



Numerical study of beam shape adaptation by anisotropic disk covering transducer or metamaterial

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Abstract – Metamaterials are intensely explored for their capabilities to modify sound beams. In addition to frequency filtering, acoustic lenses offer intriguing possibilities for shaping sound beams. For the time being, the versatility of metamaterials remains limitless. In beam-shape adaptation, however, their complexity suggests that manufacturers of transducers could benefit from combining metamaterials with more conventional materials. This paper investigates the transmission of a circumscribed beam through a stratum of anisotropic material to examine the change in beam shape after transmission. The incident sound is presumed to originate from a conventional transducer, possibly coated with a metamaterial to modify the sound field, before being transmitted through the anisotropic layer. Different incident beam shapes, such as conical-like, Gaussian, and pillar beams, are investigated. While the results are not exhaustive, they demonstrate the beam shape's adaptability.

Keywords: Beam shape adaptation, Gaussian beams, Pillar beams, Conical-like beams, Fiber-reinforced composites, Metamaterials

1 Introduction

Recently, it has been demonstrated that metasurfaces are efficient and compact structures for designing arbitrary wavefronts. Metasurfaces are planar metamaterials with a subwavelength thickness that enables wavefront sculpting by introducing gradients in the spatial wave response of these flat structures. Impedance-based holographic acoustic lenses have been developed [1] to transform the sound output from a transducer into the desired field for medical applications [2]. Bessel beams, known for their diffraction resilience, can be formed equally by a metamaterial placed before a transducer [3]. The focus can be adjusted by shifting one material to another, covering the transducer surface [4]. Acoustic gradient-index (GRIN) metasurfaces, engineered from soft graded-porous silicone rubber, also permit beam steering and focusing [5]. Acoustic metamaterials are also developed for making a Mikaelian lens [6] or coding acoustic waves by sending them through the metamaterial before transmission [7]. Certain metamaterials achieve anisotropic behavior but not in the homogenized sense of actual crystals or fiber-reinforced composites with constituents way below the involved acoustic wavelengths [8]. Embedded in-plate applications also exist in which metamaterials steer or focus beams [9].

The versatility of metamaterials remains limitless for the time being. In beam-shape modifications, however, the degree of complexity implies that transducer manufacturers might gain from merging metamaterials with traditional substances, if possible. Those substances can be crystals or fiber-reinforced composites, anisotropic but homogeneous for the involved sound waves. Emitting sound through such conventional materials can further adjust beams without sophistication, and, therefore, find applications in the above-mentioned topics of beam-shape, steering and focus adjustments in the medical field.

It is known that sound transmits through crystals in a directional-dependent manner. However, the effect on beam shape, merely due to anisotropy and not due to impedance inhomogeneity, usually is not considered. Here, we investigate the transmission of acoustic beams perpendicularly incident on an anisotropic plate. The 3D beams are modeled by a plane wave expansion based on a Fourier series approach whereby each constituting plane wave interacts with the plate under consideration.

That sound is sensitive to the anisotropic linear stiffness of composites and has been widely exploited in the past. Indeed, ultrasonic investigation of fiber-reinforced composites' stiffnesses is based on straightforward principles involving Snell's law, Newton's laws, Hooke's law, and the continuity of mechanical strains and stresses at the involved interfaces between composite layers or the surrounding

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water. Linear stiffness determination is practically achieved by exploiting direction dependence in experiments. It is, therefore, somewhat different from damage detection using C-scans or guided waves [10–13] or non-linear acoustics [14]. Breakthroughs came from the team of Dale Chimenti [15–20] and others [21–23]. A direct method to probe the stiffness of composites is based on multiple-angle incidence, done in the form of a transducer rotation along a sphere [24] or through focused sound with the help of phased array technology [25, 26] or otherwise [19, 27–29]. A significant challenge is to work on an air-coupled system [30, 31], which is not addressed in the current paper. In the last decades, the extensive use of fiber-reinforced composites forced researchers to develop novel and practical techniques to realize nondestructive testing (NDT) on those materials. Such evolution is continuing today [32–35].

Understanding sound transmission through an anisotropic layer and how an acoustic beam deforms by it can be done in the same framework as studies aimed at nondestructive testing. It has been shown before by Rehman et al. [36] that the reflected beam pattern may reveal properties of composites. They showed that reflected Gaussian beams are only sensitive to material properties if oblique incidence is considered. The current work shows that the oblique incidence is not a requirement. Transmitted beams, for normal incidence, are sensitive to composites' internal structure.

Even though the current work reveals a proof of concept and is numerical, it is essential to note what type of measurements could be performed by experimentalists for further investigations. One may, for instance, apply a needle transducer [37] as a receiver and perform a point-by-point scan of the transmitted sound beam. The needle may be piezoelectric or a metal-coated fiber tip paired with a heterodyne interferometer that senses an optical path alteration caused by the fiber tip's oscillation in the sound field [38]. Alternatively, a single-mode fiber-optic probe hydrophone (FOPH) can be used for local sound measurements [39], or Acousto-optic diffraction can also be applied to measure a sound field [40]. The latter typically comprises a global measurement of the entire beam and is insufficiently resolved for point-by-point information. Tomographic variations of acousto-optic Bragg diffraction [41–47] can approach an adaptation of acousto-optics to point-by-point measurements [48, 49] and Schlieren imaging [50, 51–56].

2 Incident beam descriptions

One may hardly imagine what sort of beams will be manufactured in future developments in metamaterials. As proof of concept, three types of 3D beams are assumed in this research: *Gaussian beams*, *pillar beams*, and *conical-like beams*.

Pillar beams look like a filled cylinders. The conical-like beams studied here have beam flanks showing exponential decay. There is a particular reason for this choice. The beauty of conical-like beams is that they possess properties similar to inhomogeneous waves. It is well known [57] that the propagation features of inhomogeneous waves in

crystals depend not only on the direction of propagation but also on the value of the inhomogeneity vector. Therefore, whereas classical homogeneous plane waves cannot reflect composites' internal symmetry for normal incidence, inhomogeneous waves are susceptible to symmetry because rotating the inhomogeneity vector (for normal incidence) must result in different propagation properties. A conical-like beam contains inhomogeneity properties for each radial direction and may reflect the fundamental in-plane symmetry at once; therefore, it is worthwhile considering.

A Gaussian beam $G(x, y, z)$, is characterized by its profile at $z = 0$ as

$$G(x, y, 0) = \exp \left[- \left[\frac{\sqrt{x^2 + y^2}}{W} \right]^2 \right], \quad (1)$$

whereas a conical-like beam $C(x, y, z)$ by

$$C(x, y, 0) = \exp \left(-\beta \left| \sqrt{x^2 + y^2} \right| \right) \quad (2)$$

and a pillar beam $P(x, y, z)$ by

$$P(x, y, 0) = \exp \left[- \left[\frac{\sqrt{x^2 + y^2}}{W} \right]^{64} \right]. \quad (3)$$

In order to compare the different beams, we enforce the same amount of energy, whence

$$\begin{aligned} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} G^2 dx dy &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} C^2 dx dy \\ &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} P^2 dx dy. \end{aligned} \quad (4)$$

Because of the symmetry of the considered profiles, i.e., each profile $K(x, y, 0) = K(R)$ with $R = \sqrt{x^2 + y^2}$ the radius, equation (4) are equivalent to

$$\int_0^{+\infty} G^2 dR = \int_0^{+\infty} C^2 dR = \int_0^{+\infty} P^2 dR \simeq B. \quad (5)$$

Therefore

$$W = \frac{\sqrt{2}}{\sqrt{\pi\beta}} \quad \text{and} \quad B = \frac{\sqrt{2\pi}}{4} W. \quad (6)$$

Modeling ultrasound beams is done by Fourier analysis, where each beam is a sum of plane waves. Therefore, the transmitted beam is the sum of all transmitted plane waves originating from the plane waves constituting the perpendicular incident beam. The interaction of plane waves with an anisotropic layer is discussed in Section 3.

3 Sound transmission

Coupling a transducer, with or without a metamaterial in between, to an anisotropic disk could be done in different ways. The current paper merely focuses on the effect caused by the anisotropic material and assumes liquid coupling. It is also assumed that the transmitted beam arises on the

opposite side of the disk in a liquid. The liquid in both cases is water, with a speed of sound of 1480 m/s and a density of 1000 kg/m³. We also assume that the disk is large enough so the reflections from its edges do not occur, an assumption that makes sense in the case of perpendicular incident bounded beams. The material is considered orthotropic, typical for fiber-reinforced composites.

The interaction of sound with an anisotropic layer is well-described in the literature [58–61]. A summary follows.

3.1 Material description

With Einstein's double suffix notation convention, the dynamics of an anisotropic material is described [20, 62] by

$$\frac{\partial \sigma_{ij}}{\partial r_j} = \rho \frac{\partial^2 u_i}{\partial t^2}. \quad (7)$$

With $i, j = 1, 2, 3$, and σ_{ij} being the stress tensor, and \mathbf{u} the particle displacement. \mathbf{r} is the position and t is time.

Hooke's law, is given by

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix} \times \begin{bmatrix} e_{11} \\ e_{22} \\ e_{33} \\ 2e_{23} \\ 2e_{13} \\ 2e_{12} \end{bmatrix}. \quad (8)$$

Orthotropy considerations [20, 62] result in

$$\begin{aligned} C_{14} = C_{24} = C_{34} = C_{15} = C_{25} = C_{35} = C_{16} = C_{26} \\ = C_{36} = C_{45} = C_{46} = C_{56} = 0. \end{aligned} \quad (9)$$

The stiffness constants of orthotropic materials are related to the Engineering constants, with Young's moduli E_{11} , E_{22} and E_{33} , the Poisson coefficients ν_{23} , ν_{13} and ν_{12} , and the shear moduli G_{23} , G_{13} and G_{12} . The connection is

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} = \begin{bmatrix} 1/E_{11} & -\nu_{12}/E_{11} & -\nu_{13}/E_{11} & 0 & 0 & 0 \\ -\nu_{21}/E_{22} & 1/E_{22} & -\nu_{23}/E_{22} & 0 & 0 & 0 \\ -\nu_{31}/E_{33} & -\nu_{32}/E_{33} & 1/E_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{23} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{31} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{12} \end{bmatrix}^{-1} \quad (10)$$

Table 1. Properties of a Carbon/Epoxy unidirectional fiber-reinforced composites plate assumed as the material used for the 3 mm disk through which each sound beam passes.

Parameter	Value
ρ [kg/m ³]	1525
E_{11} [MPa]	$119\,130 \times (1 - 0.0025i)$
E_{22} [MPa]	$8850 \times (1 - 0.03i)$
E_{33} [MPa]	$10\,000 \times (1 - 0.03i)$
ν_{23}	$0.475 \times (1 - 0.015i)$
ν_{13}	$0.275 \times (1 - 0.01i)$
ν_{12}	$0.306 \times (1 - 0.01i)$
G_{23} [MPa]	$3000 \times (1 - 0.05i)$
G_{13} [MPa]	$5000 \times (1 - 0.02i)$
G_{12} [MPa]	$5500 \times (1 - 0.03i)$

with

$$\nu_{32} = \nu_{23} \frac{E_{33}}{E_{22}}, \quad (11)$$

$$\nu_{31} = \nu_{13} \frac{E_{33}}{E_{11}}, \quad (12)$$

$$\nu_{21} = \nu_{12} \frac{E_{22}}{E_{11}}, \quad (13)$$

and

$$G_{ij} = G_{ji}. \quad (14)$$

The reverse Voigt procedure transforms the stiffness tensor C_{mn} of rank 2 to the stiffness tensor c_{ijkl} of rank 4 as $(1 \rightarrow 11)$, $(2 \rightarrow 22)$, $(3 \rightarrow 33)$, $(4 \rightarrow 23 = 32)$, $(5 \rightarrow 13 = 31)$ and $(6 \rightarrow 12 = 21)$, after which we can conveniently rotate the composite as needed for the different plane waves constituting a bounded beam. A rotation R transforms the intrinsic stiffness constants $c_{ijkl} = c_{ijkl}^I$ into stiffness constants $c_{ijkl} = c_{ijkl}^R$ in coordinates corresponding to a rotated (laboratory) system as follows:

$$c_{ijkl}^R = R_{im} R_{jn} R_{kp} R_{lq} c_{mnpq}^I, \quad (15)$$

where R_{ij} are the entries of the rotation matrix for a rotation from the intrinsic lattice coordinate system to the laboratory coordinate system.

Equation (7) then becomes

$$\rho \frac{\partial^2 u_i}{\partial t^2} = c_{ijkl} \frac{\partial^2 u_l}{\partial x_j \partial x_k}. \quad (16)$$

A plane wave solution of (16) is of the form

$$u_i = U_i \exp i[\mathbf{n}_j r_j - \omega t], \quad (17)$$

where \mathbf{n} is the wave vector. If this is entered in (16), straightforward calculations result in

$$\left[\frac{1}{\rho} c_{ijkl} n_k n_j - \omega^2 \delta_{il} \right] U_l = 0. \quad (18)$$

Equation (18) is called the Christoffel equation [20, 62]. It relates the slowness \mathbf{n}/ω and the polarization \mathbf{U} to the

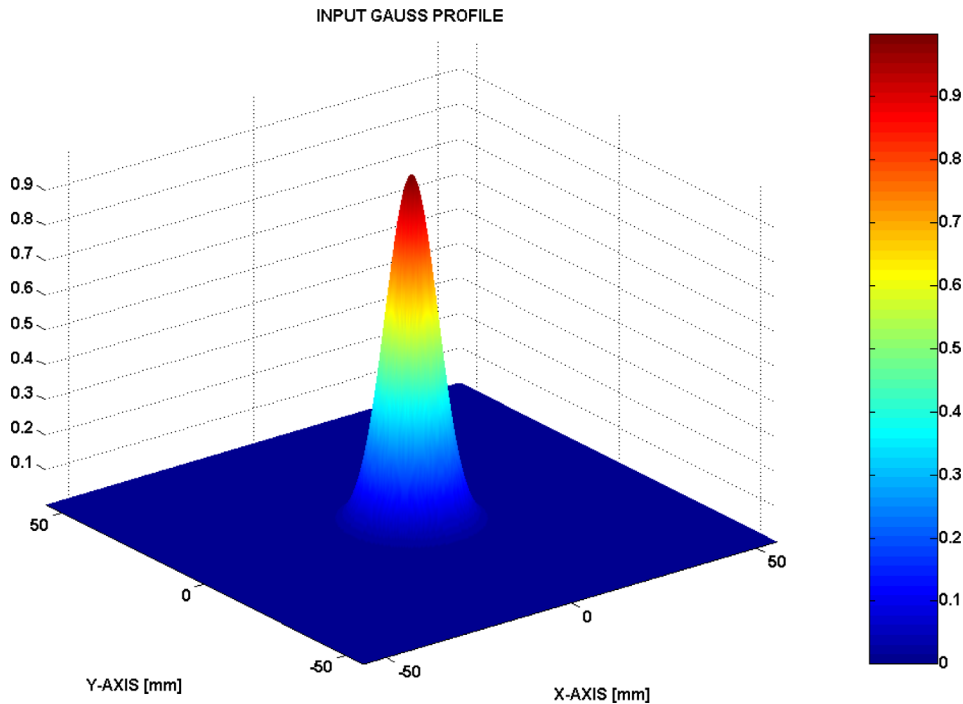


Fig. 1. Profile of an incident *Gaussian beam*, having the same amount of energy as a *conical-like beam*, shown in Figure 2, for $\beta = 100/\text{m}$.

propagation direction and is solved by assuming nontrivial solutions, followed by the determination of the corresponding eigenvectors.

3.2 Plane wave interactions with interfaces

Apart from the above determined Christoffel's equation which is comparable to a dispersion relation for isotropic media, we must account for Snell's law to determine the wave vector components along the interface, as well as the continuity condition.

3.2.a Snell's law

If sound inside the bulk of the composite laminate results from impinging plane waves (denoted by superscript "inc"), Snell's law for interfaces perpendicular to n_3 , states that

$$n_1 = n_1^{\text{inc}} \quad \text{and} \quad n_2 = n_2^{\text{inc}}. \quad (19)$$

Then, requiring nontrivial solutions, (18) leads to a sixth-degree polynomial equation of the form

$$(n_3)^6 + B_5(n_3)^5 + A_4(n_3)^4 + B_3(n_3)^3 + A_2(n_3)^2 + B_1(n_3)^1 + A_0 = 0. \quad (20)$$

Furthermore, symmetry higher than or equal to monoclinic symmetry results in

$$B_j = 0, \quad (21)$$

whence three independent solutions for n_3 are found.

3.2.b Continuity of normal stress and displacement

For a plate, continuity of normal stress and normal displacement is required along the water-solid interface,

$$u_3^{\text{water}} = u_3^{\text{solid}} \quad (22)$$

and

$$\sigma_{i3}^{\text{water}} = \sigma_{i3}^{\text{solid}}, \quad i = 1, 2, 3. \quad (23)$$

4 Numerical results

Each of the considered beams, i.e. *Gaussian*, *pillar* and *conical-like*, is decomposed into many harmonic homogeneous plane waves by a 2D Fast Fourier Transform (FFT). The FFT is taken on the interval $[-7W, 7W]^2$ on 2^{2N} equally distributed samples, for N the smallest positive integer for which

$$2^{2N} \geq 168/W \quad (24)$$

and W is expressed in [m]. The number of samples equals the number of waves in the plane wave decomposition of the considered beam, and the considered interval is sufficiently extended to ensure reliable simulations.

This paper's numerical simulations are based on calculating the transmission coefficient for every plane wave constituting the incident beam.

The considered composite has a thickness of 3 mm and consists of one layer of unidirectional material – carbon fibers in epoxy. The elastic properties of such material are

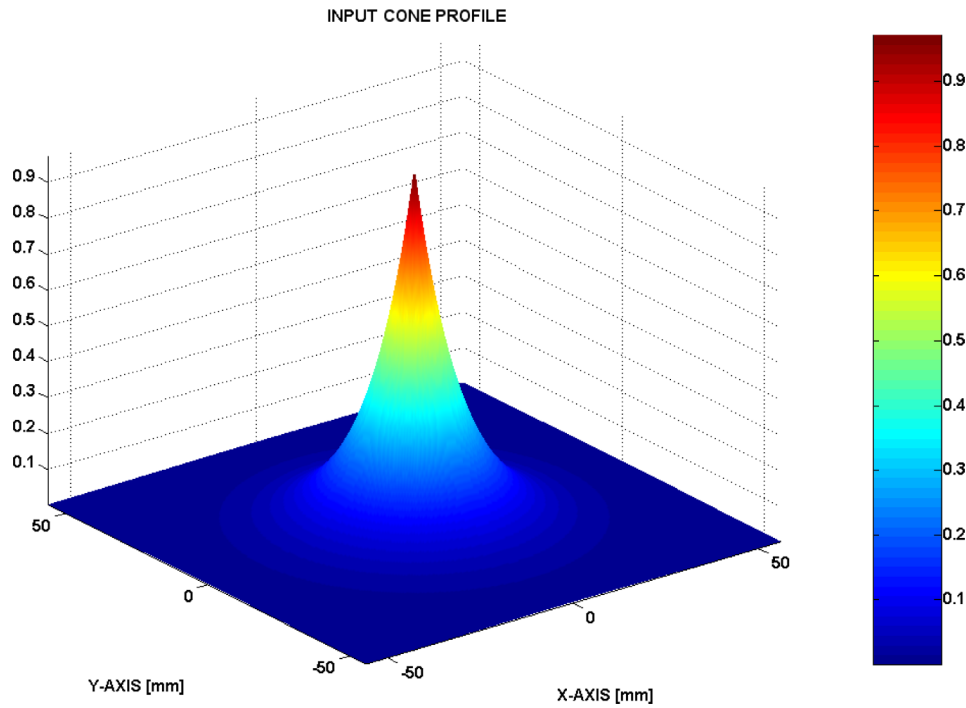


Fig. 2. Profile of a *conical-like beam* for $\beta = 100/\text{m}$. This beam shows remarkable resemblance with an inhomogeneous wave. Its amplitude diminishes exponentially from the center of the beam.

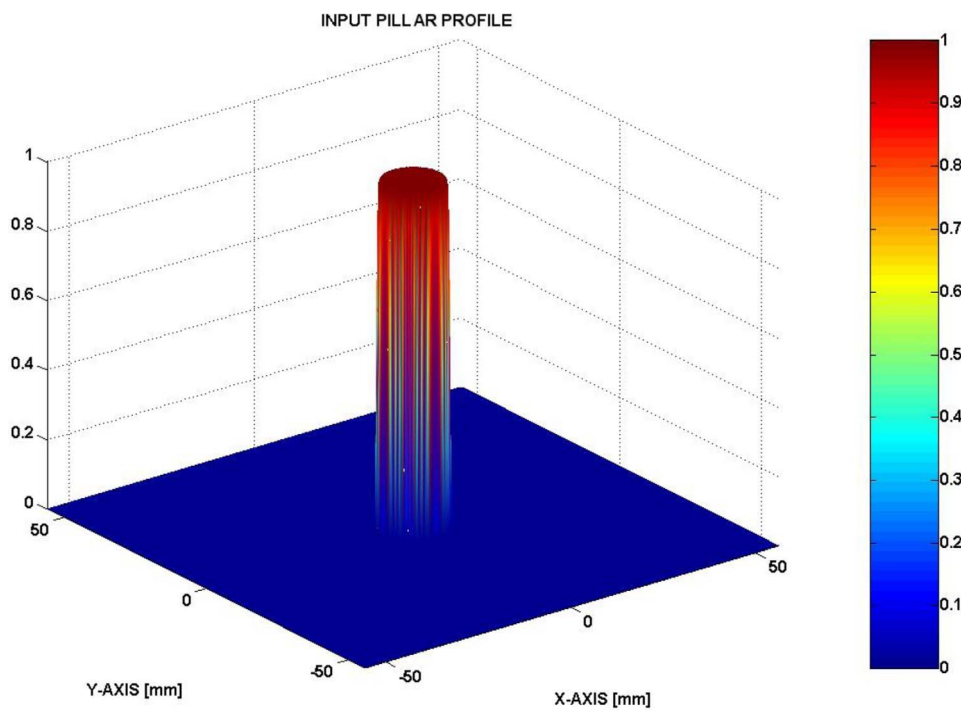


Fig. 3. Profile of an incident *pillar beam*, having the same amount of energy as a *conical-like beam*, shown in [Figure 2](#), for $\beta = 100/\text{m}$.

given in [Table 1](#). The fibers are directed in the XY -plane along the X -direction. Therefore, the material is stiffer along the X -direction than the Y -direction [10].

[Figure 1](#) shows the profile of an incident *Gaussian beam*, having the same amount of energy as a *conical-like beam*,

shown in [Figure 2](#), for $\beta = 100/\text{m}$. The comparative *pillar beam* is given in [Figure 3](#). We only discuss transmitted beam profiles because that is this paper's purpose.

[Figure 4](#) shows the transmitted beam profile for an incident *Gaussian beam* with the same amount of energy as a

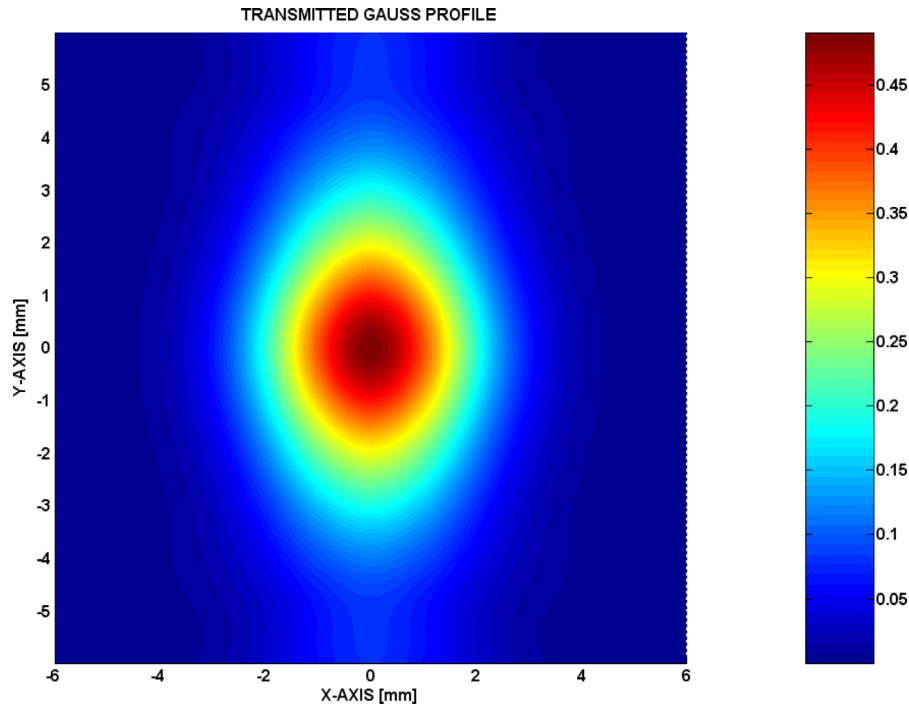


Fig. 4. Transmitted beam profile for an incident *Gaussian beam* with the same energy as a *conical-like beam* for $\beta = 400/\text{m}$. Note that the profile along the *X*-direction differs from the one along the *Y*-direction. The anisotropic disk does not significantly alter the *Gaussian beam* in any case.

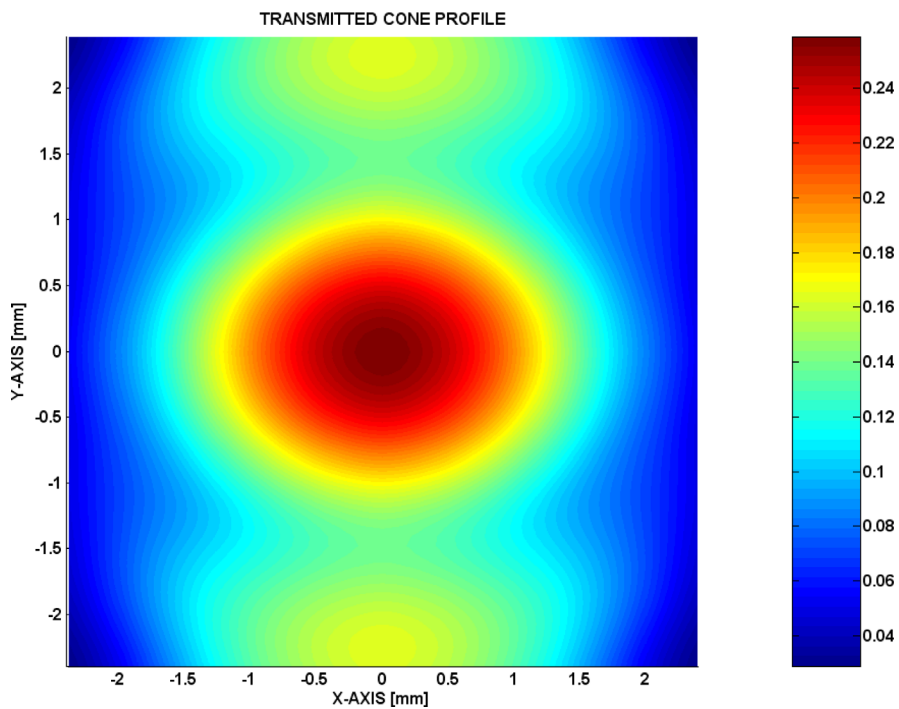


Fig. 5. Transmitted beam profile for a normal incident *conical-like beam* $\beta = 1000/\text{m}$. There is a significant difference between the profile along the *X*-direction and the *Y*-direction.

conical-like beam for $\beta = 400/\text{m}$. Note that the profile along the *X*-direction differs from the one along the *Y*-direction. A similar effect, caused by the anisotropic disk, was found for the reflected beam, though less outspoken and less useful

for the applications the topic of this paper aims at. The *Gaussian beam* is not very much influenced by the anisotropic disk, though comparing results for the other profiles are appealing.

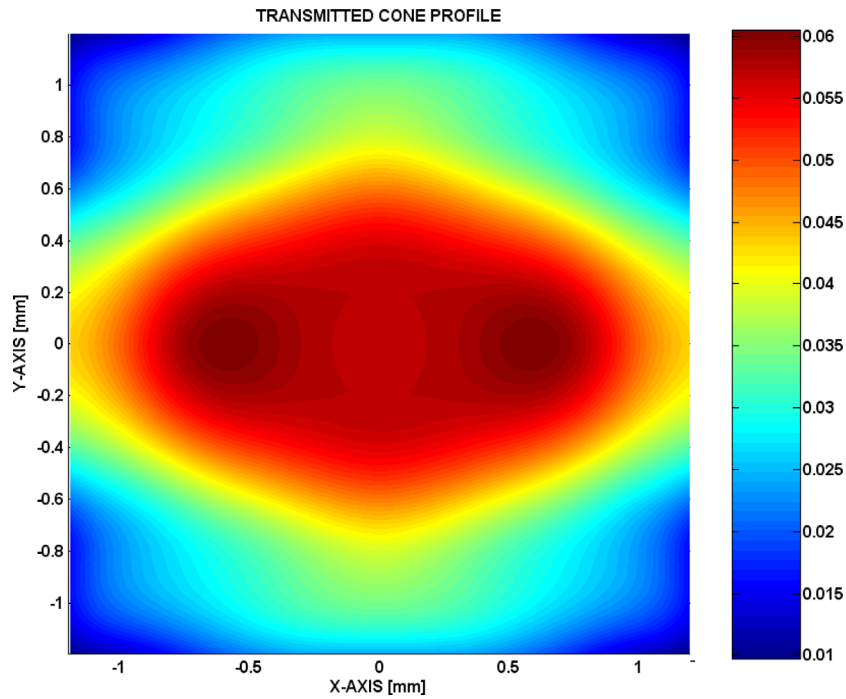


Fig. 6. Transmitted beam profile for a normal incident *conical-like beam* $\beta = 2000/\text{m}$. The pattern is much different from the one in Figure 5.

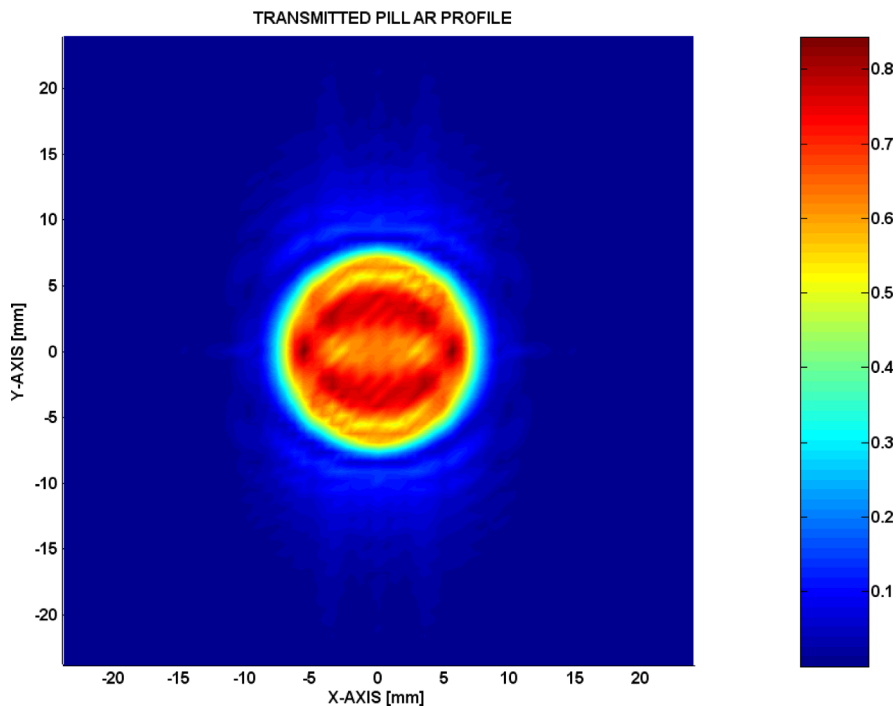


Fig. 7. Transmitted beam profile for an incident *pillar beam* having the same amount of energy as a *conical-like beam* for $\beta = 100/\text{m}$. The transmitted beam pattern is direction dependent for this type of beam, even for a relatively large diameter.

Figure 5 depicts the transmitted beam profile for a perpendicular incident *conical-like beam* $\beta = 100/\text{m}$. A significant difference exists between the profile along the X-direction and the Y-direction. Numerous simulations

show that the *conical-like beam* profile is always more sensitive than the *Gaussian beam*. The reason must be that *conical-like beams* have an exponentially decaying amplitude, similar to inhomogeneous waves [63]. Indeed, the

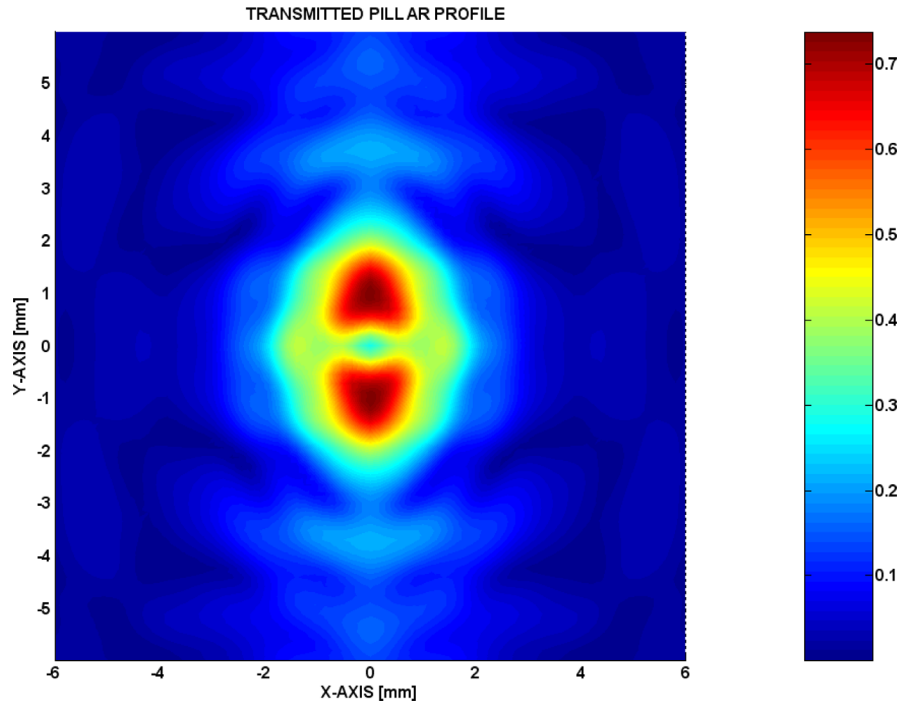


Fig. 8. Transmitted beam profile for an incident *pillar beam* with the same energy as a *conical-like beam* for $\beta = 400/\text{m}$. The pattern is more outspoken than in [Figure 7](#). The difference with [Figure 4](#) is significant.

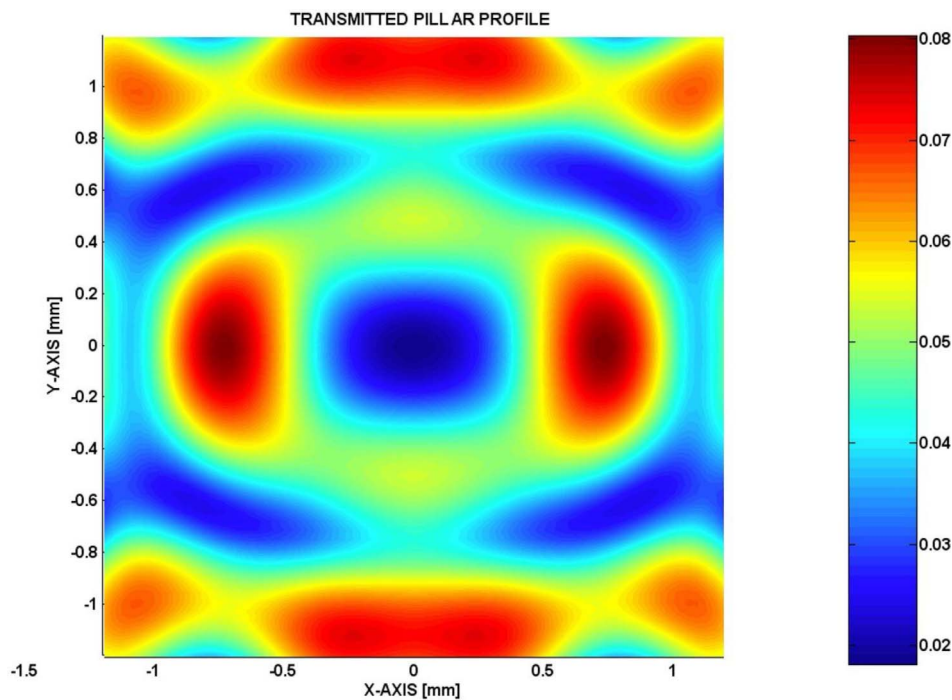


Fig. 9. Transmitted beam profile for an incident *pillar beam* having the same amount of energy as a *conical-like beam* for $\beta = 2000/\text{m}$. The transmitted beam profile has a complicated shape and depends on the direction. The difference with [Figure 6](#) is significant.

scattering and propagation of inhomogeneous waves depend also on the inhomogeneity vector's direction relative to the fiber direction [14].

If we increase the inhomogeneity β , as in [Figure 6](#), where the transmitted beam profile for a perpendicularly incident

conical-like beam, $\beta = 2000/\text{m}$, is given, we see that the pattern is much different from the one in [Figure 5](#). This is because the scattering and propagation properties of inhomogeneous waves also depend on the magnitude of the inhomogeneity β . Another reason for the latter phenomenon is

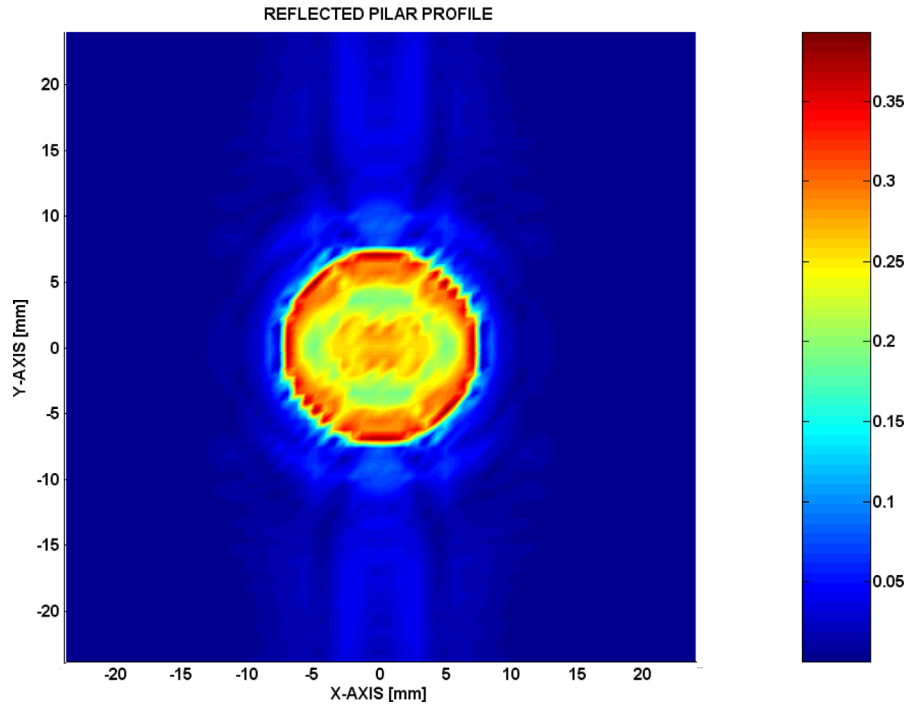


Fig. 10. Reflected beam profile for an incident *pillar beam* having the same amount of energy as a *conical-like beam* for $\beta = 100/\text{m}$. The result can be compared with [Figure 7](#) of the transmitted profile.

that narrower beams diffract much more, whence more propagation angles are involved, resulting in better sensitivity to the anisotropic elastic properties of the encountered material.

Because *conical-like beams* might be challenging to generate experimentally, we have also performed simulations for *pillar beams*, which are presumably easier to generate. We found that *pillar beams* are much more sensitive to material properties and fiber direction than classical *Gaussian beams*. [Figure 7](#) depicts the transmitted beam profile for an incident *pillar beam* with the same energy as a *conical-like beam* for $\beta = 100/\text{m}$. The transmitted beam pattern is direction-dependent for this type of beam, even for a relatively large diameter.

If we decrease the beam diameter, such as in [Figure 8](#), where the transmitted beam profile for an incident *pillar beam* having the same amount of energy as a *conical-like beam* for $\beta = 400/\text{m}$, is shown, the pattern becomes more outspoken, certainly if compared to [Figure 4](#).

In extreme situations, such as in [Figure 9](#) (to be compared with [Fig. 6](#)) where the transmitted beam profile for an incident *pillar beam* having the same amount of energy as a *conical-like beam* for $\beta = 2000/\text{m}$, is shown, the transmitted beam profile has an even more complicated shape and depends very strongly on the direction in the XY -plane.

Finally, to show the capabilities of the applied simulations, [Figure 10](#) exposes an example of a reflected field for normal incidence, matching the case of [Figure 7](#), where the transmission was shown. A study of how this reflected field would interact with the metamaterial and transducer is beyond the scope of this feasibility study for transmission.

Further investigations would require a complete investigation, including an emitting transducer, a metamaterial, and the anisotropic disk as one complex system.

4 Conclusions

The report shows that transmitted beam profiles change when transmitted through an anisotropic disk. Furthermore, *conical-like* and *pillar beams* are more influenced than *Gaussian beams*. Even though this feasibility study used one particular unidirectional fiber-reinforced composite layer as a model, depending on the requirements, the material can be tuned to match specific applications in which an anisotropic disk is placed in front of a single element transducer or a transducer covered by a metamaterial, to change transmitted beam characteristics further. Even though tunability may not be as sophisticated or flexible as a metasurface, its simplicity may add to the practical realization of specific beam profiles in a less expensive and lower level of fabrication sophistication than when only metamaterials are considered.

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