Mechanical Sensorless Speed Control of PMSMs Using an Instantaneous Complex Power Formulation

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***Abstract*—In this paper two mechanical sensorless strategies to control the rotor speed in permanent magnet synchronous motors (PMSMs) are introduced and compared. Both strategies are based on the same full feedback linearization nonlinear control strategy for the motor model formulated by using the instantaneous complex power theory. However, this strategy is transformed into mechanical sensorless in a different way. In a first case, a nonlinear reduced order observer is designed for estimating rotor position and speed, whereas in the other case a full order observer is used to obtain the rotor position and speed estimates. In a first approach, it is assumed that the load power is a known function of the mechanical variables. In the second approach, the load is assumed unknown and both observers are extended to calculate a load power estimate. In this way, two different extended observers result. The performance of both mechanical sensorless strategies are compared via simulation results.**

***Index Terms*—PMSM, Speed Control, Instantaneous Complex Power, Feedback Linearization, Nonlinear Observer**

I. Introduction

The use of Permanent Magnet Synchronous Motors (PMSMs) in electric drives is increasing in many industry applications. Among others, this motor can be found in home appliances, electric transportation and pump drivers [1], [2]. In order to obtain a high-performance electric drive, rotor position must be known with high precision [3]. For this reason, many times a position sensor is coupled to the motor axis. However, it must be noted that this sensor increases the cost and diminishes the reliability of the drive. For this reason, many researchers have proposed to use different techniques for designing mechanical sensorless controllers [4].

These model-based controllers are often designed by using a phenomenological model representing the motor. To this end, several modeling tools have been proposed. A state variables nonlinear model represented by rotor position, rotor speed and real stator currents is useful for controller design purposes. Via a nonlinear change of variables, this model can be transformed into an instantaneous power description [5]. Another technique proposes to model the motor via a complex vector theory, where stator currents and voltages are assumed to be complex variables [6], [7].

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However, as mentioned above, many times it is useful to design control strategies when the measurements of mechanical variables are not available. In such a case, the control strategies are based on an algorithm which estimates the mechanical variables (rotor position and speed) from the measurement of electric variables. This approach has two main advantages: the number of sensors is reduced and the system depends only on the more reliable electrical sensors. Since 30 years ago, several methods have been used for estimating rotor position and speed. High-gain observers based on Taylor linearization design, nonlinear high-gain observers [8] and sliding mode observers [9] can be found in the literature. It must be mentioned that full or reduced observers are used [10]. To select a full or a reduced order observer for estimating mechanical variables is an important step in PMSM sensorless controllers.

This work focuses on different issues of designing sensorless controllers for PMSMs. First, a nonlinear control strategy based on the instantaneous complex power formulation is designed. Then, this strategy is transformed into mechanical sensorless by including a nonlinear observer. The performances obtained when using two different observer algorithms are compared, first assuming the load is a known function of the mechanical variables, and latter, with unknown load power. An algorithm uses an extended nonlinear reduced order observer and the other an extended nonlinear full order observer. This comparison is used to draw some conclusions on the advantages and disadvantages of each one.

II. Complex Space Vector Model of the PMSM

A. Stationary Reference Frame Model

Using complex space vector formulation, the dynamic behavior of the PMSM is described by the following set of equations:

$$L\dot{\vec{i}}_{\alpha\beta} = -R\vec{i}_{\alpha\beta} - \vec{e}_{\alpha\beta} + \vec{v}_{\alpha\beta}, \tag{1}$$

$$J\dot{\omega} = \frac{\Re\{\vec{e}'_{\alpha\beta}\vec{i}_{\alpha\beta}\}}{\omega} - \tau_L, \tag{2}$$

where R is the stator resistance, L is the stator inductance and J is the moment of inertia of the PMSM, $\vec{v}_{\alpha\beta} = v_\alpha + jv_\beta$ and $\vec{i}_{\alpha\beta} = i_\alpha + ji_\beta$ are complex variables (denoted by the over arrow) obtained using the power invariant Clarke transform, representing the stator voltage and current of the PMSM, respectively. The load torque is denoted by τ_L, and $\vec{e}_{\alpha\beta} = e_\alpha + je_\beta$ is the back Electro Motive Force (EMF). Additionally, throughout the paper $\Re\{\}$ and $\Im\{\}$ are used to denote real part and imaginary part of a complex quantity, respectively, and $'$ stands for complex conjugate of the variable (complex conjugate transpose in case of a vector).

The back EMF is related to the rotor position θ and rotor speed ω through:

$$\vec{e}_{\alpha\beta} = j\Phi\omega e^{j\theta}, \quad (3)$$

where Φ is also a parameter of the PMSM, representing the rotor flux magnitude. If the back EMF is known, then the rotor position and speed can be obtained from (3) using:

$$\omega = |\vec{e}_{\alpha\beta}|/\Phi, \quad (4)$$
$$\theta = -j\ln(-j\vec{e}_{\alpha\beta}/|\vec{e}_{\alpha\beta}|). \quad (5)$$

B. Complex Power formulation

Using Park transformation $e^{-j\theta}$ on (1)-(2), the dynamic model of the PMSM can be transformed to a rotating reference frame, synchronous with the rotor position:

$$L\dot{\vec{i}} = -(R + jL\omega)\vec{i} - \vec{e} + \vec{v}, \quad (6)$$
$$J\dot{\omega} = \frac{\Re\{\vec{e}'\vec{i}\}}{\omega} - \tau_L, \quad (7)$$

where $\vec{i} = e^{-j\theta}\vec{i}_{\alpha\beta}$, $\vec{e} = e^{-j\theta}\vec{e}_{\alpha\beta}$ and $\vec{v} = e^{-j\theta}\vec{v}_{\alpha\beta}$. Now, defining the complex power as

$$\vec{s} = \vec{e}'\vec{i} = p + jq, \quad (8)$$

differentiating (8) with respect to time and using (6)-(7), the dynamics of the system in terms of complex power s results:

$$\dot{\vec{s}} = \left(\frac{p-p_L}{J\omega^2} - \frac{R}{L} - j\omega\right)\vec{s} + \frac{\vec{e}'\vec{v} - |\vec{e}|^2}{L}, \quad (9)$$
$$J\dot{\omega} = \frac{p - p_L}{\omega}, \quad (10)$$

where $p_L = \tau_L\omega$ is the load power.

III. Proposed Controller

A. Full Feedback Linearization

The control scheme proposed in this paper is based in full feedback linearization. To perform the full feedback linearization, a complex flat output is chosen, which prevents the appearance of zero dynamics. In this case, this output is the instantaneous complex energy of the system, defined as:

$$\vec{\xi}_1 = \underbrace{\frac{1}{2}J\omega^2}_{\xi_{1r}} + j\xi_{1i}, \quad (11)$$

where the imaginary energy, ξ_{1i}, has the following dynamics:

$$\dot{\xi}_{1i} = q. \quad (12)$$

Since (9)-(10) is a second order complex variable system, $\vec{\xi}_1$ is a flat output if control action $\vec{v}$ appears only after differentiating (11) with respect to time two times. Performing the first time derivative, it results:

$$\dot{\vec{\xi}}_1 = J\omega\dot{\omega} + j\dot{\xi}_{1i} = \underbrace{p - p_L}_{\xi_{2r}} + j\underbrace{q}_{\xi_{2i}} = s - p_L = \vec{\xi}_2, \quad (13)$$

where (10) and (12) were used. As it can be seen, $\vec{v}$ (the control variable) is not present in this result. Differentiating once more with respect to time, it results:

$$\dot{\vec{\xi}}_2 = \left(\frac{p-p_L}{J\omega^2} - \frac{R}{L} - j\omega\right)\vec{s} + \frac{\vec{e}'\vec{v} - |\vec{e}|^2}{L} - \dot{p}_L = \vec{r}, \quad (14)$$

where (9) was used, and the auxiliary complex control action of the linearized system, $\vec{r} = r_r + jr_i$, was defined. As it can be seen, $\vec{v}$ only appears after the second time derivative, therefore the output is indeed flat. From (13)-(14), the resulting linear system is:

$$\dot{\vec{\xi}}_1 = \vec{\xi}_2, \quad (15)$$
$$\dot{\vec{\xi}}_2 = \vec{r}, \quad (16)$$

which has its real and imaginary parts decoupled. Therefore, two independent control loops for this system are defined the sections III-B and III-C.

Once control action $\vec{r}$ of the linearized system is computed, from (14), control action $\vec{v}$ is computed through:

$$\vec{v} = L\frac{\vec{e}}{|\vec{e}|^2}\left[\vec{r} + \dot{p}_L - \left(\frac{p-p_L}{J\omega^2} - \frac{R}{L} - j\omega\right)\vec{s}\right] + \vec{e}. \quad (17)$$

The implementation of this linearization scheme requires knowledge of the back EMF $\vec{e}$, the rotor angular speed ω and position θ (to perform the Park transform), and the load power p_L and its first time derivative. Instead of measuring these variables, they will be estimated using observers. If the load power is a known function of the mechanical variables, only rotor position and speed need to be estimated, then the observers defined in section IV can be used. If the load power must also be estimated, then the observers of section V are recommended. The main goal in this work is to compare the observer based controllers when the load power is unknown. However, the section IV is developed to obtain some useful previous conclusions.

B. Kinetic Energy Control Loop

To control the kinetic energy, the following full state feedback controller is defined:

$$\dot{\xi}_{1r} = \xi_{2r}, \quad (18)$$
$$\dot{\xi}_{2r} = -k_1(\xi_{1r} - \xi_{1r}^*) - k_2(\xi_{2r} - \xi_{2r}^*) - k_3\xi_{3r} = r_r, \quad (19)$$
$$\dot{\xi}_{3r} = \xi_{1r} - \xi_{1r}^*, \quad (20)$$

where k_1-$k_3 \in \mathbb{R}$ are the gains of the controller, and ξ_{1r}^* and ξ_{2r}^* are the references. Additionally, ξ_{3r} is the state of an integrator, which is added to compensate steady state errors due to model uncertainties or disturbances. The references are set to:

$$\xi_{1r}^* = \frac{1}{2} J\omega^{*2}, \tag{21}$$

$$\xi_{2r}^* = 0, \tag{22}$$

where ω^* is the angular speed reference, and ξ_{2r}^* is obtained from (13), considering that in steady state $p - p_L = 0$. The gains of this linear system are chosen using pole placement to obtain the desired dynamic response.

C. Reactive Power Control Loop

The control the reactive power, the following full state feedback controller is defined:

$$\dot{\xi}_{2i} = -k_4(q - q^*) - k_5\xi_{3i} = r_i, \tag{23}$$

$$\dot{\xi}_{3i} = q - q^*, \tag{24}$$

where k_4-$k_5 \in \mathbb{R}$ are the gains of the controller and q^* is the reactive power reference. As in the energy control loop, an integrator is also added to compensate steady state errors. This is represented by state ξ_{3i}. The gains of this linear system are chosen using pole placement to obtain the desired dynamic response.

IV. Back EMF, Rotor Speed and Position Estimation

In this section two observers, a reduced order observer and a full order observer, are defined to estimate the back EMF, and the rotor speed and position, assuming the load power is a known function of the mechanical variables. They both have advantages and disadvantages, and it is up to the designer to use one or the other, depending on the available hardware.

A. Reduced Order Observer

The reduced order observer is the best option when the measured signals do not have significant high frequency noise. Features:

- It has one less state than the full order observer, and therefore it can have faster convergence.

Drawbacks:

- It amplifies high frequency noise.

1) Proposed Observer: The dynamics of $\vec{e}_{\alpha\beta}$ are obtained deriving (3) with respect to time:

$$\dot{\vec{e}}_{\alpha\beta} = \underbrace{\left(j\omega + \frac{\Re\{\vec{e}'_{\alpha\beta}\vec{i}_{\alpha\beta}\}}{J\omega^2}\right)\vec{e}_{\alpha\beta}}_{\vec{a}} + \underbrace{\left(\frac{-\vec{e}_{\alpha\beta}}{J\omega^2}\right)}_{\vec{b}} p_L. \tag{25}$$

From (4), (5) and (25) the following observer is proposed:

$$\dot{\hat{\vec{e}}}_{\alpha\beta} = \hat{\vec{a}} + \hat{\vec{b}}\hat{p}_L + g_1\vec{\varepsilon}_{\alpha\beta}, \tag{26}$$

$$\hat{\omega} = |\hat{\vec{e}}_{\alpha\beta}|/\Phi, \tag{27}$$

$$\hat{\theta} = -j\ln(-j\hat{\vec{e}}_{\alpha\beta}/|\hat{\vec{e}}_{\alpha\beta}|), \tag{28}$$

where $\vec{\varepsilon}_{\alpha\beta} = \vec{e}_{\alpha\beta} - \hat{\vec{e}}_{\alpha\beta}$,

$$\hat{\vec{a}} = \left(j\hat{\omega} + \frac{\Re\{\hat{\vec{e}}'_{\alpha\beta}\vec{i}_{\alpha\beta}\}}{J\hat{\omega}^2}\right)\hat{\vec{e}}_{\alpha\beta}, \tag{29}$$

$$\hat{\vec{b}} = \frac{-\hat{\vec{e}}_{\alpha\beta}}{J\hat{\omega}^2}, \tag{30}$$

$g_1 \in \mathbb{R}$ is a gain, and $\hat{p}_L$ is the estimated load power, which is obtained evaluating a known function of the mechanical variables using the estimated mechanical variables (for example, $\hat{p}_L = B\hat{\omega}^2$, with B a constant, or if the load torque τ_L is constant and known then $\hat{p}_L = \tau_L\hat{\omega}$). Through Lyapunov it can be found that convergence is guaranteed if $g_1 > 0$ is large enough. The demonstration is not included for space reasons, but is similar to (45)-(46) in the next section.

2) Computation of the Back EMF Estimation Error $\vec{\varepsilon}_{\alpha\beta}$: The actual back EMF $\vec{e}_{\alpha\beta}$ is not a measurable variable, therefore, $\vec{\varepsilon}_{\alpha\beta}$ cannot be directly computed. Instead, from (1) the estimated current is defined using measured variables:

$$L\dot{\hat{i}}_{\alpha\beta} = -Ri_{\alpha\beta} - \hat{e}_{\alpha\beta} + v_{\alpha\beta}. \tag{31}$$

Now $\vec{\varepsilon}_{\alpha\beta}$ can be computed subtracting (1) and (31):

$$\vec{\varepsilon}_{\alpha\beta} = \vec{e}_{\alpha\beta} - \hat{\vec{e}}_{\alpha\beta} = -L(\dot{\vec{i}}_{\alpha\beta} - \dot{\hat{\vec{i}}}_{\alpha\beta}). \tag{32}$$

To avoid computing differentiating $\vec{i}_{\alpha\beta}$ with respect to time, we first replace (31) in (32) and the resulting equation in (26). Then, the following auxiliary variable is defined:

$$\dot{\vec{\eta}} = \dot{\hat{\vec{e}}}_{\alpha\beta} + g_1L\dot{\vec{i}}_{\alpha\beta} = \hat{\vec{a}} + \hat{\vec{b}}\hat{p}_L - g_1(-Ri_{\alpha\beta} - \hat{e}_{\alpha\beta} + v_{\alpha\beta}). \tag{33}$$

Notice that solving this differential equation does not require knowledge of the time derivative of the current. Then, actual estimated back EMF is computed from:

$$\hat{\vec{e}}_{\alpha\beta} = \vec{\eta} - g_1L\vec{i}_{\alpha\beta}. \tag{34}$$

B. Full Order Observer

1) Features and Drawbacks: The full order observer is the best option when the measured signals are contaminated with high frequency noise.
Features:

- It filters out high frequency noise that can be present in the measured signals.

Drawbacks:

- It has one more state than the reduced order observer, and the maximum convergence speed is slower than that of the reduced order observer.

2) Proposed Observer: From (1) and (25), the following observer is proposed:

$$L\dot{\hat{\vec{i}}}_{\alpha\beta} = -R\hat{\vec{i}}_{\alpha\beta} - \hat{\vec{e}}_{\alpha\beta} + \vec{v}_{\alpha\beta} + g_1L\vec{\epsilon}_{\alpha\beta}, \tag{35}$$

$$\dot{\hat{\vec{e}}}_{\alpha\beta} = \hat{\vec{a}} + \hat{\vec{b}}\hat{p}_L + g_2\vec{\epsilon}_{\alpha\beta}, \tag{36}$$

where g_1, $g_2 \in \mathbb{R}$ are gains, and $\vec{\epsilon}_{\alpha\beta} = \vec{i}_{\alpha\beta} - \hat{\vec{i}}_{\alpha\beta}$.

V. Back EMF, Rotor Speed and Position and Load Power Estimation

In this section two observers are defined to estimate the back EMF, the rotor speed and position, and the load power: a reduced order observer and a full order observer. They both have advantages and disadvantages, and it is up to the designer to use one or the other, depending on the available hardware.

A. Reduced Order Observer

1) Proposed Observer: In what follows, the model will be extended. This extension includes a model for the unknown load power. It will be assumed that the power is slow varying. However, other assumptions can be used if needed and more states for representing the load power in the extended model can be added [11]. Under the above assumption, and from (25), the system to be observed is modeled by:

$$\dot{\vec{e}}_{\alpha\beta} = \vec{a} + \vec{b}\,p_L, \tag{37}$$
$$\dot{p}_L = 0. \tag{38}$$

From (4), (5), (37) and (38) the following observer is proposed:

$$\dot{\hat{\vec{e}}}_{\alpha\beta} = \hat{\vec{a}} + \hat{\vec{b}}\,\hat{p}_L + g_1\vec{\varepsilon}_{\alpha\beta}, \tag{39}$$
$$\dot{\hat{p}}_L = -al, \tag{40}$$
$$\hat{\omega} = |\hat{\vec{e}}_{\alpha\beta}|/\Phi, \tag{41}$$
$$\hat{\theta} = -j\ln(-j\hat{\vec{e}}_{\alpha\beta}/|\hat{\vec{e}}_{\alpha\beta}|), \tag{42}$$

where $\vec{\varepsilon}_{\alpha\beta} = \vec{e}_{\alpha\beta} - \hat{\vec{e}}_{\alpha\beta}$, $\hat{\vec{a}}$ is defined in (29) and $\hat{\vec{b}}$ in (30), $g_1 \in \mathbb{R}$ is a gain and al is an adaptation law to be defined. From (37)-(40) the error dynamics can be written as follows:

$$\dot{\vec{\varepsilon}}_{\alpha\beta} = \triangle\vec{a} + \triangle\vec{b}\,p_L + \hat{\vec{b}}\,\varepsilon_{p_L} - g_1\vec{\varepsilon}_{\alpha\beta}, \tag{43}$$
$$\dot{\varepsilon}_{p_L} = al, \tag{44}$$

where $\varepsilon_{p_L} = p_L - \hat{p}_L$, $\triangle\vec{a} = \vec{a} - \hat{\vec{a}}$ and $\triangle\vec{b} = \vec{b} - \hat{\vec{b}}$. To find the adaptation law al, the following candidate Lyapunov function is proposed:

$$V = \vec{\varepsilon}\,'_{\alpha\beta}\vec{\varepsilon}_{\alpha\beta} + \frac{1}{2}g_2^{-1}\varepsilon_{p_L}^2, \tag{45}$$

which is positive definite, where $g_2 \in \mathbb{R}$ is a positive gain. The adaptation law must guarantee that $\dot{V} < 0$, therefore, differentiating with respect to time it results:

$$\dot{V} = 2\Re\{[\triangle\vec{a} + \triangle\vec{b}\,p_L]\vec{\varepsilon}\,'_{\alpha\beta}\} - 2g_1|\vec{\varepsilon}_{\alpha\beta}|^2 + (2\Re\{\hat{\vec{b}}\,'\vec{\varepsilon}_{\alpha\beta}\} + g_2^{-1}al)\varepsilon_{p_L}. \tag{46}$$

For a bound estimation error, making gain g_1 large enough guarantees that (46) is negative, as long as the term that multiplies ε_{p_L} is eliminated. To do so, the following adaptation law is defined $al = -2g_2\Re\{\hat{\vec{b}}\,'\vec{\varepsilon}_{\alpha\beta}\}$. Therefore, from (40), the estimated load power results:

$$\dot{\hat{p}}_L = 2g_2\Re\{\hat{\vec{b}}\,'\vec{\varepsilon}_{\alpha\beta}\}. \tag{47}$$

2) Computation of the Back EMF Estimation Error $\vec{\varepsilon}_{\alpha\beta}$: The actual back EMF $\vec{e}_{\alpha\beta}$ is not a measurable variable, therefore, $\vec{\varepsilon}_{\alpha\beta}$ must be computed using (32). However, because of the nonlinearity of (47), the method used in section IV-A2 to avoid computing the time derivative of the current cannot be applied here. Instead, a high gain estimator is used. Assuming the current is sinusoidal, with frequency ω, its dynamics is modeled by:

$$\dot{\vec{i}}_{\alpha\beta} = j\omega\vec{i}_{\alpha\beta}. \tag{48}$$

Using this model, the following high gain estimator is proposed:

$$\dot{\hat{\vec{i}}}{}^{hg}_{\alpha\beta} = j\hat{\omega}\,\hat{\vec{i}}{}^{hg}_{\alpha\beta} + \gamma(\vec{i}_{\alpha\beta} - \hat{\vec{i}}{}^{hg}_{\alpha\beta}), \tag{49}$$

where $\gamma \in \mathbb{R}$ is a gain and $\hat{\omega}$ is defined in (41). Then, using (31) in (32) and replacing $\dot{\vec{i}}_{\alpha\beta}$ with (49), the back EMF estimation error can be computed through:

$$\vec{\varepsilon}_{\alpha\beta} = L(\gamma - j\hat{\omega})\hat{\vec{i}}{}^{hg}_{\alpha\beta} - (L\gamma + R)i_{\alpha\beta} - \hat{e}_{\alpha\beta} + v_{\alpha\beta}. \tag{50}$$

3) Summary: The reduced order observer is summarized here

$$\hat{\omega} = |\hat{\vec{e}}_{\alpha\beta}|/\Phi, \tag{51}$$
$$\hat{\theta} = -j\ln(-j\hat{\vec{e}}_{\alpha\beta}/|\hat{\vec{e}}_{\alpha\beta}|), \tag{52}$$
$$\vec{\varepsilon}_{\alpha\beta} = L(\gamma - j\hat{\omega})\hat{\vec{i}}{}^{hg}_{\alpha\beta} - (L\gamma + R)i_{\alpha\beta} - \hat{e}_{\alpha\beta} + v_{\alpha\beta}, \tag{53}$$
$$\dot{\hat{\vec{e}}}_{\alpha\beta} = \hat{\vec{a}} + \hat{\vec{b}}\,\hat{p}_L + g_1\vec{\varepsilon}_{\alpha\beta}, \tag{54}$$
$$\dot{\hat{p}}_L = 2g_2\Re\{\hat{\vec{b}}\,'\vec{\varepsilon}_{\alpha\beta}\}, \tag{55}$$
$$\dot{\hat{\vec{i}}}{}^{hg}_{\alpha\beta} = j\hat{\omega}\,\hat{\vec{i}}{}^{hg}_{\alpha\beta} + \gamma(\vec{i}_{\alpha\beta} - \hat{\vec{i}}{}^{hg}_{\alpha\beta}), \tag{56}$$

where $\hat{\vec{a}}$ and $\hat{\vec{b}}$ are defined in (29) and (30), respectively.

B. Full Order Observer

1) Proposed Observer: From (1) and (37), the following observer is proposed:

$$L\dot{\hat{\vec{i}}}_{\alpha\beta} = -R\hat{\vec{i}}_{\alpha\beta} - \hat{\vec{e}}_{\alpha\beta} + \vec{v}_{\alpha\beta} + g_1L\vec{\epsilon}_{\alpha\beta}, \tag{57}$$
$$\dot{\hat{\vec{e}}}_{\alpha\beta} = \hat{\vec{a}} + \hat{\vec{b}}\hat{p}_L + g_2\vec{\epsilon}_{\alpha\beta}, \tag{58}$$
$$\dot{\hat{p}}_L = -al, \tag{59}$$

where g_1, $g_2 \in \mathbb{R}$ are gains, $\vec{\epsilon}_{\alpha\beta} = \vec{i}_{\alpha\beta} - \hat{\vec{i}}_{\alpha\beta}$ and al is the adaptation law to be found. Subtracting (57) from (1), (58) from (37) and (59) from (38), the error dynamics results:

$$\dot{\vec{\epsilon}}_{\alpha\beta} = -\frac{R}{L}\vec{\epsilon}_{\alpha\beta} - \frac{1}{L}\vec{\varepsilon}_{\alpha\beta} - g_1\vec{\epsilon}_{\alpha\beta}, \tag{60}$$
$$\dot{\vec{\varepsilon}}_{\alpha\beta} = \triangle\vec{a} + \triangle\vec{b}\,p_L + \hat{\vec{b}}\,\varepsilon_{p_L} - g_2\vec{\epsilon}_{\alpha\beta}, \tag{61}$$
$$\dot{\varepsilon}_{p_L} = al. \tag{62}$$

To find the adaptation law al, the following candidate Lyapunov function is proposed:

$$V = \eta' P\eta + \frac{1}{2}g_3^{-1}\varepsilon_{P_L}^2, \tag{63}$$

where $g_3 \in \mathbb{R}$ is a gain, $\eta = [\vec{\varepsilon}'_{\alpha\beta}\ \vec{\epsilon}'_{\alpha\beta}]'$, and P is a positive definite matrix:

$$P = \begin{bmatrix} p_1 & -p_2 \\ -p_2 & p_1 \end{bmatrix}, \quad (64)$$

where p_1, $p_2 \in \mathbb{R}$, $p_1 > 0$, $p_2 > 0$ and $p_1^2 - p_2^2 > 0$. With these V is positive definite, and can also be written in the following form:

$$V = p_1(\vec{\varepsilon}'_{\alpha\beta}\vec{\varepsilon}_{\alpha\beta} + \vec{\epsilon}'_{\alpha\beta}\vec{\epsilon}_{\alpha\beta}) - 2p_2\Re\{\vec{\varepsilon}'_{\alpha\beta}\vec{\epsilon}_{\alpha\beta}\} + \frac{g_3^{-1}}{2}\varepsilon_{p_L}^2. \quad (65)$$

The adaptation law must guarantee that $\dot{V} < 0$, therefore, differentiating with respect to time it results:

$$\dot{V} = \Re\{2(\triangle\vec{a}' + \triangle\vec{b}' p_L)(p_1\vec{\varepsilon}_{\alpha\beta} - p_2\vec{\epsilon}_{\alpha\beta}) - \gamma_1\vec{\epsilon}'_{\alpha\beta}\vec{\varepsilon}_{\alpha\beta}\} - \gamma_2|\vec{\epsilon}_{\alpha\beta}|^2 + \frac{2p_2}{L}|\vec{\varepsilon}_{\alpha\beta}|^2 + (\Re\{2p_1\hat{\vec{b}}'\vec{\varepsilon}_{\alpha\beta} - 2p_2\hat{\vec{b}}'\vec{\epsilon}_{\alpha\beta}\} + g_3^{-1}al)\varepsilon_{p_L}, \quad (66)$$

where $\gamma_1 = 2p_1(g_2 + \frac{1}{L}) - 2p_2(\frac{R}{L} + g_1)$ and $\gamma_2 = 2p_1(g_1 + \frac{R}{L}) - 2p_2 g_2$. For a bound estimation error, making gain γ_2 large enough guarantees that (66) is negative, as long as the term that multiplies ε_{p_L} is eliminated. Since $\vec{\varepsilon}_{\alpha\beta}$ is unknown, only part of this term can be eliminated, but this is enough for convergence [12]. To do so, the following adaptation law is defined $al = 2p_2 g_3\Re\{\hat{\vec{b}}'\vec{\epsilon}_{\alpha\beta}\}$, then, the estimated load power results:

$$\dot{\hat{p}}_L = -2g_3\Re\{\hat{\vec{b}}'\vec{\epsilon}_{\alpha\beta}\}, \quad (67)$$

where $p_2 = 1$ is set, since any positive value can be chosen.

2) Summary: The full order observer is summarized here

$$\hat{\omega} = |\hat{\vec{e}}_{\alpha\beta}|/\Phi, \quad (68)$$

$$\hat{\theta} = -j\ln(-j\hat{\vec{e}}_{\alpha\beta}/|\hat{\vec{e}}_{\alpha\beta}|), \quad (69)$$

$$L\dot{\hat{\vec{i}}}_{\alpha\beta} = -R\hat{\vec{i}}_{\alpha\beta} - \hat{\vec{e}}_{\alpha\beta} + \vec{v}_{\alpha\beta} + g_1 L\vec{\epsilon}_{\alpha\beta}, \quad (70)$$

$$\dot{\hat{\vec{e}}}_{\alpha\beta} = \hat{\vec{a}} + \hat{\vec{b}}\hat{p}_L + g_2\vec{\epsilon}_{\alpha\beta}, \quad (71)$$

$$\dot{\hat{p}}_L = -2g_3\Re\{\hat{\vec{b}}'\vec{\epsilon}_{\alpha\beta}\}, \quad (72)$$

where $\hat{\vec{a}}$ and $\hat{\vec{b}}$ are defined in (29) and (30), respectively.

VI. Simulation Results

In this section the proposed controller is simulated using both the reduced and full order observers. For space reasons, only the observers of section V are simulated. The parameters of the PMSM are $L = 20.5$ mH, $R = 1.55\Omega$, $J = 2.21 \cdot 10^{-3}$ Kgm^2, $\Phi = 0.22$ Nm/A. Its nominal speed and nominal load are 1500 rpm and 7 Nm, respectively. The nominal phase current is 30 Arms.

Both control loops are designed with a settling time of 100ms and a damping of 0.707, which results in the following gains: $k_1 = 25.393 \cdot 10^3$, $k_2 = 322$, $k_3 = 973.654 \cdot 10^3$, $k_4 = 92$ and $k_5 = 4233$. The gains of both observers are obtained linearizing the observer and applying linear pole placement techniques. For the reduced order observer, the settling time is set to 40ms, the damping to 0.707, and the high gain estimator settling time to 200μs. With these parameters,

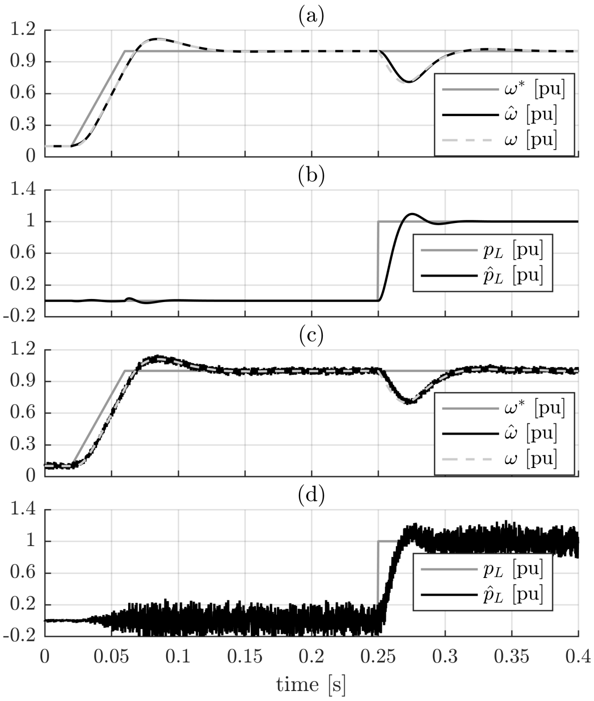

Fig. 1. Simulation with reduced order observer. a) ω^*, $\hat{\omega}$ and ω (perfect measurement). b) p_L vs $\hat{p}_L$ (perfect measurement). c) ω^*, $\hat{\omega}$ and ω (with measurement noise). d) p_L vs $\hat{p}_L$ (with measurement noise).

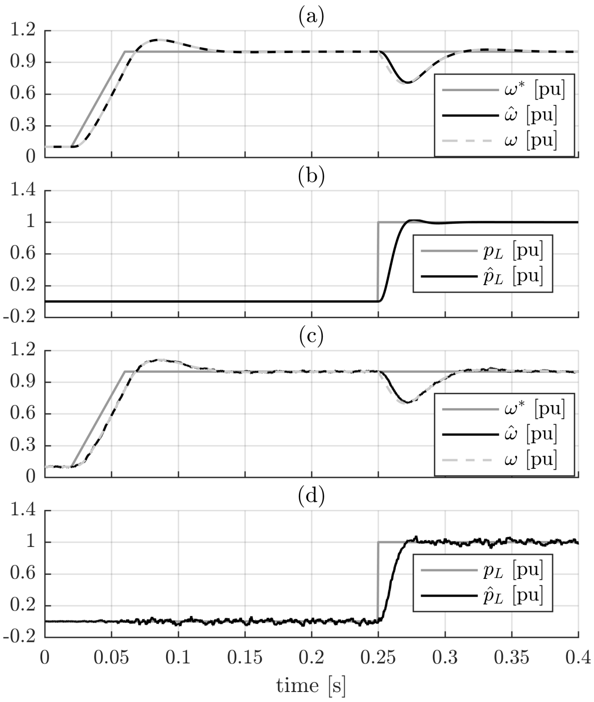

Fig. 2. Simulation with full order observer. a) ω^*, $\hat{\omega}$ and ω (perfect measurement). b) p_L vs $\hat{p}_L$ (perfect measurement). c) ω^*, $\hat{\omega}$ and ω (with measurement noise). d) p_L vs $\hat{p}_L$ (with measurement noise).

the reduced order observer gains result: $g_1 = 230$, $g_2 = 1.335$ and $\gamma = 23 \cdot 10^3$. For the full order observer the settling time is also set to 40ms and the damping to 0.707. With $p_0 = 5$ the gains result: $g_1 = 729.4$, $g_2 = -3253.5$ and $g_3 = 15.73$.

Figure 1 shows the simulation results for the controller with the reduced order observer. Two cases are analyzed: with perfect measurements (Figs. 1a-b) and with measurement noise of $\pm 0.3A$ (Figs. 1c-d). The simulation starts with the PMSM at 10% nominal speed and no load condition. At $t = 0.02$ s the speed reference is increased to nominal speed within 40 ms. As can be seen in Figs. 1a and 1c the speed converges to the reference after 100 ms in both cases, and the noise has no noticeable effect on the speed. At $t = 0.25$ s a nominal load step is introduced. As can be seen in Figs. 1b and 1d, the estimated load power converges to the actual load power within 40 ms, as expected. However, in the simulation with noise, there is significant high frequency noise present in the load power estimation. In this case, however, this does not affect the performance of the system.

The same simulations are repeated using the controller with the full order observer. Again, two cases are analyzed: with perfect measurements (Figs. 2a-b) and with measurement noise of $\pm 0.3A$ (Figs. 2c-d). The results are shown in Fig. 2. As can be seen, performance is similar, but when the measurements are contaminated with noise, the full order observer is less sensitive to this noise, as shown in the cleaner speed estimation in Fig. 2a and load power estimation in Fig. 2b.

VII. Future Works

Recently, multiphase electric machines [13], [14] and their use in industry applications are being studied. Among others, AC drives in electric propulsion systems are being developed. Multiphase motors can be used in electric and hybrid vehicles, locomotive and ships. They are also included for implementing the concept of a hybrid electric powered or more electric aircraft as a part of the more-electric aircraft initiative.

The main reason for using multiphase motors is that they can generate a magnetomotive force with lower harmonic content than that generated by threephase motors.

In future works, the authors will study the application of the proposed control strategies in AC drives containing multiphase motors.

In addition, as it is well-known EMF based observers perform in a good way at medium and high speed. For this reason, our observers will be combined with a low speed estimator for obtaining good estimates at the whole state space [4].

VIII. Conclusions

In our work two mechanical sensorless strategies for PMSMs have been compared. Both strategies are based on full feedback linearization for the motor modeled by using the instantaneous complex power theory. The strategy was implemented by using estimates of the rotor position and speed. Two cases were studied. In one of them a reduced order observer was used for estimating rotor position and speed. In the other one, a full order observer was implemented with the same purpose.

By assuming parameters and load are well-known, computational burden is diminished when the reduced order observer is used. In addition, its sensitivity to low frequency uncertainty in the measurement is lower. However, this strategy is more sensible to noise added in the measurement.

When the load was assumed to be unknown, it was necessary to include a high-gain observer for estimating the currents time derivatives, then the advantage of lower computational burden of the reduced order observer algorithm was missed.

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