TURBOMACHINERY & PUMP SYMPOSIA | HOUSTON, TX
DECEMBER 14-16, 2021
SHORT COURSES: DECEMBER 13, 2021

AN END-USER'S GUIDE TO CENTRIFUGAL PUMP ROTORDYNAMICS

William D. Marscher
CEO & Senior Consultant
Mechanical Solutions, Inc.
Whippany, NJ, USA

Bill Marscher, P.E. founded Mechanical Solutions, Inc. in 1996, which has grown to a consulting firm of 30. Bill has been an attendee since the beginning of the Pump and Turbo Symposia, and is a long-standing member of the Pump Advisory Committee. He has BS and MS degrees in Mechanical Engineering from Cornell University, and an MS in Mechanics from RPI. Bill previously worked at Worthington/ Dresser Pump, Pratt & Whitney, and CNREC. He is past president of the Society of Tribologists & Lubrication Engineers, as well as the Machinery Failure Prevention Technology society of the Vibration Institute. He is a member of the ISO TC108 Machinery Standards Committee, and is recent Vice Chair of the Hydraulic Institute Vibration Standards Committee. In 2019, he was awarded the Jack Frarey Award of the Vibration Institute.

ABSTRACT

This tutorial outlines the basics of pump rotordynamics in a form that is intended to be Machinery End User friendly. Key concepts will be defined in understandable terms, and analysis and testing options will be presented in summary form. The presentation will explain the reasoning behind the HI, ISO, and API-610 rotor and structural vibration evaluation requirements, and will summarize key portions of API-RP-684 "API Standard Paragraphs Covering Rotordynamics" as it applies to pumps.

Pump rotordynamic problems, including the bearing and seal failure problems that they may cause, are responsible for a significant amount of the maintenance budget and lost-opportunity cost at many refineries and electric utilities. This tutorial discusses the typical types of pump rotordynamic problems, and how they can be avoided in most cases by applying the right kinds of vibration analysis and evaluation criteria during the pump design and selection/ application process. Although End Users seldom are directly involved in designing a pump, it is becoming more typical that the reliability-conscious End User or his consultant will audit whether the OEM has performed due diligence in the course of pump design. In the case of rotordynamics, important issues include where the pump is operating on its curve (preferably close to BEP), how close the pump rotor critical speeds and rotor-support structural natural frequencies are to running speed or other strong forcing frequencies, how much vibration will occur at bearings or within close running clearances for expected worst case imbalance and misalignment, and whether or not the rotor system is likely to behave in a stable, predictable manner.

When and why rotordynamics analysis or finite element analysis might be performed will be discussed, as well as what kinds of information these analyses can provide to an end user that could be critical to reliable and trouble-free operation. A specific case history will be presented of a typical problematic situation that plants have faced, and what types of solution options were effective at providing a permanent fix.

INTRODUCTION

Both fatigue and rubbing wear in pump components are most commonly caused by excess rotor vibration. Sources of excess vibration include the rotor being out of balance, the presence of too great a misalignment between the pump and driver shaft centerlines, excessive hydraulic force such as from suction recirculation stall or vane pass pressure pulsations, or large motion amplified by a natural frequency resonance. Inspection of parts will often provide clues concerning the nature of damaging vibration, and may therefore suggest how to get rid of it.

For example, when casing wear is at a single clock position but around the full shaft circumference, pump/driver misalignment is a likely direct cause, although perhaps excessive nozzle loads or improperly compensated thermal growth of the driver led to the misalignment. On the other hand, if wear is at only one clock location on the shaft and full-circle around the opposing stator piece (e.g. a bearing shell or a wear ring), the likely issue is rotor imbalance or shaft bow. If wear occurs over 360 degrees of both the rotor and the stator, this implies strong asynchronous vibration, and rotordynamic instability or low flow suction recirculation should be considered as the cause.

Fortunately, there are pre-emptive procedures which minimize the chance for encountering such problems, or which help to determine how to solve such problems if they occur. These rotordynamic procedures are the subject of this tutorial.

Vibration Concepts- General

All of us know by intuition that excessive vibration can be caused by shaking forces ("excitation forces") that are higher than normal. For example, maybe the rotor imbalance is too high. Such shaking forces could be mechanically sourced (such as the imbalance) or hydraulically based (such as from piping pressure pulsations). They can even be electrically based (such as from uneven air gap in a motor, or from VFD harmonic pulses). In all these cases, high rotor vibration is typically primarily rotor increased oscillating displacement "x" in response to the shaking force "F" working against the rotor-bearing support stiffness "k". In equation form, $F = k*x$, and calculating x for a given F is known as "forced response analysis".

However, sometimes all of the shaking forces are actually reasonably low, but still excessive vibration is encountered. This can be an unfortunate circumstance during system commissioning, leading to violation of vibration specifications, particularly in variable speed systems where the chances are greater that an excitation force's frequency will equal a "natural frequency" over at least part of the running speed range. This situation is known as resonance, which amplifies the effect of the force. A key reason for performing rotordynamic analysis is to check for the possibility of resonance.

Rotordynamic testing likewise should include consideration of possible resonance. In rotor vibration troubleshooting, it is recommended to first investigate imbalance, then misalignment, and then natural frequency resonance, in that order, as likely causes, unless the specific vibration vs. frequency plot (the "spectrum") or vibration vs. time pulsations indicate other issues (some of these other issues will be discussed in some detail later). Resonance is illustrated in Figure 1.

An important concept mentioned above is the "natural frequency", which is the number of cycles per minute that the rotor or supporting structure will vibrate at if it is "rapped", like a tuning fork. Pump rotors and casings have many natural frequencies, some of which may be at or close to the operating speed range or its strong integer harmonics, thereby causing a "resonance". The vibrating pattern which results when a natural frequency is close to the running speed or some other strong force's frequency is known as a "mode shape". Each natural frequency has a different mode shape associated with it, and where this shape moves the most is generally the most sensitive, worst case place for an exciting force such as imbalance to be applied, but similarly is the best place to try a "fix" such as a gusset or some added mass.

In resonance, the "ring-out" vibration energy from previous "hits" of the force come full cycle exactly when the next hit takes place. The vibration in the next cycle will then include movement due to all hits up to that point, and will be higher than it would have been for one hit alone (the principle is the same as a child's paddle-ball). The vibration motion keeps being amplified in this way until its large motion uses up as much energy as that which is being supplied by each new hit. Unfortunately, the motion at this point is generally quite large, and is often damaging to bearings, seals, and internal running clearances (e.g. wear rings).

It is desirable that the natural frequencies of the rotor and bearing housings are well separated from the frequencies that such "dribbling" type forces will occur at. These forces most often tend to be 1x running speed (typical of imbalance), 2x running speed (typical of misalignment), or at the number of impeller vanes times running speed (so-called "vane pass" vibrations from discharge pressure pulses as the impeller vanes move past a volute or diffuser vane "cut-water").

In practice, the vibration amplification (often called "Q" as shown in Figure 1) due to resonance is usually between a factor of two and twenty five higher than it would be if the force causing the vibration was steady instead of oscillating. The level of Q depends on the amount of energy absorption, called "damping", which takes place between hits. In an automobile body, this damping is provided by the shock absorbers. In a pump, it is provided mostly by the bearings and the liquid trapped between the rotor and stator in "annular seals” like the wear rings and balance piston.

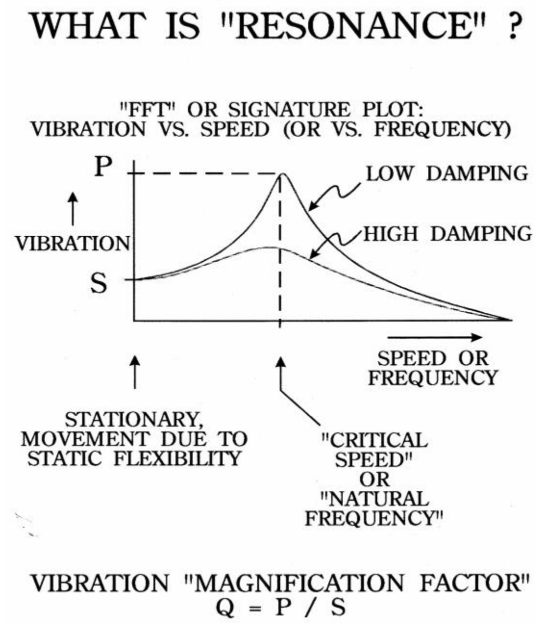

Figure 1. Illustration of Natural Frequency Resonance, and Effects of Damping

If the damping is near the point where it just barely halts oscillating motion (this is how automobile shocks are supposed to operate, to provide a smooth ride), the situation is known as “critical damping”. The ratio of the actual to the critical damping is how a rotor system’s resistance to resonant vibration is best judged. In other terms that may be more familiar, for practical values of the damping ratio, 2 times pi times the damping ratio approximately equals the logarithmic decrement or “log dec” (measures how much the vibration decays from one ring-down bounce to the next). Also, the amplification factor Q equals roughly 1/(2*damping ratio), or pi/(log dec).

One way to live with resonance (not recommended for long) is to increase the damping ratio by closing down annular seal clearances, or switching to a bearing that by its nature has more energy absorption (e.g. a journal bearing rather than an antifriction bearing). This may decrease Q to the point where it will not cause rubbing damage or other vibration related deterioration. For this reason, the API-610 Centrifugal Pump Standard does not consider a natural frequency a “critical speed” (i.e. a natural frequency of more than academic interest) if its Q is 3.3 or less. The problem with any approach relying on damping out vibration is that whatever mechanism (such as tighter wear ring clearance) is used to increase damping may not last throughout the expected life of the pump.

A counter-intuitive but important concept is the "phase angle", which measures the rotational lag between the application of a force and the vibrating motion which occurs in response to it. An example of the physical concept of phase angle is given in Figures 2 and 3. A phase angle of zero degrees means that the force and the vibration due to it act in the same direction, moving in step with one another. This occurs at very low frequencies, well below the natural frequency. An example of this is a force being slowly applied to a soft spring. Alternately, a phase angle of 180 degrees means that the force and the vibration due to it act in exactly opposite directions, so that they are perfectly out of step with each other. This occurs at very high frequencies, well above the natural frequency. This happens when the vibration forces are “mass-times-acceleration-dominated”, which happens at high frequencies, above the natural frequency closest to the excitation force frequency. Exactly at the natural frequency the phase is 90 degrees, because mass and spring forces balance leaving only damping force. Damping force is proportional to vibration velocity, and velocity lags behind oscillating force application by 90 degrees.

Knowledge of phase angle is important because it can be used together with peaks in vibration field data to positively identify natural frequencies as the dominant factor in excessive vibration, as opposed to excessive excitation forces. This identification is necessary in order to determine what steps should be taken to solve a large number of vibration problems, including rotordynamic instability problems (San Andres, 2006B).

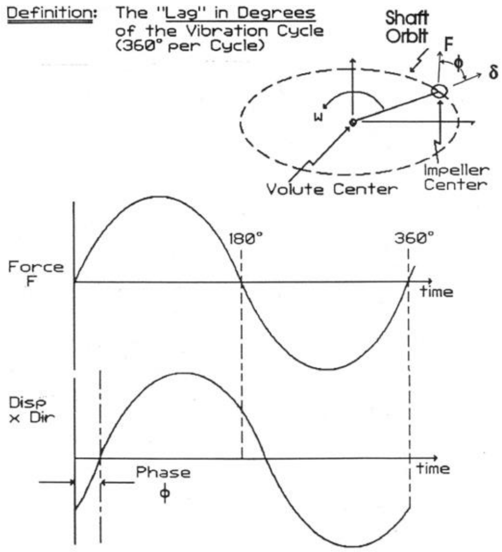

Figure 2. Definition of Phase Angle Particular to Rotors

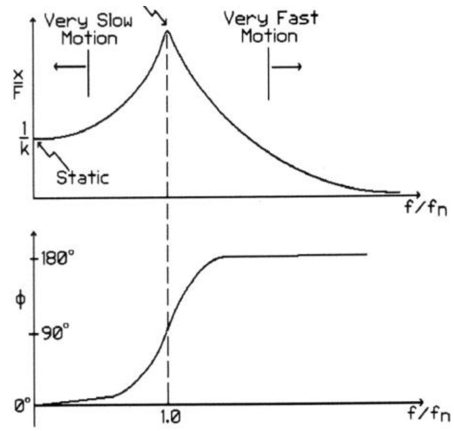

Figure 3. Relationship of Phase Angle to Frequency of Vibration Concepts

Balance

Based on End User surveys by EPRI (Electrical Power Research Institute, 1980) and others, imbalance is the most common cause of excessive vibration in machinery, followed closely by misalignment. As illustrated in Figure 4, balance is typically thought of as static (involves the center-of-mass being off-center so that the principal axis of mass distribution- i.e. the axis that the rotor would spin "cleanly" without wobble, like a top- is still parallel to the rotational centerline) and dynamic (the principal mass axis makes an angle with the rotational axis). For axially short components (e.g. a thrust washer) the difference between these two can be neglected, and only single plane static balancing is required. For components greater in length than 1/6 their diameter, dynamic imbalance should be assumed, and at least two plane balancing is required by careful specifications such as API-610 (API 2011). For rotors operating above their second critical speed (unusual for pumps), even two plane balance may not be enough because of the multiple turns in the rotor's vibration pattern, and some form of at-speed modal balancing (i.e. balancing material removal/addition location that takes into account the closest natural frequency mode shape) may be required.

When imbalance occurs, including imbalance caused by shaft bow, its shows up with a frequency of exactly 1x running speed N, as shown by the orbit and amplitude vs. frequency plot (a "spectrum") in Fig. 5. The 1xN is because the heavy side of the rotor is rotating at exactly rotating speed, and so forces vibration movement at exactly this frequency (Downham 1957, Gunter 1978, San Andres 2006B). Typically, this also results in a circular shaft orbit, although the orbit may be oval if the rotor is highly loaded within a journal bearing, or may have spikes if imbalance is high enough that rubbing is induced. ISO-1940 (ISO, 2016) provides information on how to characterize imbalance, and defines various balance Grades. The API-610 11th Edition/ ISO 13709 specification recommends ISO balance grades for various types of service. Generally, the recommended levels are between the old US Navy criterion of 4W/N (W= rotor weight in pounds mass, and N is rotor speed in RPM), which is roughly ISO G0.66, and the more typical ISO G2.5. As admitted in API-610, levels below ISO G1 are not practical in most circumstances because in removing the impeller from the balance arbor it loses this balance level, which typically requires the center of gravity to remain centered within several millionths of an inch. For loose fitting impellers, no balance requirement is given, but in practice G6.3 (about 40W/N) is used by industry. The ultimate test of balance adequacy, as well as rotordynamic behavior in general, is whether the pump vibration is within the requirements of the pump vibration standard, such as international standard ISO-10816-7 (ISO, 2017).

Next to imbalance, misalignment is the most common cause of vibration problems in rotating machinery. Misalignment is usually distinguished by two forms: offset, and angular, as discussed in detail in the excellent recent standard ANSI/ASA S2.75 (ANSI, 2017). As illustrated in Fig. 6, offset is the amount that the two centerlines are laterally separated from each other (i.e. the distance between the centerlines when extended to be next to each other). Angular is the differential crossing angle that the two shaft centerlines make when projected into each other, when viewed from first the top, and then in a separate evaluation from the side. In general, misalignment is a combination of both offset and angular misalignment. Offset misalignment requires either a uniform horizontal shift or a cosistent vertical shimming of all feet of either the pump or its driver. Angular misalignment requires a horizontal shift of only one end of one of the machines, or a vertical shimming of just the front or rear set of feet. Combined offset and angular misalignment requires shimming and/ or horizontal movement of four of the combined eight feet of the pump and its driver. In principle, shimming and/ or horizontal shifting of four feet only should be sufficient to cure a misalignment.

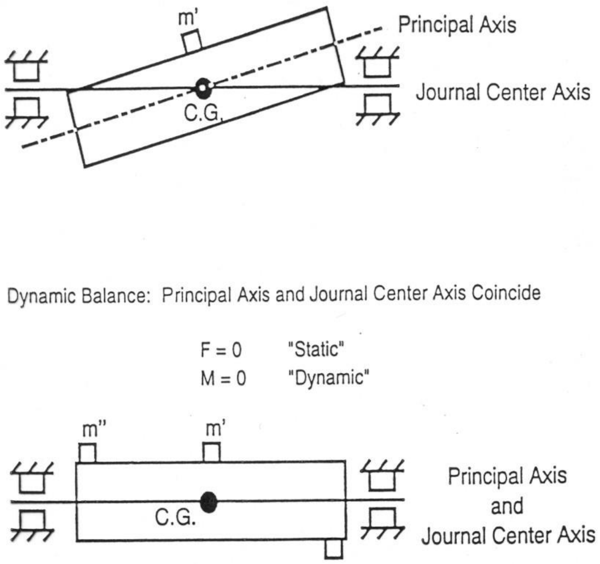

Figure 4. Static vs. Dynamic Imbalance

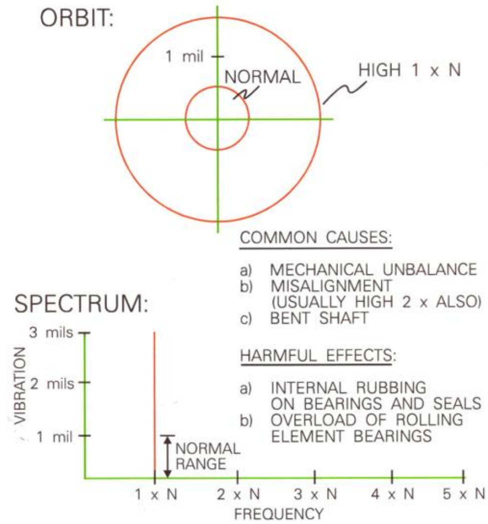

Figure 5. Imbalance Example of Orbit and FFT

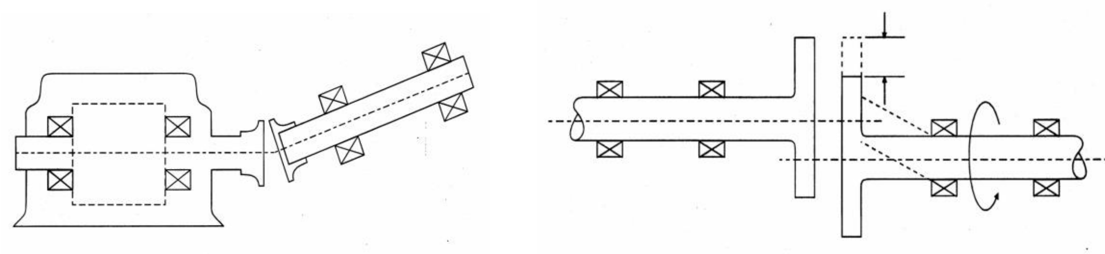

Figure 6. Illustration of Angular and Offset Misalignment

Typical requirements for offset and angular misalignment at 3600 rpm are between ½ mil and 1 mil offset, and between ¼ and ½ mil/ inch space between coupling hubs, for angular. For speeds other than 3600 rpm, the allowable levels are roughly inversely proportional to speed. However, industrial good practice (although this depends on a lot of factors including service) typically allows a maximum misalignment level of 2 mils offset or 1 mil/ inch as speed is decreased. When misalignment is a problem, it typically causes primarily 2x running speed, because of the highly elliptical orbit that it forces the shaft to run "cramped" on the misaligned side. Sometimes the misalignment load can cause higher harmonics (i.e. rotor speed integer multiples, especially 3x), and may even decrease vibration, because it loads the rotor unnaturally hard against its bearing shell. Alternately, misalignment may actually cause increased 1x vibration, by lifting the rotor out of its gravity-loaded "bearing pocket", i.e. the zone of minimum lubricant film, to result in the bearing running relatively unloaded (this can also cause shaft instability, as discussed later). Figure 7 shows a typical orbit and FFT spectrum for misalignment, in which 2x running speed is the dominant effect. This is often accompanied by relatively large axial motion, also at 2x, because the coupling experiences a non-linear "crimp" twice per revolution.

Because the rotor vibration effects from imbalance and misalignment are typically present at some combination of 1x and 2x running speed, and because studies show that imbalance and misalignment are by far the most common source of excessive pump rotor vibration (EPRI 1980), API-610 11th Edition requires that 1x and 2x running speed be accounted for in any rotordynamics analysis, and that any critical speeds close to 1x or 2x be sufficiently damped out. A damping ratio as high as 0.15 is required if a natural frequency is close to 1x or 2x running speed.

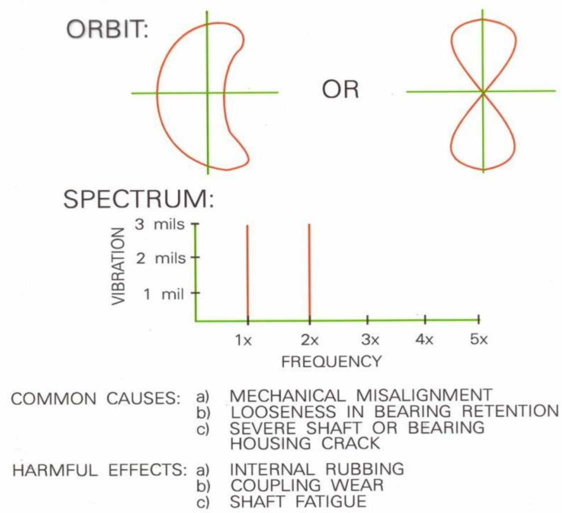

Figure 7. Misalignment Example of Shaft Orbit and FFT Spectrum

Gyroscopic Effects
Gyroscopic forces are important, and can either effectively stiffen or de-stiffen a rotor system. The key factor is the ratio of polar moment of inertia "Ip", the second mass moment taken about the rotor axis, to transverse moment of inertia "It", taken about one of the two axes through the center of mass and perpendicular to the rotor axis. This ratio is multiplied times the ratio of the running speed divided by the orbit or "whirl" speed. As shown in Fig. 8, the whirl speed is the rate of precession of the rotor, which can be "forward" (in the same direction as running speed) or "retrograde" or "backward" (opposite in direction to running speed), as dicussed by San Andres (San Andres 2006B). The whirl or precessional speed absolute value is generally less than the running speed. It is very difficult to excite backward whirl in turbomachinery because typically all forces of significance are rotating in the same direction as shaft rotation, so the forward whirl mode is typically the only one of practical concern. If the product of the inertia and speed ratio is less than 1.0, then the gyroscopic moment is de-stiffening relative to forward whirl, while if it is greater than 1.0, it tends to keep the rotor spinning about its center axis (i.e. the principle of a gyroscope) and thus contributes apparent stiffness to the rotor system, raising its forward whirl natural frequencies. It is the latter situation that designers try to achieve. In industrial pumps operating at 3600 rpm and below, gyroscopic effect is generally of secondary importance, and while it should be accounted in the rotordynamic analysis, the ratio of Ip to It does not need to be considered in any specification, and care only needs to be taken with regard to the net critical speed separation margin as a function of damping ratio or amplification factor Q.

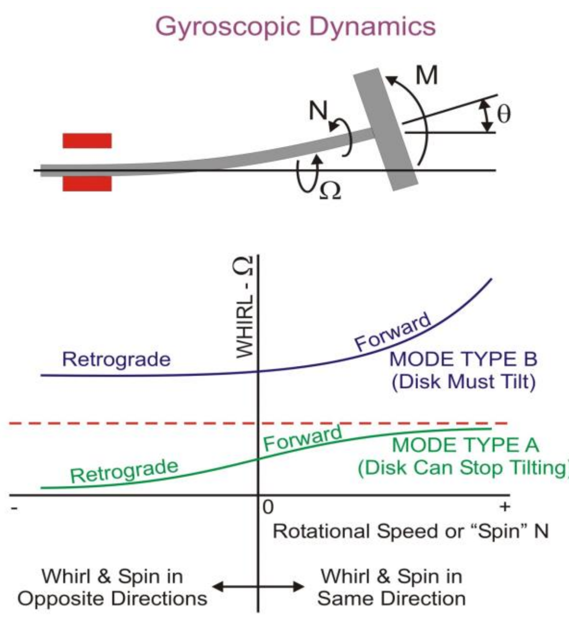

Figure 8. Illustration of Gyroscopics: Effect of Speed (Spin) on Critical Speeds (Whirl)

Rotordynamic Stability
Rotordynamic stability refers to phenomena whereby the rotor and its system of reactive support forces are able to become self-excited, leading to potentially catastrophic vibration levels even if the active, stable excitation forces are quite low (San Andres 2006, Childs 1985, Childs 2013, Marscher 2008). Instability can occur if a pump rotor's natural frequency is in the range where fluid whirling forces

(almost always below running speed, and usually about ½ running speed) can “synch-up” with the rotor whirl. This normally can occur only for relatively flexible multistage pump rotors. In addition to the “subsynchronous” natural frequency, the effective damping associated with this natural frequency must somehow drop below zero. An example of subsynchronous vibration (not always unstable) is given in Figure 9.

Cross-Coupling vs. Damping & “Log Dec”

Cross-coupled stiffness originates due to the way fluid films build up hydrodynamically in bearings and other annular close running clearances, as shown in Figure 10. The cross-coupling force vector acts in a direction directly opposite to the vector from fluid damping, and therefore many people think of it in terms of an effectively negative damping. The action of cross-coupling is very important to stability, in that if the cross-coupling force vector becomes greater than the damping vector, vibration causes reaction forces that lead to ever more vibration, in a feedback fashion, increasing orbit size until either a severe rub occurs, or the feedback stops because of fluid disruption by the large motion.

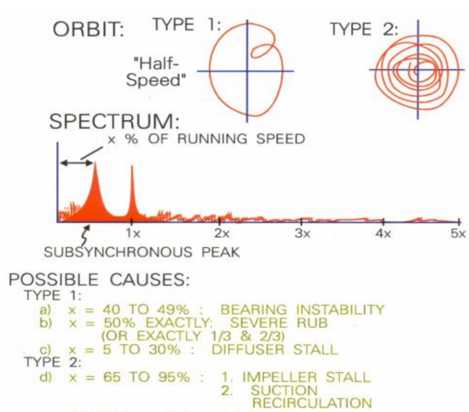

Figure 9. Subsynchronous Vibration

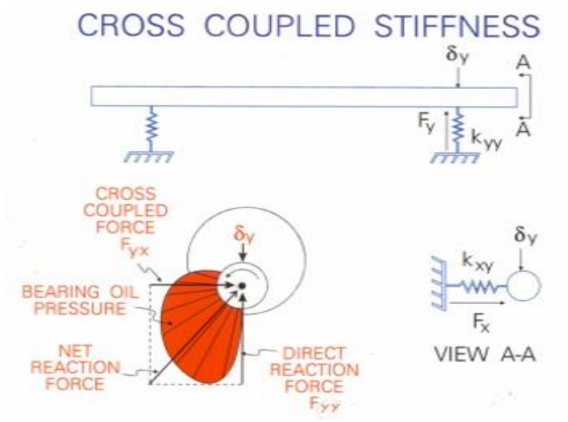

Figure 10. Cross-Coupled Stiffness

Subsynchronous Whirl & Whip

Shaft whirl is a forced response at a frequency usually below running speed, typically driven by a rotating fluid pressure field. The fluid rotational speed becomes the whirl speed of the rotor. The most common cause of whirl is fluid rotation around the impeller front or back shrouds, in journal bearings, or in the balance drum clearances. Such fluid rotation is typically about 48 percent of running speed, because the fluid is stationary at the stator wall, and rotating at the rotor velocity at the rotor surface, such that a roughly half speed flow distribution is established in the running clearance. The pressure distribution which drives this whirl is generally skewed such that the cross-coupled portion of it points in the direction of fluid rotational flow at the “pinch gap”, and can be strong. If somehow clearance is decreased on one side of the gap, due to eccentricity for example, the resulting cross-coupled force increases further, as implied by Figure 10.

As seen in Figure 10, the cross-coupled force acts perpendicular to any clearance closure. In other words, the cross-coupling force acts in the direction that the whirling shaft minimum clearance will be in another 90 degrees of whirl rotation. If the roughly half speed frequency the cross-coupled force and minimum clearance are whirling at becomes equal to a natural frequency, a 90 degree phase shift occurs, because of the excitation of resonance, as shown in Figures 2 and 3. Recall that phase shift means a delay in when the force is applied versus when its effect is reacted to. This means that the motion in response to the cross-coupling force is delayed from acting for 90 degrees worth of whirl rotation. By the time it acts, therefore, the cross-coupled force tends to act in a direction to further close the already tight minimum gap. As the gap closes in response, the cross-coupled force (which is roughly inversely proportional to this gap) increases further. The cycle continues until all gap is used up, and the rotor is severely rubbing. This process is called shaft whip, and is a dynamic instability in the sense that the process is self-excited once it initiates, no matter how well the rotor is machined, how good the balance and alignment are, etc. The slightest imperfection starts the process, and then it provides its own exciting force in a manner that spirals out of control (Marscher 1997).

The nature of shaft whip is that, once it starts, all self-excitation occurs at the unstable natural frequency of the shaft, so the vibration response frequency "locks on" to the natural frequency. Since whip begins when whirl, which is typically close to half the running speed, is equal to the shaft natural frequency, the normal 1x running speed frequency spectrum and roughly circular shaft orbit at that point show a strong component at about 48 percent of running speed, which in the orbit shows up as a loop, implying pulsating orbit growth and then decrease every other revolution. A typical observation in this situation is the "lock on" of vibration onto the natural frequency, causing whip vibration at speeds above whip initiation to deviate from the whirl's previously constant 48% (or so) percentage of running speed, becoming constant frequency instead, since the amplification of the subsynchronous motion by the whirl-resonant natural frequency now dominates all motion.

Stabilizing Component Modifications

One method of overcoming rotordynamic instability is to reduce the cross-coupling force which drives it. A complementary solution is to increase system damping to the point that the damping vector, which acts exactly opposite to the direction of the cross-coupling vector, overcomes the cross-coupling. The amount of damping required to do this is commonly determined in terms of "log dec", which is roughly 2∗pi∗damping ratio. For turbomachines including centrifugal pumps, it has been found that if the net log dec is calculated to be greater than about 0.1 then it is likely to provide enough margin versus the unstable value of zero, so that damping will overcome any cross-coupling forces which are present, avoiding rotor instability.

Typical design modifications which reduce the tendency to rotordynamic instability involve bearing and/ or seal changes, to reduce cross-coupling and hopefully simultaneously increase damping. The worst type of bearing with regard to rotordynamic instability is the plain journal bearing, which has very high cross-coupling. Other bearing concepts, with elliptical or offset bores, fixed pads, or tilting pads, tend to reduce cross-coupling, dramatically so in terms of the axially grooved and tilting pad style bearings. Another bearing fairly effective in reducing cross-coupling relative to damping is the pressure dam bearing. Even more effective and controllable, at least in principle, are the hydrostatic bearing, and actively controlled magnetic bearing. Fortunately, damping is typically so high in industrial centrifugal pumps that any bearing type, even the plain journal, results in a rotor system that usually is stable throughout the range of speeds and loads over which the pump must run. High speed pumps such as rocket turbopumps are an exception, and their rotordynamic stability must be carefully assessed as part of their design process.

Rotor Vibration Concepts Particular to Centrifugal Pumps

It is always recommended to select a pump which will typically operate close to its Best Efficiency Point (“BEP”). Contrary to intuition, centrifugal pumps do not undergo less nozzle loading and vibration as they are throttled back, unless the throttling is accomplished by variable speed operation. Operation well below the BEP at any given speed, just like operation well above that point, causes a mismatch in flow incidence angles at the impeller vanes and the diffuser vanes or volute tongues of the various stages. This loads up the vanes, and may even lead to “airfoil stalling”, with associated formation of strong vortices (miniature tornadoes) that can severely shake the entire rotor system at subsynchronous frequencies (which can result in vibration which is high, but not unbounded like a rotor instability), and can even lead to fatigue of impeller shrouds or diffuser annular walls or “strong-backs”. The rotor impeller steady side-loads and shaking occurs at flows below the onset of suction or discharge recirculation (Fraser 1985, Bowman 1990). The typical effect on rotor vibration of the operation of a pump at off-design flows is shown in Fig. 11. If a plant must run a pump away from its BEP because of an emergency situation, plant economics, or other operational constraints, at least never run a pump for extended periods at flows below the “minimum continuous flow” provided by the manufacturer. Also, if this flow was specified prior to about 1985, it may be based only on avoidance of high temperature flashing (occurring due to temperature build-up from the energy being repeatedly added to the continuously recirculating processed flow) and not on recirculation onset which normally occurs at higher flows than flashing, and should be re-checked with the manufacturer.

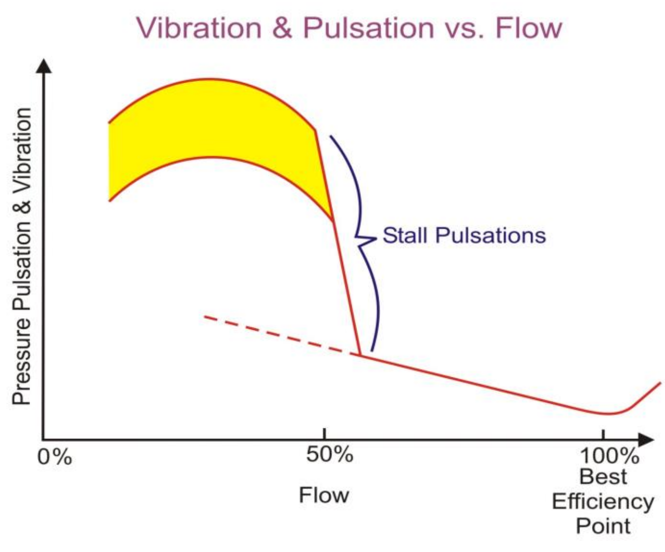

Figure 11. Effect on Vibration on Off-BEP Operation

Figure 12 shows a typical orbit and frequency spectrum due to high vane pass forces. These force levels are proportional to discharge pressure and impeller diameter times OD flow passage width, but in other respects as well are very design dependent. Vane pass forces are particularly affected by the presence (or not) of a front shroud, the flow rate versus BEP, and the size of certain critical flow gaps. In particular, these forces can be minimized by limiting “Gap A” (the “Annular” radial gap between the impeller shroud and/ or hub OD and the casing wall), and by making sure that impeller “Blade”/ diffuser vane (or volute tongue) “Gap B” is sufficiently large. Pump gapping expert Dr. Elemer Makay recommended a radial Gap A to radius ratio of about 0.01 (in combination with a shroud/ casing axial “overlap” at least 5x this long), and recommended a radial Gap B to radius ratio of about 0.05 to 0.012 (EPRI 1985). API-610 11th Edition for Centrifugal Pumps in Petrochemical Service (API 2011) makes no mention of Gap A, but recommends a minimum Gap B of 3% for diffuser pumps and 6% for volute pumps.

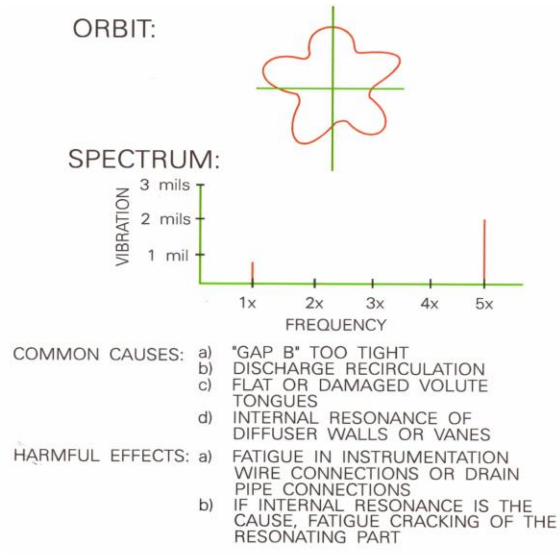

Figure 12. Vane Pass Vibration

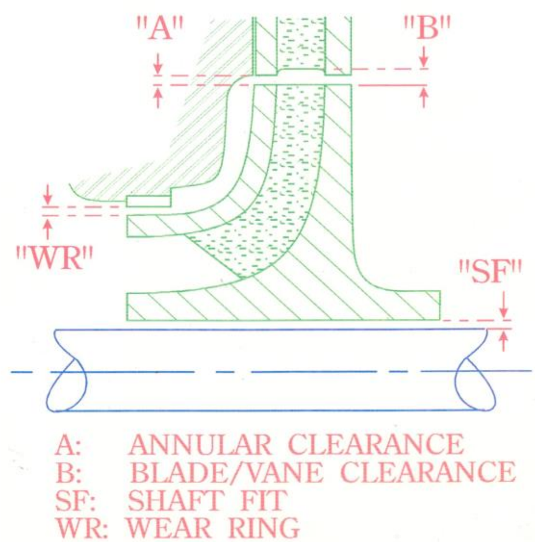

Figure 13. Various Impeller Gaps of Importance

Figure 13 illustrates Gap A and Gap B, as well as the wear ring clearance gap (discussed later) and the shaft fit-up gap (discussed above).

Fluid "Added Mass"

The fluid surrounding the rotor adds inertia to the rotor in three ways: the fluid trapped in the impeller passages adds mass directly, and this can be calculated based on the volume in the impeller passages times the pumped fluid density. However, there is also fluid around the periphery of the impellers that is displaced by the vibrating motion of the impellers. This is discussed in the literature (Blevins 1984, Marscher 2013), which show how this part of the added mass is equal to the "swept volume" of the impellers and immersed shafting, times the density of the pumped liquid. Depending upon impeller type based on the pump specific speed, this second form of added mass can be a significant percentage of the impeller metal mass. One other type of added mass, which is typically small but can be significant for high frequency vibration (such as in rocket turbopumps) or for long L/D passages (like in a canned motor pump) is the fluid in close clearances, which must accelerate to get out of the way of the vibrating rotor. The way the clearance real estate works out in a close clearance passage, the liquid on the closing side of the gap must accelerate much faster than the rotor itself in order to make way for the rotor volume. This is sometimes called "Stokes Effect", and is best accounted for by a computer program, such as the annular seal codes available from the TAMU TurboLab (San Andres 2006B).

Annular Seal "Lomakin Effect"
Annular seals (e.g. wear rings and balance drums) in centrifugal pumps and hydraulic turbines can greatly affect dynamics by changing the rotor support stiffness and therefore the rotor natural frequencies, thereby either avoiding or inducing possible resonance between strong forcing frequencies at one and two times the running speed and one of the lower natural frequencies (Childs 1982, Childs 1985, Marquette 1997, Marscher 1989). Their effect is so strong for multistage pumps that API-610 11th Edition requires that they be taken into account for pumps of three or more stages, and that their clearances be assessed for both the as-new and 2x clearance "worn" conditions. This provision by API is because the stiffness portion of this "Lomakin Effect" (first noticed by the Russian pump researcher Lomakin) is inversely proportional to radial clearance. It is also directly proportional to the pressure drop and (roughly) the product of the seal diameter and length. An illustration of how Lomakin Effect sets up is given in Figure 14.

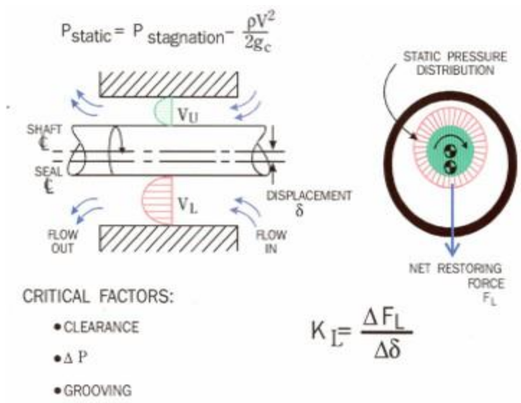

Figure 14. Illustration of the Lomakin Effect Stiffness KL in an Annular Sealing Passage

In Figure 14, Pstagnation is the total pressure upstream of the annular seal such as a wearing ring or balance drum, VU is the average gap leakage velocity in the upper (closer clearance in this case) gap and VL is the average gap leakage velocity in the lower (larger clearance in this case) gap. The parameter ρ/ gc is the density divided by the gravitational constant 386 lbm/lbf-in/sec^2. The stiffness and damping in an annular seal such as that shown in Figure 14 is provided in small part by the squeeze-film and hydrodynamic wedge effects well known to journal bearing designers. However, as shown in Fig. 14, because of the high ratio of axial to circumferential flow rates in annular liquid seals (bearings have very little axial flow, by design), large forces can develop in the annular clearance space due to the circumferentially varying Bernoulli pressure drop induced as rotor eccentricity develops, due to primarily the increased entrance loss at the clock position of the clearance that tightens as eccentricity develops (Yang & San Andres 2019, Marquette 1997). This is a hydrostatic effect rather than a hydrodynamic one, in that it does not build up a circumferential fluid "wedge" and thus does not require a viscous fluid like a journal bearing does (Marscher 1999A, San Andres 1993). In fact, highly viscous fluids like oil develop less circumferential variation in pressure drop, and therefore typically have less Lomakin Effect than a fluid like, for example, water. The Lomakin Effect stiffness within pump annular seals is not as stiff as the pump bearings, but is located in a strategically good location to resist rotor vibration, being in the middle of the pump where no classical bearing support is present.

The Lomakin Effect depends directly on the pressure drop across the seal, which for pump system flow parabolic resistance characteristic (e.g. from an orifice or a valve) results in a variation of the Lomakin support stiffness with roughly the square of the running speed. However, if the static head of the system is high compared to the discharge head, as in many boiler feed pumps for example, the more nearly constant system head results in only a small variation of Lomakin Effect with pump speed.

As rule of thumb, for short plain annular seals (e.g. ungrooved wear rings) in water, the Lomakin Effect stiffness is approximately equal to 0.4 times the pressure drop across the seal times the seal diameter times the seal length, divided by the seal diametral clearance. For grooved seals or long L/D (greater than 0.5) seals, the coefficient 0.4 diminishes by typically a factor of 2 to 10 (Marscher 1989).

The physical reason for the strong influence of clearance is that it gives the opportunity for the circumferential pressure distribution, which is behind the Lomakin Effect, to diminish through circumferential flow. Any annular seal cavity which includes circumferential grooving (e.g. "labyrinth" seals) has the same effect as increased clearance, to some degree. Deep grooves have more effect than shallow ones in this regard (San Andres 2018). If grooving is necessary but Lomakin Effect is to be maximized, grooves should be short in axial length, and radially shallow.

Impeller Forces
As an impeller moves within its diffuser or volute, reaction forces set up because of the resulting non-symmetrical static pressure distribution around the periphery of the impeller (Agostinelli 1960). These forces are normally represented by coefficients which are linear with displacement. The primary reaction forces are typically a negative direct stiffness, and a cross-coupling stiffness. Both of these forces tend to be destabilizing in situations, potentially a problem in cases where damping is low (i.e. log dec below 0.1) and where stability therefore is an issue. Their value is significant for high speed pumps such as rocket turbopumps, but is typically secondary in

industrial pump rotordynamic behavior.

Along with reactive forces (i.e. forces that change with eccentricity or vibration displacement), there are also active forces which exist independently of the impeller motion and are not affected substantially by it. These forces are "excitation forces" for the vibration. They include the 1x, 2x, and vane pass excitation forces discussed earlier (HI/ANSI 9.6.8 2014). The worst case 1x and 2x levels that should be used in a rotordynamic analysis are based on the specification's (e.g. API-610 or ISO-1940) allowable worst case imbalance force and misalignment offset and/ or angular deflections discussed earlier. The worst case zero-peak amplitude vane pass levels for an impeller are typically (in the author's experience) between five and fifty percent of the product of the pressure rise for that stage times the impeller OD times the exit flow passage width. Near BEP, the five percent value is a best guess in the absence of OEM or field test data, while close to the minimum continuous flow fifty percent is a worst case estimate (although a more likely value is 10 percen, particularly for low specific speed pumps).

Lateral Vibration Analysis of Pump Rotor Systems

Manual Methods

For certain simple pump designs, particularly single stage pumps, rotordynamic analysis can be simplified while retaining first-order accuracy. This allows manual methods, such as mass-on-spring or beam formulas, to be used. For example, for single stage double suction between-bearings pumps, simply supported beam calculations can be used to determine natural frequencies and mode shapes. Other useful simplified models are a cantilevered beam with a mass at the end to represent a single stage end-suction pump, and a simply supported beam on an elastic foundation to represent a flexible shaft multistage pump with Lomakin stiffness at each wearing ring and other clearance gaps. A good reference for these and other models is the handbook by Blevins (Blevins 1984). Other useful formulas to predict vibration amplitudes due to unbalance or hydraulic radial forces can be found in Roark (Roark 1982).

An example of how to apply these formulas will now be given for the case of a single stage between-bearings double suction pump. If the impeller mass is M, the mass of the shaft is Ms, the shaft length and moment of inertia (= pi $D^4/64$) are L and I, respectively, for a shaft of diameter D withYoung's Modulus E, the first natural frequency f_{n1} is:

$$f_{n1} = (120/pi)[(3EI)/\{L^3 (M+0.49M_s)\}]^{1/2}$$

If the whirling radius of the true center of mass of the impeller rotating at speed w radians per second relative to the bearing rotational centerline is e, then the unbalance force is simply:

$$F_{ub} = Mew^2 /g_c$$

On the other hand, if the force is independent of impeller motion (such as certain fluid forces are, approximately) the amount of vibration displacement expected at the impeller wearing rings due to force F_{ex} is:

$$X= (F_{ex} * L^3)/(48EI)$$

The simply supported beam formula can be obtained from the referenced handbooks. There are many ways to configure a pump rotor, however, and some of these cannot be adequately simulated by vibration handbook models. There is a simple method to convert the statics handbook formulas into formulas for the vibration lowest natural frequency. The method consists of using the formula for the maximum static deflection for a given shaft geometry loaded by gravity, and taking the square root of the gravitational constant (= 386 lbm/lbf-in/sec) divided by this deflection. When this is multiplied by 60/2pi, the result is a good estimate of the lowest natural frequency of the rotor. An even more simplified, though usually very approximate, procedure to estimate the lowest natural frequency is to consider the entire rotor system as a single mass suspended relative to ground by a single spring. The lowest natural frequency can then be estimated as 60/2pi times the square root of the rotor stiffness divided by the rotor mass. Make certain in performing this calculation to use consistent units (e.g. do not mix English with metric units), and divide the mass by the gravitational units constant.

Computer Methods

Shaft natural frequencies are best established through the use of modern computer software, as discussed in HI/ANSI 9.6.8 (HI 2014). Rotordynamics requires a more specialized computer program than structural vibration requires. A general purpose rotordynamics code must include effects such as 1) three dimensional stiffness (including cross-coupling) and damping at bearings, impellers, and seals as a function of fluid properties, annular passage design, speed, and load, 2) impeller and thrust balance device fluid excitation as well as response forces, and 3) gyroscopic effects, Clearly, pump rotor systems are deceptively complex. In order to make rotor vibration analysis practical, certain assumptions and simplifications are typically made, which are not perfect but are close enough to reality for practical purposes, resulting in critical speed predictions which can be expected to typically be within 5 to 10 percent of their actual values, if the analysis is performed with appropriate theory. The assumptions and simplifications usually include:

- Linear bearing coefficients, which stay constant with deflection. This can be in significant error for large rotor orbits. The coefficients for stiffness and damping are not only at the bearings, but also at the impellers and seals, and must be input as a function of speed and load.
- Linear bearing supports (e.g. bearing housings, pump, casing, and casing support pedestal).
- Perfectly tight or perfectly loose impeller and sleeve fits, except as accounted for as a worst-case unbalance.
- If flexible couplings are used, shaft coupling coefficients are considered negligible with respect to the radial deflection and bending modes, and have finite stiffness only in torsion.
- It is assumed there is no feedback between vibration and resulting response forces, except during stability analysis.

Several university groups such as the Texas A&M Turbomachinery Laboratories have pioneered the development of rotordynamics programs, and the theory behind them, including the physics of bearing, seals, and impeller secondary flows (e.g. Childs 1982, 1985, and 2013, and San Andres 1993, 2006, 2006B, and 2018). The programs available include various calculation routines for the bearing and annular seal (e.g. wear ring and balance drum) stiffness and damping coefficients, critical speed calculations, forced response (e.g. unbalance response), and rotor stability calculations. These programs include the effects of bearing and seal cross-coupled stiffness as discussed earlier. Procedures to appropriate implementation of general purchase CFD codes have also been explored (San Andres 2018, Wu and San Andres 2019).

Accounting for Bearings, Seals, and Couplings

Bearings

The purpose of bearings is to provide the primary support to position the rotor and maintain concentricity of the running clearances within reasonable limits. Pump bearings may be divided into five types (San Andres 2006 and 2006B):

1. Plain journal bearings, in which a smooth, ground shaft surface rotates within a smooth surfaced circular cylinder. The load "bearing" effect is provided by a circumferential pressure profile (“hydrodynamic wedge”) which builds between the rotating and stationary parts as rotating fluid flows circumferentially through the narrow part of the eccentric gap between the shaft journal and the cylindrical bearing insert. The eccentricity of the shaft within the journal is caused by the net radial load on the rest of the rotor (as well as witin the bearing itself) forcing the rotor to displace within the fluid gap. The build up of the hydrodynamic wedge provides a reaction force which gets larger as the eccentricity of the shaft journal increases, similar to the build-up of force in a spring as it is compressed. This type of bearing has high damping, but is the most prone to rotordynamic stability issues, due to its inherently high cross-coupling versus viscous damping.
2. Non-circular bore journal bearings, in which the bore shape is modified to increase the strength and stability associated with the hydrodynamic wedge. This includes bore shapes in which a) the bore is ovalized ("lemon bore"), b) offset bearing bores in which the upper and lower halves of the bearing shell are split and offset from each other, and c) cylindrical bores with grooves running in the axial direction (in all types of journal bearings, grooves may be provided which run in the circumferential direction, but such grooves are to aid oil flow to the wedge, not to directly modify the wedge). Types of axially grooved bearings include "pressure dam" bearings, in which the grooves are combined with stepped terraces which act to "dam" the bearing clearance flow in the direction that the highest load is expected to act, and "fixed pad" bearings, in which the lands between the grooves may be tapered so that clearances on each pad decrease in the direction of rotation. The 3-lobe “tri-land” bearing is an example of this type of bearing.
3. Tilting pad journal bearings, in which tapered, profiled pads similar to the fixed pad bearings are cut loose from the bearing support shell, and re-attached with pivots that allow the pads to tilt in a way that directly supports the load without significant reaction forces perpendicular to the load. In practice, some perpendicular loading, i.e. "cross-coupling", still occurs but is usually much less than in other types of journal bearing.
4. Externally energized bearings, which do not derive their reactive force from internal bearing fluid dynamic action, but instead operate through forces provided by a pressure or electrical source outside of the bearing shell. This includes magnetic bearings, and also includes hydrostatic bearings, in which cavities surrounding the shaft are pressurized by a line running to the pump discharge or to an independent pump. In hydrostatic bearings, as the shaft moves off center, the clearance between the shaft surface and the cavity walls closes in the direction of shaft motion, and opens up on the other side. The external pressure-fed cavities on the closing clearance side increase in pressure due to decreased leakage from the cavity through the clearance, and the opposite happens on the other side. This leads to a reaction force that tends to keep the shaft centered. Hydrostatic bearings can be designed to have high stiffness and damping, with relatively low cross-coupling, and can use the process fluid for the lubricant, rather than an expensive bearing oil system, but at the expense of delicate clearances and high side-leakage which can result in a several point efficiency decrease for the pumping system. Some hybrid bearings are now available where the leakage loss vs. support capacity is optimized.
5. Rolling element bearings, using either cylindrical rollers, or more likely spherical balls. Contrary to common belief, the support stiffness of rolling element bearings is not much higher than that of the various types of journal bearings in most pump applications. Rolling element, or “anti-friction”, bearings have certain defect frequencies that are tell-tales of whether the bearing is worn or otherwise malfunctioning. These are associated with the rate at which imperfections of the bearing parts (the inner race, the outer race, the cage, and the rolling element such as ball or cylindrical roller) interact with each other. Key parameters are the ball diameter Db, the pitch diameter Dp which is the average of the inner and outer race diameters where

they contact the balls, the number of rolling elements Nb, the shaft rotational speed N, and the ball-to-race contact angle measured versus a plane running perpendicular to the shaft axis. The predominant defect frequencies are FTF (Fundamental Train Frequency, the rotational frequency of the cage or ball "retainer", usually a little under ½ shaft running speed), BSF (Ball Spin Frequency, the rotation rate of each ball, roughly equal to half the shaft running speed times the number of balls), BPFO (Ball Pass Frequency Outer Race, usually a little less than the FTF times the number of balls), and BPFI (Ball Pass Frequency Inner Race, usually a little greater than ½ shaft running speed times the number of balls).

Annular Seals

As discussed earlier in the "Concepts" section, the typical flow-path seal in a centrifugal pump is the annular seal, with either smooth cylindrical surfaces (plain seals), stepped cylindrical surfaces of several different adjacent diameters (stepped seals), or multiple grooves or channels perpendicular to the direction of flow (serrated, grooved, or labyrinth seals). The annular sealing areas include the impeller front wear ring, the rear wear ring or diffuser "interstage bushing" rings, and the thrust balancing device leak-off annulus.

The primary action of Lomakin Effect (as discussed earlier) in annular seals is beneficial, through increased system direct stiffness and damping (Black 1976, Childs 1982, Simon and Frene 1993, and San Andres 1993) which tend to increase the rotor natural frequency and decrease the rotor vibration response at that natural frequency. However, over- reliance on Lomakin Effect can put the rotor design in the position of being too sensitive to wear of operating clearances, resulting in unexpected rotor failures due to resonance (Massey 1985). It is important that modern rotors be designed with sufficiently stiff shafts that any natural frequency which starts above running speed with new clearances remains above running speed with clearances worn to the point that they must be replaced from a performance standpoint. For this reason, API-610 requires Lomakin Effect to be assessed in both the as-new and 2x worn clearance condition.

Couplings

Couplings may provide either a rigid or a laterally low-stiffness connection between the pump and its driver. These are known as "rigid" and "flexible" couplings, respectively. Rigid couplings firmly bolt the driver and driven shafts together, so that the only flexibility between the two is in the metal bending flexure of the coupling itself. This type of coupling is common in vertical and in small end-suction horizontal pumps. In larger horizontal pumps, especially multi-stage or high-speed pumps, flexible couplings are essential because they prevent the occurrence of strong moments at the coupling due to angular misalignment. Common types of flexible couplings include gear couplings, diaphragm couplings, and disc-pack couplings (ANSI 2017, Crease 1977). All flexible couplings allow the connected shafts to kink, and radial deflection through a spacer piece between coupling hubs, but allow torsional deflection with stiffnesses that are significant, but not to the extent of rigid couplings.

In performing a rotordynamics analysis of a rigidly coupled pump and driver, the entire rotor (pump, coupling, and driver) must be analyzed together as a system. In such a model, the coupling is just one more segment of the rotor, with a certain beam stiffness and mass. In a flexibly coupled pump and driver, however, the entire rotor train usually does not need to be analyzed in a lateral rotordynamics analysis. Instead, the coupling mass can be divided in half, with half (including half the spacer) added to the pump shaft model, and the other half and the driver shaft ignored in the analysis. In a torsional analysis, the coupling is always treated as being rigid or having limited flexibility, and therefore the entire rotor system (including coupling and driver) must be included for the analysis to have any practical meaning. A torsional analysis of the pump rotor only is without value, since the rotor torsional critical speeds change to entirely new values as soon as the driver is coupled up, both in theory and in practice.

Casing and Foundation Effects

Generally, pump rotors and casings behave relatively independently of each other, and may be modeled with separate rotor dynamic versus structural models. A notable exception to this is the vertical pump, as will be discussed later. Horizontal pump casings are relatively massive, and historically have seldom played a strong role in pump rotordynamics, other than to act as a rigid reaction point for the bearings and annular seals. However, pressure on designers to save on material costs occasionally results in excessive flexibility in the bearing housings, which are cantilevered from the casing. The approximate stiffness of a bearing housing can be calculated from beam formulas given in Roark (Roark 1982). Typically, it is roughly $3EI/L^3$, where L is the cantilevered length of the bearing centerline from the casing end wall, and the area moment of inertia I for various approximate cross-sectional shapes is available from Roark. The bearing housing stiffness must be combined as a series spring with the bearing film stiffness to determine a total direct "bearing" stiffness for use in rotordynamics calculations. The following formula may be used:

$$1/k_{total} = 1/k_{housing} + 1/k_{bearing}$$

Vertical pumps generally have much more flexible motor and pump casings than comparable horizontal pumps, and more flexible attachment of these casings to the foundation. To properly include casing, baseplate, and foundation effects in such pumps, a finite element model (FEA) is required, as discussed later, and as outlined in HI/ANSI 9.6.8 (HI 2014).

Purchase Specification Recommendations with Regard to Rotordynamics

When purchasing a pump, particularly an "engineered" or "custom" as opposed to "standard" pump, it is important to properly evaluate its rotordynamic behavior, to avoid "turn-key" surprises in the field. Similarly, high risk situations benefit from such evalautons. To determine whether the risk might financially justify various levels of analysis, th reader is referred to the Risk-Uncertainty Number (UN) evalution as outlind in HI/ANSI 9.6.8 (HI 2014). OEM's may be tempted to "trust to luck" with respect to rotordynamics in order to reduce costs, unless the specification requires them to spend appropriate effort. Typically, an engineered or high-risk service pump should have the following types of analyses:

- Critical speed and mode shape: What are the natural frequency values, and are they sufficiently separated from typical "exciting" frequencies, like 1x and 2x running speed, and vane pass? (see API-610 2011, and HI 9.6.8 2014).
- Rotordynamic stability: Is there enough damping vs. cross-coupling for rotor natural frequencies, particularly those below running speed, that they will avoid becoming "self-excited"? (See API-RP-684 1992).
- Forced response: Given the closeness of any natural frequencies to exciting frequencies, and given the amount of net damping present versus the amount of allowable or likely excitation force that builds up between overhauls of the pump, will the rotor vibrate beyond its clearances, overload its bearings, or cause fatigue of the coupling? (See API-610 2011, API-RP-684 2010, and HI 9.6.8 2014).

Preferably, the specification also should require finite element analysis of structural natural frequencies for the following:

- Horizontal pump bearing housings (at least for pumps with drip pockets) and casing/ pedestal assemblies, in each case with the rotor assembly mass and water mass included (not addressed directly in API-610 2011).
- Vertical end-suction or in-line pump motor (if attached "piggy-back")/ pump casing and bearing pedestal/ pump casing (not directly addressed in API-610 2011)
- Vertical Turbine Pump (VTP) and Vertical Hi-Flow Pump (e.g. flood control) motor/ discharge head or motor/ motor stand, connected to baseplate/ foundation/ column piping/ bowl assembly.

The rotor analysis should use state-of-the-art specialized computer codes such as those available from the Texas A&M TurboLab, and should take into account annular seal (e.g. wear ring and balance device) "Lomakin Effect" rotordynamic coefficients, impeller fluid added mass, and bearing and seal "cross-coupling" coefficients that are inherent in bearings, seals, and impeller cavities. The structural analysis should include added mass effects from water inside (and for vertical turbine pumps, outside) the casing, bracketing assumptions concerning piping added stiffness and mass, and bracketing assumptions concerning foundation/ baseplate interface stiffness. Common bracketing assumptions for piping are that the pipe nozzles are held perfectly rigid in one analysis, and is assumed to be completely free to move in a second analysis. Sometimes the piping is included to at least the first hanger or support, and is then assumed pinned at this location. The only "guaranteed" accurate analysis is to include all piping and reasonable estimates for support stiffness, but this is usually considered cost-prohibitive. For the foundation, typical bracketing assumptions are that the baseplate edge is simply supported (i.e. like on knife edges, fixed vertically but able to pivot) all around its periphery in one analysis, and fully fixed around the periphery in another analysis. For improved accuracy, at least average flexural properties for the floor and subfloor should be included under or as part of the baseplate. As with the piping, however, the only reasonable-certaintyof-accuracy analysis is to include the entire floor, key other masses on the floor, and all floor pillars and supports, with the assumption of usually a simple support for the outer periphery of the floor, where it meets outside walls of the room or cavity below, such as a sump. Usually, but not always, such floor detail does not substantially change the results and is considered cost-prohibitive. Such detail is particularly important to include, however, when the floor stiffness is less than 10x that of the pump discharge head (vertical pumps) or support pedestal (horizontal pumps), or if floor natural frequencies are predicted to be within +/-20% of running speed.

A counter-intuitive aspect of lateral rotordynamics analysis is how press-fit components (such as possibly coupling hubs, sleeves, and impellers) are treated. For the case of a slip fit/ keyed connection, it is easy to appreciate that only the mass but not the stiffness of these components should be included. However, even if the press-fit is relatively tight, it has been found by researchers (including the author) that the stiffening effect is typically small. Obviously if the press fit is high enough, the parts will behave as a single piece, but typically such a heavy press is beyond maintenance practicality. Therefore, standard practice in rotordynamic analysis is to ignore the stiffening effect of even press-fitted components, as discussed and recommended in API-RP-684. The author's approach in such cases typically is to analyze the rotor in a bracketing fashion, i.e. do the analysis with no press fit, and re-do it with the full stiffening of a rigid fit-up, with inspection of the results to assure that no resonances will exist at either extreme, or anywhere in between. In the case of torsional analysis, the rule changes, however. API-RP-684 introduces the concept of penetration stiffness, where the full torsional rigidity of a large diameter shaft attached to a small diameter shaft is not felt until some "penetration length" (per a table in API-RP-684) inside the larger diameter part. Of greater consequence, in most cases in the author's experience, is the slip between the shaft and fit-up components such as impellers, balancing disks or drums, and sleeves. If the shaft fit is a medium to high level of press-fit, then no slip between the shaft and component is assumed, although the API-684 criteria can be applied for a modest added torsional flexibility. If the shaft fit is a light press and/ or loose fit with a key, the shaft is assumed able to twist over a length equal to 1/3 its diameter (API estimates 1/3 the key engagement length, instead), until the key is fully engaged. While this D/3 procedure is approximate and dependent upon key dimensioning and keyway fit-up, practice has shown that it typically results in an excellent agreement between analysis predictions and torsional critical speed test results.

Although other specifications such as the Hi/ANSI 9.6.4 (2018), or ISO 10816-7 (2017) provide some guidelines for vibration measurement and acceptance levels, there is not a great deal of guidance in most pump specifications concerning rotordynamic analysis. The new HI/ANSI 9.6.8 (2014) and API-610 11th Edition (2011) are exceptions, along with API RP-684 (2010). API-610 discusses lateral analysis in detail in Section 8.2.4 and Annex I. This specification requires that any lateral rotordynamic analysis report include the first three natural frequency values and their mode shapes (plus any other natural frequencies that might be present up to 2.2x running speed), evaluation based on as-new and 2x worn clearances in the seals, fluid added-mass used for the rotor as well as effect of mass of the stationary supports, stiffness and damping used for all bearings and annular seals, and that the report list any assumptions which needed to be made in the rotor model. It discusses that resonance problems are to evaluated based on damping as well as critical speed/ running speed separation margin, and provides Figure I.1 to tie the two together (the bottom line is that there is no separation margin concern for any natural frequency with a damping ratio above 0.15, i.e. log dec of at least 0.94). It also gives criteria for comparison to test stand intentional imbalance test results. It requests test results in terms of a "Bode plot". This is a plot of log vibration vs. frequency combined with phase angle vs. frequency, as shown by example in Figure 3 of this tutorial. As will be recalled, this plot identifies and verifies the value of natural frequencies and shows their amplification factor.

One of the more notable novel aspects of API-610 as well as HI/ANSI 9.6.8 is that they recommend that there are a number of situations for which lateral rotordynamics analysis is over-kill, and therefore its cost can be avoided. These situations are when the new pump is identical or very similar to an existing pump, or if the rotor is "classically stiff". The basic definition of "classically stiff" is that its first dry critical speed (i.e. assuming Lomakin Stiffness is zero) is at least 20 percent above the maximum continuous running speed (and 30 percent above if the pump might ever actually run dry). Also, as discussed earlier, in addition to API-610, API also provides a useful "Tutorial on the API Standard Paragraphs Covering Rotordynamics ...", as API Publication RP-684, which provides some insight and philosophy behind the specifications for pumps, as well as compressors and turbines. In addition to similar insights, HI/ANSI 9.6.8 provides recommended specification paragraph templates.

Torsional Vibration Analysis of Pump and Driver Rotor Assemblies
API-610 11th Edition, as well as the API-RP-684 Recommended Practice, and HI/ANSI 9.6.8 also provide requirements and recommendations for torsional analysis. As discussed earlier, lateral rotordynamics can often be analyzed without including other pumping system components such as the driver. However, torsional vibration of the pump shaft and sometimes the vibration of the pump stationary structure as well are system-dependent, because the vibration natural frequencies and mode shapes will change depending on the mass, stiffness, and damping of components other than those included inside the pump itself. Therefore, API-610 and HI 9.6.8 require that the entire train be analyzed during a torsional analysis, with the exception of the case of a torsionally soft hydraulic coupling.

Although torsional vibration problems are not common in pumps, complex pump/driver trains have potential for torsional vibration problems. Problems can be checked for by calculation of the first several torsional critical speeds and of the forced vibration response of the system due to excitations during start-up transients, steady running, trip, and motor control transients. The forced response should be in terms of the sum of the stationary plus oscillating shear stress in the most highly stressed element of the drivetrain, usually the minimum shaft diameter at a keyway.

In pump lateral rotordynamics, it is important to account for added mass, as discussed earlier. In torsional dynamic analysis, fluid inertia is much less important, as discussed by Marscher et al (2013). Typically, tests on practical impellers lead to determination of a rotor torsional added mass of at most one or two percent. The precise prediction of this added mass requires lab or field testing, or CFD, and is seldom worth the effort. The practice of including all fluid mass within the impeller passages for torsional added mass is a gross over-estimate, and should be avoided. The reason for the minimal impact of the fluid within the impeller on impeller rotary inertia is that, as the impeller oscillates, fluid moves easily in and out of the suction and discharge, and is not forced to rotate with the impeller.

Generally (not always), calculation of the first three torsional modes in a pumping system is sufficient to cover the expected forcing frequency range. To accomplish this, the pump assembly must be modeled in terms of at least three flexibly connected relatively rigid bodies: the pump rotor, the coupling hubs (including any spacer), and the driver rotor. If a flexible coupling (e.g. a disc coupling) is used, the coupling stiffness will be on the same order as the shaft stiffnesses, and must be included in the analysis. Good estimates of coupling torsional stiffness, which is usually (but not always) relatively independent of speed or steady torque, are listed in the coupling catalog data. Often a range of stiffness for a given size is availabl, by modifying the coupling flexible element.

If a gear box is involved, each gear must be separately accounted for in terms of inertia and gear ratio. The effect of the gear ratio is to increase effective rotary inertia and torsional stiffness of faster (geared up) portions of the train relative to the slower ("reference") rotor in the train, The ratio of the increase is the square of the ratio of the high speed to the reference speed. In a very stiff rotor system, the flexibility of the gear teeth may need to be accounted as well, as part of the rotor system's torsional flexibility.

If the pump or driver rotor is not at least several times as stiff torsionally the shaft connecting the rotor to the coupling (the "stub shaft"), then the individual shaft lengths and internal impellers should be included in the model. In addition, any press fits or slip fits with keys should have a "penetration factor" assessed for the relatively thinner shaft penetrating the larger diameter shaft such as a coupling hub, impeller hub, or motor rotor core. API-684 recommends this be1/3 the length of the hub key, but the author has had better experience using 1/3 the diameter of the thinner shaft, which is added to the length of the thinner shaft. For a sleeve attached to a shaft with a key, for example, this decreases the effective stiffening effect of the sleeve by 1/3 shaft diameter on *each* end of the sleeve. In addition, API-684 provides Table 2-1, which gives additional penetration factors when a shaft diameter changes, under the assumption that the thinner shaft does fully "recognize" extra stiffness of its larger diameter until an edge effect occurs. An example of this penetration factor is 0.107 for a shaft diameter step-up of 3.0, i.e. the smaller diameter shaft increases in length by 0.107 diameters. This is approximately correct, but is generally a very small effect that is often ignored.

Methods of manually calculating the first several torsional natural frequencies are given in Blevins (1984). However, in the case that a resonance is predicted, the torsional calculations must include the effects of system damping, which is difficult to assess accurately manually. Therefore, to determine the shaft stresses, a detailed numerical procedure should be used, such as Finite Element Analysis (FEA), which can calculate stresses during forced response and transients. These stresses can limit the life of the shafting when the system is brought up to speed during start-up, unexpectedly trips out, or runs steadily close to a resonance. Even with FEA, however, a good estimate of the system damping and of the frequencies and magnitudes of all of the significant excitation forces is required. API-610 (2011) paragraph 5.9.2.2 gives a list of the minimum types of oscillating torques that must be included in such an analysis. This includes 1x and 2x N for either shaft of a geared train, and the number of poles times slip frequency for an induction motor or during start-up of a synchronous motor (e.g. 2x slip starts at 120 Hz at initial start-up of a synchronous 2-pole motor, and then decays to zero as the motor comes up to speed).

Excitation harmonics are (as a minimum) n/2 x running speed for reciprocating engines, where n is an integer multiplier of running speed. Often the strongest torsional harmonics of a reciprocating engine are "half-harmonics" of the number of pistons times running speed (e.g. 3-1/2x for a 7 cylinder diesel), or strong ½ running speed (e.g. due to a mistuned cylinder) and its harmonics for a 4-cycle engine. For a VFD, API-610 requires evaluation at 1x line frequency and 2x line frequency, as well as n x RPM, where n is an integer defined by the drive and/ or motor manufacturer. Older VFD's had strong torsional harmonics at 6x, 12x, 18x, and sometimes 24x running speed. The 6x harmonics were due to the way the electrical sine wave driving the motor was simulated by the typical VFD, which was done in 6 steps across the generated line frequency sine wave. However, modern adjustable speed drives, or pulse-width-modulated VFD's, have relatively weak harmonics, which are often neglected at the recommendation of the drive or motor OEM.

The opportunity for resonance is typically displayed in a Campbell Diagram of natural frequency vs. running speed, in which speed range is shown as a shaded vertical zone, and excitations are shown as "sunrays" emanating from the origin (0, 0 point) of the plot. An example of a Campbell Diagram is provided in Figure 15. API requires that each of these forcing frequencies miss natural frequencies by at least +/- 10 percent, or else that a forced response stress and Goodman Diagram fatigue analysis is performed to prove that a possible resonance will not fatigue the shaft, within a sufficient factor of safety (usually with a worst case of at least 2). It is important that the shaft stresses evaluated in this manner include stress concentrations at highly stressed location. Typically, these stress concentrations (e.g. keyways) are equal to or less than 3.0.

The lowest torsional mode is the one most commonly excited in pump/driver systems, and most of the motion in this mode occurs at the pump impeller(s). In this situation, the primary damping is from energy expended by the pump impellers when they operate at slightly higher and lower instantaneous rotating speeds due to the vibratory torsional motion. A rough estimate of the amount of this damping is the relationship derived by Marscher (2002, 2008):

$$\text{Damping} = 2*(\text{Rated Torque})*(\text{Evaluated Frequency})/(\text{Rated Speed})^2$$

To determine the frequencies at which large values of vibratory excitation torque are expected, and the value of the torque occurring at each of these frequencies, the pump torque at any given speed and capacity can be multiplied by a zero-to-peak amplitude "per unit" factor "p.u.". The p.u. factor at important frequencies (as listed above) can be obtained from motor and control manufacturers for a specific system, and is typically about 0.01 (or up to 0.05 for wastewater or slurry pumps) of the steady operating torque at the condition of interest, zero-to-peak. Unsteady hydraulic torque from the pump is also present at frequencies equal to 1x and 2x running speed, and usually more importantly at "vane pass", i.e. the running speed times the number of impeller vanes. At these frequencies, the p.u factor is typically a maximum of about 0.01 for 1x and 2x, and between 0.01 and 0.05 for vane pass, with the higher values being more typical of off-BEP (Best Efficiency Point) operation. Typically, this value is supplied to the analyst by the OEM, but in the author's opinion, values of less than P.U. 0.005 at 1x, 2x, and vane pass should not be accepted.

Judgment on the acceptability of the assembly's torsional vibration characteristics should be based on whether the forced response shaft stresses are below the fatigue limit by a sufficient factor of safety, at all operating conditions. As mentioned earlier, the minimum recommended factor of safety is 2, as evaluated on an absolute worst case basis (including the effects of all stress concentrations, e.g. from key ways, normally about 2.7x) on a Goodman Diagram, for a carefully analyzed rotor system. API-610 (2011) and API-RP-684 (2010) provide no recommendations for this safety factor. It is also important to simultaneously account for worst case bending and axial thrust stresses during a forced response fatigue analysis, using for example von Mises equivalent stress.

Vertical Pump Rotor Evaluation

A common form of vertical pump is the vertical turbine pump, or VTP, which is very different from other pumps because of its less stringent balancing, shaft straightness, and motor shaft alignment tolerances, because of its long flexible casing and the casing's flexible attachment to ground, and because of the peculiar spaghetti-like lineshafting which connects the motor to the below-ground liquid-end impellers of the "bowl assembly" of the pump. However, like other pumps, it is the bearing loads and the bearing and wear ring clearances where problems are likely to occur.

The flexibility of the VTP structure and shafting result in many closely spaced modes within the range of frequencies for which strong exciting forces are expected. An average of one mode per 100 cpm is not unusual for deepwell VTP's. VTP pumps also exhibit nonlinear shaft dynamics because of the large shaft excursions which occur in the lightly loaded long length/diameter ratio bearings, as explained below.

An important element of VTP shaft vibrations is the strong effect of axial thrust on the impellers, causing a roughly 10 percent increase in shaft natural frequencies, as discussed by Kovats (1962) and Blevins (1984), and providing a restoring moment which tends to suppress lateral vibrations in a non-linear fashion, as explained by Blevins. Another important factor is the statistical character of the support provided by any given lineshaft bearing at any point in time. If the bearings behaved consistently and linearly, FEA could be used to accurately predict the lineshaft modes. However, the normally lightly loaded lineshaft bearings exhibit a rapid, nonlinear increase in bearing stiffness as the lineshaft gets close to the bearing wall. Given the flexibility of the lineshaft and the relatively weak support provided by the pump casing "column piping", and given the relatively large assembly tolerances and misalignments in the multiple lineshaft bearings of these machines, the contribution of each bearing to the net rotordynamic stiffness is a nearly random and constantly changing situation, as explained conceptually in Fig. 16. The result is that in practice there is no single value for each of the various theoretically predicted shaft natural frequencies, but rather the natural frequencies of the lineshafting and shaft in the bowl assembly must be considered on a time-averaged nonlinear basis.

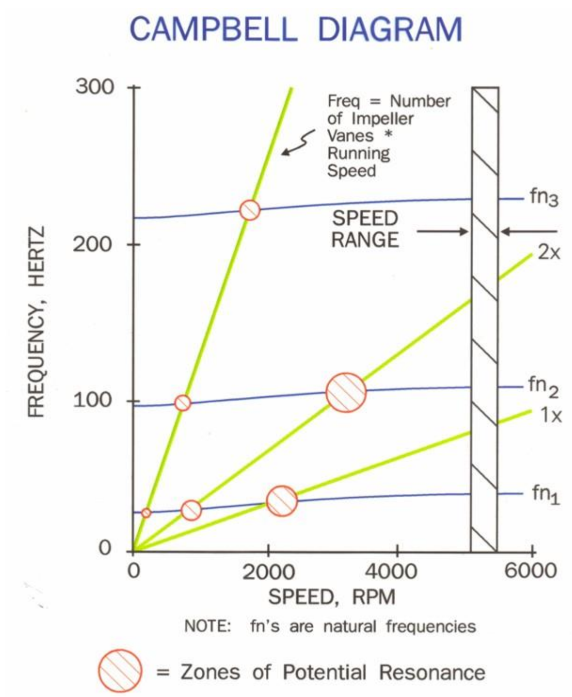

Figure 15. Typical Campbell Diagram, Showing Torsional Stiffness Increase with Load

Methods of Analysis and Test for Vertical Pumps

An important advance in the experimental study of VTP pumps was the development some years ago of the underwater proximity probe by a major instrumentation supplier. Studies reported in the literature which have made use of such probes to observe actual shaft motion during various conditions of interest include Marscher (1986, 1990), and Spettel (1985). A useful simplified method of predicting lineshaft reliability with a worst-case model known as the "jumprope" model has been reported by Marscher (1986).

The concept is to model the lineshaft vibrational motion and loads in the worst-case limit by the deflection and end-support forces associated with a whirling jumprope, with the addition of axial thrust and Euler beam-bending

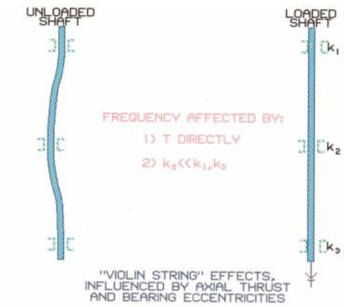

Figure 16. Vertical Pump Lineshaft Rotor Behavior

stiffness effects. The deflection of such a jumprope may be calculated by a quasi-static analysis, based on a concept called D'Alembert's Principle with the end conditions set equal to the radius of the circular path of the "hands" (bearing walls) controlling the "rope" (shaft), and the load per unit length at each point along the rope equal to the local displacement, times the mass per unit length, times the square of the rotational frequency. The deflections predicted by this model are worst case, regardless of the value of or linearity of the bearing stiffness, if the circular orbit of the end conditions is set equal to the diametral clearance of the lineshaft bearings, and if the rotor deflection slope within each bearing is set equal to the bearing diametral clearance divided by the bearing length. The latter condition is the so-called "encastre" condition, studied by Downham (1957), and Yamamoto (1954).

It is the encastre condition which ultimately limits the shaft deflection and stresses, and the bearing loads, both by limiting the slope of the shaft, and by changing the end support condition of a shaft length in the analysis from "simple" (i.e. knife edge) to fixed. Compared to the load caused by the whirling shaft mass in this condition, minimal bearing forces are caused by initial unbalance, misalignment, or bends in the shaft, which is why liberal tolerances on these are commercially acceptable. For relatively stiff lineshafting such as in most reactor coolant pumps, the jumprope model gives answers which are too conservative to be useful, but for the majority of VTP's it gives a quick method of confirming that shaft stresses and bearing loads are acceptable even in the presence of worst case whirl.

Vertical Pump Combined Rotordynamic and Structural Vibration Pre-Installation Analysis

In general, VTP vibrations of the stationary structure, the lineshafting, and the pump and motor rotors should be done simultaneously, using finite element analysis (FEA). The goal of such analysis is to determine at least all natural frequencies and mode shapes up to 2.5 times running speed. The components in such a model are best represented mathematically in considerable detail, as follows:

- Include foundation mass and stiffness within a radial distance (measured from the center of the pump base) at least equal to the height of the top of the motor relative to the level of attachment of the baseplate to the floor.
- Include piping details important to modal mass and stiffness, such as hangers, bulkheads, and expansion joints, and (if the piping is rigidly attached) all piping and its enclosed fluid within a spherical zone of radius (relative to the center of mass of the pump/motor) equal to at least the height of the top of the motor relative to the level of attachment of the baseplate to the floor.
- Include the mass (and location of center-of-mass) of the close-coupled motor and magnetic drive (if so equipped), and of the discharge head or motor stand.
- Include any pedestal, discharge head, and motor stand stiffness, including variations between the piping in-line and perpendicular directions, taking particularly into account the effects of coupling access or stuffing box access windows.
- Include the below-ground column piping and bowl assembly, the fluid in and immediately around the column piping and bowl assembly (See Blevins 1984), any column piping stiffeners or supports, and any shaft enclosure tubing.
- Include the mass of all pump impellers, and attach them to the pump casing through their bowl bearings and (if impellers are shrouded) the wear ring Lomakin Effect stiffness, both direct and cross-coupled, and damping. Also include effective added mass for fluid inside and around the impellers and lineshafting, per HI 9.6.8 (2014).
- Include all other rotating component masses and effective assembled flexibility for the motor coupling, and the motor rotor.
- Include the lineshaft bearing stiffnesses, both direct and cross-coupled, based on available data. If data is lacking, the author's experience for typical VTP bearings is that they provide stiffness in proportion to diameter, such that stiffness equals approximately 10000 lbf/inch per inch of diameter for soft plastic or rubber bearings, and an estimated 100,000 lbf/inch per inch of diameter for metal (such as bronze) lineshaft bearings. Never forget during analysis "what if", however, that the stiffness of lineshaft bearings is highly nonlinear, since they are more like "bumpers" than bearings.
- Separate calculations for shaft natural frequencies and vibration amplitudes should be performed for at least three situations: minimum stiffness at all bearings and seals, most probable stiffness at all bearings and seals, and maximum stiffness at all bearings and seals.
- Include a Forced Response Analysis if a possibility of a shaft resonance is predicted. In calculating forced response, include as minimum forces worst-case unbalance in each impeller and in the motor rotor and drive rotors or motor coupling halves, worst case misalignment across the drive or motor coupling, and worst case unsteady or rotating hydraulic forces on each impeller.

- Include torsional as well as lateral, axial, and mixed vibration modes in all analyses. If flexible couplings are used, a reasonable estimate must be made of the coupling torsional, lateral, and axial stiffnesses, which are usually listed as catalog data. It should be assumed that the thrust and radial bearings and annular seals provide no torsional constraint or stiffness.

If a pump is low in horsepower, often analysis will be bypassed. This always entails some degree of risk, since small pumps can resonate just as easily as large pumps. In general, if such an approach is taken, it is the author's experience that it is cost-effective in the long-run to analyze all pumps of 100 HP or greater in the manner described, prior to installation. ANSI/HI 9.6.8 (2014) gives a practical methodology for trading analysis cost vs. risk and uncertainty. In addition, the following should be considered "danger flags", increasing the need for detailed analysis:

- Vibration specifications requiring less than 0.20 ips peak.
- Variable speed units.
- Pumps that tie into headers that look like flutes (because they may acoustically act like flutes!)
- Flexibly supported pillow block bearings on U-Joint drive shafting.
- All equipment mounted to a particularly flexible foundation (foundation mass less than 5x the weight of the total weight of the supported equipment, or foundation flexural stiffness less than 10x that of the vertical pump discharge head or horizontal pump pedestal).
- Lack of pipe supports close to the pump, when piping is hard-coupled to the pump.

Case History: Multistage Pump Changed from Baseload to Cycling Service
A Northeastern power plant had experienced chronic boiler feed pump failures for eight years, since the unit involved had been switched from base load to modulated load. The longest that the turbine-driven pump had been able to last between major rotor element overhauls was 5 months. The worst wear was seen to occur on the inboard side of the pump. The turbine was not being damaged. The pump OEM had decided on the basis of detailed vibration signature testing and subsequent hydraulic analysis that the internals of the pump were not well enough matched to part-load operation, and proposed replacement of the rotor element with a new custom-engineered design, at a very substantial cost. Although the problem showed some characteristics of a critical speed, both the OEM and the plant were sure that this could not be problem, because a standard rotordynamics analysis performed by the OEM had shown that the margin of safety between running speed and the predicted rotor critical speeds was over a factor of two. However, the financial risk associated with having "blind faith" in the hydraulics and rotor dynamic analyses was considerable. In terms of OEM compensation for the design, and the plant maintenance and operational costs associated with new design installation, the combined financial exposure of the OEM and the plant was about $800,000.

Impact vibration testing by the author using a cumulative time averaging procedure discussed in the references (Marscher 1999B) quickly determined that one of the rotor critical speeds was far from where it was predicted to be over the speed range of interest, as shown in Figure 17, and in fact had dropped into the running speed range. Further testing indicated that this critical speed appeared to be the sole cause of the pump's reliability problems. "What-if" iterations using a test-calibrated rotor dynamic computer model showed that the particular rotor natural frequency value and rotor mode deflection shape could best be explained by insufficient stiffness in the driven-end bearing. This was demonstrated by the "Critical Speed Map" of Figure 17. The bearing was inspected and found to have a pressure dam clearance far from the intended value, because of a drafting mistake, which was not caught each time the bearing was repaired or replaced. Installation of the correctly constructed bearing resulted in the problem rotor critical speed shifting to close to its expected value, well out of the operating speed range. The pump has since run years without need for overhaul.

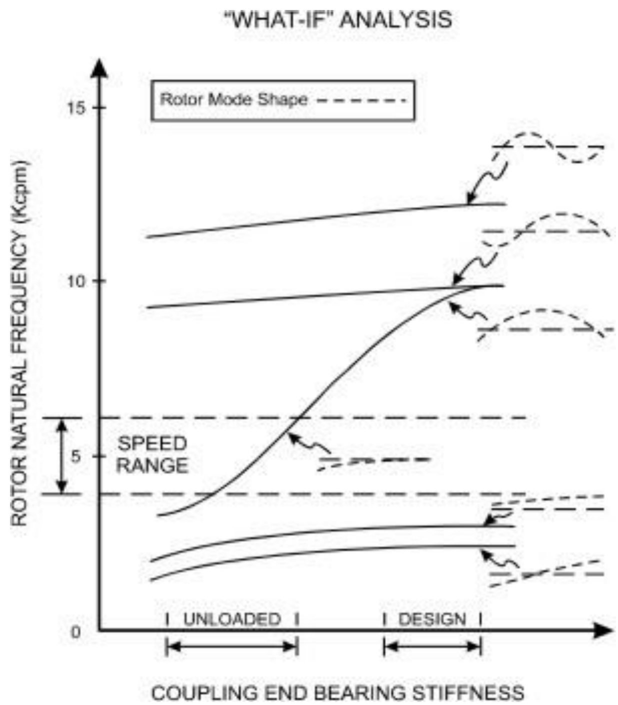

Figure 17. Variation of Rotor Critical Speeds with IB Bearing Stiffness

CONCLUSIONS

Pump rotordynamics can appear complex. The purpose of this tutorial has been to provide a "jump-start" in the rotordynamic evaluation process, so End Users can either learn to do it themselves, or carry on intelligent review of analyses performed on their behalf by OEM's or rotordynamic consultants. Final tips:

- Analyze rotors "up front", before installation, and preferably before purchase. If there is not an in-house group to do this, hire a third party consultant, or make it part of the bidding process that the manufacturer must perform such analysis in a credible manner, and report the results to you in accordance with API-610 and/or HI/ANSI 9.6.8 guidelines and requirements. In addition, there are many "ballpark" checks and simple analyses that you, as a non-specialist, can do for yourself, as outlined in this tutorial.
- Be very careful about the size of the pump purchased versus what is truly needed for the plant process pumping system. Do not buy significantly over-sized pumps that then must spend much of the time operating at part load, unless they are accompanied with an appropriately sized recirculation system.
- In the case of rotordynamics analysis, the use of computerized tools are much more likely to result in the correct conclusions than more traditional approximate techniques. Including details such as fluid added-mass and Lomakin Effect is essential.

NOMENCLATURE

BEP= best efficiency operating point of the pump
c= radial clearance in the sealing gaps (in or mm)
C= damping constant (lbf-s/in or N-s/mm)
D= shaft diameter (in or mm)
E= elastic modulus or Young's modulus (psi or N/mm2)
F= force (lbf or N)
FEA= finite element analysis
FRF= Frequency response function
f= frequency (cycles per second, Hz)
f_n= natural frequency (cycles per second, Hz)
g_c= gravitational unit (386 in/s2 or 9800 mm/s2)
I= area moment of inertia (in4 or mm4)
K= spring constant (lbf/in or N/mm)
L= shaft length (in or mm)
M= mass (lbm or kg)
N= shaft rotational speed (revolutions per min, rpm)

T= time (s)
V= vibration velocity amplitude, peak (in/s or mm/s)
X= vibration displacement, peak (mils or mm)
A= acceleration of vibration (in/s^2 or mm/s^2)

REFERENCES & BIBLIOGRAPHY

Agostinelli, A., Nobles, D., Mockridge, C.R., 1960, An Experimental Investigation of Radial Thrust in Centrifugal Pumps, J. Engrg. for Power, Trans ASME, v.82.

ANSI-ASA S2.75, 2017, Shaft Alignment Methodology, Washington D.C.

API 610, 11th Ed., 2011, Amer. Petroleum Instit., Washington D.C.

API-RP-684, 1st Ed., 2010, Amer. Petroleum Instit., Washington D.C.

Arghir, M., and Frene, J., 2004, "A Bulk-Flow Analysis of Static and Dynamic Characteristics of Eccentric Circumferentially Grooved Liquid Annular Seals," ASME J. Trib., 126 (2), pp. 316–325.

Arndt, N., Acosta, A. J., Brennen, C. E., Caughey, T. K., 1979, "Rotor-Stator Interaction in a Diffuser Pump," NASA Tech Briefs, MFS-26092 .

Black, H.F., 1979, Effects of Fluid-Filled Clearance Spaces on Centrifugal Pump Vibrations. 8th Turbomachinery Symposium, Texas A&M Univ.

Blevins, R.D., 1984, Formulas for Natural Frequency and Mode Shape, Robert Krieger Publishing Co., Malabar FL.

Bolleter, U., 1988, "Blade Passage Tones of Centrifugal Pumps, Vibrations, Vol. 4, No. 3, pp. 8-13.

Bowman, D., Marscher, W., and Reid, S., 1990, Pump Rotor Critical Speeds: Diagnosis and Solutions, Proc. International Pump Symposium, Texas A&M University.

Childs, D.W., 1982, Finite Length Solutions for Rotordynamic Coefficients of Turbulent Annular Seals, ASME/ASLE Lubrication Conf., ASME Paper 82-LUB-42 .

Childs, D.,W. and Moyer, D., 1985, Vibration Characteristics of the HPO Turbopump of the SSME,ASME J. of Engineering for Gas Turbines and Power, v. 107, pp. 152-159.

Childs, D. W., 2013, Turbomachinery Rotordynamics With Case Studies, 1st ed.,
Minter Spring Pubs., College Station, TX, Chap. 7. 2.

Crease, A.B., 1977, Design Principles and Lube of Gear Couplings, Int'l Conf on Flexible Couplings, IMechE.

Dodd, V.R., 1974, Total Alignment, The Petroleum Publishing Co., Tulsa, OK.

Downham, E., 1957, Theory of Shaft Whirling, No.5 - Influence of Plain Bearings on Shaft Whirling, The Engineer .

Electrical Power Research Institute, 1980, Recommended Design Guidelines for Feedwater Pumps in Large Power Generating Plants, Ed. E. Makay et al, EPRI CS-1512

Fraser, W.H., 1985, *Centrifugal Pump Hydraulic Performance and Diagnostics, Pump Handbook,* McGraw-Hill.

Gunter, E.J., and Li, D.F., 1978, Unbalance Response Analysis of Dual Rotor Systems, University of Virginia Report No. UVA/643092/MAE81/138, Charlottesville, VA.

Hergt, P., Krieger, P., 1970, Radial Forces in Centrifugal Pumps with Guide Vanes, Proc IMechE, v.184 Pt 3N.

HI/ANSI Standards 9.6.8, 2014, Dynamics of Pumping Systems, The Hydraulic Institute, Parsippany NJ.

HI/ANSI Standards 9.6.4, 2016, Rotodynamic Pumps – Vibration, The Hydraulic Institute, Parsippany NJ.

Ha, T. W., and Choe, B. C., 2012, "Numerical Simulation of Rotordynamic Coefficients for Eccentric Annular-Type-Plain-Pump Seal Using CFD Analysis," J. Mech. Sci. Technology, 26 (4), pp. 1043–1048.

Hirs, G. G., 1973, "A Bulk-Flow Theory for Turbulence in Lubricant Films," ASME J. Lubrication Tech. 95 (2), pp. 137–145.

International Standards Organization (ISO), 2016, ISO-1940 for Machinery Rigid Rotor Balance, Geneva Switzerland

International Standards Organization (ISO), 2017, ISO-10816-7 Vibration Standards for Centrifugal Pumps, Geneva Switzerland

Jery, B., Acosta, A., Brennan, C., Caughey, T., 1984, Hydrodynamic Impeller Stiffness, Damping, and Inertia in the Rotordynamics of Centrifugal Flow Pumps, Proc Rotordynamic Instability Problems in High Speed Turbomachinery, Texas A&M Univ.

Kovats, A., 1962, Vibration of Vertical Pumps, ASME Journal of Engineering for Power.

Lu, X., San Andres, L., 2018, Leakage and Rotordynamic Force Coefficients of a Three-Wave (Air in Oil) Wet Annular Seal: Measurements and Predictions, ASME Journal of Engineering for Gas Turbines and Power 141 (3)

Marquette, O.R., Childs, D.W., and San Andres, L., 1997, Eccentricity effects on the rotordynamic coefficients of plain annular seals: theory versus experiment, ASME Journal of tribology 119 (3), pages 443-447

Marscher, W.D., 1986, The Effect of Fluid Forces at Various Operation Conditions on the Vibrations of Vertical Turbine Pumps, Proc. IMechE, Radial Loads and Axial Thrusts on Pumps.

Marscher, W.D., 1987, The Relationship Between Pump Rotor System Tribology and Appropriate Vibration Specifications for Centrifugal Pumps, Proc IMechE 3rd Euro Congress on Fluid Machinery for the Oil & Petrochemical Industries, Hague, Netherlands.

Marscher, W.D., 1989, Analysis and Test of Multistage Pump Wet Critical Speeds, ASLE/ASME Joint Trib. Conf., Ft. Lauderdale, FL.

Marscher, W.D., 1990, The Effect of Variable Frequency Drives on Vibration Problems in Vertical Pumps, Proc. Water & Wastewater Conference, Barcelona, Spain.

Marscher, W D, 1997, Vibration Chapter and Predictive Maintenance Chapter, STLE Tribology & Lubrication Handbook: STLE Park Ridge, IL.

Marscher, W.D., 1999A, Bearing Application and Lubrication Chapter, Modern Marine Engineer's Manual 3rd Edition, Cornell Maritime Press: Centreville, MD.

Marscher, W.D., 1999B, The Determination of Rotor Critical Speeds While Machinery Remains Operating through Use of Impact Testing, IMAC Conf. Orlando FL, SEM.

Marscher, W.D., 2002, Avoiding Failures in Centrifugal Pumps, Tutorial in Proc. TAMU International Pump Symp, Houston, TX.

Marscher, W.D.; Onari, M.M.; Boyadjis, P.A., et al, 2008, Pump Mechanical Behavior & Vibration, Chapter in Centrifugal Pump Handbook, 4th Edition, McGraw-Hill, Edited by Paul Cooper et al

Marscher, W.D., Boyadjis, P.A., Gamarra, J., and Gruener, J., 2013, Effect of Component Interference Fit and Fluid Density on the Lateral and Torsional Natural Frequencies of Pump and Turbomachinery Rotor Systems, Lecture in Proc. TAMU International Pump Symposium, Houston, TX

Massey, I., 1985, Subsynchronous Vibration Problems in High Speed Multistage Centrifugal Pumps, Proc 14th Turbomachinery Symposium, Turbomachinery Laboratories, Texas A&M Univ.

Ohashi, H., Sakurai, A., Nishihama, J., 1988, "Influence of Impeller and Diffuser Geometries on the Lateral Fluid Forces of Whirling Impeller, N89-22907, TAMU Instab. Conf., pp. 285-305.

Roark, R.J., & Young, W.C., 1982, *Formulas for Stress and Strain*, McGraw-Hill, NY.

San Andres, L., 1993, Dynamic force and moment coefficients for short length annular seals, ASME J. of Tribology 115 (1), 61-70

San Andres, L., 1993, Effect of shaft misalignment on the dynamic force response of annular pressure seals, ASME Tribology

Transactions 36 (2), 173-182

San Andres, L., Soulas, T., Fayolle, P., 2002, A Bulk-Flow Model of Angled Injection Lomakin Bearings, Journal of Engineering for Gas Turbines and Power 129 (1), 195-204

San Andres, L., 2006, Hydrodynamic fluid film bearings and their effect on the stability of rotating machinery, Design and Analysis of High Speed Pumps (pp. 10-1 – 10-36), Educational Notes RTO-EN-AVT-143, Paper 10. Neuilly-sur-Seine, France

San Andrés, L., 2006B, Introduction to Pump Rotordynamics, In Design and Analysis of High Speed Pumps (pp. 9-1 – 9-26). Educational Notes RTO-EN-AVT-143, Paper 9. Neuilly-sur-Seine, France: RTO

San Andrés, L., Wu, T., Maeda, H., and Ono, T., 2018, "A Computational Fluid Dynamics Modified Bulk Flow Analysis for Circumferentially Shallow Grooved Liquid Seals," ASME J. Eng. Gas Turbines Power, 140 (1).

Simon, F., Frene, J., 1993, Analysis for Incompressible Flow in Annular Pressure Seals, J. of Trib., Trans ASME, 115 (1), pp.203-205.

Spettel, T., 1985 (April), The Dynamics of a Vertical Pump, Orbit Magazine, Bently Nevada Corp.

Thomson, W.T., 1981, *Theory of Vibration with Applications*, Prentice-Hall.

Wu, T, San Andrés, L., 2019, Pump Grooved Seals: A Computational Fluid Dynamics Approach to Improve Bulk-Flow Model Predictions, Journal of Engineering for Gas Turbines and Power 141 (10).

Yamamoto, T., 1954, On the Critical Speeds of a Shaft, Memoirs of the Faculty of Engineering, Nagoya University, Nagoya Japan,v. 6.

Yang, J., San Andrés, L., 2019, On the Influence of the Entrance Section on the Rotordynamic Performance of a Pump Seal With Uniform Clearance: A Sharp Edge Versus a Round Inlet, J. Engineering for Gas Turbines and Power 141(3)

Zirkelback, N., San Andres, L., 1996, Bulk-flow model for the transition to turbulence regime in annular pressure seals
Tribology Transactions 39 (4), 835-842

ACKNOWLEDGEMENTS

The author would like to thank Mechanical Solutions, Inc. for its support of the author in the construction of this tutorial. In particular, Paul Boyadjis, Eric Olson, Maki Onari, Dan Owens, and Chris Hurrell contributed portions of the analysis results. Special thanks to Mike Marscher and Brian Fletcher who authored much the graphics. Many thanks also to the manufacturers and users of pumps worldwide who have employed MSI in the pursuit of the many interesting design challenges, analyses, and troubleshooting testing that provided valuable background and experiences. In addition, the author would like to express their gratitude to the many key individuals in the pump industry (both OEM's and End Users) with whom he has had the pleasure to work with over the years on various projects.