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Logical Pluralism and Interpretations of Logical Systems

Abstract. Logical *pluralism* is a general idea that there is more than one correct logic. Carnielli and Rodrigues [2019a] defend an epistemic interpretation of the paraconsistent logic *N4*, according to which an argument is valid in this logic just in case it necessarily preserves evidence. The authors appeal to this epistemic interpretation to briefly motivate a kind of logical pluralism: "different accounts of logical consequence may preserve different properties of propositions". The aim of this paper is to study the prospect of a logical pluralism based on different interpretations of logical systems. First, we give our analysis of what it means to interpret a logic — and make some hopefully useful distinctions along the way. Second, we present what we call an *interpretational logical pluralism*: there is more than one correct logic and a logic is correct only if it has some adequate interpretation. We consider four variants of this idea, bring up some possible objections, and try to find plausible solutions on behalf of the pluralist. We will argue that interpretations of logical systems provide a promising — albeit not unproblematic — route to logical pluralism.

Keywords: interpretations of logical systems; logical pluralism; pure and applied logics; collapse argument

1. Introduction

Logical *pluralism* is the general idea that there is more than one correct logic; logical *monism* is the opposed stance according to which only one logic is correct. Over the last few years, logical pluralism has been developed in many different ways. Some authors, for example, have suggested that there is a plurality of correct logics because there are

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various different kinds of truth bearers [Russell, 2008]. Others have suggested that there is a plurality of correct logics because there are various legitimate choices as to which are the logical constants of natural language [Varzi, 2002], or alternatively, various ways to make precise the meaning of these constants [Kouri Kissel, 2018]. Finally, some authors have claimed that there is a plurality of correct logics because there are various ways to make precise the meaning of the vernacular predicate "follows deductively from" [Beall and Restall, 2006; Cook, 2010]. Despite the differences, all these approaches share two features: (i) they assume that every logical system has a privileged or canonical purpose, which is to give an account of deductive validity in the language we use to communicate and make science, that is, natural language; consequently, (ii) they understand the logical pluralism thesis along the following lines:

(Canonical pluralism) There is more than one correct logic; a logic is correct only if it gives an accurate account of validity in natural language.

Many of the pluralist proposals in the literature are exposed to, or designed to avoid, an objection known as the *collapse argument*. In a nutshell, the objection is that logic is normative, in the sense that it gives us norms for reasoning. However, if two or more different logics are all correct with respect to a certain domain of discourse, then — the objection goes—only the stronger one will be normatively relevant in the abovementioned sense. Hence, as far as normativity is concerned, there is no significant difference between saying that more than one logic is correct and saying than only one is. Pluralism 'collapses' into monism. We will come back to this argument in Section 3.

Recently, there has been an interesting discussion about the interpretation of logical systems. It has focused on paraconsistent logic, but its philosophical import is not restricted to this family of systems.[1] Carnielli and Rodrigues [2019a] have defended an epistemic interpretation of the paraconsistent logic *N4*. According to the authors, this logic preserves evidence: an argument $A_1, A_2, \ldots / B$ is valid in *N4* just in case it is necessary that, if there is evidence for each of the premises, there is also evidence for the conclusion. This works as a justification of paraconsistency, for in real life it is not unusual to find conflicting evidence with respect to some claim A, that is, evidence for both A and $\neg A$. (For

[1] A logic is *paraconsistent* if it invalidates the principle of Explosion: $A, \neg A / B$. For detailed discussions of the notion of paraconsistency, see [Barrio and Da Ré, 2018].

example, think of two reliable tests for a medical disease, where one of the tests gives a 'positive' and the other a 'negative'.) In such cases, we should not conclude that we have evidence for any arbitrary claim B, so the principle of Explosion fails. Carnielli and Rodrigues [2019a, p. 21] appeal to their epistemic interpretation of *N4* to briefly motivate a kind of logical pluralism:

> [D]ifferent accounts of logical consequence may preserve different properties of propositions. We do not think that logical consequence has to be always identified with preservation of truth. Thus, classical [...] and paraconsistent logics may be interpreted as being concerned with preservation of, respectively, truth [...] and availability of evidence.

As the discussion proceeded, some authors pointed out that Carnielli and Rodrigues' interpretation of *N4* faces some difficulties, to which we will turn in Section 2.

The aim of this paper is to study the prospect of a logical pluralism based on different interpretations of logical systems. Firstly, we give our analysis of what it means to interpret a logic; along the way, we make a hopefully useful distinction between three senses in which a logical system can be interpreted: the application of the system, the interpretation of its metatheory and, lastly, the philosophical background assumed. Secondly, we present the following reading of the pluralist thesis:

(Interpretational pluralism) There is more than one correct logic; a logic is correct only if it has some adequate interpretation.

The position does not require that each correct logic gives an accurate account of validity in natural language. Also, notice that the above template only provides necessary conditions for correctness — viz. having an adequate interpretation needs not be enough for a logic to be correct. We consider four variants of the thesis, each providing sufficient conditions for correctness, and claim that at least two of them are interesting. We bring up some possible objections against each of the variants, one of them being a close relative of the collapse argument. Finally, we try to find plausible solutions on behalf of the pluralist. We will argue that interpretations of logical systems provide a promising — albeit not unproblematic — route to logical pluralism.

The structure of the paper is as follows. In Section 2, we tackle the notion of *interpretation*. In Section 3, we briefly exemplify how the collapse objection against logical pluralism works. Finally, in Section 4 we turn to interpretational pluralism.

2. On the notion of *interpretation*

The notions of 'interpretation' and 'philosophical interpretation' of a logical system have featured prominently in some recent articles [e.g. Arenhart, 2021; Barrio and Da Ré, 2018; Carnielli and Rodrigues, 2019a; Lo Guercio and Szmuc, 2018]. However, there does not seem to be a clear consensus as regards what exactly these notions mean. Barrio and Da Ré make some clarifying remarks in this respect; their position is summarised by Arenhart [2021, p. 7] as follows:

> Pure logic concerns the systems of logic understood as mathematical objects of study (...) Applied logic selects a context of application, such as the study of electrical circuits, computer programming, inference in natural language, among others [...] [E]ven if we select a single field of application, such as inference in natural language, a system of logic may have different interpretations.

To illustrate the last claim, here are two of the examples given by Barrio and Da Ré. First, suppose you use Kleene's weak three-valued logic to study reasoning in natural language. Then, you can interpret the third truth-value either as 'meaningless' [Bochvar, 1938] or as 'off-topic' [Beall, 2016]; in both cases, validity will be understood as truth preservation. Now, suppose that you use paraconsistent logic *LP*. You can interpret the third value as a glut ('both true and false') [Priest, 2006b] or as a gap ('neither true nor false') [Beall and Ripley, 2004]; in the first case, validity will be understood as truth preservation; in the second case, as non-falsity preservation.

While we think that the remarks made by Barrio and Da Ré are essentially correct, we consider useful making some further clarifications. Next, we sketch a picture where there are three senses in which a logical system can be interpreted.

Pure logics are abstract mathematical objects — no surprise here. In what follows, we let a pure logic be any pair $\langle \mathcal{L}, \Rightarrow \rangle$, where $\mathcal{L}$ is a formal language and $\Rightarrow \subseteq \mathcal{P}(\mathcal{L}) \times \mathcal{P}(\mathcal{L})$.[2] The relation $\Rightarrow$ may be specified using a proof theory or a formal semantics.

[2] This definition is very permissive: (i) it allows systems with single and systems with multiple conclusions or premises; (ii) it allows nontransitive, nonreflexive and non-monotonic systems, and (iii) it allows that $\Rightarrow$ is not invariant under substitution. We are as liberal as possible because a discussion of what counts as a pure logic is beyond the scope of this study; however, none of our arguments hinge on this issue.

Given a pure logic, we can assign a meaning to the sentences of its language; in other words, we can take these sentences to be talking about certain things. Thus, for example, the language of Boolean propositional logic is sometimes read as talking about electrical circuits, and the language of Lambek calculus as talking about certain grammatical structures. We call such assignments of meaning *applications*, and assume the following concise definition:

- *Application*: To apply a pure logic $\langle \mathcal{L}, \Rightarrow \rangle$ to a domain of discourse D is to provide a (maybe non-mechanical) procedure to translate claims from $\mathcal{L}$ into claims from D and *vice versa*.

To illustrate, suppose you want to apply classical propositional logic, hereafter $\langle \mathcal{L}_{CP}, \Rightarrow_{CP} \rangle$, to natural language in general (viz. not restricted to some specific domain such as physics, mathematics, etc.). Then, you assume the standard translation procedure, according to which 'or' is read mostly as '$\vee$', 'but' is read mostly as '$\wedge$', sentences without so-called connectives are replaced by propositional parameters, and so on and so forth. As Priest [2006a] notes, this procedure is not spelled out in much detail in general, but it gives you enough procedural knowledge to move from natural language to $\mathcal{L}_{CP}$ and *vice versa*.

Hence, we have our first sense in which a pure logic may be interpreted: we apply the logic to a given domain of discourse (thus specifying an intended reading of the logical constants). However, this sense of interpretation will not be our main concern throughout the paper. Rather, we will focus on the kind of interpretations that give rise to Barrio and Da Ré's examples above. Pure logics are specified using some metatheoretical machinery; often, a proof system or a formal semantics. We can *interpret* the elements of this machinery, that is to say, we can assign them a meaning. So, for another example, Carnielli and Rodrigues [2019a] propose a two-valued non-compositional semantics for the paraconsistent logic *N4*, where the semantic values '1' and '0' are interpreted as, respectively, 'there is evidence for' and 'there is no evidence for' (and the system is paraconsistent, so there can be evidence for A and for $\neg A$ at the same time). This, together with the above example about the possible interpretations of *LP*, gives us that paraconsistent logics, in general, have received at least three different interpretations of their metatheories[3]:

[3] Here we just consider the 3-valued version of *LP*. There are other alternatives:

Logic	Author(s)	Semantic Values	Validity
LP	Priest [2006b]	'just true', 'just false', 'both true and false'	truth preservation
LP	Beall and Ripley [2004]	'just true', 'just false', 'neither true nor false'	non-falsity preservation
N4	Carnielli and Rodrigues [2019a]	'there is evidence for', 'there is no evidence for'	evidence preservation

Table 1. Interpretations of the metatheory in paraconsistent logics

As Table 1 suggests, the way we interpret the semantic values of a metatheory constrains the way we can interpret validity. For example, suppose you take Priest's reading of the semantic values of *LP* and, moreover, you assume that a valuation function represents an assignment of meaning to the non-logical vocabulary of the language. Then, you have good reasons to say that an argument is valid in *LP* just in case it necessarily preserves truth, and, on the other hand, you cannot reasonably say that valid arguments in *LP* preserve falsity (for you interpret value '0' as 'false' and there are *LP*-valid arguments which do not preserve value '0'). In general, we will consider the interpretation of the semantic values and the interpretation of validity as two related but not identical aspects of the interpretation of the metatheory. Next, a succinct formulation:

- *Interpretation of the metatheory*: To interpret the metatheory of a pure logic $\langle \mathcal{L}, \Rightarrow \rangle$ is to associate the elements of this metatheory (e.g. the semantic values and the property of validity) with a meaning.

For concision, we will sometimes refer to interpretations of the metatheory as *metatheoretical interpretations*. It is worth making a couple of remarks. First, the application and the metatheoretical interpretation of a logic constrain each other. For a simple example, suppose that one applies classical logic to natural language. If one interprets the value 1 as 'true' and the value 0 as 'false', then one cannot reasonably translate '$\vee$' as 'and'; also, if one *does* translate '$\vee$' as 'and', one seems bound to interpret the value 1 as 'false' and the value 0 as 'true'. In any case, we

for example Priest [2006b] also provides a version with two values and a relational semantics.

consider that the distinction between application and metatheoretical interpretation is useful, for given one and the same application, a logic might have many different metatheoretical interpretations — let the case of *LP* in Table 1 serve as an example.

A second remark is that, even though the meta-theoretical interpretation often includes an assignment of meaning to the semantic values, this needs not be the case, for it depends on the structure of the theory. For instance, inferentialist approaches may not interpret the semantic values but, rather, the notion of validity and some other notions such as the rules of inference. Thus, Carnielli and Rodrigues [2019b] interpret the rules of *N4* as evidence-preserving, and they read "A holds" as "there is evidence for the truth of A" and "$\neg A$ holds" as "there is evidence for the falsity of A".

Another logic that admits more than one metatheoretical interpretation is the paracomplete and paraconsistent *FDE*. Under its original interpretation, advanced by Belnap [1977], *FDE* is a logic to reason with potentially inconsistent and incomplete information states: the idea is that a proposition A has the value 'both' when a database has been informed that A is true, and it has also been informed that A is false; A has the value 'none' when the database has received no information about A. The logic *FDE* determines what follows from the information received, so for example, if the database is told that A is true, A is false, and B is true, then it has also been told that $A \wedge B$ is true and that $A \wedge B$ is false. Recently, another interpretation of *FDE* was proposed by Beall [2019], under a dialetheist influence.[4] Beall claims that *FDE* is the "one true logic" for it makes room for reasoning in a very general way: it can deal with both gaps and gluts. Table 2 summarizes the two readings.

Now, the fact that many interpretations have been proposed for the metatheory of a given logic, certainly does not imply that all the interpretations proposed work. We assume the following:

- *Adequate metatheoretical interpretation*: Let $\langle \mathcal{L}, \Rightarrow \rangle_D$ be the result of applying the pure logic $\langle \mathcal{L}, \Rightarrow \rangle$ to the domain D. A metatheoretical interpretation I of $\langle \mathcal{L}, \Rightarrow \rangle_D$ is adequate just in case all the resulting predictions hold.

For example, many authors think that Belnap's metatheoretical interpretation of *FDE* is adequate to capture told-true preservation.

[4] *Dialetheism* is the thesis that some contradictions are true.

Author	Semantic values	Validity
Belnap [1977]	'just told true', 'just told false', 'told both true and false', 'told neither true nor false'	told-true preservation
Beall [2019]	'just true', 'just false', 'both true nor false' 'neither true nor false'	truth preservation

Table 2. Metatheoretical interpretations of *FDE*

The notion of 'prediction' that we appeal to is a broad one. It is usually not a prediction about the future, such as predictions in empirical sciences. It is just a set of sentences that are implied by the theory and its interpretation, most of which were not part of the initial motivating data for the theory. In general, one can depart from a logical system with some axioms and rules, which may seem reasonable under some interpretation. This structure will naturally imply certain theorems. Those new theorems under the same interpretation (the 'predictions'), may be unreasonable, and therefore one can reject the original system. For example, one can take classical logic to capture valid arguments from natural language; therefore, every classically valid argument is interpreted as intuitively valid. But as we know, this 'predicts' that arguments such as Explosion are also intuitively valid — which does not seem to be the case. So, one can be tempted to reject the interpretation of classical logic as a theory of intuitive validity, for it seems inadequate. In any case, as in science, the results will often not be conclusive: contradictory evidence can usually be saved by changing the auxiliary hypotheses. For a more developed theory about predictions in logic, see the recent work of Martin and Hjortland [2021].

Moreover, checking the adequacy of an interpretation is not an easy task. Logical theories can be used not only to study truth preservation in so-called pathological contexts (e.g. semantic paradoxes), but also to study other disputed concepts such as grounding, justification, or explanation. We take for granted that there *are* correct theories about those concepts. But as usual in philosophy, authors rarely reach an agreement. This is a general metaphilosophical problem, and not a specific problem for our notion of adequacy, which only needs some theories to be correct.

Bear in mind that logics can always be inadequately interpreted. To give a silly example, imagine that classical logic is interpreted as capturing the notion of truth-preservation. Suppose that we take its usual truth-tables but interpret '1' as 'false' and '0' as 'true'. Then, we obtain an inadequately interpreted logic: it claims, for example, that 'it is raining and it is not raining' is always true, while 'it is raining or it is not raining' is always false.

The epistemic interpretation of *N4* defended by Carnielli and Rodrigues faces some difficulties that call doubts on its adequacy. First Lo Guercio and Szmuc [2018] and then also Arenhart [2021] remarked that *N4* validates the principle of Adjunction, viz. $A, B/A \wedge B$. However, this is not an evidence preserving rule, at least with the usual conception of evidence. For usually there might be evidence for A, and evidence for $\neg A$, but no evidence for $A \wedge \neg A$ (e.g., if we take 'evidence' as the idea that the proposition has $\frac{1}{2}$ or higher probability).

So, this is the second sense in which a pure logic may be interpreted: we assign a meaning to the elements of its metatheory. The third and last sense concerns the philosophical account we give of the notions involved in, or related to, the previous two senses of interpretation. This philosophical account can play an important role in giving reasons for or against a particular metatheoretical interpretation. Consider, for example, the logic *LP*. Take the gappy interpretation, defended by Beall and Ripley [2004]. The authors endorse the philosophical thesis that gappy sentences are assertable; thus, they claim that *LP* preserves not only non-falsity but also — and more importantly — assertability.[5] Take now the glutty interpretation. You can claim that *LP* preserves assertability, without having to commit to the non-standard view that gappy sentences are assertable; however, you will be promoting a non-standard view on truth, namely, that some sentences are both true and false. These differences are summarised in Table 3. As the table shows, Priest's and Beall and Ripley's approaches to *LP* differ not only in the interpretation of the metatheory, but also in the philosophical backgrounds.

- *Philosophical background*: Let $\langle \mathcal{L}, \Rightarrow \rangle_{D,I}$ be the result of applying the pure logic $\langle \mathcal{L}, \Rightarrow \rangle$ to the domain D under the metatheoretical interpretation I. To give a philosophical background for $\langle \mathcal{L}, \Rightarrow \rangle_{D,I}$ is to

[5] The thesis that there are gappy sentences and, moreover, gappy sentences are assertable, is called by the authors *analetheism*. So, their article is a defence of an analetheic interpretation of *LP*.

Author(s)	Metatheoretical interpretation	Philosophical background
Priest [2006b]	Glutty. Validity preserves truth	Truth is sufficient for assertability Validity preserves assertability
Beall & Ripley [2004]	Gappy. Validity preserves non-falsity	Truth or non-falsity is sufficient for assertability. Validity preserves assertability

Table 3. Interpretations of *LP*

give a philosophical account of the notions involved in, or relevantly related to, the application of $\langle \mathcal{L}, \Rightarrow \rangle$ to D under I.

Thus, for example, the philosophical background of the usual semantics for classical logic might include a story about what it amounts for a sentence to be 'true' or 'false'. The above definition is quite vague of course, but we think good enough to make the distinction between the three senses of interpretation useful.

Here is another example of divergence in philosophical backgrounds. It concerns intuitionistic logic. The constructivist approach [Brouwer, 1908] says that classical mathematics is wrong and that it should be replaced by intuitionistic mathematics. More generally, it claims that intuitionistic logic should be the logic for reasoning in general. This view is also shared by more contemporary authors such as Dummett [1975], although Dummett also used different examples such as the indeterminacy of the past and the future. The epistemic reading [Heyting, 1956][6] says that classical mathematics is not necessarily wrong, but it is based on weak argumentative steps. We should be able to draw a difference between solid constructive reasoning and more abstract platonistic reasoning. Broadly speaking, both views can be summarised in Table 4. As the table shows, constructivism and epistemicism coincide with respect to the interpretation of intuitionistic metatheory, but they disagree on the philosophical level. There is a much deeper discussion about the meaning of intuitionistic logic that we cannot cover here; for what matters to our argument, it is important to observe that the logic admits two different philosophical backgrounds.

[6] In this paper, Heyting distinguishes between the 'logic of being' (classical logic) and 'the logic of knowledge' (intuitionistic logic), and he seems to suggest that both approaches can coexist. In other papers, he has a more incompatibilist attitude towards classical logic.

Metatheoretical interpretation		Philosophical background	
Semantic values	Validity		
‘assertable’, ‘refutable’	assertability preservation	Constructivism	Truth and assertability coincide
		Epistemicism	Truth and assertability do not coincide

Table 4. Interpretations of intuitionistic logic

It is also important to remark that philosophical backgrounds are often necessary in order to determine whether an interpretation is adequate or not. For example, a logic for assertability can be developed in different ways, depending on what assertability is taken to be (non-falsity, truth, etc.). We need the philosophical background to judge the adequacy of the interpretation.

In this section we have presented three senses in which a pure logic may be interpreted: you can determine the application of the logic (thus specifying the intended reading of the logical constants), you can interpret the elements of the metatheory (such as the semantic values and the property of validity), and you can characterize the relevant notions philosophically associated with validity (e.g. ‘truth’, ‘assertability’, ‘preservation’, etc.). We do not intend to have given a clear-cut distinction that can be mechanically applied. Moreover, there is a degree of holism in interpreting a logical theory, and we do not assume that the three aspects of interpretation are always independent. However, we hope that the distinction throws some light upon the debate on ‘interpretations’ and ‘philosophical interpretations’ of logics.

To finish, we should briefly mention that according to Arenhart [2021], the notion of philosophical interpretation of a logical system is often misleading and, thus, should be avoided. The author advances two main complaints, but they are both related to the epistemic interpretation of paraconsistency in Carnielli and Rodrigues [2019a], and none of them affects the claims we made so far.

The first complaint is that, according to Carnielli and Rodrigues, a philosophical interpretation seems to be something that holds fixed across all logical systems of a given kind; in particular, if the dialetheist interpretation of paraconsistency is correct, all paraconsistent logics should be interpreted in dialetheic terms, and if, alternatively, the epistemic interpretation of paraconsistency is correct, then all paraconsistent logics should be interpreted in an epistemic fashion. Such a view is implausible, for it “is in conflict with the best practices of most paraconsistent logicians” (p. 8). Now, our proposal is certainly not exposed to this objection. What we call interpretation of the metatheory is clearly something that we assign to particular logical systems, and thus our view is compatible with there being some paraconsistent logics subject to a dialetheic interpretation, and some subject to an epistemic one.

The second complaint made by Arenhart is that the talk of philosophical interpretations tends to reverse the ‘order of priorities’ that guides the practice of philosophical logic. Carnielli and Rodrigues suggest — or so Arenhart claims — that we first propose some paraconsistent system as the logic of deductive reasoning, and then we use dialetheism to ‘interpret’ paraconsistency and make our proposal plausible. According to Arenhart, the order goes the other way around: we have first some independent motivation for dialetheism, and then we argue that, since dialetheism is true, the logic of deductive reasoning must be paraconsistent. Be that as it may, this objection also leaves our proposal untouched, for we do not presuppose any particular order of priorities in the motivating data for a logical system. We do not assume that a logician first embraces a paraconsistent logic and then starts looking around for semantic theses that would convince folks to join them. We only distinguish between the different senses in which a logical system can be correlated with (interpreted in, translated to) a natural language or other suitable domain of application.

Lastly, we note that the question of whether the notion of ‘interpretation’ should or should not be used is to some extent a mere terminological issue. If someone thinks that the notion of ‘interpretation’ is somehow vitiated, then they can restrict themselves to talk about ‘applications’ of logical systems. This person, however, will have to acknowledge that there are different senses in which a logical system can be applied; to accept this is already to accept the kind of distinctions we are proposing.

3. The collapse argument

In this section we provide a brief — though quite general — explanation of how the collapse argument against logical pluralism works. The main idea behind the objection is, remember, that logic is normative, in the sense that it gives us norms for reasoning. The normativity of logic is often spelled out in terms of so-called *bridge principles*; these are claims that connect facts about entailment with epistemic norms.[7] The kind of bridge principle most common in the literature has the following form:[8] "if A entails B then [specific epistemic requirement]". Two examples:

(Wo−) If A entails B, you ought to (not believe A or not disbelieve B).[9]

(Wo+) If A entails B, you ought to (not believe A or believe B).

The premise that logic is normative amounts, in this approach, to the claim that there is at least one acceptable bridge principle. The collapse argument does not depend on which principle is the right one. Next, a reconstruction:

(1) (*Pluralism*) Let $\langle \mathcal{L}_1, \Rightarrow_1 \rangle$ and $\langle \mathcal{L}_2, \Rightarrow_2 \rangle$ be two distinct pure logics. Suppose that they both have an adequate interpretation to model validity in the vernacular.
(2) (*Normativity*) If A entails B, then [specific epistemic requirement].
(3) Let A/B be an argument in the vernacular. Let $\mathcal{L}_i(A)$ be the translation of the vernacular sentence A into $\mathcal{L}_i$. Suppose that $\mathcal{L}_1(A) \Rightarrow_1 \mathcal{L}_1(B)$ but $\mathcal{L}_2(A) \not\Rightarrow_2 \mathcal{L}_2(B)$.
(4) A entails B. [By (1), (3), and the definition of adequacy.]
(5) [Specific epistemic requirement]. [By (2) and (4).]
(6) To derive the [specific epistemic requirement], no fact about $\langle \mathcal{L}_2, \Rightarrow_2 \rangle$ was used as a premise. And
(7) A logic is normatively relevant to obtain an epistemic requirement only if some fact about this logic must be used as a premise to derive the requirement. Hence,
(8) $\langle \mathcal{L}_2, \Rightarrow_2 \rangle$ is not normatively relevant to obtain [specific epistemic requirement].

Now, suppose that $\langle \mathcal{L}_1, \Rightarrow_1 \rangle$ is stronger than $\langle \mathcal{L}_2, \Rightarrow_2 \rangle$; by this we mean that, for every vernacular argument, if the argument's $\mathcal{L}_2$-translation is $\mathcal{L}_2$-valid, then its $\mathcal{L}_1$-translation is $\mathcal{L}_1$-valid. Thus, we can run repeatedly

[7] The talk about bridge principles and also the tags below are taken from the widely cited though unpublished paper by MacFarlane [2004].

[8] For simplicity, we focus on single-premised arguments. Nothing hinges on this.

[9] It is plausible to say that Beall and Restall [2006] endorse (Wo−). The authors characterise the normativity of logic by claiming that "if an argument is valid, then you somehow go wrong if you accept the premises but reject the conclusion" (p. 16). Beall and Restall's proposal is one of the most discussed variants of logical pluralism in the literature.

the above sequence to conclude that there is no epistemic requirement for which $\langle \mathcal{L}_2, \Rightarrow_2 \rangle$ is normatively relevant. As far as normativity is concerned, there is no significant difference between saying that these two logics are both correct and saying that only $\langle \mathcal{L}_1, \Rightarrow_1 \rangle$ is. Pluralism between them 'collapses' into monism.

There have been various proposals to answer the collapse argument. Most of them rely on relativising the normatively relevant logic to some plausible parameter. Some authors choose this parameter to be the context [Caret, 2017; Kouri Kissel, 2018], some choose it to be the epistemic goal or purpose [Blake-Turner and Russell, 2021], and yet other choose it to be the domain of inquiry (physics, economics, etc.) [Hjortland, 2017]. To evaluate the success of any of these options is beyond the aim of this study. In the next section, we will analyze this argument in the context of an interpretation-based kind of logical pluralism.

4. Interpretational pluralism

For starters, we should make clear that whenever we speak of logical pluralism, we have in mind a plurality of *applied* logics, as, first, the fact that there are many pure logics is trivially true, and hence uninteresting; and second, pure logics are just mathematical objects, so, as they stand, they are not the kind of thing that can be correct or incorrect in a relevant sense. In what follows, then, 'correct logic' should be read as 'correct applied logic' but we will drop the qualification 'applied' for brevity.

As we anticipated in Section 1, interpretational pluralism (hereafter, IP) is the idea that there is more than one correct logic, and a logic is correct only if it has some adequate interpretation. We will discuss four increasingly demanding variants of this idea. Each makes the idea more precise and provides sufficient conditions for correctness.

The first variant leaves things almost as they were:

(IP–1) There is more than one correct logic; a logic is correct just in case it has some adequate metatheoretical interpretation.

So, for example, suppose that *FDE* has an adequate interpretation to model electric circuits, and linear logic an adequate interpretation to model grammatical structures. Then, (IP–1) is true.

This variant is compatible with *application pluralism*, or the idea that different logics are good or fruitful for different domains. Application pluralism is often regarded as uninteresting [Cook, 2010; Eklund, 2020;

Priest, 2006a; Steinberger, 2019], and rightly so. As a matter of fact, logical systems are applied to a variety of fields, including computer science, artificial intelligence, electrical circuits, and natural language grammar. Nowadays, nobody would deny that different logics can be good or fruitful in their respective fields of application. Logical pluralism, however, is supposed to be a contentious philosophical thesis. Thus, applications alone do not provide an interesting form of logical pluralism. The problem extends to (IP–1), since it admits cases of application pluralism. For instance, suppose that LP has an adequate interpretation as a logic for fictional contexts (such as the story 'Sylvan's Box' [Priest, 1997]), while classical logic has an adequate interpretation as the logic of the real world; thus, IP–1 is true — but this does not sound as a bold claim, since the logics involved have different domains of application.

Another way to spell out the issue is as follows. Most — if not all — variants of logical pluralism in the literature relativise the correctness of logics to some parameter (choice of truth bearers, choice or meaning of logical constants, among others). Depending on the parameter chosen, the resulting pluralist thesis is more or less interesting (or contentious, or original). The issue with (IP–1) is that:

(Triviality threat) The notion of interpretation, taken as a parameter to which we relativise the correctness of logics, does not guarantee by its own an interesting form of logical pluralism.

In the face of it, we need to look for additional conditions that a logic should satisfy to count as correct. Where to find these conditions? Well, presumably, among the features traditionally ascribed to *logic*. Beall and Restall [2006] describe three such features: normativity, necessity, and formality. Here we do not discuss them at depth (this would take us far from the scope of this paper); we just give a rough sketch of how we understand them. In a nutshell, a logic is *necessary* just in case, whenever it validates an argument, the argument necessarily preserves truth from premises to conclusion. The notion of formality is more cumbersome, for as MacFarlane [2000, p. 239] convincingly argues, it has no univocal meaning settled within the tradition. We follow the mainstream and say that a logic is *formal* just in case it is indifferent with respect to content or subject matter; this implies, at the very least, that the logic applies to any domain of discourse whatsoever (so, e.g., a logic that only works for sentences about physics is not formal). Observe that if logics are formal, the pluralism in question cannot be domain-relative: all the correct logics apply to all the domains. So, formality rules out the 'uninteresting' kind of application-based pluralism that we mentioned above. Lastly, we assume the characterisation of normativity we advanced in Sections 1 and 3: a logic is *normative* just in case it gives us — via some acceptable bridge principle — norms for reasoning. Beall and Restall take these three properties as essential for a logic to be correct — though, of course, the authors' understanding of these notions needs not be identical to ours in all respects. Then, the second variant of interpretational pluralism we consider is:

(IP–2) There is more than one correct logic; a logic is correct just in case it has some adequate metatheoretical interpretation, it is formal, necessary, and normative.

It is noteworthy that (IP–2) is different from Beall and Restall's pluralism. The authors ask each correct logic to precisify a pretheoretical notion of validity as truth preservation. In contrast, we do not presuppose that each correct logic should be understood in terms of truth-preservation; it must preserve truth, but this can be a secondary feature of the theory.

What would a witness of (IP–2) look like? Consider classical logic, hereafter $\langle \mathcal{L}_C, \Rightarrow_C \rangle$. Those who defend classical logic would normally claim that this logic is formal, for it applies to every possible content.[10] Let us grant also the plausible — though not undisputed — assumption that it captures truth-preserving arguments; it follows that it satisfies necessity. It only remains to check whether it is normative. The answer is presumably "yes", and the reason is that truth is a normative concept, at the very least in the following sense: if you *know* that some claim is true, then you should believe this claim. Thus, $\langle \mathcal{L}_C, \Rightarrow_C \rangle$ can be reasonably associated with some principle along the lines:[11]

$$\text{If } \mathcal{L}_C(A) \Rightarrow_C \mathcal{L}_C(B) \text{, you ought to (not believe } A \text{ or believe } B)$$

(where $\mathcal{L}_C(A)$ is, as before, the translation of the vernacular sentence A into $\mathcal{L}_C$). Hence, classical logic would count as a witness of (IP–2).

[10] This is not a universal position. For instance, domain-relative logical pluralists may claim that classical logic is good for some domains but defective for others. And even some logical monists who favour a non-classical system may admit that classical logic is good for some domains (e.g. decidable ones), though surely not for all.

[11] This is just an example; any other principle that could be reasonably associated with a truth-preserving logic would lead to the collapse-style objection that we outline below.

In contrast, there are many (interpreted) logics that do not qualify as witnesses of (IP–2); we give three examples. First, a paraconsistent logic applied only to understand paradoxical short stories would not count as formal, because it only applies to one domain of discourse. Second, from a classical perspective, connexive logics [see Wansing, 2020] are not truth preserving, because they have more validities than classical logic (e.g. in a connexive logic, usually $A \to B$ implies $\neg(A \to \neg B)$). Lastly, if a certain logic (e.g. *FDE*) is applied just to study some abstract mathematical structures (e.g. billatices), then the resulting theory does not seem to be normative in any interesting sense.

So, what other logics would count as a witness of (IP–2)? A natural suggestion points to intuitionistic logic, hereafter $\langle \mathcal{L}_I, \Rightarrow_I \rangle$; this system is favoured by various pluralistic proposals [see, e.g., Beall and Restall, 2006; Caret, 2021; Carnielli and Rodrigues, 2019a; Kouri Kissel, 2018]. Suppose that intuitionistic logic has an adequate interpretation; for example, it captures demonstrability-preserving arguments. There is also consensus that it is formal. Since by assumption classical logic preserves truth, and intuitionistic logic is weaker than classical logic, it follows that intuitionistic logic preserves truth as well, and thus satisfies necessity. Lastly, since — again — intuitionistic logic is weaker than classical logic, then, whatever bridge principle can be reasonably associated with classical logic, the 'same' principle can be associated with intuitionistic logic[12]; following our previous example, we have

$$\text{If } \mathcal{L}_I(A) \Rightarrow_I \mathcal{L}_I(B), \text{ you ought to (not believe } A \text{ or believe } B)$$

Hence, intuitionistic logic can be regarded as normative, and qualifies as a witness of (IP–2).

However, the above line of thought gives rise to an objection very much resembling the collapse argument:

(1) Both $\langle \mathcal{L}_C, \Rightarrow_C \rangle$ and $\langle \mathcal{L}_I, \Rightarrow_I \rangle$ are correct. (In particular, $\langle \mathcal{L}_C, \Rightarrow_C \rangle$ has an adequate interpretation as the logic of truth-preservation, and $\langle \mathcal{L}_I, \Rightarrow_I \rangle$ as the logic of demonstrability-preservation.)
(2) If $\mathcal{L}_I(A) \Rightarrow_I \mathcal{L}_I(B)$, you ought to (not believe A or believe B). [By normativity of intuitionistic validity.]

[12] We say 'the same' to simplify the exposition. Strictly speaking, we are talking about two instances of a unique schematic principle of the form: "if A/B is valid in such and such logic, then [specific epistemic requirement]".

(3) If $\mathcal{L}_C(A) \Rightarrow_C \mathcal{L}_C(B)$, you ought to (not believe A or believe B). [By normativity of classical validity]
(4) Let A/B be such that $\mathcal{L}_C(A) \Rightarrow_C \mathcal{L}_C(B)$ but $\mathcal{L}_I(A) \not\Rightarrow_I \mathcal{L}_I(B)$
(5) You ought to (not believe A or believe B) [By (3) and (4).]
(6) To derive that you ought to (not believe A or believe B), no fact about $\langle \mathcal{L}_I, \Rightarrow_I \rangle$ was used as a premise. And
(7) A logic is normatively relevant to obtain an epistemic requirement only if some fact about this logic must be used as a premise to derive the requirement. Hence,
(8) $\langle \mathcal{L}_I, \Rightarrow_I \rangle$ is not normatively relevant to obtain that you ought to (not believe A or believe B)

We can generalise the argument just as before, to conclude that there is no case in which $\langle \mathcal{L}_I, \Rightarrow_I \rangle$ is normatively relevant for some epistemic requirement of the form "You ought to (not believe A or believe B)".

This collapse-style problem does not only affect classic and intuitionistic logic. Consider any two logics such that one is stronger than the other. If we can reasonably associate the stronger logic with some bridge principle P, then we can also reasonably associate the weaker logic with the 'same' bridge principle — the reason is that, if P is acceptable for some set of arguments, it must be also acceptable for any subset thereof. But if we associate both logics with the same bridge principle, then only one of them is normatively relevant. A succinct formulation:

(COLLAPSE THREAT) Consider two logics such that one is stronger than the other. Whatever the intended interpretation of the weaker logic is, if the stronger logic *is* normative, then the weaker might become normatively irrelevant.

The defendant of (IP–2) has a *prima facie* plausible answer at their disposal. They can argue that we did not associate intuitionistic logic with the adequate kind of bridge principle. Granted, if some principle can be reasonably associated with classical logic, then it can also be reasonably associated with intuitionistic logic — for the simple reason that the latter is weaker than the former. However, it does not follow that the principle in question is appropriate to capture the sense in which intuitionistic logic is normative. Indeed — the argument goes — intuitionistic logic is meant to regulate our beliefs about demonstrability, and hence its normativity should be spelled out in terms of some principle as

> If $\mathcal{L}_I(A) \Rightarrow_I \mathcal{L}_I(B)$, you ought to (not believe that A is demonstrable, or believe that B is demonstrable).

No such principle is available to classical logic, because classical logic validates the schema $\neg\neg A/A$, but — according to the pluralist — the fact that $\neg\neg A$ is demonstrable does not guarantee that A is demonstrable as well. Hence, suppose that intuitionistic logic (this time associated with the new bridge principle) and classical logic are the witnesses of (IP–2). If we try to run the collapse argument, we fail: there are many epistemic norms of the form "You ought to (not believe that A is demonstrable or believe that B is demonstrable)" for which we will not be able to conclude that intuitionistic logic is normatively irrelevant.

Though this solution has some intuitive appeal, we do not find it entirely satisfying. The main problem is that the agent ends up with something similar to a Moorean paradox: they will believe the conclusion, but at the same time they will not believe that it is demonstrable. This is certainly not in the spirit of most intuitionistic logicians (according to whom provability *is* truth). Even for the most pluralistic intuitionistic logicians, this is unsatisfying: which is the normativity of their notion of proof? It seems that it becomes almost irrelevant for our actual belief management.[13]

In response, the defendant of (IP–2) can give up intuitionistic logic and look for another candidate for their pluralism. One promising option is Kleene's weak three-valued logic *WK*, or equivalently $\langle \mathcal{L}_{WK}, \Rightarrow_{WK} \rangle$. This logic has an interpretation that has been considered adequate to characterise arguments with consistent premises where the content of the conclusion is included in the content of the premises [Ciuni and Carrara, 2019]. *WK* can be regarded as formal, for its intended application is not usually restricted to any particular domain of discourse. Besides, *WK* is weaker than classical logic, so, under the assumption that the latter preserves truth, the former preserves truth as well, and thus satisfies necessity. Lastly, it can be regarded as normative, for it can be taken to regulate our beliefs about content inclusion in arguments, and thus it can be reasonably associated with some principle such as

> If $\mathcal{L}_{WK}(A) \Rightarrow_{WK} \mathcal{L}_{WK}(B)$, and A is consistent, you ought to believe that the content of B is included in the content of A.

No such principle is available to classical logic, because classical logic validates the schema $A/A \vee B$, but — according to the pluralist — this

[13] A similar problem might affect other logics of truth-connected concepts such as evidence or assertability. However, the precise scope of this objection is yet to be determined.

schema does not guarantee inclusion of content. Hence, if we take *WK* and classical logic as our witnesses of (IP–2), and we try to run the collapse argument, we fail again: there will be many epistemic norms of the form "You ought to believe that the content of B is included in the content of A" for which we will not be able to conclude that *WK* is normatively irrelevant.

This line of argument certainly avoids the Moorean paradox mentioned above: there is nothing awkward in believing the conclusion of an argument, and not believing that the content of the conclusion is included in the content of the premises. However, the proposal still faces a problem. When we say that a logic gives us norms for reasoning, we usually assume that the norms at issue concern reasoning *with the very constituents* of the arguments that we assess in the logic (e.g. (Wo+) tells that you ought not to believe the premises of a valid argument and not believe its conclusion). Here, however, *WK* only regulates our reasoning *about a certain property* of these constituents (more precisely, the property of preserving content). Hence, even if *WK* satisfies the normativity constraint that (IP–2) imposes, it does so in a quite weak way. The problem is more general, and it reminds us of the triviality threat that affected (IP–1). Consider any logic applied to a certain domain D; we can always say that the logic is normative over our commitments in D; but in many cases, this falls short of our intuitive understanding of what the normativity of logic is. For another example, suppose you apply classical propositional calculus to model electrical circuits. You can say, of course, that the resulting theory is normative over certain beliefs of yours (e.g. that circuits can be in such and such states), but the normativity involved does not seem to be what we intuitively call the normativity of *logic*.

To ensure a more substantive form of normativity, we move on to another variant of interpretational pluralism:

(IP–3) There is more than one correct logic; a logic is correct just in case it has some adequate metatheoretical interpretation, it is formal, it is necessary, and it is normative for some epistemically relevant attitudes directed towards the constituents of the arguments assessed.

With this reformulation, we avoid weaker kinds of normativity, such as those involved in the proposals above. Let us take classical logic as one plausible witness of (IP–3). Is there another one? One promising candidate is *grounding* logic. Sometimes it has been proposed as a logic for non-causal explanation [e.g. Correia, 2014]. Let $\langle \mathcal{L}_G, \Rightarrow_G \rangle$ be the grounding logic, and A/B a vernacular argument. The idea is that if $\mathcal{L}_G(A) \Rightarrow_G \mathcal{L}_G(B)$, then A explains B. Grounding logic can be regarded as formal, for its application as the logic of non-causal explanation is not restricted to any particular domain of discourse. Besides, it is weaker than classical logic, so, under the assumption that the latter preserves truth, the former preserves truth as well. Lastly, grounding logic can be regarded as normative, for we can plausibly associate it with some bridge principle along these lines:

If $\mathcal{L}_G(A) \Rightarrow_G \mathcal{L}_G(B)$, you are allowed to explain B using A.

Notice that *explaining* can be plausibly regarded as an epistemically relevant attitude, and it is directed towards the very constituents of the arguments assessed in the logic. No such principle is available to classical logic, because, for example, classical logic validates the schema $\neg\neg A/A$, but — according to the pluralist — the truth of $\neg\neg A$ cannot in general explain A. Hence, if we assume that classical and grounding logic are the witnesses of (IP–3), and we try to run the collapse argument, we fail yet again: there are many epistemic norms of the form "You are allowed to explain B using A" for which we will not be able to conclude that the grounding logic is normatively irrelevant.

We think that (IP–3) constitutes a sufficiently demanding, and thus philosophically interesting, form of logical pluralism.[14] Now, does (IP–3) capture the intuitive understanding of the normativity of logic? We do not commit to a positive answer. Some people may think that (IP–3) is too liberal, and that a logic is normative only if it regulates certain *specific* (or 'privileged') attitudes, such as belief and acceptance. Hence, we move to the last and most strict variant of interpretational pluralism we consider in this paper:

[14] To be clear, the need for more demanding versions of pluralism is not based on the failure of adequacy. As we mentioned above, determining the adequacy of an interpretation is as difficult as determining the correctness of a philosophical theory. But for what matters, grounding logic is a good candidate for a logic of explanation, and *WK* is a good candidate for a logic of content inclusion (also, as we will argue later, probability logic is a good candidate for a logic of acceptance). We don't argue explicitly in favor of those specific facts, for we think this is out of the scope of this paper. The corresponding versions of logical pluralism are perfectly reasonable. We are just concerned about the weakness of those versions of logical pluralism from a normative point of view.

(IP–4) There is more than one correct logic; a logic is correct just in case it has some adequate metatheoretical interpretation, it is formal, it is necessary, and it is normative for our belief in, or acceptance of, the constituents of the arguments assessed.

Now, can (IP–4) still be achieved? We do not have a definite answer to this. We provide a suggestion here. Suppose that some probabilistic logic (where Adjunction fails) is the logic for evidence. Now, which will be the normativity of this logic? One may use the following rather common-sensical principle of evidential permission:

(Permission) If there is evidence in favour of A, you might accept A.

In light of this approach, it might be rational to accept A_1, A_2, $\ldots$, A_n but reject $A_1 \wedge A_2 \wedge \cdots \wedge A_n$. Indeed, in cases such as the Preface paradox, one might accept A_1, A_2, $\ldots$, A_n, and $\neg(A_1 \wedge A_2 \wedge \cdots \wedge A_n)$. This is not inconsistent in some probabilistic logics. Can this be compatible with classical logic? Some authors have taken this route. For example, Stalnaker [1984] distinguishes between acceptance and belief. One might accept a contradictory set of sentences, but one cannot believe it. Therefore, there might be a logic for belief (e.g. classical logic) and a logic for acceptance (e.g. probabilistic logic). This can count as a case of (IP–4) under our perspective.

Consider whether the position is vulnerable to the Collapse Argument. Suppose that you accept and believe A_1, A_2, $\ldots$, A_n but you do not know whether to accept or believe $A_1 \wedge A_2 \wedge \cdots \wedge A_n$. Classical logic tells you that you cannot believe the conjuncts and disbelieve the conjunction; while probabilistic logic tells you that accepting the conjuncts and the negation of the conjunction is rational. The solution now is straightforward: you do not believe the negated conjunction, but you accept it. This requires a clear distinction between belief (involuntary, consistent, truth-aimed) and acceptance (voluntary, research-aimed). This case shows a possible application of an interpretational logical pluralism in our sense. Moreover, it involves only normative concepts related to intentional attitudes. The logic for acceptance is not just an analysis of the argument, as it has direct consequences for what to do.

It is hard to know if this solution can be extended to other cases. Moreover, many philosophers would reject the idea that acceptance and belief can go in different directions; for example, they claim that both notions aim at truth. Accepting something and believing the contrary may seem inadequate. Following this idea, (IP–4) would be less promising.

However, other authors think that acceptance has a pragmatic component, and accepting something that one takes as strictly speaking false is not necessarily incorrect, as it happens with idealizations [see, e.g., Stalnaker, 1984]. To conclude, looking at the previous examples (with grounding, explanation, content inclusion, etc.), we think that trying to find epistemic notions which do not *only* depend on truth is one of the main ways of achieving an interesting version of logical pluralism.

A final remark. All variants of IP considered in the paper are compatible with a kind of pluralism about logical norms. Indeed, the examples we propose for (IP–2)–(IP–4) all rely on the idea that different logics can be associated with bridge principles regulating different attitudes (e.g. attribution of content-inclusion in the case of *WK*, explanation in the case of grounding logic). It is also worth mentioning that there has been an interesting debate lately on whether the notion of validity is normative on its own — in which case bridge principles capture its normativity — or only in a derivative way — in which case bridge principles follow from facts about validity together with general epistemic principles for belief [see, e.g., Field, 2015; Russell, 2020; Tajer, 2020]. All variants of IP remain neutral in this debate. As we stated above, the assumption that logic is normative, in our sense, amounts to the claim that for each correct logic there is at least some acceptable bridge principle — it does not matter what the particular grounds are for accepting the principle in question.

5. Conclusions

In this paper, we have studied the prospect of a logical pluralism based on interpretations of logical systems. First, we analysed the very notion of interpretation, taking some examples from the history of logic and distinguishing three senses in which a logical system can be interpreted. We then considered four variants of an interpretation-based pluralism. According to our view, the position needs to find a balance between a trivial pluralism, where the range of "correct logics" includes systems with non-normative interpretations, and a more substantial pluralism, where two normative interpretations can work at the same time. The problem with this latter approach is that most normative concepts are interconnected (for example, provability is naturally connected to truth), leading the position to a possible reformulation of the Collapse Argument. We hope that this paper has helped to clarify the role that philosophical interpretations can have in the discussion about logical pluralism and to point out some of the problems that every interpretation-based logical pluralism must solve.

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