$\begin{bmatrix} \vec{H}_{SS} & \vec{H}_{SC} \\ \vec{H}_{CS} & \vec{H}_{CC} \end{bmatrix} = \begin{bmatrix} 1.0905\angle 161° & 4.0090\angle 39° \\ 2.7863\angle 24° & 16.5618\angle 44° \end{bmatrix} \text{mil pp/g}$$

Table 1 shows calculated ICs using the conversion equations from Equations (15), (17), and (25 to (28), based on the known ICs from either the multiplane or static-couple balance model, and from the combined models. It can be seen that the calculated ICs are almost the same as those calculated directly from the definitions, with tiny differences due to cumulative errors in calculation. Here one can see that ICs from a real case verify the developed conversion equations above.

Using either Equation (1) with multiplane balance model or Equation (4) with static-couple balance model, required balance weights to offset the initial vibration at two planes can be determined. The former approach yields the following balance weights $\vec{W}_1 = 0.49 \text{ gram } \angle 208°,\ \vec{W}_2 = 0.18 \text{ gram } \angle 112°$

The latter approach yields the weights as follows:

$$\vec{W}_S = 0.25 \text{ gram } \angle 187°,\ \vec{W}_C = 0.27 \text{ gram } \angle 228°$$

Note that

$$\vec{W}_1 = \vec{W}_S + \vec{W}_C,\ \ \vec{W}_2 = \vec{W}_S - \vec{W}_C$$

The above two sets of weights are identical. Among available weights and holes, the final weights and their orientations were chosen as below:

$$\vec{W}_1 = 0.5 \text{ gram } \angle 202.5°,\ \ \vec{W}_2 = 0.2 \text{ gram } \angle 112.5°$$

1X vibration level was reduced from around 6 mil pp to less than 1 mil pp after placing the above weights.

Table 1 Calculated ICs from conversion equations on a rotor kit example

Conversion equation	Calculated results (mil pp/g)
Equation (15a)	$\vec{H}_{SS} = 1.1001\angle 161°$
Equation (15b)	$\vec{H}_{CS} = 2.8092\angle 23°$
Equation (15c)	$\vec{H}_{CC} = 16.5553\angle 44°$
Equation (15d)	$\vec{H}_{SC} = 4.0168\angle 39°$
Equation (17a)	$\vec{h}_{11} = 11.3406\angle 43°$
Equation (17b)	$\vec{h}_{21} = 7.8802\angle 218°$
Equation (17c)	$\vec{h}_{12} = 9.2180\angle 223°$
Equation (17d)	$\vec{h}_{22} = 4.8622\angle 58°$
Equation (25a)	$\vec{h}_{12} = 9.2181\angle 223°$
Equation (25b)	$\vec{h}_{22} = 4.8479\angle 57°$
Equation (25c)	$\vec{H}_{CC} = 16.5552\angle 44°$
Equation (25d)	$\vec{H}_{SC} = 4.0143\angle 39°$
Equation (26a)	$\vec{h}_{12} = 9.2180\angle 223°$
Equation (26b)	$\vec{h}_{22} = 4.8766\angle 58°$
Equation (26c)	$\vec{H}_{SS} = 1.0909\angle 161°$
Equation (26d)	$\vec{H}_{CS} = 2.8041\angle 23°$
Equation (27a)	$\vec{h}_{11} = 11.3396\angle 43°$
Equation (27b)	$\vec{h}_{21} = 7.8688\angle 219°$
Equation (27c)	$\vec{H}_{CC} = 16.5557\angle 44°$
Equation (27d)	$\vec{H}_{SC} = 4.0193\angle 39°$
Equation (28a)	$\vec{h}_{11} = 11.3396\angle 43°$
Equation (28b)	$\vec{h}_{21} = 7.8917\angle 218°$
Equation (28c)	$\vec{H}_{SS} = 1.1093\angle 161°$
Equation (28d)	$\vec{H}_{CS} = 2.8142\angle 23°$

EXCEL SPREADSHEETS FOR IC CONVERSIONS

The above IC conversions can be implemented through Excel spreadsheets for quick and easy evaluation. Figure 5 shows the IC conversions between static-couple and multiplane balance models with Excel spreadsheets to cover Equations (13) to (18). These values correspond to the above rotor kit experimental case. One just needs to enter known ICs in the left column cells, and the corresponding ICs will be given immediately. For example, entering in the left column cells 4 ICs for multiplane balance model:

$$\vec{h}_{11},\ \vec{h}_{21},\ \vec{h}_{12},\ \text{and}\ \vec{h}_{22}$$

will generate 4 ICs immediately for static-couple model:

$$\vec{H}_{SS},\ \vec{H}_{CS},\ \vec{H}_{SC},\ \text{and}\ \vec{H}_{CC}$$

in the right column cells as shown in Figure 5. The rest three groups function in the same way.

Note that the last two groups are used to convert ICs for the same static-couple model, but expressed with sensitivity to either static/couple vibration vectors or individual probe vibration vectors. When dealing with static or couple pair weight only, one may find that using

$$\vec{h}_{1,S}\ \text{and}\ \vec{h}_{2,S}$$

or

$$\vec{h}_{1,C}\ \text{and}\ \vec{h}_{2,C}$$

is much easier without having to convert between static/couple vibration vectors and individual probe vibration vectors.

	From Probe 1&2 ICs due to W1 & W2					To S & C ICs due to Ws & Wc			
	h11	h21	h12	h22	Converted to -->	Hss	Hcs	Hsc	Hcc
Sensitivity (mil pp/g)	11.3405	7.8777	9.2189	4.8643		1.1001	2.8092	4.0168	16.5553
phase lag (deg)	43	218	223	58		161	23	39	44
	From S & C ICs due to Ws & Wc					To Probe 1 & 2 ICs due to W1 & W2			
	Hss	Hcs	Hsc	Hcc	Converted to -->	h11	h21	h12	h22
Sensitivity (mil pp/g)	1.0905	2.7863	4.009	16.5618		11.3406	7.8802	9.2180	4.8622
phase lag (deg)	161	24	39	44		43	218	223	58
	From S&C ICs due to Ws & Wc					To Probe 1 & 2 ICs due to Ws & Wc			
	Hss	Hcs	Hsc	Hcc	Converted to -->	h1,s	h2,s	h1,c	h2,c
Sensitivity (mil pp/g)	1.0905	2.7863	4.009	16.5618		2.1233	3.6602	20.5585	12.5729
phase lag (deg)	161	24	39	44		45	192	43	226
	From Probe 1 & 2 ICs due to Ws & Wc					To S&C ICs due to Ws & Wc			
	h1,s	h2,s	h1,c	h2,c	Converted to -->	Hss	Hcs	Hsc	Hcc
Sensitivity (mil pp/g)	2.1233	3.6602	20.5585	12.5729		1.0905	2.7863	4.0090	16.5618
phase lag (deg)	45	192	43	226		161	24	39	44
Note: Enter data only in left column cells, if applicable									

Figure 5 IC conversions between static-couple and multiplane balance models via Excel spreadsheets

Figure 6 shows the IC conversions from combined multiplane and static-couple balance models via Excel spreadsheets. Those values also correspond to the above rotor kit experimental case. One just needs to enter known ICs in the left column cells, and the corresponding ICs will be given automatedly. For example, entering in the left column cells 4 ICs from combined balance models:

$$\vec{h}_{11},\ \vec{h}_{21},\ \vec{H}_{SS},\ \text{and}\ \vec{H}_{CS}$$

will generate other 4 ICs immediately:

$$\vec{h}_{12},\ \vec{h}_{22},\ \vec{H}_{CC},\ \text{and}\ \vec{H}_{SC}$$

in the right column cells as shown in Figure 6. The rest three groups function in the same way, and they have different combinations.

	From ICs due to W_1 & W_S					To ICs due to W_2 & W_C			
	h_{11}	h_{21}	H_{SS}	H_{CS}	Converted to -->	h_{12}	h_{22}	H_{CC}	H_{SC}
Sensitivity (mil pp/g)	11.3405	7.8777	1.0905	2.7863		9.2181	4.8479	16.5552	4.0143
Phase lag (deg)	43	218	161	24		223	57	44	39
	From ICs due to W_1 & W_C					To ICs due to W_2 & W_S			
	h_{11}	h_{21}	H_{CC}	H_{SC}	Converted to -->	h_{12}	h_{22}	H_{SS}	H_{CS}
Sensitivity (mil pp/g)	11.3405	7.8777	16.5618	4.009		9.2180	4.8766	1.0909	2.8041
Phase lag (deg)	43	218	44	39		223	58	161	23
	From ICs due to W_2 & W_S					To ICs due to W_1 & W_C			
	h_{12}	h_{22}	H_{SS}	H_{CS}	Converted to -->	h_{11}	h_{21}	H_{CC}	H_{SC}
Sensitivity (mil pp/g)	9.2189	4.8643	1.0905	2.7863		11.3416	7.8688	16.5557	4.0193
Phase lag (deg)	223	58	161	24		43	219	44	39
	From ICs due to W_2 & W_C					To ICs due to W_1 & W_S			
	h_{12}	h_{22}	H_{CC}	H_{SC}	Converted to -->	h_{11}	h_{21}	H_{SS}	H_{CS}
Sensitivity (mil pp/g)	9.2189	4.8643	16.5618	4.009		11.3396	7.8917	1.1093	2.8142
Phase lag (deg)	223	58	44	39		43	218	161	23
Note: Enter data only in left column cells, if applicable									

Figure 6 IC conversions from combined static-couple and multiplane balance models via Excel spreadsheets

Case 1

The first case is to demonstrate how to apply the developed IC conversion equations between the two balance models when ICs for one balance model are known. In this case, ICs are known for the multiplane balance model from previous testing. High vibration predominantly due to the 1X component was observed from proximity probes on this hydrogen-cooled generator, driven by a steam turbine, as shown in Figure 7. The 2-pole generator was running at 3600 rpm and rotating clockwise when viewed from turbine to generator. The two generator bearings were named as bearing #5 (drive-end) and bearing #6 (non-drive-end). A pair of X-Y probes was installed at 45-degree left and right at bearing #5 while another pair of X-Y probes was installed at 60-degree left and 30-degree right at bearing #6, as shown in Figure 7.

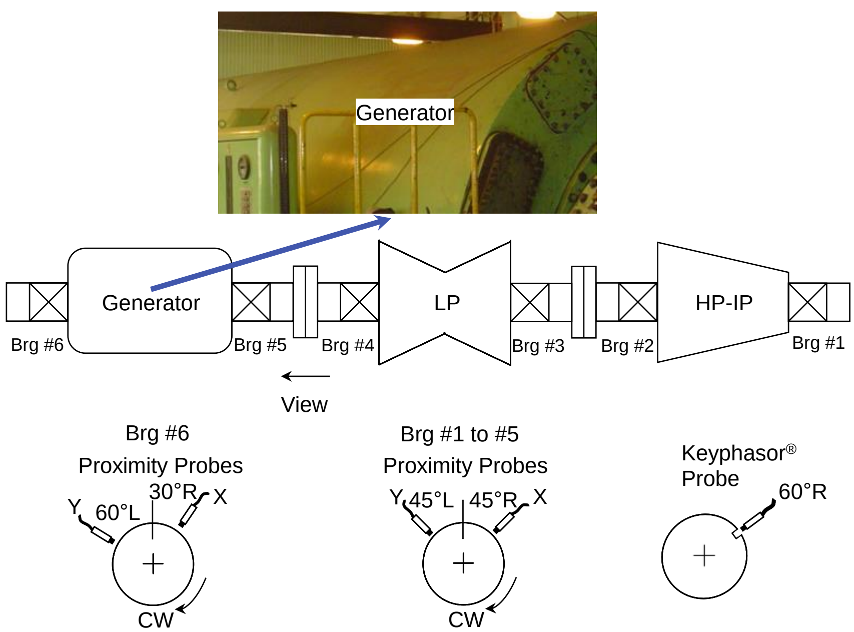


Figure 7 Machine train diagram of steam turbine generator in Case 1

1X vibration amplitudes were higher on Y-probes than on X-probes at the two bearings on the generator. Balance calculations were therefore conducted on Y-probes only. To match the same nomenclature and subscripts here in the paper, probes and weight plane at bearing #5 is denoted as “1” while those at bearing #6 is denoted as “2”.

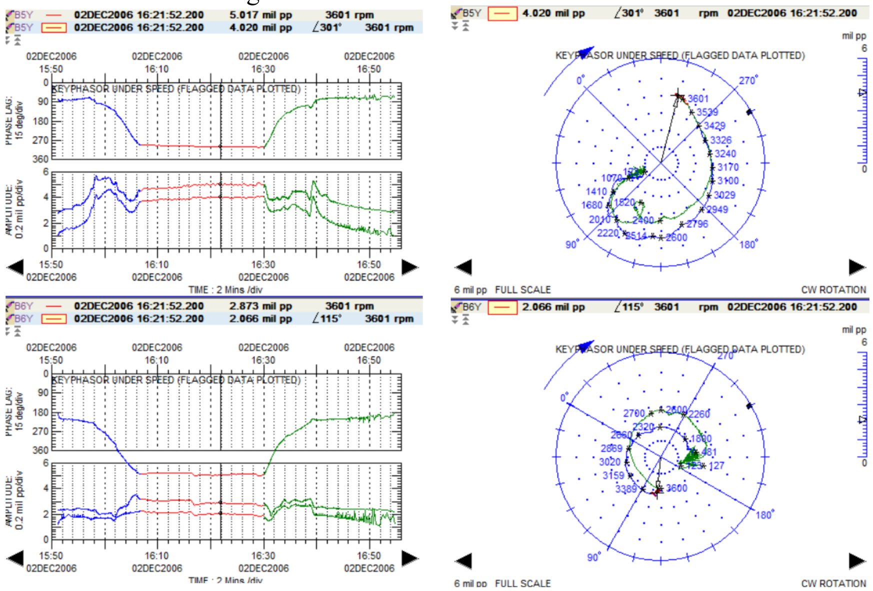


Figure 8 Direct and 1X trend plots (left) and 1X polar plots (right) at generator DE and NDE Y-probes before balance

Figure 8 shows both trend (direct and 1X) and 1X polar plots. Note that curves are colored differently with blue and green being startup and coast-down, and red being steady-state (rated speed) conditions. As shown in Figure 8, Y-probe readings from bearings #5 and #6 at rated speed 3600 rpm were

$$\vec{A}_1^{(0)} = 4.02 \text{ mil pp } \angle 301°, \ \ \vec{A}_2^{(0)} = 2.066 \text{ mil pp } \angle 115°$$

The previous ICs for the multiplane balance model were given by

$$\begin{bmatrix} \vec{h}_{11} & \vec{h}_{12} \\ \vec{h}_{21} & \vec{h}_{22} \end{bmatrix} = \begin{bmatrix} 0.2353\angle 133° & 0.1474\angle 307° \\ 0.1474\angle 294° & 0.1049\angle 117° \end{bmatrix} \text{mil pp/oz}$$

where $\vec{h}_{11}$, $\vec{h}_{12}$, $\vec{h}_{21}$, and $\vec{h}_{22}$ as shown in Equation (1) in which synchronous vibration vectors were defined as original ones from the two Y-probes (“1” was referenced to 45-degree left, and “2” was referenced to 60-degree left) while weights at both ends were all referenced to 45-degree left. Balance plane radius where weights were placed was about 10 inches with the one at bearing #5 slightly larger than that at bearing #6 (about 1% difference). Note that radius difference between the two weight planes would not affect the validity of all the IC conversion equations given in the paper. Weight planes at bearing #5 and bearing #6 had 44 and 36 holes for weight placement, respectively.

Using Equation (1), the required balance weights at two planes appeared to be

$$\vec{W}_1 = 23 \text{ oz } \angle 299°, \ \ \vec{W}_2 = 28 \text{ oz } \angle 259°$$

Placement of the above weight amounts at the two planes were not feasible based on available empty holes from the weight map. IC data was then studied to find a better resolution.

ICs for static and couple weights were calculated based on known $\vec{h}_{11}$, $\vec{h}_{12}$, $\vec{h}_{21}$, and $\vec{h}_{22}$ values without placing static or couple pair trial weights. Note that the Y-probe at bearing #6 was not parallel to the Y-probe at bearing #5. In order to evaluate static and couple effects better, the synchronous vector at bearing #6 as though it were measured by a proximity probe at 45-degree left, needed to be known, and $\vec{h}_{11}$, $\vec{h}_{12}$, $\vec{h}_{21}$, and $\vec{h}_{22}$ needed to be applicable to this change. Although the above-mentioned synchronous vector at bearing #6 could be determined by using vectors from both X and Y probes (virtual probe rotation), $\vec{h}_{11}$, $\vec{h}_{12}$, $\vec{h}_{21}$, and $\vec{h}_{22}$ might not match the newly defined vector. Therefore, the original vector was used as the new vector except its phase being lagged 15 degrees more. Thus, the two vibration vectors referenced to 45-degree left became

$$\vec{A}_1^{(0)} = 4.02 \text{ mil pp } \angle 301°, \; \vec{A}_2^{(0)} = 2.066 \text{ mil pp } \angle(115+15)°$$

and the IC matrix with both vibration and weight vectors referenced to 45-degree left became

$$\begin{bmatrix} \vec{h}_{11} & \vec{h}_{12} \\ \vec{h}_{21} & \vec{h}_{22} \end{bmatrix} = \begin{bmatrix} 0.2353\angle 133° & 0.1474\angle 307° \\ 0.1474\angle(294+15)° & 0.1049\angle(117+15)° \end{bmatrix} \text{mil pp/oz}$$

Figure 9 shows calculated ICs for static and couple pair weights from known influence vectors $\vec{h}_{11}$, $\vec{h}_{12}$, $\vec{h}_{21}$, and $\vec{h}_{22}$ used for the multiplane balance model, without having to physically place static or couple trial weights. The direct couple influence vector $\vec{H}_{CC}$ was the most sensitive one (0.3172 mil pp/oz ∠131°), indicating appropriate couple pair weight would effectively reduce the current synchronous vibration level, especially to bearing #5 ($\vec{h}_{1,C} = 0.3822$ mil pp/oz $\angle 131°$). Static weights appeared not to be sensitive to 1X vibration vectors at the running speed for this generator, as shown in Figure 9.

	From Probe 1&2 ICs due to W1 & W2					To S & C ICs due to Ws & Wc			
	h11	h21	h12	h22	Converted to -->	Hss	Hcs	Hsc	Hcc
Sensitivity (mil pp/oz)	0.2353	0.1474	0.1474	0.1049		0.0262	0.0655	0.0650	0.3172
phase lag (deg)	133	309	307	132		160	136	132	131
	From S & C ICs due to Ws & Wc					To Probe 1 & 2 ICs due to W1 & W2			
	Hss	Hcs	Hsc	Hcc	Converted to -->	h11	h21	h12	h22
Sensitivity (mil pp/oz)									
phase lag (deg)									
	From S&C ICs due to Ws & Wc					To Probe 1 & 2 ICs due to Ws & Wc			
	Hss	Hcs	Hsc	Hcc	Converted to -->	h1,s	h2,s	h1,c	h2,c
Sensitivity (mil pp/oz)	0.026154	0.065525	0.065005	0.317207		0.0900	0.0430	0.3822	0.2522
phase lag (deg)	160.1368	136.0427	131.5479	130.5138		143	302	131	310
	From Probe 1 & 2 ICs due to Ws & Wc					To S&C ICs due to Ws & Wc			
	h1,s	h2,s	h1,c	h2,c	Converted to -->	Hss	Hcs	Hsc	Hcc
Sensitivity (mil pp/oz)									
phase lag (deg)									
Note: Enter data only in left column cells, if applicable									

Figure 9 IC conversions from multiplane to static-couple balance model via Excel spreadsheets in Case 1

The current static and couple vibration vectors were as follows:

$$\vec{S}^{(0)} = 1.003 \text{ mil pp } \angle 292°, \; \vec{C}^{(0)} = 3.035 \text{ mil pp } \angle 304°$$

Using Equation (4) by setting $\vec{W}_S = 0$ and neglecting $\vec{H}_{SC}$ effect, the required couple weights were calculated as follows:

$$\vec{W}_C = \frac{0 - 3.035 \text{ mil pp } \angle 304°}{0.3172 \text{ mil pp/oz } \angle 131°} = 9.6 \text{ oz } \angle 353°$$

or

$$\vec{W}_1 = 9.6 \text{ oz } \angle 353°, \quad \vec{W}_2 = 9.6 \text{ oz } \angle 173°$$

Based on available weights and holes on the two balance planes as well as the above estimation, the following chosen weights as shown in Figure 10

$$\vec{W}_1 = 10.6 \text{ oz } \angle 347.7°, \quad \vec{W}_2 = 10.6 \text{ oz } \angle 165°$$

would yield 1X vibration amplitudes of about 0.2 and 0.7 mil pp at bearing #5 and bearing #6, predicted from the original multiplane IC matrix.

After placing the above weights, 1X vibration at bearings # 5 and #6 were reduced to 0.2 and 0.4 mil pp respectively, as shown in Figure 11.

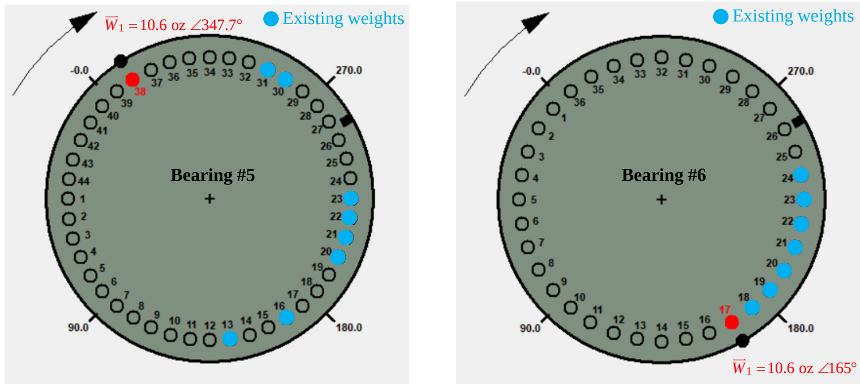


Figure 10 Weight placement in Case 1

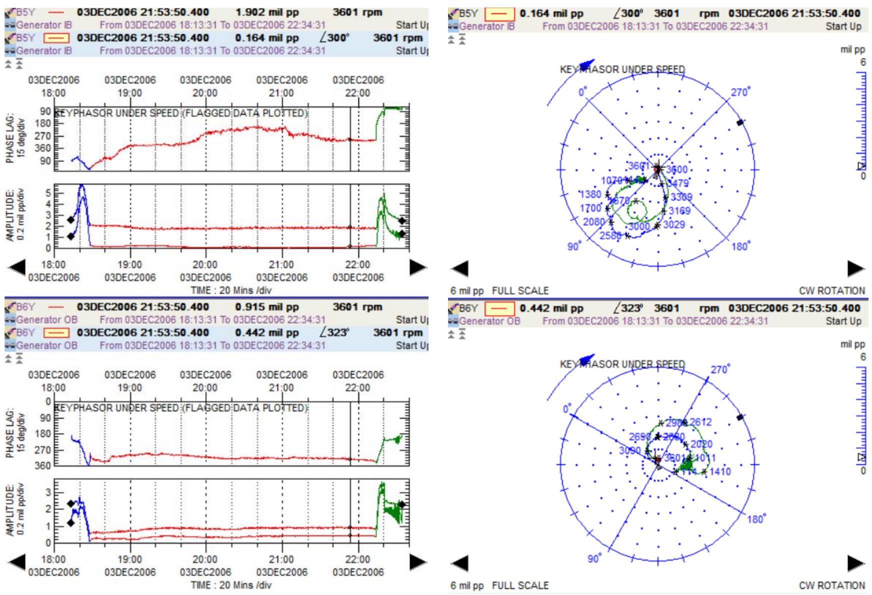


Figure 11 Direct and 1X trend plots (left) and 1X polar plots (right) at generator DE and NDE Y-probes after balance

Case 2

In this example, high synchronous 1X vibration due to unbalance was observed via proximity probes on an air-cooled generator driven by a gas turbine, as shown in Figure 12. The base load full power output is approximately 100 MW. The gas turbine was composed of low-pressure booster, high pressure core, and power turbine. The 2-pole generator was directly coupled to a power turbine with a spool at rated speed of 3600 rpm. The machine was viewed from the gas turbine to the generator and therefore the generator was considered rotating in the counter-clockwise direction. The 9-meter long generator rotor weighted about 26,500 kg and was supported by two elliptical journal bearings.

Vibration was monitored by XY pairs of non-contacting proximity probes mounted at 45-degree right (X-probe) and 45-degree left (Y-probe) relative to the 0-degree vertical reference. The alert alarm was set as 3.0 mil pp at rated speed of 3600 rpm. As usual, for this counter-clockwise shaft rotation machine, X-directional amplitude was higher than its corresponding Y-directional amplitude. Therefore, X-directional vibration was used to calculate its amplitude and phase lag as well as the corresponding influence data. Generator Drive end (DE) and non-drive end (NDE) are referred to as bearing 1 and bearing 2, respectively, for vibration readings and IC data from X-probes.

The initial run without weight placement at rated speed of 3600 rpm as indicated in Figure 13, showed the following stabilized 1X vibration vectors from X-probes as below

$$\vec{A}_1^{(0)} = 3.301 \text{ mil pp } \angle 143°, \quad \vec{A}_2^{(0)} = 1.998 \text{ mil pp } \angle 107°$$

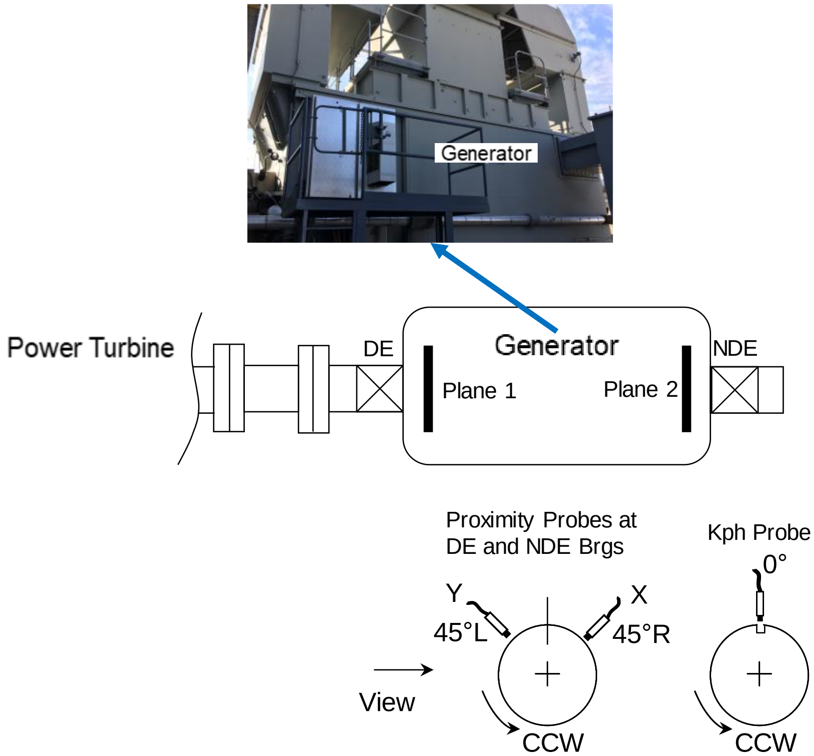


Figure 12 Machine train diagram of generator rotor balance in Case 2

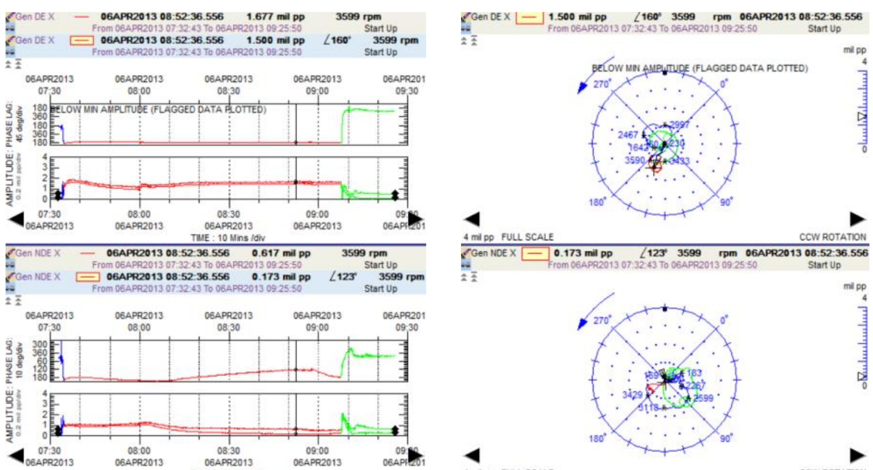


Figure 13 Direct and 1X trend plots (left) and 1X polar plots (right) at generator DE and NDE Y-probes before balance

This type of machine had been balanced previously at two different sites. At one site, a static pair weight (same amount of weights placed in the same orientation at both plane 1 and plane 2) was used at that time to reduce vibration level at both bearing 1 and bearing 2 satisfactorily. And its static pair IC data at rated speed of 3600 rpm was known for probe 1 (DE-X probe) and probe 2 (NDE-X probe), respectively as below:

$$\vec{h}_{1,S} = 0.1941\angle 95° \text{ mil pp/oz}$$
$$\vec{h}_{2,S} = 0.1206\angle 31° \text{ mil pp/oz}$$

At another site, a couple pair weight (same amount of weights placed in the opposite orientation at plane 1 and plane 2) was used at that time to reduce vibration level at the second critical speed of around 3000 rpm. And its couple pair IC data at rated speed of 3600 rpm was also computed for probe 1 (DE-X probe) and probe 2 (NDE-X probe), respectively as below:

$$\vec{h}_{1,C} = 0.0712\angle 24° \text{ mil pp/oz}$$
$$\vec{h}_{2,C} = 0.0296\angle 118° \text{ mil pp/oz}$$

It can be noticed that vibration was higher than 3 mil pp only at generator DE. Was it possible to place weights only at DE fan ring, i.e., plane 1?

To answer this question, influence coefficients $\vec{h}_{11}$ and $\vec{h}_{21}$ need to be obtained first. Based on the conversion equations described in the early part of this paper, all influence coefficients in any format, including $\vec{h}_{11}$ and $\vec{h}_{21}$, can be computed easily using a developed Excel spreadsheet, as shown in Figure 14.

From Probe 1 & 2 ICs due to Plane 1/2 weight						To static & couple ICs due to static/couple weight			
	h11	h21	h12	h22	Converted to --->	Hss	Hcs	Hsc	Hcc
sensitivity (mil/oz)									
phase lag (deg)									
From static & couple ICs due to static/couple weight						To Probe 1 & 2 ICs due to Plane 1/2 weight			
	Hss	Hcs	Hsc	Hcc	Converted to --->	h11	h21	h12	h22
sensitivity (mil/oz)	0.134854	0.0890168	0.0375885	0.0394957	3	0.1137353	0.0628374	0.0918499	0.0613328
phase lag (deg)	71.303232	132.5059	47.127502	2.0490953		77.785229	44.603743	116.49824	17.055871
From static & couple ICs due to static/couple weight						To Probe 1 & 2 ICs due to static/couple weight			
	Hss	Hcs	Hsc	Hcc	Converted to --->	h1,s	h2,s	h1,c	h2,c
sensitivity (mil/oz)									
phase lag (deg)									
					2				
From Probe 1 & 2 IC due to static/couple weight						To static & couple IC due to static/couple weight			
	h1,s	h2,s	h1,c	h2,c	Converted to --->	Hss	Hcs	Hsc	Hcc
sensitivity (mil/oz)	0.1941	0.1206	0.0712	0.0296	1	0.134854	0.0890168	0.0375885	0.0394957
phase lag (deg)	95	31	24	118		71.303232	132.5059	47.127502	2.0490953
Note: Enter data only in left column cells, if applicable									

Figure 14 IC conversions from static-couple to multiplane balance model via Excel spreadsheets in Case 2

The corresponding solution of Plane 1 weight only by using the above obtained ICs $\vec{h}_{11}$ and $\vec{h}_{21}$ based on $\vec{A}_1^{(0)}$ and $\vec{A}_2^{(0)}$ with

$$\vec{W}_1 = 17.5 \text{ oz}\angle 240°, \quad \vec{W}_2 = 0$$

would yield predicted 1X vibration vectors at bearing1 and bearing 2 as below

$$\vec{A}_1 \approx 1.33 \text{ mil pp } \angle 150°, \quad \vec{A}_2 \approx 0.90 \text{ mil pp } \angle 109°$$

Therefore, the above calculated weights were added at Plane 1 only, as shown in Figure 15.

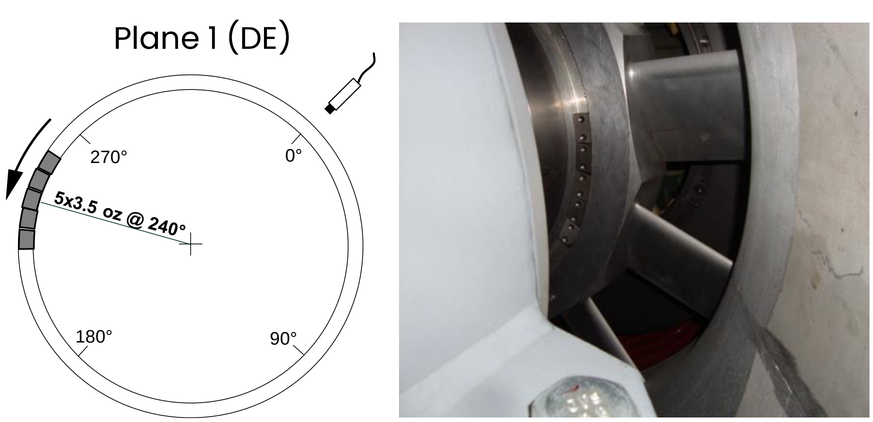
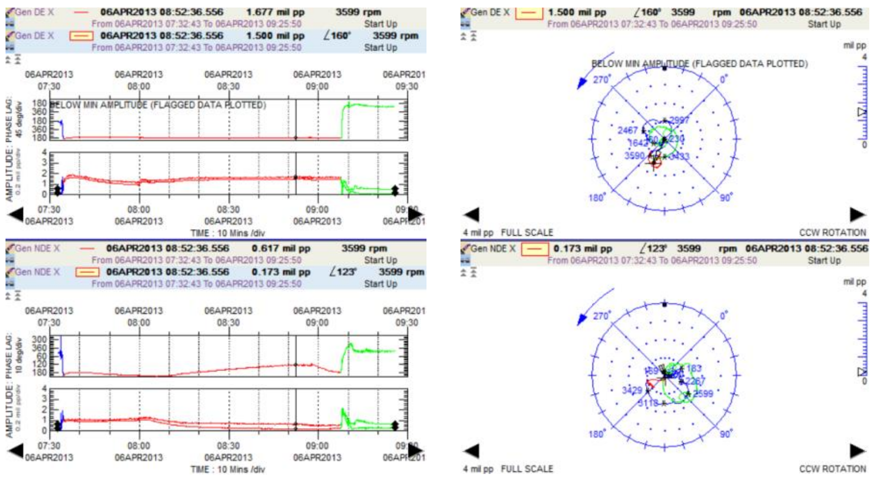


Figure 15 One-shot balance weights at generator DE fan ring (Plane 1)

The one-shot balancing with weights at Plane 1 only reduced vibration at both ends effectively. Figure 16 shows direct and 1X trend plots as well as 1X polar plots at DE and NDE X-probes after placement of 5 weights at DE fan ring plane. Stabilized 1X vibration vectors measured by X-probes as shown in Figure 16 after balance became

$$\vec{A}_1^{(0)} = 1.500 \text{ mil pp } \angle 160°, \quad \vec{A}_2^{(0)} = 0.173 \text{ mil pp } \angle 123°$$

which are very close to the predicted 1X vibration vectors.



(a) (b)

Figure 16 Direct and 1X trend plots (left) and 1X polar plots (right) at DE and NDE X-probes after the one-shot balancing

Note that the static pair ICs $\vec{h}_{1,S}$ and $\vec{h}_{2,S}$ as well as the couple pair ICs $\vec{h}_{1,C}$ and $\vec{h}_{2,C}$ at rated speed were not directly obtained from the same machine. These influence vectors were generated from other machines with the same design. However, they appeared to be accurate enough for predicted vibration response. Certainly, if these ICs had been generated from the identical machine, the results might have been more accurate.

Case 3

This is a steam turbine generator with rated speed of 3600 rpm and power output of approximately 45 MW. It is composed of high pressure (HP), double-flow low pressure (LP) turbines and a hydrogen-cooled generator. HP, LP, and generator rotors are each supported by two journal bearings, numbered in order from the turbine to the generator. When viewed in Figure 17 from the turbine towards the generator, the machine train is seen to be rotating in the clockwise direction. Both the steam turbine and the generator

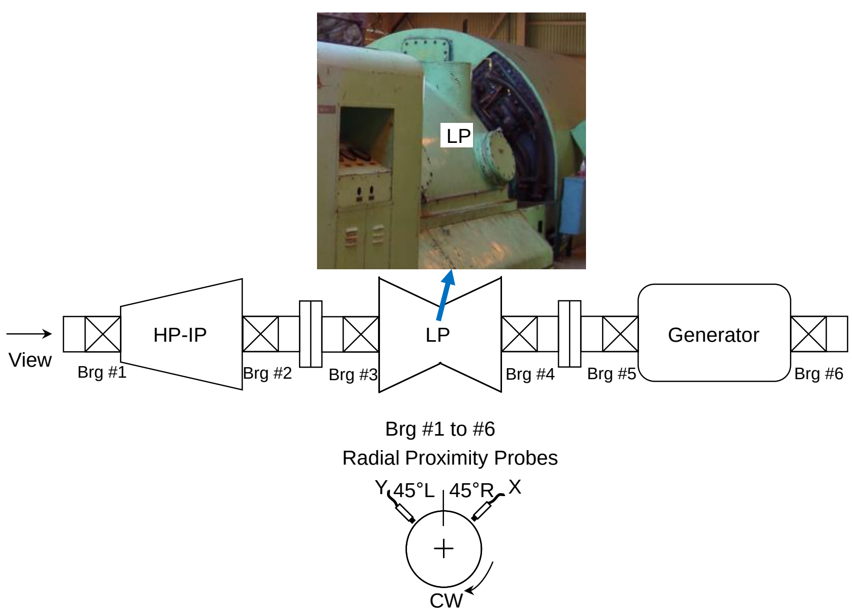


Figure 17 Machine train diagram of LP rotor balance in Case 3

vibrations are measured by XY pairs of non- contacting proximity probes mounted at 45-degree left (Y-probe) and 45-degree right (X-probe) relative to the 0-degree vertical reference at each bearing from Bearing #1 to #6.

At rated speed of 3600 rpm, the highest overall vibration reading on the symmetric double-flow LP rotor was approximately 2.7 mil pp with the 1X component of about 2.3 mil pp. Balancing was requested in order to further reduce the vibration, though considered to be optional with this level of amplitude. Figure 18 shows bearings #3 and #4 and their X and Y proximity probes as well as weight access ports prior to casing assembly.

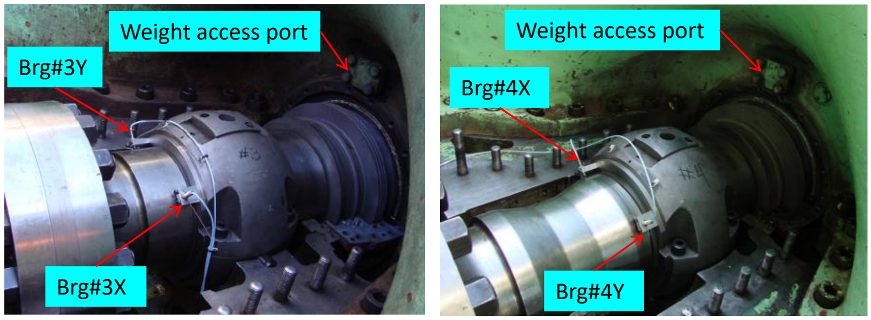


Figure 18 Balance plane access and probe locations on LP rotor

1X vibration amplitudes were higher on Y-probes than on X-probes at the two bearings on the LP rotor. Balance calculations were therefore conducted on Y-probes only. To use the same nomenclature and subscripts for the equations used earlier, probes and weight plane at bearing #3 is denoted as "1" while those at bearing #4 is denoted as "2". ICs were calculated based on the stabilized steady-state base load condition of about 45 MW. As shown in Figure 19, Y-probe readings at bearings #3 and #4 in this condition were

$$\vec{A}_1^{(0)} = 2.327 \text{ mil pp } \angle 253°, \quad \vec{A}_2^{(0)} = 1.835 \text{ mil pp } \angle 100°$$

If expressed in static and couple components as shown in Eq. 2, they were equivalent to

$$\vec{S}^{(0)} = 0.542 \text{ mil pp } \angle 203°, \quad \vec{C}^{(0)} = 2.024 \text{ mil pp } \angle 265°$$

As shown in the polar plots (Figure 19), the LP rotor ran between the first (translational) and the second (pivotal) modes. The second mode contributed more to the vibration readings.

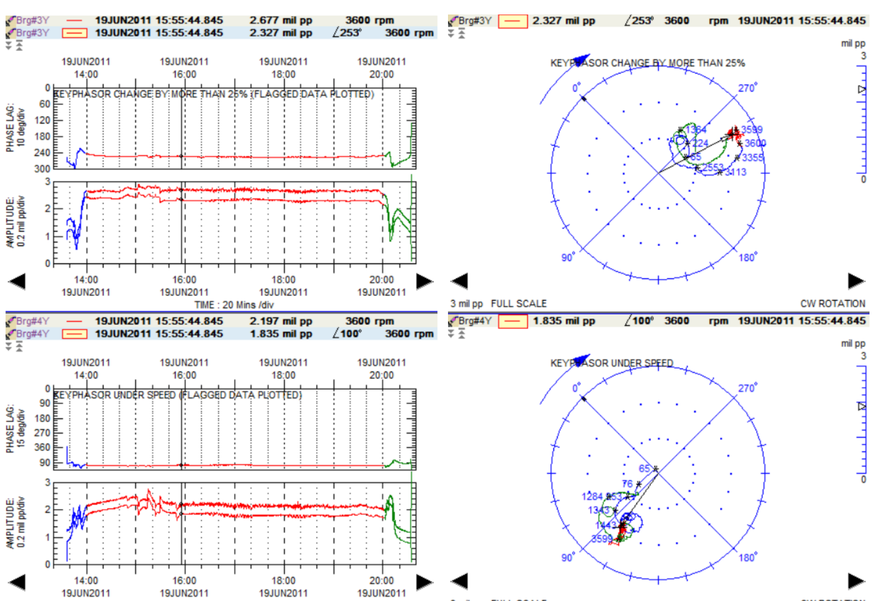


Figure 19 Direct and 1X trend plots (left) and 1X polar plots (right) at LP DE and NDE Y-probes before balance

Figure 20 shows balance weight maps at bearings #3 and #4. The radius from the center of the shaft to balance holes was about 12.625 inches. There were 24 holes for each balance plane, and they were numbered in order against shaft rotation. Holes 12 to 16 at balance plane 3, and holes 11 to 15 at balance plane 4 were already occupied with weights. Due to time restrictions and difficulty in accessing weight planes in the field, trial weight placement was only performed at bearing #3 balance plane. The Keyphasor probe was located 45-degree left, in the same orientation as all the Y-probes. The trial weight was attempted to reduce the current vibration level, and if vibration would be in the desired low level after the trial weight placement no further action would be needed. The plan was to place a trial weight of about 10 ounces at around 15 degrees against the shaft rotation relative to the Y-probe. However, it was mistakenly believed that hole 1 at both balance planes was aligned with the Keyphasor notch. Therefore, the existing weight of 9.48 ounces on hole 14 at balance plane 3 was taken out (equivalent of adding the same weight at hole 2). This action increased the vibration level to above 3.5 mil pp at bearing #3. Examining the Keyphasor pulse indicated that the Keyphasor notch was aligned in the middle between holes 14 and 15. Therefore, the trial weight as shown in Figure 20 was actually

$$\vec{W}_1^{(1)} = 9.48 \text{ oz } \angle 172.5°, \quad \vec{W}_2^{(1)} = 0$$

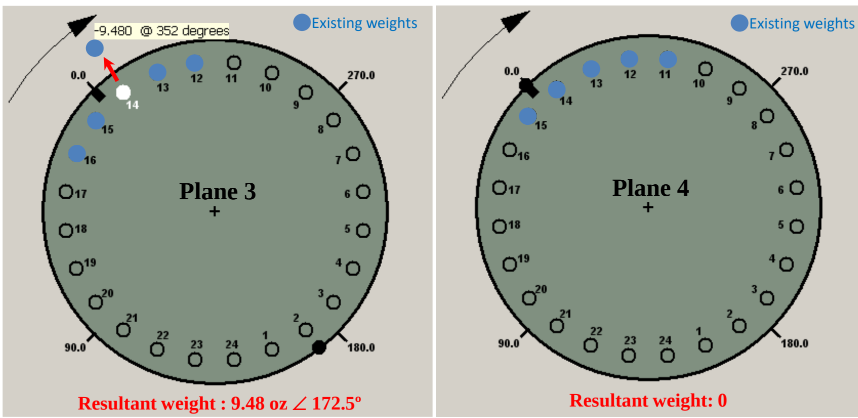


Figure 20 First weight placement (weight removal) from balance Plane 3 on LP rotor

with the corresponding vibration vectors

$$\vec{A}_1^{(1)} = 3.527 \text{ mil pp } \angle 256°, \quad \vec{A}_2^{(1)} = 2.647 \text{ mil pp } \angle 77°$$

Using Equation (23) with the assumption that cross effects $\vec{H}_{SC}$ and $\vec{H}_{CS}$, are close to zero for this symmetric LP rotor, static and couple ICs can be estimated as

$$\vec{H}_{SS} \approx \frac{\vec{A}_1^{(1)} + \vec{A}_2^{(1)} - \left(\vec{A}_1^{(0)} + \vec{A}_2^{(0)}\right)}{\vec{W}_1^{(1)} + \vec{W}_2^{(1)}} = 0.0896 \text{ mil pp /oz } \angle 158°$$

$$\vec{H}_{CC} \approx \frac{\vec{A}_1^{(1)} - \vec{A}_2^{(1)} - \left(\vec{A}_1^{(0)} - \vec{A}_2^{(0)}\right)}{\vec{W}_1^{(1)} - \vec{W}_2^{(1)}} = 0.2390 \text{ mil pp /oz } \angle 68°$$

To offset both static and couple vibration components, the required static and couple pair weights would be

$$\vec{W}_S = \frac{0 - \vec{S}^{(0)}}{\vec{H}_{SS}} = 6.0 \text{ oz } \angle 225°, \quad \vec{W}_C = \frac{0 - \vec{C}^{(0)}}{\vec{H}_{CC}} = 8.5 \text{ oz } \angle 17°,$$

which are equivalent to the weights at two planes as below:

$$\vec{W}_1 = \vec{W}_S + \vec{W}_C = 4.3 \text{ oz } \angle 336°, \quad \vec{W}_2 = \vec{W}_S - \vec{W}_C = 14.1 \text{ oz } \angle 209°$$

This would require, at balance Plane 3, weight placement on hole 13 where an existing weight was there already. Since it was not feasible to place the above weights, an alternative was needed. Note that the main vibration was composed of the couple component, and sensitivity of couple IC $\vec{H}_{CC}$ was about 2.7 times of that of static IC $\vec{H}_{SS}$. Therefore, placement of the couple pair weight appeared to be more effective if it could be implemented. This will also demonstrate to see if such estimated $\vec{H}_{CC}$ was accurate and effective enough to balance the couple vibration component.

To offset the couple vibration component based on the estimated IC data, the desired couple weight placement $\vec{W}_C = 8.5$ oz $\angle 17°$ would need insertion of a weight on hole 15 or 16 at balance Plane 3 plus a weight on hole 3 or 4 at balance Plane 4. However, since holes 15 and 16 at Plane 3 were already occupied with the existing weights, two available weights (5.62 ounces on hole 11 and 8.36 ounces on hole 17) were placed as shown in Figure 21, resulting in an equivalent weight of 10.073 ounces at 3.6 degrees relative to the Y-probe at Plane 3. A weight plug of 9.95 ounces was placed on hole 3 at Plane 4. Thus, the correction weights were as follows:

$$\vec{W}_1^{(2)} = 10.073 \text{ oz } \angle 3.6°, \quad \vec{W}_2^{(2)} = 9.95 \text{ oz } \angle 187.5°$$

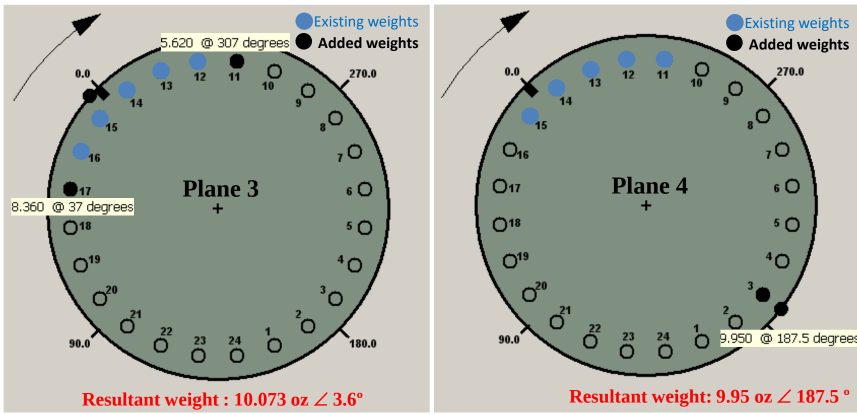


Figure 21 First weight placement (weight removal) from balance Plane 3 on LP rotor

which were approximately equivalent to the couple pair weight $\vec{W}_C \approx 10$ oz $\angle 6°$. The corresponding vibration vectors with the above weights, as shown in Figure 22, were

$$\vec{A}_1^{(2)} = 0.287 \text{ mil pp } \angle 228°, \quad \vec{A}_2^{(2)} = 0.895 \text{ mil pp } \angle 210°$$

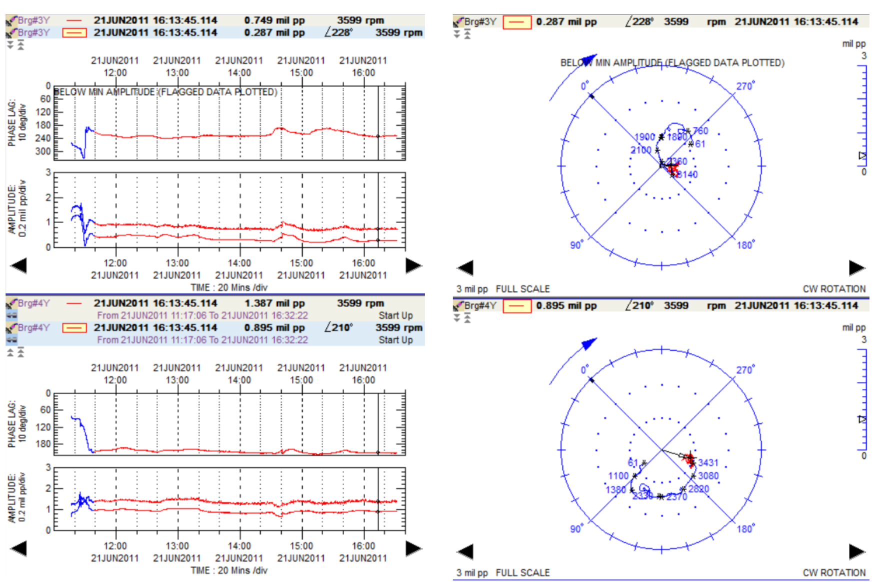


Figure 22 Direct and 1X trend plots (left) and 1X polar plots (right) at LP DE and NDE Y-probes after balance

which were equivalent to combination of static and couple components

$$\vec{S}^{(2)} = 0.586 \text{ mil pp } \angle 214°, \quad \vec{C}^{(2)} = 0.314 \text{ mil pp } \angle 22°$$

At such a low vibration level, no further balancing would be needed.

Its multiplane ICs can be computed as below

$$\begin{bmatrix}\vec{h}_{11} & \vec{h}_{12}\\ \vec{h}_{21} & \vec{h}_{22}\end{bmatrix} = \begin{bmatrix}\vec{A}_1^{(1)}-\vec{A}_1^{(0)} & \vec{A}_1^{(2)}-\vec{A}_1^{(0)}\\ \vec{A}_2^{(1)}-\vec{A}_2^{(0)} & \vec{A}_2^{(2)}-\vec{A}_2^{(0)}\end{bmatrix}\begin{bmatrix}\vec{W}_1^{(1)} & \vec{W}_1^{(2)}\\ \vec{W}_2^{(1)} & \vec{W}_2^{(2)}\end{bmatrix}^{-1} = \begin{bmatrix}0.1276\angle 90^\circ & 0.0927\angle 225^\circ\\ 0.1262\angle 228^\circ & 0.1305\angle 97^\circ\end{bmatrix} \text{ mil pp / oz}$$

It can be seen that $\vec{h}_{11}$ is very close to $\vec{h}_{22}$ while $\vec{h}_{12}$ is very close to $\vec{h}_{21}$ in both sensitivity and phase, as expected from Equation (21) for this symmetric rotor. The corresponding ICs for static-couple balance model can be computed easily using the developed Excel spreadsheet, as shown in Figure 23.

	From Probe 1&2 ICs due to W_1 & W_2					To S & C ICs due to Ws & Wc			
	h11	h21	h12	h22	Converted to -->	Hss	Hcs	Hsc	Hcc
Sensitivity (mil pp/oz)	0.1276	0.1262	0.0927	0.1305	1	0.0963	0.0218	0.0152	0.2188
phase lag (deg)	90	228	225	97		149	37	264	72
	From S & C ICs due to Ws & Wc					To Probe 1 & 2 ICs due to W_1 & W_2			
	Hss	Hcs	Hsc	Hcc	Converted to -->	h11	h21	h12	h22
Sensitivity (mil pp/oz)									
phase lag (deg)									
	From S&C ICs due to Ws & Wc					To Probe 1 & 2 ICs due to Ws & Wc			
	Hss	Hcs	Hsc	Hcc	Converted to -->	h1,s	h2,s	h1,c	h2,c
Sensitivity (mil pp/oz)									
phase lag (deg)									
	From Probe 1 & 2 ICs due to Ws & Wc					To S&C ICs due to Ws & Wc			
	h1,s	h2,s	h1,c	h2,c	Converted to -->	Hss	Hcs	Hsc	Hcc
Sensitivity (mil pp/oz)									
phase lag (deg)									
Note: Enter data only in left column cells, if applicable									

Figure 23 IC conversions from multiplane to static-couple balance model via Excel spreadsheets in Case 3

The above static and couple ICs can be considered as *true* values as they are converted from multiplane balance model without any assumption. The cross-effects $\vec{H}_{SC}$ and $\vec{H}_{CS}$ are very small compared with direct static and couple influence coefficients $\vec{H}_{SS}$ and $\vec{H}_{CC}$. Table 2 shows static and couple ICs using the one trial weight method versus the true values via the two trial runs. The IC data from the one trial weight method is very close to the true values with less than 10% difference in sensitivity and less than 10 degrees in phase. This can be considered well acceptable for field balancing. As can be seen in this case, using the estimated IC data from the one trial weight method yields satisfactory balancing results. In this example, only one trial weight was applied at one of the two balance planes to obtain both static and couple ICs on this symmetric rotor.

Table 2 Static and couple ICs obtained from one trial weight method versus the true values via the two trial runs on a symmetric LP rotor

Static/Couple ICs	ICs from one trial weight run	Compared with True ICs
$\vec{H}_{SS}$ (mil pp /oz)	$0.0896\angle 158^\circ$	$0.0963\angle 149^\circ$
$\vec{H}_{CC}$ (mil pp /oz)	$0.2390\angle 68^\circ$	$0.2188\angle 72^\circ$

Case 4

This case is to demonstrate how to apply the developed conversion equations in real field balance tasks to obtain all ICs once IC data for one type of weight is known, without having to perform two complete trial weight runs. It is often possible that IC data from a static or couple pair weight is already known from previous record. But based on current vibration data and the previous known static or couple IC data, a static or couple pair weight may not produce satisfactory results. Therefore, a trial weight from Plane 1 or 2 may be implemented. No further trial weight run is needed to obtain all ICs. One can use the conversion equations given in this paper to obtain all ICs in different formats for calculation. Having less trial runs in the field is beneficial and critical for cost saving and production needs.

In this example, high 1X vibration due to unbalance was observed via proximity probes on an air-cooled generator driven by a gas turbine, as shown in Figure 12, a similar machine as in Case 2.

The initial run without weight placement at rated speed of 3600 rpm showed the following stabilized 1X vibration vectors from X-probes as below (see Figure 24)

$$\vec{A}_1^{(0)} = 3.033 \text{ mil pp } \angle 243°, \quad \vec{A}_2^{(0)} = 1.430 \text{ mil pp } \angle 37°$$

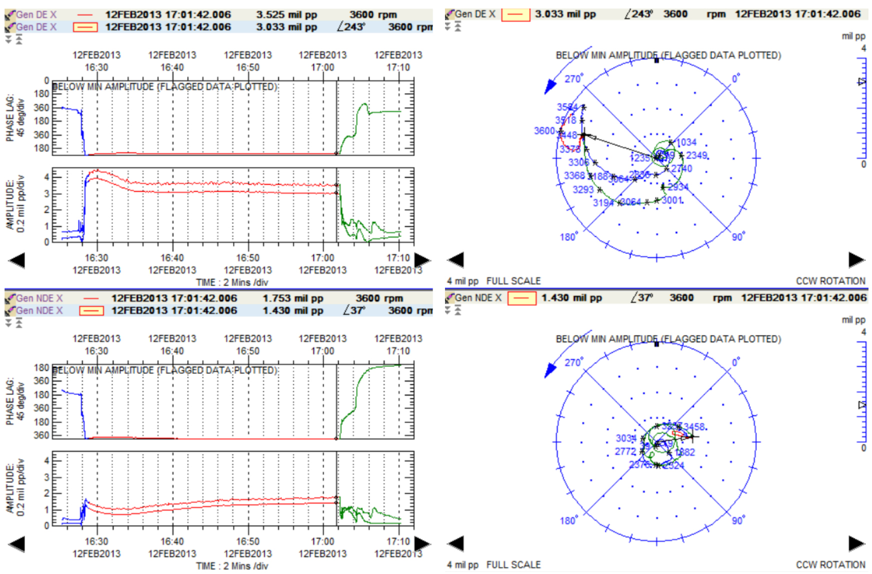


Figure 24 Direct and 1X trend plots (left) and 1X polar plots (right) at generator DE and NDE X-probes before balance

A machine with the same design had been balanced previously at a different site. A static pair weight (same amount of weights placed in the same orientation at both Plane 1 and Plane 2) was used at that time to reduce vibration level at both bearing 1 and bearing 2 satisfactorily. And its static pair IC data at rated speed of 3600 rpm was known as below:

$$\vec{h}_{1,S} = 0.1653\angle 100° \text{ mil pp/oz}, \quad \vec{h}_{2,S} = 0.1149\angle 33° \text{ mil pp/oz}$$

Based on vibration vectors $\vec{A}_1^{(0)}$ and $\vec{A}_2^{(0)}$ along with static pair ICs $\vec{h}_{1,S}$, and $\vec{h}_{2,S}$ (static weight pair effects to DE and NDE X-probes), it was impossible to further reduce 1X vibration below 2 mil pp at both ends by placing a pair of static weight. Therefore, weight placement only at Plane 1 was considered in an attempt to reduce vibration and obtain IC data for further trim balancing if needed.

There are two fan ring planes where balance weights can be placed through access covers near DE and NDE bearings, as indicated in Figure 12. Each weight plug is approximately 3.5 ounces (100 grams). Since 1X vibration phase lag is referenced to the 45-degree right for X-probes, weight orientation is also referenced to the 45-degree right.

According to the polar plot in Figure 24, 4 weight plugs were centered at 5 degrees against shaft rotation relative to the X-probe when Keyphasor probe at the top dead center was just aligned to the Keyphasor notch, at Plane 1, i.e., generator DE end balance plane. No weight was placed at Plane 2. Figure 25 shows the corresponding weight map. The weight vectors were as below:

$$\vec{W}_1^{(1)} = 14 \text{ oz}\angle 5°, \quad \vec{W}_2^{(1)} = 0$$

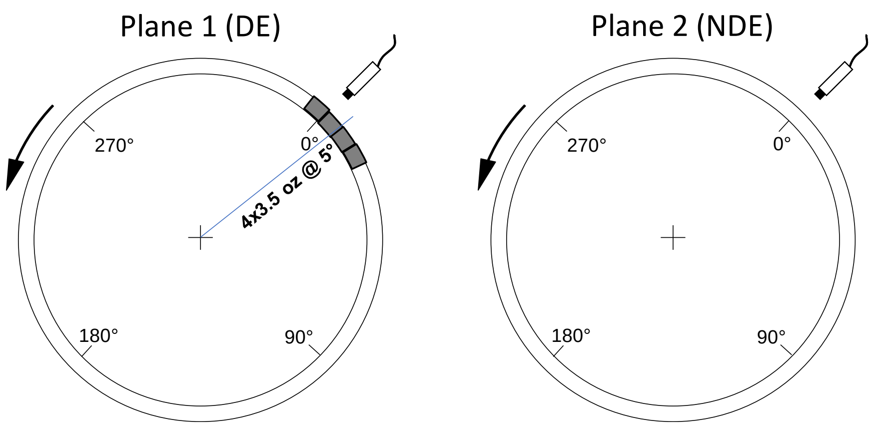


Figure 25 Weight placement at Plane 1 only

The corresponding vibration response with the above weights is shown in Figure 26. The stabilized 1X vibration vectors at 3600 rpm from X-probes were as below

$$\vec{A}_1^{(1)} = 1.685 \text{ mil pp } \angle 235°, \quad \vec{A}_2^{(1)} = 2.252 \text{ mil pp } \angle 34°$$

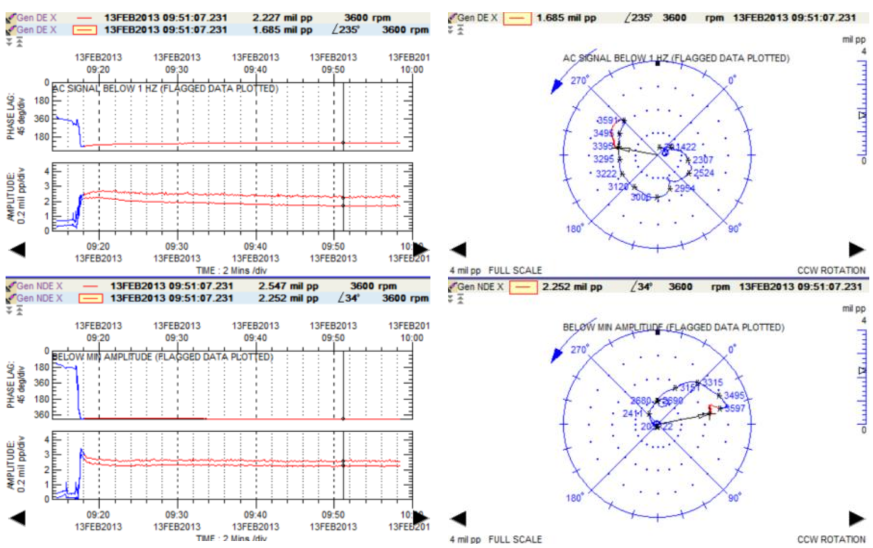


Figure 26 Direct and 1X trend plots (left) and 1X polar plots (right) at generator DE and NDE X-probes corresponding to weight placement in Figure 25

Though vibration at bearing 1 was reduced to below 2 mil pp, vibration at bearing 2 was increased to above 2 mil pp. The target was to reduce 1X vibration to less than 2 mil pp at both bearings.

The corresponding ICs from Plane 1 can be computed directly as below:

$$\vec{h}_{11} = \frac{\vec{A}_1^{(1)} - \vec{A}_1^{(0)}}{\vec{W}_1^{(1)}} = 0.0989\angle 68° \text{ mil pp/oz}$$

$$\vec{h}_{21} = \frac{\vec{A}_2^{(1)} - \vec{A}_2^{(0)}}{\vec{W}_1^{(1)}} = 0.0591\angle 24° \text{ mil pp/oz}$$

Normally one would proceed to place trial weights at Plane 2 to obtain $\vec{h}_{12}$ and $\vec{h}_{22}$. However, using IC conversion equations along with previously obtained $\vec{h}_{1,S}$ and $\vec{h}_{2,S}$, $\vec{h}_{12}$ and $\vec{h}_{22}$ can be obtained via Excel spreadsheets (see Figure 27) as below:

$$\vec{h}_{12} = 0.0964\angle 132° \text{ mil pp/oz}$$

$$\vec{h}_{22} = 0.0571\angle 42° \text{ mil pp/oz}$$

	From Probe 1&2 ICs due to W_1 & W_2					To S & C ICs due to W_s & W_c			
	h_{11}	h_{21}	h_{12}	h_{22}	Converted to -->	H_{ss}	H_{cs}	H_{sc}	H_{cc}
Sensitivity (mil pp/oz)									
phase lag (deg)									
	From S & C ICs due to W_s & W_c					To Probe 1 & 2 ICs due to W_1 & W_2			
	H_{ss}	H_{cs}	H_{sc}	H_{cc}	Converted to -->	h_{11}	h_{21}	h_{12}	h_{22}
Sensitivity (mil pp/oz)									
phase lag (deg)									
	From S&C ICs due to W_s & W_c					To Probe 1 & 2 ICs due to W_s & W_c			
	H_{ss}	H_{cs}	H_{sc}	H_{cc}	Converted to -->	$h_{1,s}$	$h_{2,s}$	$h_{1,c}$	$h_{2,c}$
Sensitivity (mil pp/oz)									
phase lag (deg)									
	From Probe 1 & 2 ICs due to W_s & W_c					To S&C ICs due to W_s & W_c			
	$h_{1,s}$	$h_{2,s}$	$h_{1,c}$	$h_{2,c}$	Converted to -->	H_{ss}	H_{cs}	H_{sc}	H_{cc}
Sensitivity (mil pp/oz)	0.1653	0.1149			1	0.1176	0.0802		
phase lag (deg)	100	33				73	141		
Note: Enter data only in left column cells, if applicable									

2

	From ICs due to W_1 & W_s					To ICs due to W_2 & W_c			
	h_{11}	h_{21}	H_{ss}	H_{cs}	Converted to -->	h_{12}	h_{22}	H_{cc}	H_{sc}
Sensitivity (mil pp/g)	0.0989	0.0591	0.1176	0.0802	3	0.0964	0.0571	0.0485	0.0568
Phase lag (deg)	68	24	73	141		132	42	21	3
	From ICs due to W_1 & W_c					To ICs due to W_2 & W_s			
	h_{11}	h_{21}	H_{cc}	H_{sc}	Converted to -->	h_{12}	h_{22}	H_{ss}	H_{cs}
Sensitivity (mil pp/g)									
Phase lag (deg)									
	From ICs due to W_2 & W_s					To ICs due to W_1 & W_c			
	h_{12}	h_{22}	H_{ss}	H_{cs}	Converted to -->	h_{11}	h_{21}	H_{cc}	H_{sc}
Sensitivity (mil pp/g)									
Phase lag (deg)									
	From ICs due to W_2 & W_c					To ICs due to W_1 & W_s			
	h_{12}	h_{22}	H_{cc}	H_{sc}	Converted to -->	h_{11}	h_{21}	H_{ss}	H_{cs}
Sensitivity (mil pp/g)									
Phase lag (deg)									
Note: Enter data only in left column cells, if applicable									

Figure 27 IC conversions from combined multiplane and static-couple balance models via Excel spreadsheets in Case 4

The corresponding multiplane solution using the above obtained ICs $\vec{h}_{11}$, $\vec{h}_{21}$, $\vec{h}_{12}$ and $\vec{h}_{22}$, based on $\vec{A}_1^{(1)}$ and $\vec{A}_2^{(1)}$, with

$$\vec{W}_1 = 7 \text{ oz}\angle 135°, \quad \vec{W}_2 = 14 \text{ oz}\angle 250°$$

would yield predicted 1X vibration vectors at bearing1 and bearing 2 as below

$$\vec{A}_1 \approx 1.20 \text{ mil pp } \angle 253°, \quad \vec{A}_2 \approx 1.89 \text{ mil pp } \angle 21°$$

Therefore, the above calculated weights were added at Plane 1 and Plane 2, as shown in Figure 28.

The final vibration readings at Bearing 1 and Bearing 2 are as below, very close to the predicted response above.

$$\vec{A}_1 = 1.876 \text{ mil pp } \angle 240°, \quad \vec{A}_2 = 1.913 \text{ mil pp } \angle 26°$$

Figure 28 shows corresponding direct and 1X trend plots along with 1X polar plots at generator DE and NDE X-probes after balance.

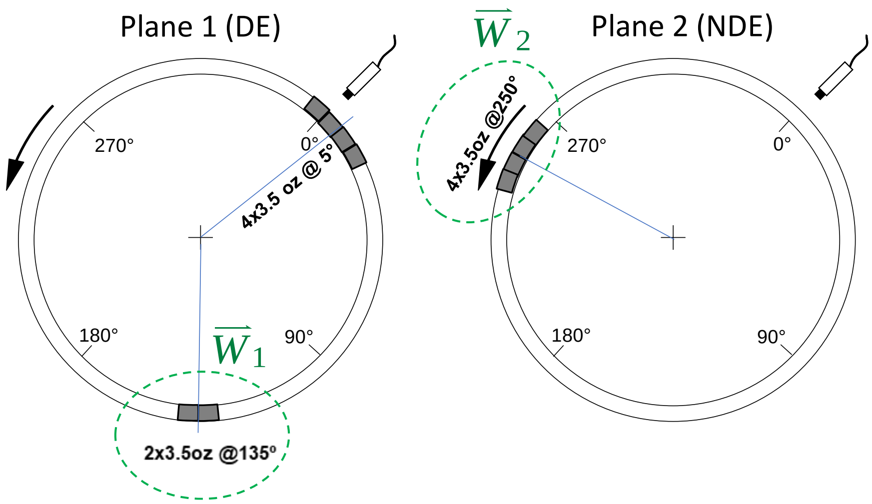


Figure 27 Additional weight placement of 7 oz at Plane 1 and 14 oz at Plane 2

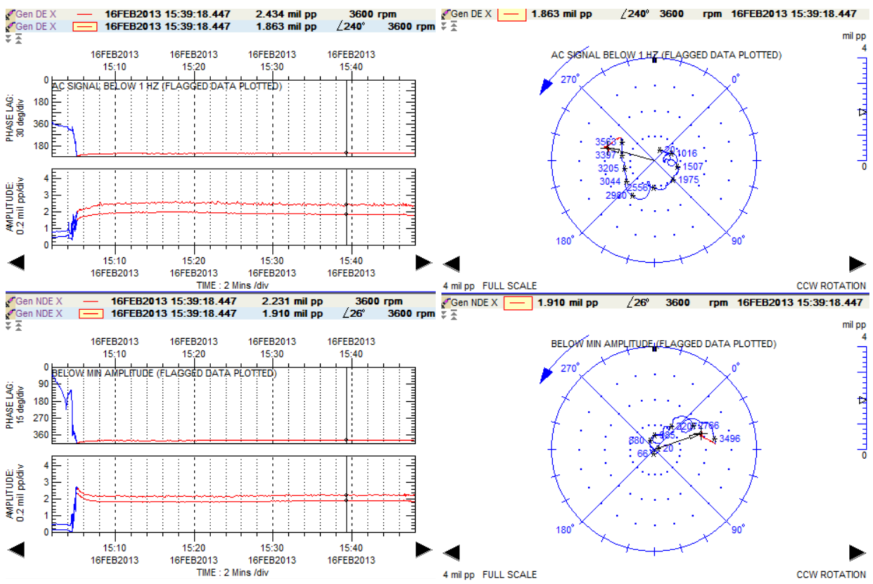


Figure 28 Direct and 1X trend plots (left) and 1X polar plots (right) at generator DE and NDE X-probes after balance

Note that the static pair IC data at rated speed was not directly obtained from the same machine. These ICs were generated from a machine with the same design. However, they appeared to be accurate enough for predicted vibration response. Certainly, if the static pair ICs had been generated from the identical machine, the results might have been more accurate.

CONCLUSIONS AND DISCUSSION

There are two different balance models for the two-plane balancing: static-couple and multiplane. For the static-couple balance model, cross effects between static pair weight and couple vibration response as well as between couple pair weight and static vibration response are introduced. Thus, there are two different formats of ICs. This paper gives all possible six IC conversions from one model to the other as well as from the two mixed models, as long as any two sets of ICs are known. These IC conversion equations hold true for any rotor, either symmetric/anti-symmetric or asymmetric in geometry or vibration modes. They have been developed analytically and confirmed by experimental results. These IC conversions can be implemented easily via Excel spreadsheets as shown in the current paper.

When dealing with a symmetric rotor with assumed symmetric or anti-symmetric vibration modes, cross effects between static pair weight and couple vibration response or coupled pair weights and static response are very minimal and can be neglected. In this special case, one trial weight run can yield both static and couple ICs close to the true values.

Four cases presented here demonstrate how these IC conversions can be used in real balancing tasks in the field with the developed Excel spreadsheets. Use of these IC conversions can reduce trial weight runs in the field, thus reducing plant cost significantly while achieving satisfactory balancing results and bringing rotating machines in safe operation quickly.

It should be noted that cross effects among a multibody turbomachinery could exist sometimes, especially for those connected with rigid couplings. The author expressed string cross effects between a generator rotor and an IP turbine rotor that one end or static/couple pair weight on either generator or IP turbine balance planes would significantly affect vibrations at all four bearings. Successful balancing would involve more than two balance planes, and one can even use a combination of static/couple pair weight among two or more adjacent bodies in a multiplane balance program. However, the current introduced conversion equations still hold within the concerned body supported by two bearings, with or without the cross effects due to any adjacent body. A seemingly symmetric rotor such as generator rotor exhibits non-zero $\vec{H}_{CS}$ and $\vec{H}_{SC}$ just because of the adjacent bodies connected.

NOMENCLATURE

$\vec{A}_1$	=1X synchronous vibration vector measured by probe 1
$\vec{A}_2$	=1X synchronous vibration vector measured by probe 1
$\vec{C}$	= Couple vibration vector (180 degrees out-of-phase)
$\vec{C}_C$	= Couple vibration vector due to couple pair weight
$\vec{C}_S$	= Couple vibration vector due to static pair weight
$\vec{H}_{CC}$	= Couple vibration influence vector due to couple pair weight
$\vec{H}_{CS}$	=Couple vibration influence vector due to couple pair weight
$\vec{H}_{SC}$	= Static vibration influence vector due to couple pair weight
$\vec{H}_{SS}$	= Static vibration influence vector due to static pair weight
$\vec{h}_{11}$	= Probe 1 vibration influence vector due to Plane 1 weight
$\vec{h}_{12}$	= Probe 1 vibration influence vector due to Plane 2 weight
$\vec{h}_{21}$	= Probe 2 vibration influence vector due to Plane 1 weight
$\vec{h}_{22}$	= Probe 2 vibration influence vector due to Plane 2 weight
$\vec{h}_{1,C}$	= Probe 1 vibration influence vector due to couple pair weight
$\vec{h}_{2,C}$	= Probe 2 vibration influence vector due to couple pair weight
$\vec{h}_{1,S}$	= Probe 1 vibration influence vector due to static pair weight
$\vec{h}_{2,S}$	= Probe 2 vibration influence vector due to static pair weight
$\vec{S}$	= Static vibration vector (in-phase)
$\vec{S}_C$	= Static vibration vector due to couple pair weight
$\vec{S}_S$	= Static vibration vector due to static pair weight
$\vec{W}_1$	= Weight vector at balance Plane 1
$\vec{W}_2$	= Weight vector at balance Plane 2
$\vec{W}_C$	= Couple pair weight vector (weight at two ends 180 degrees out-of-phase; the vector aligned with the weight at Plane 1)
$\vec{W}_S$	= Static pair weight vector (weight at two ends in-phase)
$\Delta\vec{A}_{1,C}$	= $\vec{A}_1$ due to couple pair weight - initial $\vec{A}_1$
$\Delta\vec{A}_{2,C}$	= $\vec{A}_2$ due to couple pair weight - initial $\vec{A}_2$
$\Delta\vec{A}_{1,S}$	= $\vec{A}_1$ due to static pair weight - initial $\vec{A}_1$
$\Delta\vec{A}_{2,S}$	= $\vec{A}_2$ due to static pair weight - initial $\vec{A}_2$
$\Delta\vec{C}_C$	= $\vec{C}$ due to couple pair weight - initial $\vec{C}$
$\Delta\vec{C}_S$	= $\vec{C}$ due to static pair weight - initial $\vec{C}$
$\Delta\vec{S}_C$	= $\vec{S}$ due to couple pair weight - initial $\vec{S}$

$\Delta \vec{S}_S \quad = \vec{S}$ due to static pair weight - initial $\vec{S}$

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