Journal of Hydrology 450–451 (2012) 150–158

Contents lists available at SciVerse ScienceDirect

Journal of Hydrology

journal homepage: www.elsevier.com/locate/jhydrol

Analytical study of hydraulic and mechanical effects on tide-induced head fluctuation in a coastal aquifer system that extends under the sea

Luis Guarracino [a,*], Jesús Carrera [b,1], Enric Vázquez-Suñé [b,1]

[a] CONICET, Facultad de Cs. Astronómicas y Geofísicas, Universidad Nacional de La Plata, Paseo del Bosque s/n, La Plata 1900, Argentina
[b] Instituto de Diagnóstico Ambiental y Estudios del Agua, Consejo Superior de Investigaciones Científicas, C/Jordi Girona 18, 08034 Barcelona, Spain

ARTICLE INFO

Article history:
Received 11 November 2011
Received in revised form 7 April 2012
Accepted 8 May 2012
Available online 16 May 2012
This manuscript was handled by Philippe Baveye, Editor-in-Chief, with the assistance of Magdeline Laba, Associate Editor

Keywords:
Tide-induced heads
Loading efficiency
Costal aquifer
Submarine aquifer
Analytical solution

SUMMARY

Head response to sea level fluctuations has been extensively used for characterizing coastal aquifers. When the aquifer is semiconfined and extends for a certain distance D under the sea, head response results from the superposition of two types of effects: hydraulic (i.e., ground water flow connection through aquifer and aquitard) and mechanical (induced by tidal loading onto the sea floor). Solutions are available for this problem that has been analyzed before, but only for D zero or infinity. These solutions do not allow analyzing aquifer systems that extends for a finite D, or identifying them, which is critical for coastal aquifer management. We derive an exact analytical solution that describes separately the mechanical and hydraulic effects. The proposed analytical solution is a generalization of most of existing analytical solutions. A simpler approximate analytical solution is also obtained for soft aquitards with low permeabilities. We find that the impact of the hydraulic component of the aquitard and the mechanical effects in the total head fluctuation at the shoreline is significant, but not very sensitive to the properties of the aquitard. The amplitude of these fluctuations relative to that of the sea tide ranges approximately between 1 (small D) and 0.5 (large D). This implies that aquifer penetration under the sea can indeed be identified if it is below a certain threshold, beyond which the system responds as if D was infinity. Surprisingly, the time lag is close to zero regardless of hydraulic parameters of the aquifer system.

1. Introduction

The study of the dynamic relationship between sea tides and coastal groundwater using analytical models has received much attention since the 1950s. Aquifer head fluctuations are important for two main reasons. First, gradient fluctuations cause an increase in local dispersivity (Dentz and Carrera, 2005, 2007; Cirpka and Attinger, 2003). This increase causes the freshwater-saltwater mixing zone to broaden (Abarca et al., 2007) and enhances mixing, which activates chemical reactions (Sanford and Konikov, 1989; Rezaei et al., 2005). Second, tide-induced fluctuations can be used to derive aquifer parameters in coastal regions (e.g. Jha et al., 2003). Jacob (1950) and Ferris (1951) were the first to derive analytical equations for describing tide-induced head fluctuations in an aquifer that ends at the coastline. These equations predict that head fluctuations are dampened and shifted in time with respect to sea level fluctuations. The phase shift and the exponent of the dampening are identical and depend on hydraulic diffusivity (Carr and van der Kamp, 1969). Therefore, it is not surprising that the response to sea level fluctuations, possibly coupled with pumping test data, has been widely used to estimate hydraulic parameters of coastal aquifers (e.g. Pandit et al., 1991; Jha et al., 2003; Banerjee et al., 2008; Carol et al., 2009).

The hydraulic diffusivity obtained by matching the phase shift and amplitude reduction of head data with respect to tides is important in itself because diffusivity is an excellent indicator of connectivity (Knudby and Carrera, 2006). Slooten et al. (2010) showed that tidal response inland is most sensitive to hydraulic conductivity near the shore. Therefore, by performing this analysis at many observation wells, one can infer which areas are well connected to the sea (i.e., those with high diffusivity). In fact, tidal response data coupled with pump tests have been used to derive spatially varying maps of transmissivity (Alcolea et al., 2007, 2009).

Tidal response data can ideally be used to derive also the offshore distance at which a confined aquifer is connected to the sea (Li and Chen, 1991). The resulting information is of basic importance for coastal aquifer management because pumping should concentrate in areas that are far from, or poorly connected with, the sea. However, while the approach sounds exciting, difficulties arise in practice. For instance, the hydraulic diffusivity derived from amplitude damping is often different to that derived from phase shift for

* Corresponding author. Tel.: +54 221 4236593.
E-mail addresses: luisg@fcaglp.unlp.edu.ar (L. Guarracino), jesus.carrera@idaea.csic.es (J. Carrera), enric.vazquez@idaea.csic.es (E. Vázquez-Suñé).
[1] Tel.: +34 934095410.

0022-1694/$ - see front matter © 2012 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.jhydrol.2012.05.015

confined and semiconfined aquifers (e.g. Trefry, 1999; Zhou, 2008). Several factors contribute to difficulties. First, the connection to the sea in (semi) confined aquifers does not occur at the shore, but at a seawards distance, which is usually unknown. Second, part of the tidal wave is absorbed by leakage into the aquitard. Third, part of the signal is caused not by hydraulic connection, but by mechanical loading. Additionally, though not discussed here, heterogeneity or inland fluctuations in recharge or pumping also add complexity to the problem (Townley, 1995; Trefry, 1999; Slooten et al. (2010); Monachesi and Guarracino, 2011). These three factors have been the subject of some research, as discussed below.

The effect of leakage on tide-induced head fluctuations in a horizontal confined aquifer overlain by a semipermeable layer has been studied by many researchers. Jiao and Tang (1999) developed an analytical solution for an aquifer system that ends at the coastline assuming that the leakage is linearly proportional to the difference in head between the two formations (i.e., neglecting the dampening effect of elastic storage at the aquitard). Based on the same hypotheses Li and Jiao (2001a) obtained an analytical solution for an aquifer system that extends a certain distance under the sea. They conclude that leakage from the offshore portion of the aquitard tends to increase the groundwater fluctuation, while leakage from the inland portion tends to decrease the fluctuation. As a result, the amplitude reduction exponent is not identical to the phase shift.

A more realistic approximation to leakage can be obtained by acknowledging the effect of the elastic storage of the aquitard, which damps the water flux across the aquitard driven by head gradients, but causes water fluxes driven by mechanical loading. Sea level fluctuations imply fluctuations in the loading exerted by the sea over the medium. According to Terzaghi's theory (Terzaghi, 1954), an increase in loading will be initially reflected in an increase in water pressure which will be slowly dissipated, and absorbed by the porous matrix, as water flows away. This effect is easily represented as a sink/source term proportional to the time derivative of the total stress (Bear, 1972) which in this case is proportional to the seawater level. The resulting mathematical description consists of two coupled boundary-value problems that describe water flow subject to periodic elastic compressions and expansions in the aquifer and the aquitard.

Li and Jiao (2001b) obtained analytical solutions of the two coupled boundary-value problems for an aquifer system that terminates at the coastline. More recently, Li et al. (2008) solved this problem for an aquifer system that extends infinitely under the sea and showed that the elastic storage of the aquitard can significantly enhance the tidal head fluctuation in the aquifer. Shortly afterwards, Geng et al. (2009) considered the case of a single confined aquifer where its submarine outlet is covered by a layer with properties dissimilar to the aquifer. These works make it apparent that the effects of mechanical loading and aquifer connection to the sea need to be taken into account. The question remains, however, as to whether seawards connectivity can be identified. As discussed above, connection to the sea is a critical issue for coastal aquifer management. It would be desirable to tell whether an observed fluctuation is caused by hydraulic connection to the sea or by tidal loading. Existing analytical solutions do not allow this separation.

In this context, the objective of our work is to derive an exact analytical solution to describe tide-induced head fluctuation in an aquifer system that extends a finite distance under the sea, so as to derive under which conditions such distance can be identified from head measurements.

2. Mathematical model and analytical solution

We consider a coastal aquifer system consisting of an unconfined aquifer, a confined aquifer and an aquitard (semipermeable layer) between them. The unconfined aquifer ends at the coastline and both the deep aquifer and the aquitard extend over a finite distance D under the sea, as shown in Fig. 1. This set-up is an extension of those by Li and Jiao (2001b) and Li et al. (2008), who considered, $D = 0$ and $D = \infty$, respectively. For the mathematical description of the problem we consider a Cartesian coordinate system where the x-axis is positive landward with its origin at the coastline in the middle point of the semipermeable layer. The z-axis is vertical, positive upward, with its origin on the x-axis.

The following assumptions are made in order to derive an analytical solution: (a) all the layers are homogeneous and isotropic and extend landward infinitely; (b) water flow is vertical in the aquitard and horizontal in the confined aquifer (e.g. Neuman and Witherspoon, 1969; Li and Jiao, 2002); (c) the water table fluctuation in the unconfined aquifer is neglected (e.g. Jiao and Tang, 1999; Li et al., 2008); (d) spatial variations in salinity can be neglected when computing head fluctuations induced by sea level variations (Ataie-Ashtiani et al., 2001; Slooten et al., 2010). The assumption (c) is supported by numerous field studies (e.g. White and Roberts, 1994; Millham and Howes, 1995). Unconfined aquifers usually have large specific yields which dump tidal effects so that tidal fluctuations in unconfined aquifers can be negligible compared to those in confined aquifers. Under the above assumptions, the mathematical model of groundwater response to tidal fluctuations can be written as a system of two coupled boundary-value problems for the aquitard and the confined aquifer.

Water flow in the aquitard (semipermeable layer) is assumed to be vertical so no derivatives with respect to x are considered in equations and no boundary conditions are required on vertical boundaries. In this case, water flow is described by the following boundary-value problem in terms of equivalent freshwater head $h_1(x,z,t)$:

$$S_{S1}\frac{\partial h_1}{\partial t} = K_1\frac{\partial^2 h_1}{\partial z^2}, \quad -\infty < t < \infty, \quad 0 \leqslant x < \infty, \quad -\frac{b_1}{2} < z < \frac{b_1}{2} \tag{1}$$

$$S_{S1}\frac{\partial h_1}{\partial t} = K_1\frac{\partial^2 h_1}{\partial z^2} + S_{S1}L_{e1}\frac{dh_S}{dt}, \quad -\infty < t < \infty, \quad -D \leqslant x < 0, \quad -\frac{b_1}{2} < z < \frac{b_1}{2} \tag{2}$$

$$h_1(x, b_1/2, t) = 0, \quad -\infty < t < \infty, \quad 0 \leqslant x < \infty \tag{3}$$

$$h_1(x, b_1/2, t) = h_S(t) = Ar_\rho\cos(\omega t), \quad -\infty < t < \infty, \quad -D \leqslant x < 0 \tag{4}$$

$$h_1(x, -b_1/2, t) = h(x,t), \quad -\infty < t < \infty, \quad -D \leqslant x < \infty \tag{5}$$

where S_{S1} is specific storage (L^{-1}); K_1, hydraulic conductivity (LT^{-1}); L_{e1}, tidal loading efficiency (–); b_1, thickness of the layer (L); h_S, tidal sea level (L); A, tidal amplitude (L); r_ρ, density ratio (–), that is, $r_\rho = \rho_s/\rho_f$, where ρ_s and ρ_f are the densities of seawater and freshwater, respectively; ω, tidal angular velocity (T^{-1}); and h is the equivalent freshwater head (L) in the confined aquifer. The mechanical effect generated by the elastic compression and expansion of the semipermeable layer due to sea-tide loading is modelled by Eq. (2). For a detailed description of this equation and involved parameters we refer to van der Kamp and Gale (1983).Water flow in the confined aquifer is described by the following boundary-value problem in terms of head $h(x,t)$:

$$S\frac{\partial h}{\partial t} = T\frac{\partial^2 h}{\partial x^2} + K_1\frac{\partial h_1}{\partial z}\bigg|_{z=-b_1/2}, \quad -\infty < t < \infty, \quad 0 \leqslant x < \infty \tag{6}$$

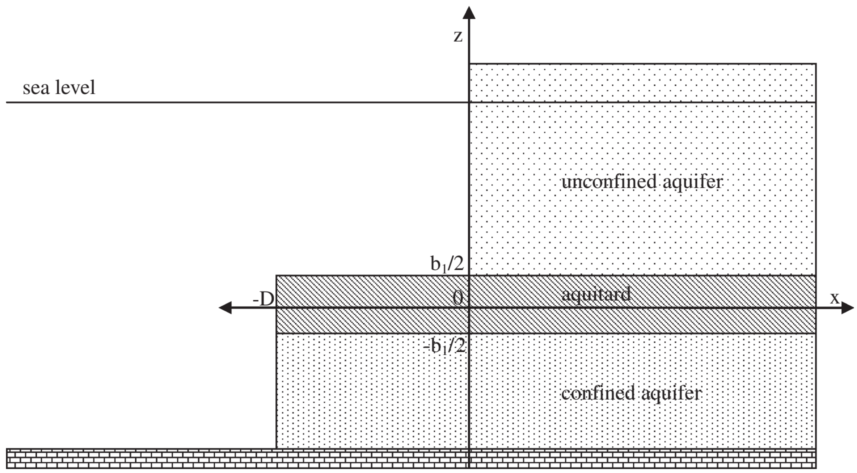

Fig. 1. Schematic of the coastal aquifer system extending under the sea.

$$S\frac{\partial h}{\partial t} = T\frac{\partial^2 h}{\partial x^2} + K_1\frac{\partial h_1}{\partial z}\bigg|_{z=-b_1/2} + SL_e\frac{dh_S}{dt}, \quad -\infty < t < \infty, \quad -D \leqslant x < 0 \tag{7}$$

$$\lim_{x\to\infty}\frac{\partial h}{\partial x} = 0, \quad -\infty < t < \infty \tag{8}$$

$$h(-D,t) = h_S(t) = Ar_\rho\cos(\omega t) \quad -\infty < t < \infty \tag{9}$$

$$\lim_{x\to 0^+} h(x,t) = \lim_{x\to 0^-} h(x,t), \quad -\infty < t < \infty \tag{10}$$

$$\lim_{x\to 0^+}\frac{\partial h}{\partial x} = \lim_{x\to 0^-}\frac{\partial h}{\partial x}, \quad -\infty < t < \infty \tag{11}$$

where S is the storativity (–); T, the transmissivity ($L^2\,T^{-1}$); and L_e is the tidal loading efficiency (–). Eq. (7) describes the mechanical effect on the confined aquifer. The boundary condition (8) expresses that tidal induced water flux tends to zero far inland. The hydraulic connection between seawater and groundwater is established by the Dirichlet boundary condition (9). Eqs. (10) and (11) state the continuities of water head and water flow at the coastline, respectively.In order to compare the analytical solution with previous studies we use the aquifer and aquitard tidal propagation parameters a (L^{-1}) and a_1 (L^{-1}), respectively, and the leakage number u (–) which are defined as (Li and Jiao, 2001b; Li et al., 2008):

$$a = \sqrt{\frac{\omega S}{2T}}, \quad a_1 = \sqrt{\frac{\omega S_{S1}}{2K_1}}, \quad u = \frac{K_1}{\omega S b_1}. \tag{12}$$

The meaning of a (and a_1) is well known since Carr and van der Kamp (1969); $1/a$ is the characteristic dampening distance defined as the distance at which the amplitude of head fluctuation decays by a factor of $1/e \sim 0.37$, so that hydraulic fluctuations virtually disappear for distances inland around $3/a$. Similarly direct hydraulic connection (leakage) across the semipermeable layer can be neglected if $b_1 \gg 1/a_1$.Moreover, water fluxes produced by mechanical loading are relevant over a thickness of the order of $1/a_1$. The meaning of u is more subtle. It compares the leakage produced across the aquitard by a change Δh in the sea level ($K_1\Delta h/b_1$) with the flux needed during a tidal cycle to produce the same head change in the aquifer ($S\omega\Delta h$), so that the leakage component can be neglected when $u \ll 1$. This discussion suggests using tidal dampening factors of the aquifer and aquitard, a and a_1, together with the dimensionless leakage factor u (plus loading efficiencies) as characteristic variables of the problem. As we shall see, this is not the most convenient option.

The analytical solution in the confined aquifer can be expressed as the sum of three components:

$$h(x,t) = h_h(x,t) + h_{h1}(x,t) + h_m(x,t) \tag{13}$$

where h_h is the hydraulic component caused by direct connection between seawater and groundwater along the aquifer, h_{h1} the hydraulic component caused by indirect hydraulic connection through the aquitard, and h_m the mechanical component induced by tidal loading onto the sea floor. The above components have the following expressions (see appendix for the derivation):

$$h_h(x,t) = Ar_\rho g(x+D,t,0) \tag{14}$$

where $g(x,t,\varphi) = e^{-apx}\cos(\omega t - aqx - \varphi)$, and

$$h_{h1}(x,t) = Ar_\rho C_{e1} f(x,t,\varphi_1) \tag{15}$$

$$h_m(x,t) = Ar_\rho[C_e f(x,t,\varphi) - C_{e1} f(x,t,\varphi_1)] \tag{16}$$

where C_e and φ are the comprehensive tidal efficiency and phase shift defined by Li et al. (2008) for the aquifer system; C_{e1} and φ_1 the comprehensive tidal efficiency and phase shift when loading effects are neglected ($L_{e1} = L_e = 0$); and

$$f(x,t,\varphi) = \begin{cases} g(0,t,\varphi) + \frac{1}{2}g(2D+x,t,\varphi) - g(D+x,t,\varphi) - \frac{1}{2}g(-x,t,\varphi), & -D \leqslant x < 0 \\ \frac{1}{2}g(x,t,\varphi) + \frac{1}{2}g(2D+x,t,\varphi) - g(D+x,t,\varphi), & 0 \leqslant x < \infty. \end{cases} \tag{17}$$

The comprehensive tidal efficiency and phase shift are defined as follows (Li et al., 2008):

$$C_e = \left|\frac{\varepsilon}{(p+iq)^2}\right|, \qquad \varphi = -\arg\left(\frac{\varepsilon}{(p+iq)^2}\right) \tag{18}$$

where

$$p = \sqrt{\sqrt{(1+uI_c)^2 + (uR_c)^2} + uR_c} \tag{19}$$

$$q = \sqrt{\sqrt{(1+uI_c)^2 + (uR_c)^2} - uR_c} \tag{20}$$

$$\varepsilon = 2iL_e + 2u(1+i)a_1b_1\frac{1 + L_{e1}\{\cosh[(1+i)a_1b_1] - 1\}}{\sinh[(1+i)a_1b_1]} \tag{21}$$

with

$$R_c = \mathrm{Re}\{(1+i)a_1b_1\coth[(1+i)a_1b_1]\}$$
$$I_c = \mathrm{Im}\{(1+i)a_1b_1\coth[(1+i)a_1b_1]\}. \quad (22)$$

These equations are somewhat involved, so their meaning is not immediately grasped. For this purpose, in the next section we explore some limiting cases which allow verifying our solution by comparison with other published solutions. This comparison also sheds some light on the physical interpretation of our solution.

3. Limiting cases and comparison with existing solutions

We consider three sets of limiting cases. First we analyze the asymptotic limits of very small and very large D. Second, we analyze the asymptotic limits corresponding to high and low permeability and specific storage of the aquitard.

3.1. Negligible and infinite extent of the aquifer under the sea

The proposed analytical solution is a generalization of the solutions obtained by Li and Jiao (2001b) and Li et al. (2008) which consider, respectively, zero and infinite extensions of the aquifer system under the sea. These analytical solutions describe two limit cases where head fluctuations are exclusively induced by hydraulic connection (Li and Jiao, 2001b) and by hydro-mechanical effects (Li et al., 2008).

Case 1: If D is very small ($D \simeq 0$), it is easy to check that $f(x,t,\varphi) \simeq 0$ and Eq. (13) leads to:

$$h(x,t) = h_h(x,t) = Ar_\rho e^{-apx}\cos(\omega t - aqx), \quad 0 \leqslant x < \infty \quad (23)$$

which is essentially the analytical solution obtained by Li and Jiao (2001b) except for the density ratio, r_ρ, which they assumed to be 1. It is important to remark that the amplitude of the head fluctuation is dampened by a factor e^{-apx} with $p \geqslant 1$ (see Eq. (19)). Notice that the ap term appears in all the dampening terms in Eqs. (14)–(16). Therefore, the dampening distance that was originally defined for a single confined aquifer, has to be redefined as $1/ap$. Also note that time–lags between sea tide and head fluctuation predicted by (23) and the classical solution obtained by Ferris (1951) differ a factor q. Fig. 2 shows how p and q change with dimensionless thickness of the aquitard (a_1b_1) and the leakage parameter u. Values of p and q increase with both a_1b_1 and u, resulting in more dampened amplitudes and larger time–lags.

Case 2: If D is very large ($D \to \infty$), head fluctuations produced far offshore will be dampened at the shore. That is, Eq. (13) becomes:

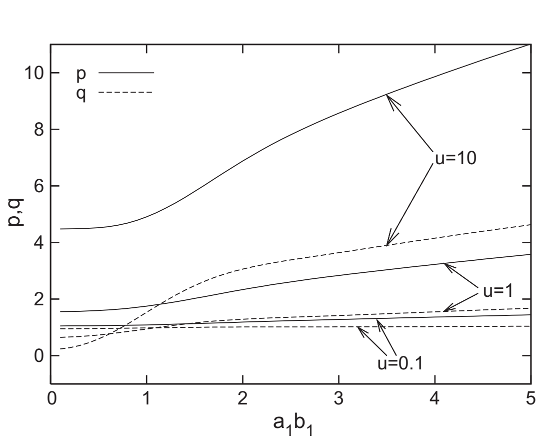

Fig. 2. Changes of p and q coefficients with dimensionless aquitard thickness a_1b_1 for different values of leakage parameter u.

$$h(x,t) = h_1(x,t) + h_m(x,t) = \begin{cases} \frac{Ar_\rho C_e}{2} g(x,t,\varphi), & 0 \leqslant x < \infty \\ Ar_\rho[g(0,t,\varphi) - \frac{1}{2}g(-x,t,\varphi)], & -\infty < x < 0 \end{cases} \quad (24)$$

which is exactly the same equation obtained by Li et al. (2008) for an aquifer extending infinitely under the sea.

Comparison of these two extreme cases provides some insight into the nature of the three components of head fluctuations. We discuss only the inland portion of the solution ($0 \leqslant x \leqslant \infty$) where observation wells are usually located. When $D = 0$, the amplitude of freshwater head in the aquifer at the sea outlet is equal to that of the tide (Ar_ρ), which is dampened inland by a factor e^{-apx} and shifted aqx in time (Li and Jiao, 2001b). Notice that using the distance to the outlet ($x + D$), instead of x, the solution to this case is exactly the aquifer hydraulic component h_h (Eq. (14)). The other extreme case ($D = \infty$) will occur when apD is large (say $apD > 3$). In this case, sea level fluctuations at the outlet are dampened by energy losses in the aquifer (represented by tidal propagation parameter a), which may be enhanced by fluxes into the aquitard (whose effect is represented by factor p). As it can be observed in Fig. 2, p may be much larger than 1, so that the role of the aquitard can be important in shortening the distance where outlet fluctuations are relevant. In this case, the amplitude at the coast is reduced by $C_e/2$. Since C_e is typically around 1, this implies an approximately 1/2 reduction of the sea level amplitude.

3.2. Asymptotic limits of aquitard parameters

Case 3: Impermeable confining layer ($k_1 \sim 0$) and infinite extent of the aquifer under the sea ($D = \infty$)

The most immediate simplification of (24) is the case of impermeable confining layer derived by van der Kamp (1972). Letting $u = 0$ in (19)–(22) yields $p = 1$, $q = 1$ and $\varepsilon = 2iL_e$. Substituting these values back in (18) we have $C_e = L_e$ and $\varphi = 0$, so that (24) becomes

$$h(x,t) = \begin{cases} \frac{Ar_\rho L_e}{2} e^{-ax}\cos(\omega t - ax), & 0 \leqslant x < \infty \\ Ar_\rho L_e[\cos(\omega t) - \frac{1}{2}e^{ax}\cos(\omega t + ax)], & -\infty < x < 0 \end{cases} \quad (25)$$

which is the analytical solution obtained by van der Kamp (1972) for rρ = 1. Notice that the main difference between this solution and (24) lies in the dampening and phase shift factor. The p and q factors represent the contribution from the semipermeable layer. Therefore, they should be equal to 1 whenever leakage across the aquitard (k1/b1) is small compared to the storage capacity of the aquifer during a tidal cycle (sω).

Case 4: Impermeable confining layer ($k_1 \sim 0$) and rigid aquifer ($L_e = 0$)

Assuming that $L_e = 0$ and that the permeability of the confining layer is very low (u<<1) then the aquitard contribution becomes small and we recover Eq. (14) with $p = q = 1$. This is the traditional solution obtained by Jacob (1950) and Ferris (1951) but with ($x + D$) instead of x.

Case 5: Stiff aquitard ($S_{s1} \sim 0$)

A simpler analytical solution can be obtained for very stiff aquitards (negligible S_{S1}, i.e., $a_1 \to 0$). Asymptotic approximations of (19)–(22) yields $p = \sqrt{\sqrt{1+u^2}+u}, q = \sqrt{\sqrt{1+u^2}-u}$ and $\varepsilon = 2(u + iL_e)$. Substituting these values back in (18) we have:

$$C_e = \sqrt{\frac{u^2 + L_e^2}{u^2+1}}, \qquad \varphi = -\arg\left(\frac{u(L_e-1)}{u^2+L_e}\right). \quad (26)$$

The analytical solution obtained with the above expressions of the comprehensive tidal efficiency and tidal phase shift is identical to the solution derived by Li and Jiao (2001a) for a leaky aquifer system ignoring the effect of the elastic storage of the semipermeable layer. The equality of both expressions can be verified by replacing (26) in (16) and rewriting expressions (13) in terms of $\cos(\omega t - aqx)$ and $\sin(\omega t - aqx)$.

Case 6: Soft aquitards with low permeabilities

The confining layer of coastal aquifers often consists of materials deposited under submarine conditions during the Holocene. That is, they tend to be very fine (i.e., low K_1) and soft (i.e., high S_{S1}). Therefore, it is reasonable to assume that a_1 will be large ($1/a_1 \ll b_1$), but not infinite, whereas u will be very small. If $a_1 b_1$ is very large, then $R_c = I_c = a_1 b_1$ and $\varepsilon = 2iL_e + 2(i+1)ua_1 b_1 L_{e1}$. Substituting these into (18)–(20) yields

$$C_e = \frac{\sqrt{[\gamma^2 L_{e1} + (1+\gamma)(L_e + \gamma L_{e1})]^2 + \gamma^2 (L_e - L_{e1})^2}}{\gamma^2 + (1+\gamma)^2}$$
$$\varphi = -\arg\left(\frac{\gamma(L_e - L_{e1})}{\gamma^2 L_{e1} + (1+\gamma)(L_e + \gamma L_{e1})}\right) \tag{27}$$
$$p = \sqrt{\sqrt{(1+\gamma)^2 + \gamma^2} + \gamma}, \quad q = \sqrt{\sqrt{(1+\gamma)^2 + \gamma^2} - \gamma}$$

where $\gamma = a_1 b_1 u = \sqrt{\frac{S_{s1} K_1}{2\omega S^2}}$.

The main implication of the analytical solution given by (27) is that the aquifer response to tides no longer depends on u and a_1 separately, but only on γ, which simplifies the analysis. The dimensionless number γ represents the amount of water released from elastic storage in the aquitard during each tidal cycle, relative to the storage capacity of the aquifer. Therefore, it is a sorptivity parameter similar to the one controlling early time water uptake by aquifers in Pulse Tests (Hsieh et al., 1981; Papadopoulos et al., 1973) or early time solute diffusion into immobile matrices during tracer tests in fractured media (Medina and Carrera, 1996; Carrera et al., 1998). Finally, a rather simple solution can be obtained by assuming $L_e = L_{e1}$. Substituting the above values of p and q into Eq. (21) for ε and Eq. (18) for C_e and φ yields $C_e = L_e$ and $\varphi = 0$. Therefore, the solution becomes:

$$h(x,t) = Ar_p[\frac{L_e}{2}(g(x,0) + g(2D+x,0)) + (1-L_e)g(D+x,0)]. \tag{28}$$

4. Discussion of the proposed solution

In order to analyze groundwater fluctuations induced by sea tides for different extensions of the aquifer system under the sea, we design the following hypothetical example. The aquifer system consists of a confined sand aquifer overlain by a 5 m confining silt layer. The hydraulic and elastic parameters of the silt and sand layers are assumed to be $S_{S1} = 0.0001$ 1/m, $K_1 = 0.001$ m/h, $L_{e1} = 1$, $S = 0.0002$, $T = 20$ m^2/h and $L_e = 0.9$. On the other hand, the sea tide is assumed to be semidiurnal (period of 12.4 h) with an amplitude of 1 m and density ratio $r_p = 1.025$. Using these values, we obtain the following parameters for the analytical solution: $a = 0.15917 \times 10^{-2}$ 1/m, $a_1 = 0.15917$ 1/m, $u = 1.9735$, $p = 2.1868$, $q = 0.8345$, $C_e = 0.9760$, $C_{e1} = 0.7141$, $\varphi = -2.789510^{-2}$, $\varphi_1 = -0.9395$.

Fig. 3 displays the temporal evolution of sea tide, total head fluctuation h, aquifer's hydraulic component h_h, aquitard's hydraulic component h_{h1} and mechanical component h_{h1} at the coastline ($x = 0$) for extensions of 100 and 1000 m of the aquifer system under the sea. It can be observed that for $D = 100$ m, the amplitudes of h_{h1} and h_m are small and the main contribution to the total head fluctuation comes from the h_h component. Conversely, for $D = 1000$ m, the amplitude of h_h is highly attenuated and the total head h is basically determined by h_{h1} and h_m. This simple test illustrates the importance of decoupling hydraulic and mechanical effects to better understand the origin of head fluctuation and flow dynamics in the aquifer system.

In order to perform a more general analysis of the effect of hydraulic parameters on tide-induced fluctuations, we compute amplitudes and time lags of the total head fluctuation and its components in a piezometer located at the coastline. The maximum heads can be obtained by evaluating the analytical expressions (13)–(16) at the point where the temporal derivatives are zero. The time values t_h, t_{h1}, t_m and t_t for which the respective heads h_h, h_{h1}, h_{hm} and h reach the first maximum have the following expressions:

$$t_h = \frac{aq(D+x)}{\omega} \tag{29}$$

$$t_{h1} = \frac{1}{\omega}\tan^{-1}\left\{\frac{s(0,\varphi_1) + s(2D,\varphi_1) - 2s(D,\varphi_1)}{c(0,\varphi_1) + c(2D,\varphi_1) - 2c(D,\varphi_1)}\right\} \tag{30}$$

$$t_m = \frac{1}{\omega} \times \tan^{-1}\left\{\frac{C_e[s(0,\varphi) + s(2D,\varphi) - 2s(D,\varphi)] - C_{e1}[s(0,\varphi_1) + s(2D,\varphi_1) - 2s(D,\varphi_1)]}{C_e[c(0,\varphi) + c(2D,\varphi) - 2c(D,\varphi)] - C_{e1}[c(0,\varphi_1) + c(2D,\varphi_1) - 2c(D,\varphi_1)]}\right\} \tag{31}$$

where $c(D,\varphi) = e^{-apD}\cos(aq(x+D) - \varphi)$ and $s(D,\varphi) = e^{-apD}\sin(aq(x+D) - \varphi)$. The sea tide has a maximum at $t = 0$ (Eq. (9)), then the above equations describe the time–lag between the sea tide and the tide-induced in the observation point. In the following we analyze sequentially the three head components for varying apD using the parameters defined at the beginning of this section. We then analyze the effects of $a_1 b_1$, u, and γ. Fig. 4 displays the maximum heads (half amplitude) and time lags of h_h, h_{h1}, h_m and h at the coastline ($x = 0$) as a function of the dimensionless extension of the aquifer system under the sea apD. The hydraulic component of the confined aquifer h_h goes to 0 as the aquifer system increases seaward. That is, the aquifer hydraulic connection is dampened as D grows, both because of head attenuation in the aquifer (tidal propagation parameter a) and because of flux into the aquitard (factor p). Meanwhile, the aquitard hydraulic component h_{h1} tends to an asymptotic constant value $Ar_\rho C_{e1}/2$. The maximum head of the mechanical component h_m increases from 0 to $Ar_\rho\sqrt{C_e^2 + C_{e1}^2 + 2C_e C_{e1}\cos(\varphi - \varphi_1)}/2$ when apD increases. Finally, the total fluctuation h decreases from Ar_ρ to the asymptotic value $Ar_\rho C_e/2$, that corresponds to the amplitude of h for an infinite extension of the aquifer system as derived by Li et al. (2008). Note that h is almost constant from $apD = 2$, even though the individual components have not yet reached its asymptotic values. That is, the decrease in h_h is compensated by the increase in h_{h1} and h_m for $apD > 2$. From the analysis of maximum head fluctuations we can conclude that amplitudes of h which are approximately half of the sea tide amplitude do not indicate a good hydraulic connection between the seawater and the confined aquifer. On the contrary, they may suggest that the outlet is far away. When the aquifer system extends a distance greater than about $3apD$ head fluctuations are originated by leakage trough the aquitard and loading effects. Overall, the semi amplitude of total head, h, is approximately

$$A_h = Ar_\rho\left[\frac{C_e}{2} + \left(1 - \frac{C_e}{2}\right)e^{-2apD}\right] \simeq \frac{Ar_\rho}{2}(1 + e^{-2apD}). \tag{32}$$

The time–lags between the first maximum of the sea tide and the maximum amplitudes of h_h, h_{h1}, h_m and h are estimated from expressions (28)–(31). The time–lag of h_h increases linearly with apD while time–lags of h_{h1} and h_m go respectively to the constant asymptotic values $-\varphi_1/\omega$ and $-\frac{1}{\omega}\tan^{-1}\left[\frac{C_e\sin\varphi - C_{e1}\sin\varphi_1}{C_e\cos\varphi - C_{e1}\cos\varphi_1}\right]$. A surprising observation is that time–lag of the total fluctuation h is relatively small and goes rapidly to an asymptotic value near to zero ($-\varphi/\omega$). This implies that the leakage of the aquitard and mechanical effects tend to synchronize the phase of the total head h with

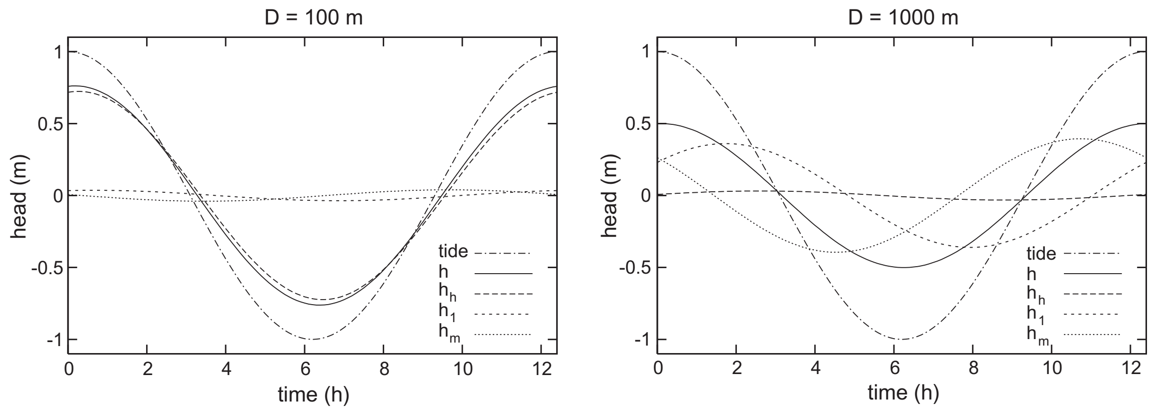

Fig. 3. Tide-induced head components at the coastline for two different extensions of the aquifer system under the sea.

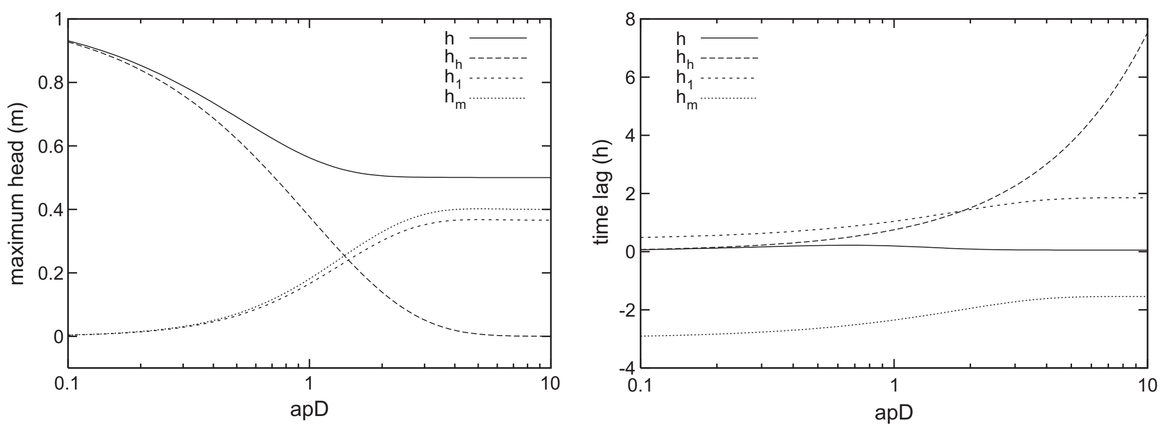

Fig. 4. Changes of maximum heads fluctuations and time–lags at the coastline with dimensionless extension under the sea *apD*. Notice (1) the amplitude of *h* fluctuations is approximately equal to that of h_h for $apD < 0.1$, and about 0.5 (actually, $r_pC_e/2$) for $apD > 2$; (2) the time lag is close to zero for all *apD*.

the sea tide. If these effects are not included in the analysis, the time–lag would linearly increase with *apD* according to Eq. (28).

The dimensionless thickness of the aquitard (a_1b_1) influences the total head h via h_1 and h_m components. Fig. 5 displays the changes of maximum head amplitudes and time–lags with the dimensionless extension under the sea *apD* for two extreme values of the dimensionless thickness of the aquitard a_1b_1. For $a_1b_1 = 0.1$, the aquitard is virtually non-existent and water flow takes place through the top of the aquifer giving as a result a maximum value of the hydraulic component h_{h1}. In this case, the mechanical component h_m reflects basically loading effects in the confined aquifer. On the other hand, when $a_1b_1 = 5$ the hydraulic component h_{h1} is negligible and the mechanical component h_m reaches its maximum value. For $apD > 3$ the total head fluctuation h is basically originated by mechanical effects. Note that even though the mechanical and hydraulic components show significant variations with the thickness of the aquitard, the total head fluctuation is only lightly influenced by this parameter, as shown by Eq. (32). A similar conclusion can be obtained from the analysis of time–lags. The time–lag of total head fluctuation is not very sensitive to the thickness of the aquitard and it is almost in phase with the sea tide.

Fig. 6 shows how the maximum heads and time–lags change with the leakage parameter u. For $u = 0.1$, the hydraulic component h_1 is small while the mechanical component h_m reaches large values. As expected, this situation is similar to the one described in Fig. 4 for a large aquitard thickness. The maximum head curves for $u = 10$ are almost identical to the ones obtained for small aquitard thicknesses, except for the mechanical component h_m which now includes the loading effects of both confined aquifer and aquitard.

Finally, we test the accuracy of the approximated analytical solution obtained for a soft aquitad with low permeability. The coefficients given by Equations (28)–(31) are expressed in terms of $\gamma = a_1b_1u$ and are valid for large values of a_1b_1 and small values of u. Fig. 7 shows the maximum head and time–lag of total head fluctuation as a function of the dimensionless extension under the sea *apD* obtained with the exact and approximate analytical solutions for $a_1b_1 = 2$ and $u = 0.5$ ($\gamma = 1$). As the figure clearly shows, the predicted values by both analytical solutions are in excellent agreement. Therefore, the proposed approximate solution can be used in many practical situations where the conditions imposed on a_1b_1 and u are naturally satisfied.

5. Conclusions

We have derived an analytical solution to study hydraulic and mechanical effects on tide-induced head fluctuations in a costal aquifer system that extends a finite distance under the sea, with the ultimate aim of analyzing whether this distance can be estimated from head observations at piezometers near the coast. The mechanical effect is originated by the fluctuations of sea level that compress and expand elastically the offshore portion of the aquifer–aquitard system. The hydraulic components reflect leakage across the aquitard and flow through the aquifer. The derived analytical solution is a generalization of those by Li and Jiao

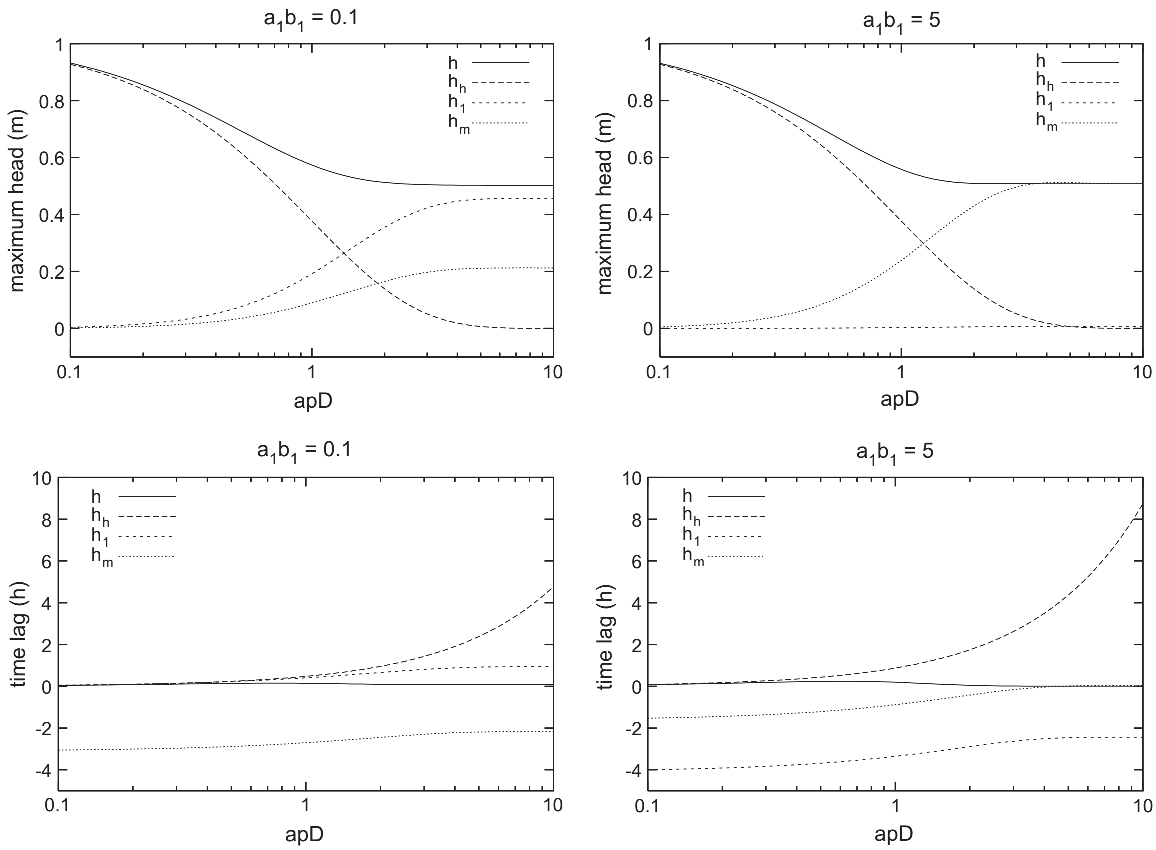

Fig. 5. Maximum heads and time–lags at the coastline versus dimensionless extension under the sea *apD* for different thickness of the aquitard.

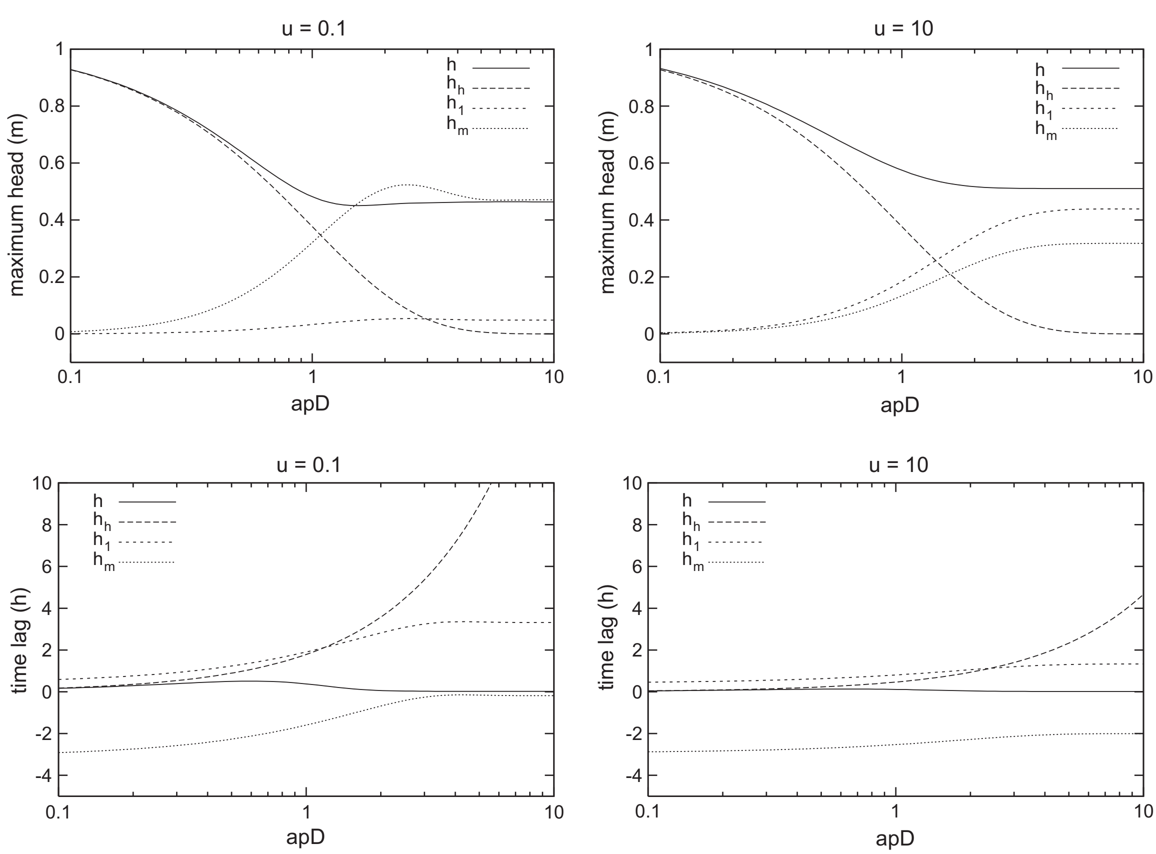

Fig. 6. Maximum heads and time–lags versus dimensionless extension under the sea *apD* for different values of the leakage parameter *u*.

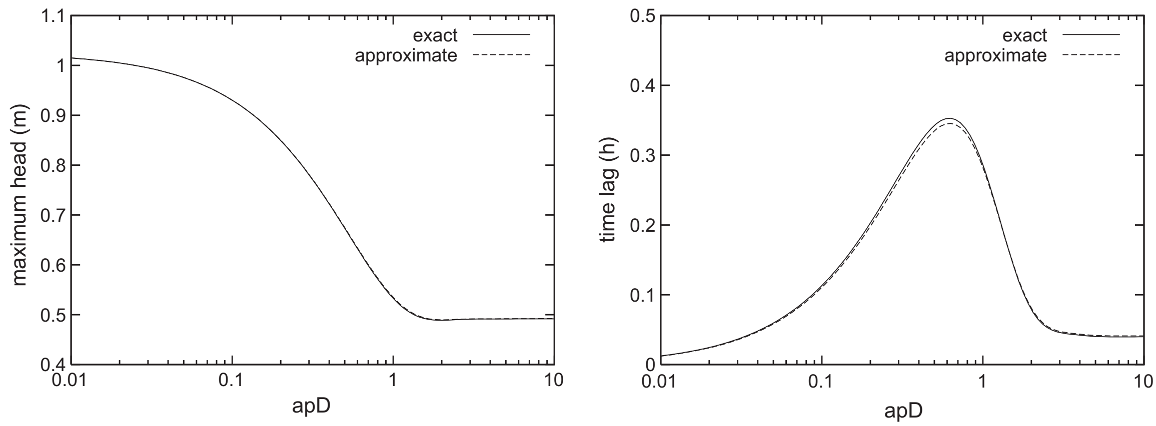

Fig. 7. Maximum head and time–lag versus dimensionless extension under the sea *apD* for $a_1b_1 = 2$ and $u = 0.5$ predicted by exact and approximated analytical solutions.

(2001b) and Li et al. (2008) which consider, respectively, zero and infinite extensions of the aquifer system under the sea. We have also derived closed-form expressions for maximum amplitudes and for time lags between sea tide and tide-induced head fluctuations.

Asymptotic analysis of the solution, which facilitated comparison with existing analytical solutions, and hypothetic studies show that the hydraulic component of the aquitard and mechanical effects increase the amplitude of the total groundwater fluctuation observed inland and tends to synchronize its phase with the sea tide. These results indicate that large amplitudes of tide-induced head fluctuations cannot be associated with good hydraulic connection between the seawater and the confined aquifer. In fact, the sensitivity analysis points out that it is very hard to identify the offshore distance of the deep aquifer outlet. The hydro-mechanical effect is significant for intermediate and large extensions of aquifer system under the sea and should be included in the estimation of hydraulic parameters from tide-induced groundwater fluctuations.

Identification of the distance of the aquifer outlet to the sea is only possible when *apD* is smaller than 3. Otherwise, the aquifer response is independent of *D* (as if *D* was infinite), which was the case analyzed by Li et al. (2008). In this case, the amplitude of the fluctuations at the shore is $Ar_\rho C_e/2$ (approximately $A/2$), and the phase shift is zero. The latter is useful to test the validity of the hypotheses. If one observes a large phase shift, then it can be concluded that our assumptions are not applicable. Hydraulic properties can be derived if additional wells are available inland because the amplitude of fluctuations is dampened by a factor of $\exp(-apx)$, whereas the phase is shifted by *aqx*.

On the other hand, if the amplitude is larger than one half of that of tides, then one can estimate *apD* quite accurately. Since *ap* can be obtained from the further reduction of amplitude inland, we conclude that *D* can be obtained whenever a second piezometer is available. Again, this situation can be validated by actual data because the phase shift at the coastline should be close to zero.

Appendix A

The solutions of differential problems (1)–(11) are obtained from complex forms of the involved equations. Note that boundary conditions (4) and (9) can be written as $h_s(t) = \mathrm{Re}\{Ar_\rho e^{i\omega t}\}$, then we assume that h_1 and h can be expressed as

$$h_1(x,z,t) = \mathrm{Re}\{Ar_\rho Y(x,z)e^{i\omega t}\} \tag{A1}$$

$$h(x,t) = \mathrm{Re}\{Ar_\rho X(x)e^{i\omega t}\} \tag{A2}$$

where $Y(x,z)$ and $X(x)$ are complex functions. Substituting (A1) into (1)–(5) we obtain the following differential problem:

$$\frac{\partial^2 Y}{\partial z^2} = i\omega\frac{S_{S1}}{K_1}Y, \quad 0 \leqslant x < \infty, \quad -\frac{b_1}{2} < z < \frac{b_1}{2} \tag{A3}$$

$$\frac{\partial^2 Y}{\partial z^2} = i\omega\frac{S_{S1}}{K_1}(Y - L_{e1}), \quad -D \leqslant x < 0, \quad -\frac{b_1}{2} < z < \frac{b_1}{2} \tag{A4}$$

$$Y(x, b_1/2) = 0, \quad 0 \leqslant x < \infty \tag{A5}$$

$$Y(x, b_1/2) = 1, \quad -D \leqslant x < 0 \tag{A6}$$

$$Y(x, -b_1/2) = X(x), \quad -D \leqslant x < \infty. \tag{A7}$$

The solution of (A3)–(A7) is:

$$Y(x,z) = \frac{X(x)\sinh[(1+i)a_1b_1(0.5 - z/b_1)]}{\sinh[(1+i)a_1b_1]}, \quad 0 \leqslant x < \infty, \quad -\frac{b_1}{2} < z < \frac{b_1}{2} \tag{A8}$$

$$Y(x,z) = L_{e1} + \frac{(1 - L_{e1})\sinh[(1+i)a_1b_1(0.5 + z/b_1)]}{\sinh[(1+i)a_1b_1]} + \frac{(X(x) - L_{e1})\sinh[(1+i)a_1b_1(0.5 - z/b_1)]}{\sinh[(1+i)a_1b_1]}, \quad -D \leqslant x < 0, \quad -\frac{b_1}{2} < z < \frac{b_1}{2}. \tag{A9}$$

Similarly, substituting (A2) into (6)–(11) we obtain the following differential problem:

$$\frac{\partial^2 X}{\partial x^2} = \frac{i\omega S}{T}X(x) - \frac{K_1}{T}\left.\frac{\partial Y}{\partial z}\right|_{z=-b_1/2}, \quad 0 \leqslant x < \infty \tag{A10}$$

$$\frac{\partial^2 X}{\partial x^2} = \frac{i\omega S}{T}(X(x) - L_e) - \frac{K_1}{T}\left.\frac{\partial Y}{\partial z}\right|_{z=-b_1/2}, \quad -D \leqslant x < 0 \tag{A11}$$

$$\lim_{x\to\infty}\frac{\partial X}{\partial x} = 0 \tag{A12}$$

$$X(-D) = 1 \tag{A13}$$

$$\lim_{x\to 0^+}X(x) = \lim_{x\to 0^-}X(x) \tag{A14}$$

$$\lim_{x\to 0^+}\frac{\partial X}{\partial x} = \lim_{x\to 0^-}\frac{\partial X}{\partial x}. \tag{A15}$$

The solution to (A10)–(A15) is

$$X(x) = \frac{\varepsilon}{2(p+iq)^2}[e^{-a(p+iq)x} + e^{-a(p+iq)(2D+x)}] + [1 - \frac{\varepsilon}{(p+iq)^2}]e^{-a(p+iq)(D+x)}, \quad 0 \leqslant x < \infty \tag{A16}$$

$$X(x) = \frac{\varepsilon}{2(p+iq)^2}[2 - e^{-a(p+iq)x} + e^{-a(p+iq)(2D+x)}] + [1 - \frac{\varepsilon}{(p+iq)^2}]e^{-a(p+iq)(D+x)}, \quad -D \leqslant x < 0 \tag{A17}$$

where $p + iq = \sqrt{2i + 2u(1+i)a_1 b_1 \coth[(1+i)a_1 b_1]}$.

Finally, substituting (A8) and (A9) in (A1) we obtain the analytical solution for head fluctuations in the semipermeable layer; and substituting (A16) and (A17) in (A2) we obtain the solution for the confined aquifer given by (13)–(16).

References

Abarca, E., Carrera, J., Sánchez-Vila, X., Dentz, M., 2007. Anisotropic dispersive Henry problem. Adv. Water Resour. 30 (4), 913–926.

Alcolea, A., Castro, E., Barbieri, M., Carrera, J., Bea, S., 2007. Inverse modeling of coastal aquifers using tidal response and hydraulic tests. Ground Water 45 (6), 711–722.

Alcolea, A., Renard, P., Mariethoz, G., Bertone, F., 2009. Reducing the impact of a desalination plant using stochastic modeling and optimization techniques. J. Hydrol. 365 (3–4), 275-288377-389.

Ataie-Ashtiani, B., Volker, R.E., Lockington, D.A., 2001. Tidal effects on ground-water dynamics in unconfined aquifers. Hydrol. Proc. 15 (4), 655–669.

Banerjee, P., Sarwade, D., Singh, V.S., 2008. Characterization of an island aquifer from tidal response. Environ. Geol. 54 (4), 901–906.

Bear, J., 1972. Dynamics of Fluids in Porous Media. American Elsevier, New York.

Carol, E.S., Kruse, E.E., Pousa, J.L., Roig, A.R., 2009. Determination of heterogeneities in the hydraulic properties of a phreatic aquifer from tidal level fluctuations: a case in Argentina. Hydrogeol. J. 17 (7), 1727–1732.

Carr, P.A., van der Kamp, G.S., 1969. Determining aquifer characteristics by the tidal method. Water Resour. Res. 5 (5), 1023–1031.

Carrera, J., Sánchez-Vila, X., Benet, I., Medina, A., Galarza, G., Guimerá, J., 1998. On matrix diffusion: formulations, solution methods and qualitative effects. Hydrogeol. J. 6 (1), 178–190.

Cirpka, O.A., Attinger, S., 2003. Effective dispersion in heterogeneous media under random transient flow conditions. Water Resour. Res. 39 (9), 1257. http://dx.doi.org/10.1029/2002WR001931.

Dentz, M., Carrera, J., 2005. Effective solute transport in temporally fluctuating flow through heterogeneous media. Water Resour. Res. 41, W08414. http://dx.doi.org/10.1029/2004WR003571.

Dentz, M., Carrera, J., 2007. Mixing and spreading in stratified flow. Phys. Fluids 19, 017107. http://dx.doi.org/10.1063/1.2427089.

Ferris, J.G., 1951. Cyclic fluctuations of water level as a basis for determining aquifer transmissibility. Int. Assoc. Sci. Hydrol. Publ. 33, 148–155.

Geng, X., Li, H., Boufadel, M.C., Li, S., 2009. Tide-induced head fluctuations in a coastal aquifer: effects of the elastic storage and leakage of the submarine outlet-capping. Hydrogeol. J. 17 (5), 1289–1296.

Hsieh, P.A., Tracy, J.V., Neuzil, C.E., Bredehoeft, J.D., Silliman, S.E., 1981. A transient laboratory method for determining the hydraulic properties of 'Tight' rock – I. Theory. Int. J. Rock Mech. Min. Sci. Geomech. Abstr. 18, 245–252.

Jacob, C.E., 1950. Flow of groundwater. In: Engineering Hydraulics. Wiley, New York, pp. 21–386.

Jha, M.K., Kamii, Y., Chikamori, K., 2003. On the estimation of phreatic aquifer parameters by the tidal response technique. Water Resour. Manage. 17, 69–88.

Jiao, J.J., Tang, Z.H., 1999. An analytical solution of groundwater response to tidal fluctuation in a leaky confined aquifer. Water Resour. Res. 35 (3), 747–751.

Knudby, C., Carrera, J., 2006. On the use of apparent hydraulic diffusivity as an indicator of connectivity. J. Hydrol. 329 (3–4), 377–389.

Li, G., Chen, C., 1991. Determining the length of confined aquifer roof extending under the sea by the tidal method. J. Hydrol. 123 (1–2), 97–104.

Li, H., Jiao, J.J., 2001a. Tide-induced groundwater fluctuation in a coastal leaky confined aquifer system extending under the sea. Water Resour. Res. 37 (5), 1165–1171.

Li, H., Jiao, J.J., 2001b. Analytical studies of groundwater-head fluctuation in a coastal confines aquifer overlain by a semi-permeable layer with storage. Adv. Water Resour. 24 (5), 565–573.

Li, H., Jiao, J.J., 2002. Analytical solutions of tidal groundwater flow in coastal two-aquifer system. Adv. Water Resour. 25 (4), 417–426.

Li, G., Li, H., Boufadel, M.C., 2008. The enhancing effect of the elastic storage of the seabed aquitard on the tide-induced groundwater head fluctuation in confined submarine aquifer systems. J. Hydrol. 350 (1–2), 83–92.

Medina, A., Carrera, J., 1996. Coupled estimation of flow and solute transport parameters. Water Resour. Res. 32 (10), 3063–3076.

Millham, N.P., Howes, B.L., 1995. A comparison of methods to determine K in a shallow coastal aquifer. Ground Water 33 (1), 49–57.

Monachesi, L., Guarracino, L., 2011. Exact and approximate analytical solutions of groundwater response to tidal fluctuations in a theoretical inhomogeneous coastal confined aquifer. Hydrogeol. J. 19 (7), 1443–1449.

Neuman, S.P., Witherspoon, P.A., 1969. Theory of flow in a confined two aquifer system. Water Resour. Res. 5 (4), 803–816.

Pandit, A., El-Khazen, C.C., Sivaramapillai, S.P., 1991. Estimation of hydraulic conductivity values in a coastal aquifer. Ground Water 29 (2), 175–180.

Papadopoulos, I.S., Bredehoeft, J.D., Cooper, H.H., 1973. On the analysis of slug test data. Water Resour. Res. 9 (4), 1087–1089.

Rezaei, M., Sanz, E., Raeisi, E., Ayora, C., Vázquez-Suñé, E., Carrera, J., 2005. Reactive transport modeling of calcite dissolution in the fresh-saltwater mixing zone. J. Hydrol. 311 (1–4), 282–298.

Sanford, W.E., Konikov, L.F., 1989. Simulation of calcite dissolution and porosity changes in saltwater mixing zones in coastal aquifers. Water Resour. Res. 25 (4), 655–667.

Slooten, L.J., Carrera, J., Castro, E., Fernández-García, D., 2010. A sensitivity analysis of tide-induced head fluctuations in coastal aquifers. J. Hydrol. 393 (3-4), 370–380.

Terzaghi, K., 1954. Theoretical Soil Mechanics. Wiley, New York.

Townley, L.R., 1995. The response of aquifers to periodic forcing. Adv. Water Resour. 18 (3), 125–146.

Trefry, M.G., 1999. Periodic forcing in composite aquifers. Adv. Water Resour. 22 (6), 645–656.

van der Kamp, G., 1972. Tidal fluctuations in a confined aquifer extending under the sea. In: Proc. 24th Int. Geol. Congr. pp. 101–106.

van der Kamp, G., Gale, J.E., 1983. Theory of earth tide and barometric effects in porous formations with compressible grains. Water Resour. Res. 19 (2), 538–544.

White, J.K., Roberts, T.O.L., 1994. The significance of groundwater tidal fluctuations. In: Wilkson, W.B. (Ed.), Groundwater Problems in Urban Areas. Inst. of Civ. Eng, London, pp. 31–42.

Zhou, X., 2008. Determination of aquifer parameters based on measurements of tidal effects on a coastal aquifer near Beihai, China. Hydrol. Proc. 22 (16), 3176–3180.