Applied Acoustics 73 (2012) 842–848

Contents lists available at SciVerse ScienceDirect

Applied Acoustics

journal homepage: www.elsevier.com/locate/apacoust

Technical Note

A simplified two-dimensional acoustic diffusion model for predicting sound levels in enclosures

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A R T I C L E I N F O

Article history:
Received 14 March 2011
Received in revised form 10 February 2012
Accepted 20 February 2012
Available online 15 March 2012

Keywords:
Diffusion model
Kantorovich method
Steady-state sound field prediction

A B S T R A C T

A new concept for the enclosure-acoustic prediction derived from the mathematical theory of diffusion was proposed some years ago [J. Picaut et al., Acustica 1997]. This model has been applied to predict the sound level distribution in rooms of simple geometries with good accuracy and a relatively low calculation time. However, in situations related with (optimal) acoustic design, the need to evaluate multiple simulations may increase the computational cost. The aim of this work is to provide an approximately equivalent two-dimensional diffusion model achieving similar results with a significant reduction of the execution time. The proposed simplified model is obtained by means of the Kantorovich method. Comparisons of numerical simulations performed with the full diffusion model and the software CATT-Acoustic® are presented to show the efficiency of the simplified diffusion model.

1. Introduction

Ray-tracing method is one of the most well known and most effective techniques for estimating the propagation characteristics in rooms with complex geometries. This approach has been successfully applied to a large number of practical situations [1,2]. However, the calculation time increases with the complexity of the room shape or when several sound sources are considered.

A new method for predicting the sound field in rooms was developed by Picaut et al. [3] some years ago. This model may be considered as an extension of the classical theory of reverberation to non-diffuse sound fields and can be easily solved by means of the finite element method for arbitrary geometries and non-homogeneous boundary conditions. This *acoustic diffusion model* (ADM) has been applied in several situations concerning indoor noise prediction in empty, coupled and fitted rooms taking into account both low and high absorption on interior surfaces [4–9] and atmospheric attenuation [10]. The corresponding studies have demonstrated that the ADM presents lower accuracy than ray tracing technique. It is worth noting that both models are energy-based methods, which ignore complex wave phenomena such as interference and diffraction, so they are considered to be valid for high frequency range only, where the acoustic wavelength is much smaller than the room dimensions. The main advantage of the ADM compared with the ray-tracing based method is the reduction in the computation time. Despite this, in those cases where it is necessary to perform a large number of simulations to evaluate different scenarios (i.e. in optimal design situations) [11], the use of the ADM may increase substantially the calculation time.

In this paper, a quasi-equivalent two-dimensional formulation of the ADM, called *simplified acoustic diffusion model* (SADM), is presented. This approach is proposed for predicting the sound field in rooms with approximately flat ceilings. However, this type of geometry represents a quite general class of rooms. The main interest of the proposed method is the considerable reduction in computation time with respect to the ADM although maintaining a similar accuracy. Both diffusion models are compared in terms of the steady-state sound pressure level (SPL) distributions and evaluated using CATT-Acoustic® v8.0 which is based on the ray-tracing technique. Numerical simulations are then presented taking into consideration different configurations.

2. Acoustic diffusion model (ADM)

In this section, a brief explanation of the ADM, originally proposed by Picaut et al. [3], is introduced. This model describes the non-uniform reverberant sound field in rooms by making use of a mathematical analogy between the scattering of sound by surfaces with diffuse reflections and the diffusion of particles in a diffusive medium. Following this assumption, it is possible to obtain the stationary sound energy density $w(\mathbf{r})$ corresponding to the reverberant field, for a room of volume V_r, as the solution of the following governing system [5,9]:

$$D\nabla^2 w(\mathbf{r}) - \sigma w(\mathbf{r}) + q(\mathbf{r}) = 0 \text{ in } V_r, \tag{1}$$

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http://dx.doi.org/10.1016/j.apacoust.2012.02.012

$$D\frac{\partial w(\mathbf{r})}{\partial \mathbf{n}} + Acw(\mathbf{r}) = 0 \text{ on } \partial V_r, \tag{2}$$

where ∇^2 is the Laplace operator, D is a diffusion coefficient, σ is a coefficient of volumetric absorption, $q(\mathbf{r})$ is the source sound power per unit volume, $\mathbf{r}$ is the position vector corresponding to an arbitrary point, $\mathbf{n}$ is the exterior normal to the boundaries, A is an absorption factor of the interior surfaces and c is the speed of sound. The terms involved in the previous equations adopt different expressions in accordance with the selected configurations to be analyzed. Accordingly, the diffusion coefficient D can be expressed as [8],

$$D = \begin{cases} \frac{c\lambda_r}{3} & \text{for empty rooms,} \\ \frac{c\lambda_r\lambda_{fitt}}{3(\lambda_r + \lambda_{fitt})} & \text{for fitted rooms.} \end{cases} \tag{3}$$

For empty rooms, D considers the morphology of the room with interior surfaces area S_r through the respective mean free path $\lambda_r = 4V_r/S_r$. If a sub-volume V_{fitt} of V_r contains scatter objects (fittings), the diffusion by fittings is described by the mean path length for a sound ray between two collisions with objects $\lambda_{fitt} = 4/(S_{fitt}\, n_{fitt})$, where S_{fitt} represents the surface area of the object and n_{fitt} is the number of scattering objects per unit volume in V_{fitt}. Thus, the diffusion coefficient D for fitted rooms is obtained from a combination of the diffusely-reflecting surfaces of the room and the scattering obstacles within the room [8]. The subscripts r and $fitt$ denote the variable referring to the room and the scattering objects, respectively.

In addition, it is possible to include mixed specular and diffuse reflections on the room boundaries by performing an empirical adjustment of the diffusion coefficient. In particular, for rooms with pure specular reflections, D is adjusted to the value D_a by introducing a correction factor $K = 5$ so that $D_a = K \times D$ [8] (of course, in the case of completely diffuse reflections, $K = 1$).

The absorption term in empty rooms $\sigma = mc$ takes into account the atmospheric attenuation, where m is the absorption coefficient of air [10]. In the case of fitted rooms, the absorption term $\sigma = mc + c\alpha_{fitt}/\lambda_{fitt}$ is obtained from the sum of the absorption contribution of the room and the fittings, respectively, where α_{fitt} is the absorption coefficient of the obstacles located in the room [8].

Eq. (2) corresponds to the boundary conditions on interior surfaces. The absorption factor (A) takes different forms depending on the absorption coefficient $\alpha(\mathbf{r})$ of the considered surface [6,9]:

$$A = \begin{cases} \frac{\alpha(\mathbf{r})}{4} & \text{for low absorption } (\alpha < 0.2), \\ -\frac{\ln[1-\alpha(\mathbf{r})]}{4} & \text{for high absorption } (\alpha \geq 0.2). \end{cases} \tag{4}$$

Another expression of this factor can be found in Ref. [9]. The frequency dependence is taken into account through the absorption coefficients of the room surfaces and the obstacles within the room. The total sound pressure level (SPL) is determined by adding the contributions of direct and reverberant fields, the last one being obtained from the numerical solution of the preceding equations. The resulting total sound field is then expressed as [5,8],

$$\mathrm{SPL}(\mathbf{r}) = 10\log_{10}\left\{\rho c\left[\int_{V_S}\frac{q(\mathbf{r})}{4\pi r^2}\mathrm{e}^{-r/\lambda_{fitt}}\mathrm{d}V_S + cw(\mathbf{r})\right]\Big/P_{ref}^2\right\}, \tag{5}$$

where $r = ||\mathbf{r} - \mathbf{r}_s||$ denotes the distance from a receiver point to an arbitrary point of the source $\mathbf{r_s}$ in the subdominant V_s, ρ is the air density and $P_{\text{ref}} = 2 \times 10^{-5}$ Pa. The first term in expression (5) is formulated in a general form. In this paper, only point sources are considered, hence the source term is defined as $q(\mathbf{r}) = \Sigma W_s\, \delta(\mathbf{r} - \mathbf{r}_s)$, being W_s the stationary acoustic power.

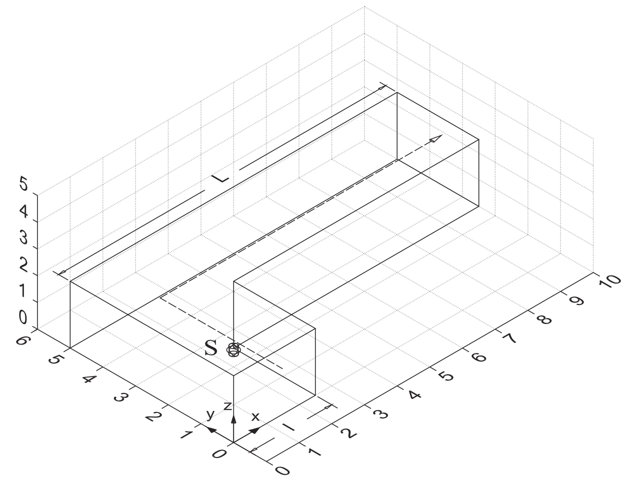

Fig. 1. Geometry of the modelled L-shaped enclosure and the location of the source: the model is parameterized by the length ratio $\beta = L/l$. The source–receiver distance is taken along the dashed line (units in m).

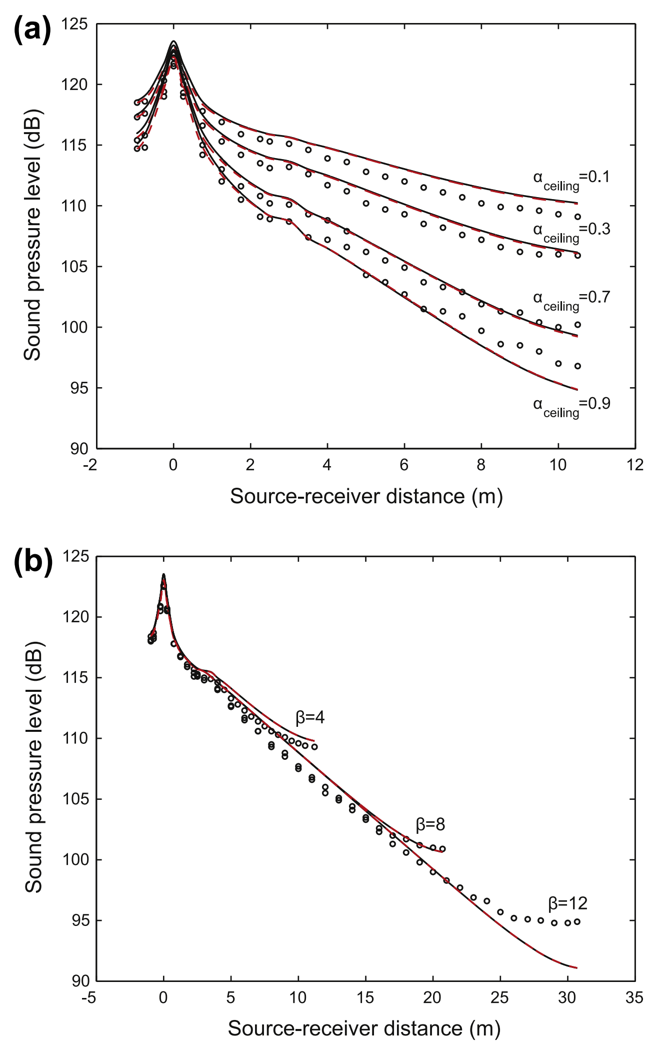

Fig. 2. SPL distribution as a function of the source–receiver distance in the L-shaped enclosure shown in Fig. 1 for different values of $\alpha_{ceiling}$ and $\beta = 4$ (a) and for different values of β and $\alpha_{ceiling} = 0.1$ (b): Ray tracing technique (○), ADM (—) and SADM (---).

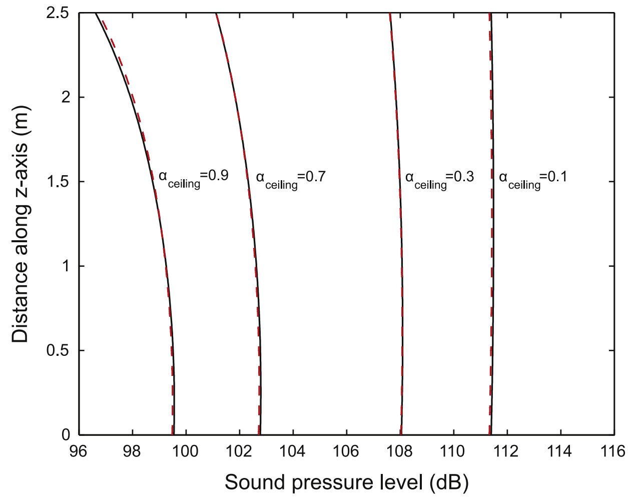

Fig. 3. SPL distribution along z-axis at position (7,3.75) m for different values of $\alpha_{ceiling}$ in the L-shaped enclosure shown in Fig. 1: ADM (—) and SADM (---).

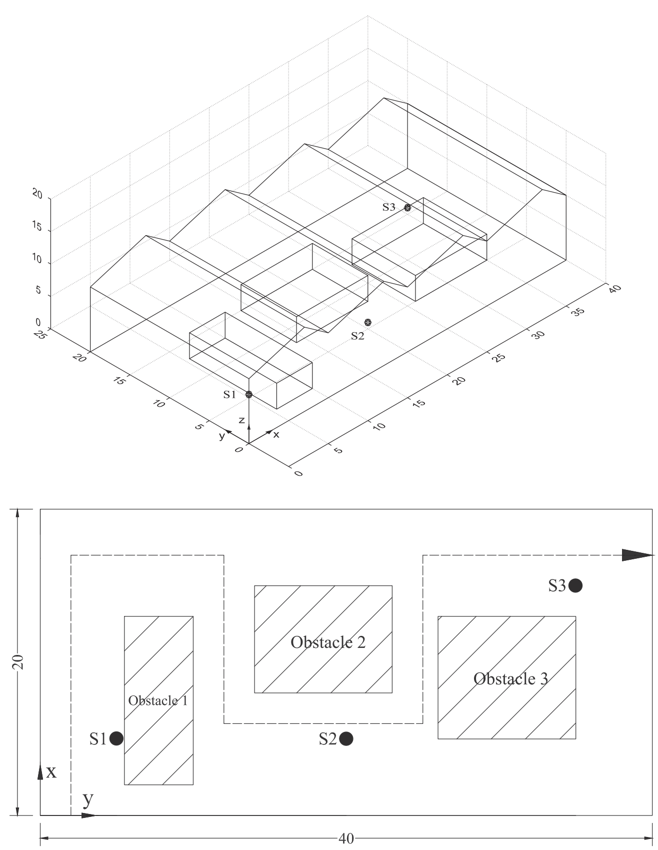

Fig. 4. 2D and 3D geometries of the hypothetical factory and the locations of the sound sources. The SPL distribution is evaluated along the dashed line (units in m).

3. Simplified acoustic diffusion model (SADM)

The full diffusion model (ADM) may be approximately reduced to a simplified two-dimensional diffusion equation by means of the Kantorovich method [12,13]. This is a well known technique for the dimensional reduction of differential equations. It occupies a position intermediate between the exact solution and the solution which is obtained by means of the methods of Ritz and Galerkin [12].

According to this technique, the reverberant energy density $w(\mathbf{r})$ is represented approximately as the product of two functions, one corresponding to the variation in the domain related to the horizontal plane and the other considering the variation in height. Following this methodology, the reverberant energy density can be expressed as,

$$w(\mathbf{r}) = P(x, y) \times Z(z), \tag{6}$$

where $Z(z)$ is a function selected "a prori" in order to approximate the vertical variation and $P(x,y)$ remains as an unknown function. In this study, the vertical variation of the reverberant energy density is approximated by means of the following second order polynomial:

$$Z(z) = 1 + a_1 z + a_2 z^2. \tag{7}$$

The corresponding coefficients a_1 and a_2 are determined from the boundary conditions defined in the two extreme planes (floor and ceiling),

$$D\frac{dZ(z)}{dz} = \pm AZ(z). \tag{8}$$

Now, substituting the approximate expression (6) into Eq. (1), a "residual function" $\varepsilon(\mathbf{r},w)$ is obtained. Obviously, this function is zero when w is the exact solution. In the same way, the integral of the product between the residual function and an arbitrary function ψ must be zero for the exact solution of the problem,

$$\int \varepsilon(\mathbf{r}, w)\psi(\mathbf{r})\, dV_r = 0. \tag{9}$$

This is called orthogonality condition (between ε and ψ) [12]. According to the Kantorovich method, the function ψ is taken in the form:

$$\psi(\mathbf{r}) = Z(z)\phi(x, y), \tag{10}$$

where $Z(z)$ is the preselected function adopted in Eq. (7) and ϕ is another arbitrary function. Substituting Eq. (10) into Eq. (9) and reordering the resulting expression, the following integral is obtained:

$$\int_\Omega \phi(x, y)\left[\int_0^H \varepsilon(\mathbf{r}, w)Z(z)dz\right]dxdy = 0, \tag{11}$$

where the domain Ω represents the horizontal plane (2D geometry) of the considered room and H is the height of the room. Considering the arbitrary character of ϕ it is clear from the last equation that the expression between brackets must be zero. Accordingly,

$$\int_0^H \varepsilon(\mathbf{r}, w)Z(z)dz = 0. \tag{12}$$

Thus, from Eq. (1) and expression (6), the above equation can now be written as,

$$\int_0^H DZ(z)^2 dz \nabla_p^2 P + \int_0^H D(\frac{d^2Z(z)}{dz^2}Z(z))dz\, P - \int_0^H \sigma Z(z)^2 dz\, P + \int_0^H qZ(z)dz = 0, \tag{13}$$

where ∇_p^2 is the Laplace operator in the plane. Operating in the same way with Eq. (2) the reduced boundary condition is obtained. Thus, the simplified acoustic diffusion model (SADM) may be expressed as,

$$D_{Z1}\nabla_p^2 P + (D_{Z2} - \sigma_Z)P + q_Z = 0 \quad \text{in } \Omega, \tag{14}$$

$$D_{Z1}\frac{\partial P}{\partial \mathbf{n}} + A_Z cP = 0 \quad \text{on } \partial\Omega, \tag{15}$$

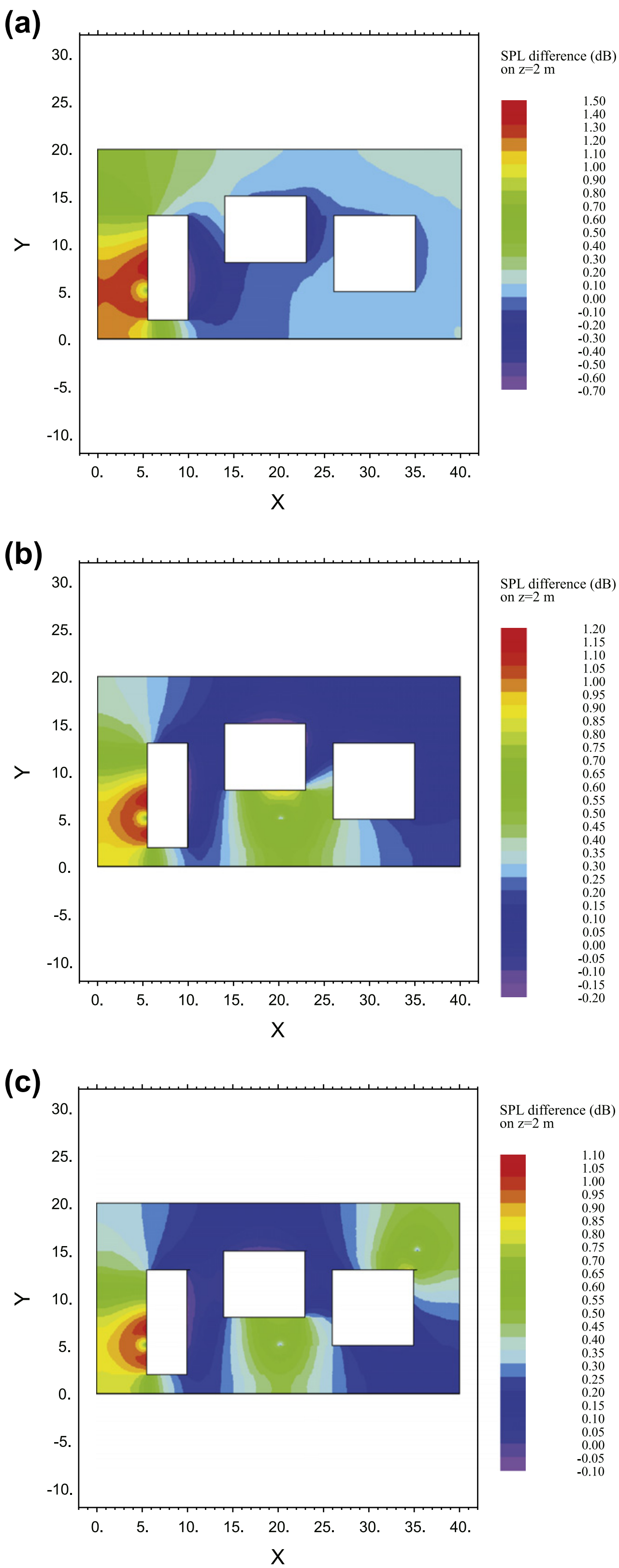

Fig. 5. Differences between the ADM and the SADM in terms of the SPL distribution at 2 m above the floor level for S1 active (a), S1 and S2 active (b) and S1, S2 and S3 active (c).

where $\partial\Omega$ is the perimeter of the horizontal plane of the considered room. From Eqs. (14) and (15) the following definitions have been made,

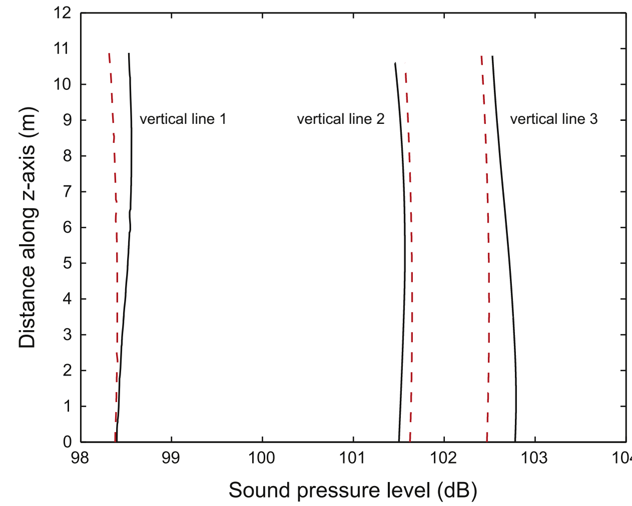

Fig. 6. SPL distribution along the *z*-axis. Vertical line 1 passing through (4, 16) m, vertical line 2 passing through (15,3) m and vertical line 3 passing through (37, 12) m in the hypothetical factory shown in Fig. 4. Only one source (S1) is considered: ADM (—) and SADM (---).

$$D_{Z1} = \int_0^H DZ(z)^2 \mathrm{d}z, \tag{16}$$

$$D_{Z2} = \int_0^H D\left(\frac{\mathrm{d}^2 Z(z)}{\mathrm{d}z^2} Z(z)\right) \mathrm{d}z, \tag{17}$$

$$\sigma_Z = \int_0^H \sigma Z(z)^2 \mathrm{d}z, \tag{18}$$

$$q_Z = \int_0^H qZ(z) \mathrm{d}z, \tag{19}$$

$$A_Z = \int_0^H AZ(z)^2 \mathrm{d}z. \tag{20}$$

Of course, once P is obtained as the solution of this problem, the approximated reverberant energy density is given by expression (6) and the acoustic field (SPL) is obtained by means of expression (5). The present derivation assumes the ceiling to be flat. However, from a pragmatic point of view, the methodology may also be applied for non-planar ceilings if at least they can be approximated as a flat surface, in fact taking an equivalent constant height. A numerical example for a non-flat ceiling is presented in the next section.

4. Numerical applications

In this section, several enclosures are studied in order to compare the proposed two-dimensional acoustic diffusion model (SADM) with the full one (ADM). Additionally, both models are evaluated with the ray tracing technique implemented in the software CATT-Acoustic®. The diffusion models are solved by means of the finite element method using the software Flex-PDE®. In the first two examples, the reflections on the surfaces are considered completely diffuse. Nevertheless, in the last example, both totally diffuse and specular reflections are evaluated by means of the corresponding modification in the diffusion coefficient D according to the stated in Section 2. The corresponding absorption factor (A) is used according to the absorption coefficient (α) adopted in each room surface.

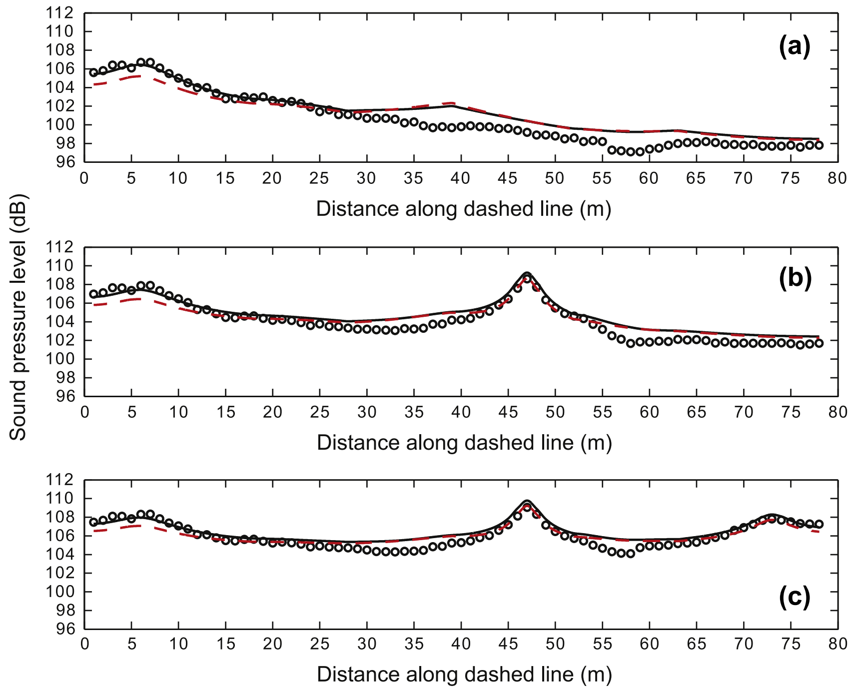

Fig. 7. SPL distribution as a function of the distance along the dashed line in the hypothetical factory shown in Fig. 4 for S1 active (a), S1 and S2 active (b) and S1, S2 and S3 active. (c) Ray tracing technique (○), ADM (—) and SADM (---).

Some preliminary simulations were performed to estimate the required number of sound rays in the ray tracing software in order to obtain a correct prediction of the sound field. All the numerical results are presented in terms of SPL distribution and are evaluated at the octave frequency-band of 1000 Hz.

4.1. L-shape enclosure

An L-shaped enclosure with varying dimension and varying absorbing coefficients on the ceiling is considered (Fig. 1). This configuration was proposed by Le Bot and Bocquillet [14] for calculating the acoustic pressure fields in rooms by means of an integral formulation. Width and height are equal to $l = 2.5$ m and the length L may vary according to the non-dimensional parameter $\beta = L/l$, characterizing the shape of the room. An omnidirectional source with a power level of 120 dB is located at ($x = 1.25$ m, $y = 1.25$ m and $z = 1.7$ m). The sound pressure level (SPL) distribution is evaluated along a horizontal line centered inside the room. The absorption coefficients corresponding to the walls and the floor are taken as 0.1 and the atmospheric absorption is $m = 0.0007$ m^{-1}.

Results of the comparison among the ADM, the SADM and the ray tracing technique are presented in Fig. 2. Fig. 2a compares the SPL for a fixed shape with $\beta = 4$ and four different absorption coefficients for the ceiling ($\alpha_{ceiling} = 0.1$, 0.3, 0.7 and 0.9). The ray tracing simulation was performed with a number of sound rays of 6×10^4, 10×10^4, 50×10^4 and 100×10^4 for $\alpha_{ceiling} = 0.1$, 0.3, 0.7 and 0.9, respectively. Fig. 2b compares the SPL for a uniform absorption coefficient ($\alpha = 0.1$) and three different values of the length ratio ($\beta = 4$, 8 and 12). The ray tracing simulation was performed with a number of rays of 6×10^4, 10×10^4 and 10×10^4 for $\beta = 4$, 8 and 12, respectively. Both graphics show a negligible difference between the diffusion models and a good agreement against the ray tracing software (<2 dB) except for $\beta = 12$ where the maximum error is about 3.8 dB (Fig. 2b). In this case, several receptors near the end of the enclosure appear to be underestimated by the diffusion models. This behavior is inherent to the diffusion model in long rooms where the sound prediction is not accurate near the boundary [4,15].

Fig. 3 illustrates a comparison between the diffusion models in terms of the SPL variation on the vertical axis (z) at position ($x = 7$ m and $y = 3.75$ m) for different values of $\alpha_{ceiling}$. The results show an excellent agreement between both models with a maximum discrepancy of 0.1 dB.

The employed average computation times, for the chosen octave band, are of the order of 200 s for the ray tracing model, 10 s for the ADM and less than 1 s for the SADM.

4.2. Enclosure with non-flat ceiling and several obstacles

This configuration is similar to that proposed by Le Bot and Bocquillet [14]. The enclosure represents a hypothetical factory with a non-planar ceiling and absorbing obstacles (Fig. 4). Three omnidirectional sources (S1, S2 and S3), each one with a sound power

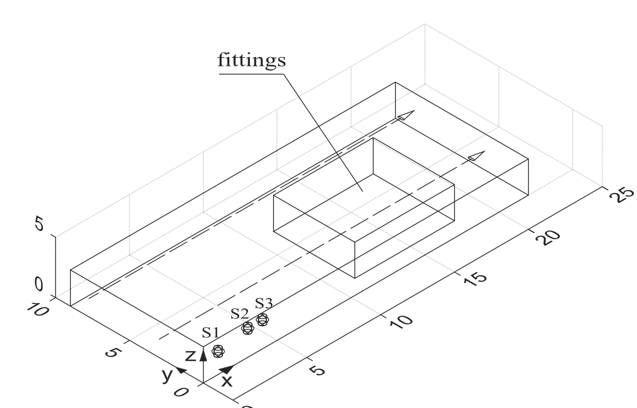

Fig. 8. Geometry of the modelled enclosure with a fitted zone and the location of the sound sources. The SPL distribution is evaluated along the dashed lines (units in m).

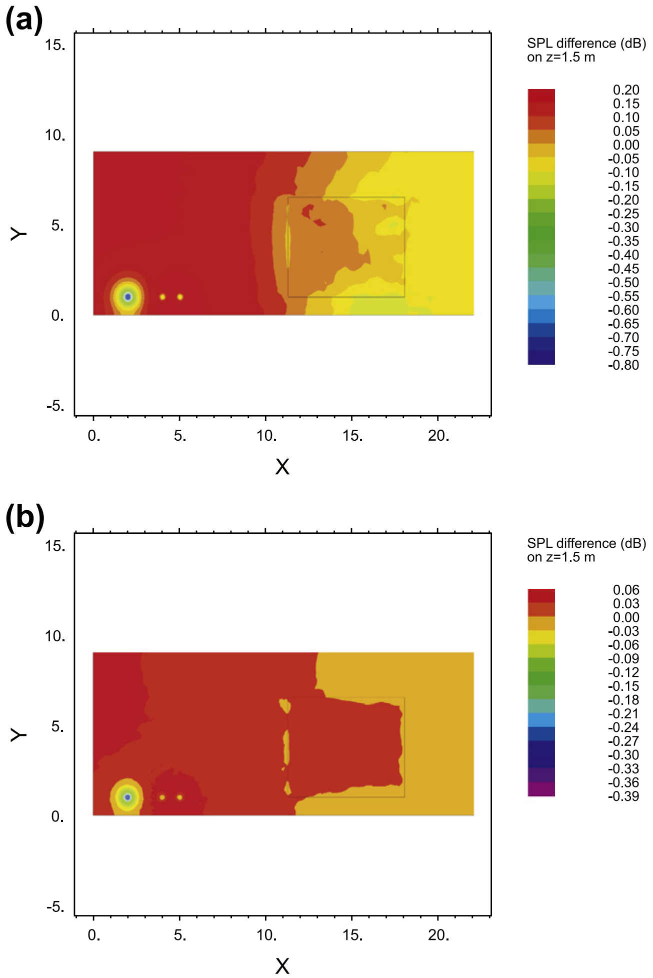

Fig. 9. Differences between the ADM and SADM in terms of the SPL distribution at 1.5 m above the floor level for diffuse surface reflections (a) and specular surface reflections (b).

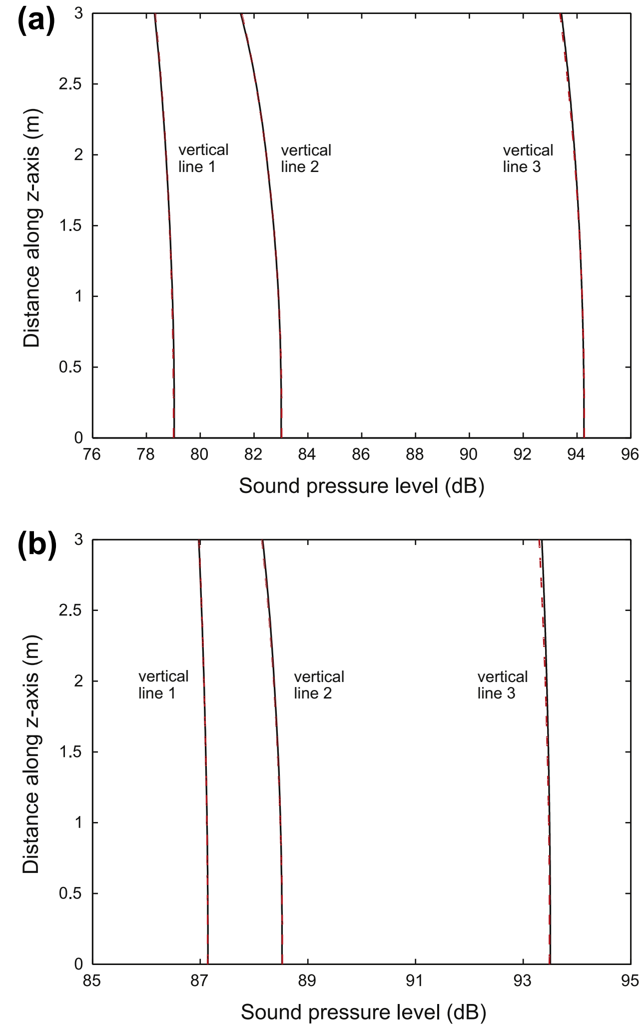

Fig. 10. SPL distribution along the *z*-axis. Vertical line 1 passing through (20,7) m, vertical line 2 passing through (15,5) m and vertical line 3 passing through (4,7) m in the enclosure with a fitted zone shown in Fig. 8 for diffuse surface reflections (a) and specular surface reflections (b): ADM (—) and SADM (---).

level of 120 dB, are considered. They are located at (x = 5 m, y = 5 m and z = 1 m), (x = 20 m, y = 5 m and z = 1 m) and (x = 35 m, y = 15 m and z = 1 m), respectively. The selected absorption coefficients of the different surfaces are: 0.05 for floor and ceiling, 0.02 for walls and 0.2, 0.3 and 0.35 for obstacle 1, 2 and 3, respectively. The atmospheric attenuation is the same as in the previous example.

For the SADM implementation, the room ceiling is modeled as a flat surface with a mean height of 11 m according to the limitation mentioned in Section 3.

Fig. 5 shows a comparison between the ADM and SADM in terms of the SPL distribution on a plane at z = 2 m for one source (S1) active (Fig. 5a), two sources (S1 and S2) active (Fig. 5b) and three sources (S1, S2 and S3) active (Fig. 5c). Despite the approximation adopted for the height in the SADM, the results are sufficiently accurate with a negligible error in the far field and a maximum error of 1.5 dB at a region near the source S1 (Fig. 5a).

Fig. 6 illustrates a comparison of the SPL vertical variation obtained with the ADM and SADM along three receiver paths: vertical line 1 passing through (x = 4 m and y = 16 m), vertical line 2 passing through (x = 15 m and y = 3 m) and vertical line 3 passing through (x = 37 m and y = 12 m). Only one source (S1) is considered. In all the cases the SADM model has a good accuracy with a maximum error of about 0.3 dB.

Comparisons of the SPL distributions, along a horizontal line 2 m above the floor level, among the diffusion models and the ray tracing technique are presented in Fig. 7. Evaluation is performed for S1 active (Fig. 7a), S1 and S2 active (Fig. 7b) and S1, S2 and S3 active (Fig. 7c). In all the cases, the number of rays for the ray tracing simulations was set to 10×10^4. Results show that both diffusion models produce very similar results against the ray tracing model with a maximum error of about 2.5 dB. In particular, an increased level of accuracy is obtained as more sources are considered due to the greater diffusivity of the reverberant sound field (i.e., see Fig. 7c).

The average computation times, for the selected octave band, are of the order of 300 s for the ray tracing model, 25 s to the ADM and less than 1 s for the SADM.

4.3. Enclosure with a fitted zone

The enclosure shown in Fig. 8 is analyzed. A fitted zone is included inside the room with a volume of $6.75 \times 5.5 \times 3$ m^3. Three omnidirectional sources are considered: source 1 (S1) at position (x = 2 m, y = 1 m and z = 0.5 m), source 2 (S2) at position (x = 4 m, y = 1 m and z = 1 m) and source 3 (S3) at position (x = 5 m, y = 1 m and z = 1 m). The corresponding sound power levels are: 105, 102 and 102 dB, respectively. The selected absorption coefficients are: 0.02 for floor, 0.5 for ceiling, 0.08 for walls and 0.3 for

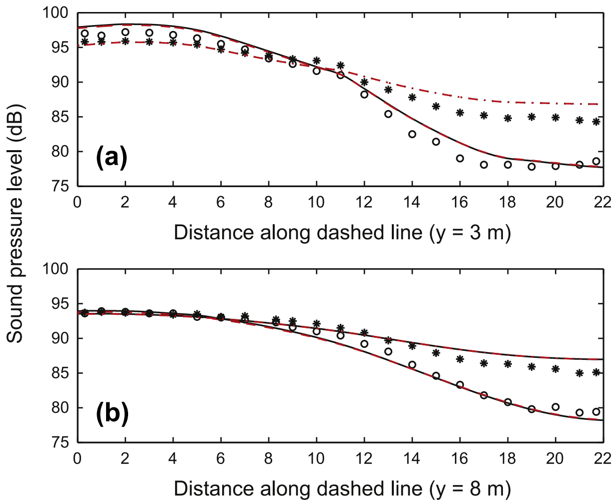

Fig. 11. SPL distribution as a function of the distance along the dashed lines at position $y = 3$ m (a) and $y = 8$ m (b) in the enclosure with a fitted zone shown in Fig. 8: Ray tracing technique with diffuse (○) and specular (∗) surface reflections, ADM with diffuse (—) and specular (---) surface reflections and SADM with diffuse (---) and specular (-.-.-) surface reflections.

fittings. The fitting zone is simulated in the software CATT-Acoustic considering 30 rectangular blocks distributed uniformly with a mean path length equal to that adopted in the diffusion model ($\lambda_{fitt} = 2.37$ m). The complete ray tracing simulation was performed with 300×10^4 sound rays.

Fig. 9 shows a comparison between the diffusion models in terms of the SPL distribution on a plane at $z = 1.5$ m considering both pure diffuse (Fig. 9a) and pure specular (Fig. 9b) reflections on the room surfaces. Fig. 10 shows the SPL differences along the vertical axis (z) in three different positions: vertical line 1 passing through ($x = 20$ m and $y = 7$ m), vertical line 2 passing through ($x = 15$ m and $y = 5$ m) and vertical line 3 passing through ($x = 4$ m and $y = 7$ m). Comparisons are performed for diffuse (Fig. 10a) and specular (Fig. 10b) reflections. Both graphics illustrate a close fit between the diffusion models with a maximum discrepancy of the order of 0.8 dB.

Fig. 11 shows a comparison against the ray tracing method for two horizontal lines ($z = 1.5$ m) located at $y = 3$ m (Fig. 11a) and $y = 8$ m (Fig. 11b). The diffusion models exhibit a good agreement with the ray tracing technique with a maximum error of about 2.5 dB.

The average computation times, for the selected octave band, are around 900 s for the ray tracing model, 30 s for the ADM and 1 s for the SADM.

5. Conclusions

A two-dimensional acoustic diffusion model (SADM) has been proposed as a simplification of the full diffusion model (ADM) for predicting the distribution of sound pressure levels in enclosures. Some numerical comparisons among both diffusion models and a well-known ray tracing technique have been conducted to study the effectiveness of the present approach.

From the results, a close similarity between the ADM and SADM is observed in terms of the SPL distribution along both horizontal and vertical directions with maximum differences generally less than 1 dB. The numerical comparisons with the ray tracing technique show consistent results with mean errors of the order of 0.2–0.7 dB.

Computing times for the ADM are at least 10 times lower than those of the ray tracing software while the SADM is around 10–30 times faster than the ADM. This becomes a significant computational advantage when a large number of simulations for optimal design are needed. Thus, for example, if an optimal iterative process is intended and 250 calculations are adopted to evaluate different combinations of design variables, the resulting computation time would be 5000 s (∼1.5 h) for the implementation of the ADM, while only 250 s (less than 5 min) for the SADM. In that sense, the SADM is well-suitable to use in the preliminary stages of a design. The application of this methodology in the framework of an optimal acoustic design will be presented in a future work.

Acknowledgements

The authors would like to thank the support of Secretaría de Ciencia y Tecnología of Universidad Tecnológica Nacional and CONICET.

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