Accepted to Phys. Fluids 10.1063/5.0128809

Comment on "Faraday waves in a Hele-Shaw cell" [Phys Fluids 30, 042106 (2018)]

A. Boschan,[1, a)] M. Noseda,[1] M. A. Aguirre,[1] and M. Piva[1]

Grupo de Medios Porosos, Departamento de Física, Facultad de Ingeniería, Universidad de Buenos Aires, Paseo Colón 850, 1063 Buenos-Aires, Argentina.

(Dated: 11 November 2022)

[a)] abosch@fi.uba.ar

Accepted to Phys. Fluids 10.1063/5.0128809

We propose improved dimensionless variables and scaling law to describe the height of Faraday waves in a vertically vibrating Hele-Shaw cell. In comparison with those suggested by Li *et al.*[1], the influence of the liquid depth d on the wave height H is disregarded, and the critical acceleration F_c, at which the Faraday instability is triggered, is now taken into account. We support our approach with results from an additional set of experimental data, that includes the measurement of F_c, and encompasses the parameter range studied by Li *et al.*[1]. Our proposal is based on the following arguments:

1) The term $\tanh(kd)$ (here $k = \frac{2\pi}{\lambda}$, and λ is the wavelength) controls the crossover from shallow to deep water waves regimes[2], becoming asymptotic to unity as d exceeds a certain liquid depth d_{cross} (for fixed λ). In the deep water waves regime, d is large enough so that the waves don't interact with the bottom of the Hele-Shaw cell, and H and λ become independent of d. Specifically, we understand that this condition is met in the work of Li *et al.*[1] (for which $d \geq 10$ mm). In fact, assuming the crossover occurs as late as $kd \approx 2$ (with $\tanh(2) = 0.964$), and considering that λ ranges from 15 mm to 25 mm (as reported in Fig. 10 of Li *et al.*[1]), then, the condition 4.7 mm$< d_{cross} <$8 mm is satisfied in their work. On the other hand, no consistent increase of H with d can be inferred from Table III of Li *et al.*[1] (for constant f and F). Moreover, it is evident in previous works by some of the same authors, in equal, or very similar, experimental conditions (cf. Fig. 5 of Li, Yu, and Liao[3] and Fig. 2 of Li, Xu, and Liao[4]). Indeed, in those articles, it is stated that H is independent of d for $d \geq 10$ mm. Finally, from previous works by other authors, that employed a cell gap of 1.87 mm (instead of 1.7 mm from Li *et al.*[1]), and for $f=$ 24 Hz, it was shown how H and λ became independent of d for $d \sim 6$ mm (cf. Fig. 5 of Martino *et al.*[5], and Fig. 6.12 from Barba-Maggi[6]).

We conclude that, in the work of Li *et al.*[1], H is independent of d, and then, it is not correct to include d in the dimensionless variables (and scaling law) for H.

2) As Faraday waves are only triggered if the vibration acceleration F exceeds a critical value F_c, and vanish for $F < F_c$, their amplitude is more likely to be controlled by a reduced parameter $(F - F_c)$ than simply by F. This parameter was used by Perinet *et al.*[7] to study the amplitude of Faraday waves, and by other authors[8,9] to study the transition from order to disorder in Faraday wave patterns.

In view of these arguments, we propose a scaling law with two dimensionless variables Π_1 and Π_2, four independent variables H, f, g, $(F - F_c)$, and two independent physical dimensions L and T, as follows:

$$\Pi_1 = \frac{Hf^2}{g}, \Pi_2 = \frac{F - F_c}{g} \tag{1}$$

$$\Pi_1 = \Phi(\Pi_2) \Rightarrow \frac{Hf^2}{g} = \Phi\left[\frac{F - F_c}{g}\right] \tag{2}$$

To compare both approaches, we have performed experiments for f = 16, 20, 25, 30 and 35 Hz, replicating the liquid depths explored by Li *et al.*[1] (d =10, 20, 30 mm). The Hele-Shaw cell used is that reported in Martino *et al.*[5], while all other experimental conditions reproduced those of Li *et al.*[1]. The experimental procedure was the following: the value of F was increased gradually from F=0 (flat free surface) up to the stationary waves regime, which is triggered at a first critical value $F = F_c^+$. At this point, H increases sharply from zero to a finite value. We continue increasing F in steps of 0.1 m/s^2, letting the system stabilize for one minute before each measurement of H, up to the non-stationary waves regime. In this regime, the waves are no longer sub-harmonic, and do not possess well-defined period, height or wavelength. We then decrease F down, in steps of 0.1 m/s^2, until the instability vanishes sharply at $F = F_c(< F_c^+)$ (this second critical value is the one reported in our comment[10]), and the free surface is again flat. Data points are an average of five measurements for each parameter set. This way of determining F_c is similar to that employed in Rajchenbach, Leroux, and Clamond[10]. We refer the reader to that article, and to Perinet *et al.*[7], on the hysteretic behavior of Faraday waves. The acceleration F ranged from 8 to 25 m/s^2 in our experiments. Note that, as in Li *et al.*[1], the analysis is restricted to the 2D profiles of the waves.

Fig. 1 shows data points from Li *et al.*[1] along with ours, using the dimensionless variables suggested by the former. The solid line is the scaling law of Li *et al.*[1]. Note that data points appear dispersed, and clearly segregated by the value of d, as they do in Fig. 4 of Li *et al.*[1]. Those furthest away from the scaling of Li *et al.*[1] correspond to relatively small or large values of F. For comparison, a data collapse plot using our dimensionless variables is shown in Fig. 2. Note that the hollow data points in Fig.2 correspond to the same measurements as those of Fig.1, however, using our dimensionless variables, they are not segregated by the value of d, showing thus a good data collapse. Dispersion of data is observed as $\frac{(F-F_c)}{g}$ vanishes. Unluckily, in the work of Li *et al.*[1], F_c was not considered, so we cannot include their data points in Fig. 2. The solid line is a fit by our scaling law, in the form of a

Accepted to Phys. Fluids 10.1063/5.0128809

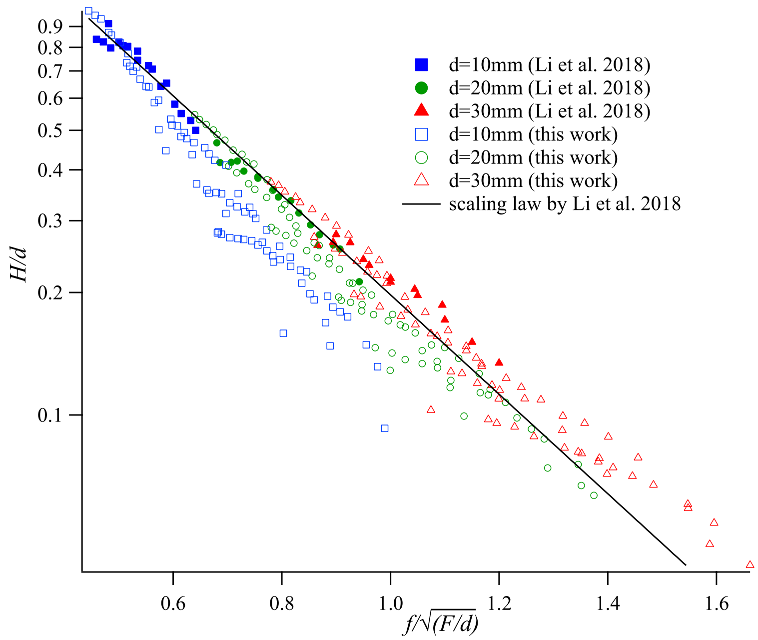


FIG. 1. Data collapse using the dimensionless variables of Li *et al.*[1] (cf. with Fig. 4 from that work). Full symbols correspond to experimental data from Li *et al.*[1] (f ranges from 18 to 24 Hz), while hollow symbols represent experimental data from this work (f ranges from 16 to 35 Hz). The solid line is the scaling law of Li *et al.*[1].

power-law function:

$$\frac{Hf^2}{g} \propto \left[\frac{F - F_c}{g}\right]^n \tag{3}$$

The fit yields a value of n = 0.34. Since Faraday waves belong to the particular class of phenomena for which an oscillatory instability is triggered only when the critical value of the control parameter is attained[11,12], we find that a power law dependence of H on $(F - F_c)$ is physically meaningful. Note that, in our scaling, H varies inversely with f^2, as opposed to the scaling of Li *et al.*[1], for which $log(H/d) \propto -f$ (Eq. 42 from that work). In the shallow water regime, our scaling law is likely to be invalid because it doesn't take into account a possible variation of H with d. The determination of appropriate dimensionless variables and scaling law in the crossover from shallow to deep water waves regimes shall be the subject of further research.

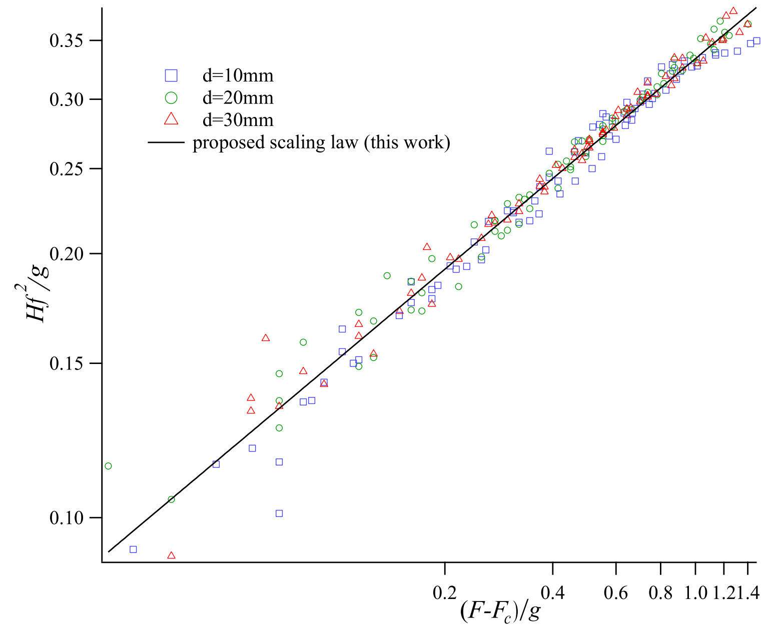


FIG. 2. Data collapse of the dimensionless wave height $\frac{Hf^2}{g}$ as a function of reduced acceleration $\frac{(F-F_c)}{g}$ for the experimental data from this work. Each hollow symbol in this figure corresponds to one in Fig. 1; both arise from the same measurement. Note the absence of segregation by the value of d. The solid line is a fit by the scaling law $\frac{Hf^2}{g} \propto \left[\frac{F-F_c}{g}\right]^n$, yielding $n = 0.34$. Unluckily, F_c was not considered in the work of Li *et al.*[1], so we cannot include their data points in this figure.

We are indebted to Dr. G. Gauthier for fruitful discussions.

REFERENCES

[1] J. Li, X. Li, K. C. B., Xie, and S. Liao, "Faraday waves in a Hele-Shaw cell," Phys. Fluids **30**, 042106 (2018).

[2] T. B. Benjamin and F. Ursell, "The stability of the plane free surface of a liquid in vertical periodic motion," Proc. R. Soc. A **225**, 505515 (1954).

[3] X. Li, Z. Yu, and S. Liao, "Observation of two-dimensional Faraday waves in extremely shallow depth," Phys. Rev. E **92**, 033014 (2015).

[4]X. Li, D. Xu, and S. Liao, "Observations of highly localized oscillons with multiple crests and troughs," Phys. Rev. E **90**, 031001 (2014).

[5]R. Martino, A. Boschan, D. B. Maggi, G. Bongiovanni, J. Geminard, and M. Piva, "Sediment motion induced by Faraday waves in a Hele-Shaw cell," Phys. Rev. E **101**, 043112 (2020).

[6]D. Barba-Maggi, "Dinámica de suspensiones concentradas sometidas a vibración mecánica," PhD. Thesis (2020), http://bibliotecadigital.fi.uba.ar/files/show/278.

[7]N. Perinet, C. F. Beas, J. Chergui, D. Juric, and S. Shin, "Histeretic Faraday waves," Phys. Rev. E **93**, 063114 (2016).

[8]N. Tufillaro, R. Ramshankar, and J. Gollub, "Order-disorder transition in capillary ripples," Phys. Rev. Lett. **62**, 422 – 425 (1989).

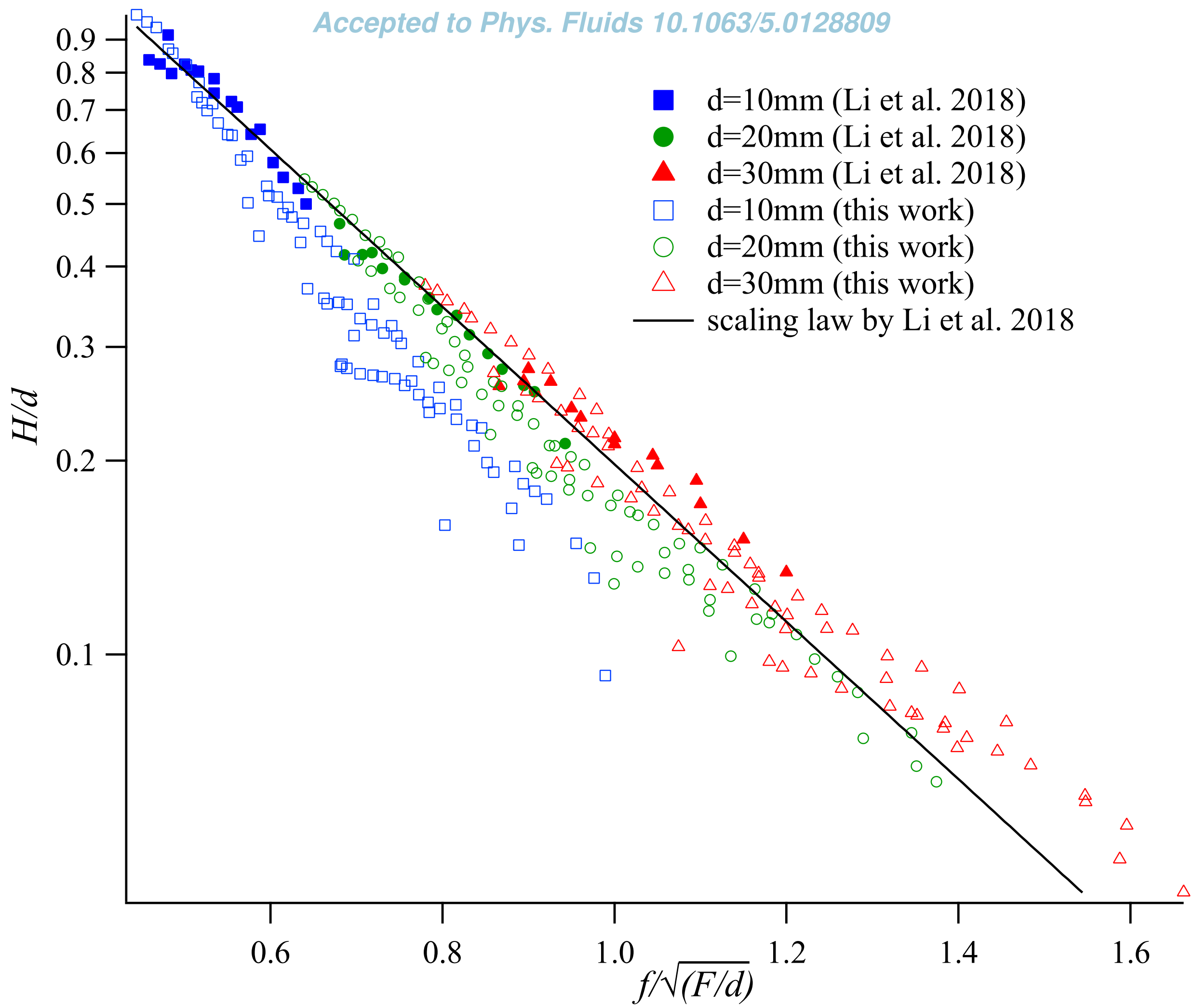
[9]I. Shani, G. Cohen, and J. Fineberg, "Localized instability on the route to disorder in Faraday waves," Phys. Rev. Lett. **104**, 184507 (2010).

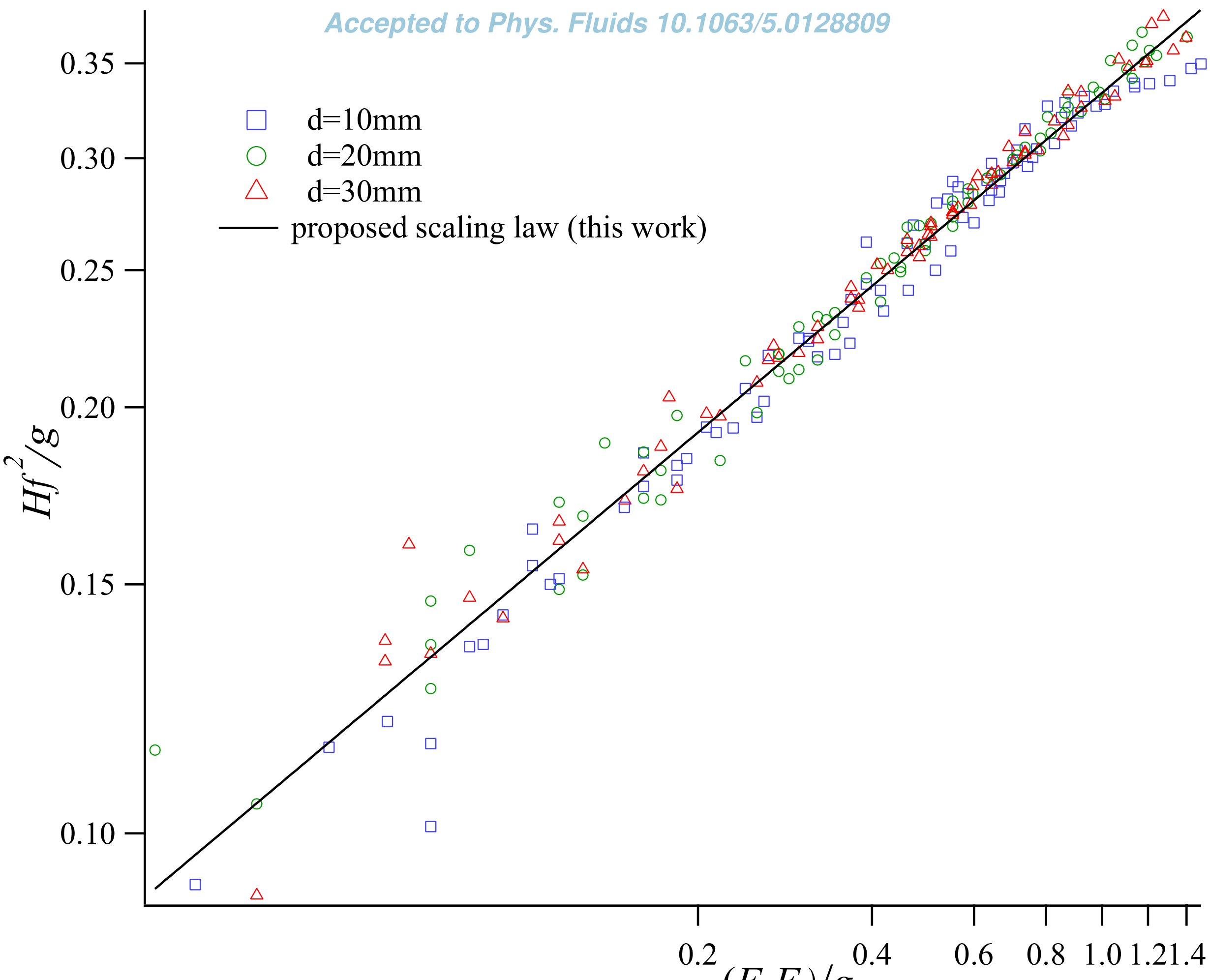
[10]J. Rajchenbach, A. Leroux, and D. Clamond, "New standing solitary waves in water," Phys. Rev. Lett. **107**, 024502 (2011).

[11]P. Fleury, "Phase Transitions, Critical Phenomena, and Instabilities," Science **211**, 125–31 (1981).

[12]V. Steinberg, J. Fineberg, E. Moses, and I. Rehberg, "Pattern selection and transition to turbulence in propagating waves," Phys. D: Nonlinear Phenomena **37(1-3)**, 359–383 (1989).

Accepted to Phys. Fluids 10.1063/5.0128809




0.35
0.30
0.25
0.20
0.15
0.10
Hf^2/g
d=10mm
d=20mm
d=30mm
proposed scaling law (this work)
0.2
0.4
0.6
0.8
1.0
1.2
1.4
$(F-F_c)/g$