Efficient Domination In Fuzzy Graphs and Intuitionistic Fuzzy Graphs in Strong and weak forms

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Abstract: This work defines the concepts of strong efficient dominating set and intuitionistic fuzzy graph. We also introduce an intuitionistic fuzzy graph and a strong efficient dominating number of fuzzy graphs. The strong efficient dominant number in fuzzy graphs has some limitations that are studied, and intuitionistic fuzzy graphs are derived..

Keywords: Strong dominating set, strong efficient dominating set and strong efficient dominating number, efficient dominating set.

1. INTRODUCTION

From the fuzzy relations given by Zadeh, Kafmann introduces the idea of fuzzy graph. Despite the fact that Rosenfeld provided a further, more detailed description that included fuzzy vertices, fuzzy edges, and various fuzzy analogues of key graph theory ideas including routes, cycles, connectedness, and others. Tharmar et al., 2023 investigated the applications of r-neighbourhood spaces using relations. A. Somasundaram, S. Somasundaram, and A. Somasundaram propose the notions of independent dominance, total domination, and connected domination of fuzzy graphs as an idea of domination in fuzzy graphs.In fuzzy graphs, C. Natarajan and S.K. Ayyaswamy establish strong (weak) domination. Atanassov put forward the initial definition of intuitionistic fuzzy graphs. R. Parvathi and G. Tamizhendhi looked into the idea of dominance in intuitionistic fuzzy graphs. The concepts of an effective strong dominant set of fuzzy graphs and an intuitive fuzzy.

2. PRELIMINARIES

A fuzzy graph with V as the underlying set is a pair $G(\sigma,\mu)$ where $\sigma:V\rightarrow[0,1]$ is a fuzzy subset, $\mu:V\times V\rightarrow[0,1]$ is a fuzzy relation on the fuzzy subset σ, such that $\mu(uv)\le\sigma(u)\wedge\sigma(v)$ for all $u,v\in V$.Two nodes x and y are said to be neighbor if $\mu(uv)>0$.

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The underlying crisp graph of the fuzzy graph G:(σ,μ) is denoted as $G^*(\sigma^*, \mu^*)$ where $\sigma^* = \{u \in V / \sigma(u) > 0\}$ and $\mu^* = \{(uv) \in V \times V / \mu(uv) > 0\}$.

A fuzzy graph $G(\sigma, \mu)$ with the underlying set V, the order of G is defined and denoted by $O(G) = \sum_{u \in V} \sigma(G)$ and size of G is define and denoted by $S(G) = \sum_{u,v \in V} \mu(uv)$.

Let $G(\sigma, \mu)$ be a fuzzy graph. The degree of a node is defined as $d(u)=\Sigma_{v \neq u,\, v \in V} \mu(u, v)$.An edge in a fuzzy graph G: (σ,μ) is said to be an strong edge if $\mu(uv) = \sigma(u) \wedge \sigma(v)$.

A fuzzy graph $G(\sigma, \mu)$ is said to be a complete fuzzy graph if $\mu(uv) = \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$.A fuzzy graph G: (σ,μ) is said to be a strong fuzzy graph if $\mu(uv) = \sigma(u) \wedge \sigma(v)$ for all $uv \in E$.

Let $G(\sigma, \mu)$ be a fuzzy graph and u be a node in G then there exist a node v such that (u, v) is a strong arc then we say that u dominates v.

Let $G(\sigma, \mu)$ be a fuzzy graph. A set D of V is said to be fuzzy dominating set of G if every v ε V-D there exit u ε D such that u dominates v.

A fuzzy dominating set D of a fuzzy graph G is called minimal fuzzy dominating set of G, if every node v ε D, D-{v} is not a fuzzy dominating set.

The fuzzy dominating number γ(G) of the fuzzy graph G is the minimum cardinality taken over all minimal fuzzy dominating set o f G.

An intuitionistic fuzzy graph (IFG) is of the form G=(V,E) , where $V = \{v_1, v_2, \ldots, v_n\}$ such that $\mu_1 : V \to [0,1], \gamma_1 : V \to [0,1]$ denote the degree of membership and non-member ship of the element $v_i \in V$ respectively and $0 \le \mu_1(v_i) + \gamma_1(v_i) \le 1$ for every $v_i \in V, (i = 1,2,..n)$. $E \subseteq V \times V$ where $\mu_2 : V \times V \to [0,1],\ and\ \gamma_2 : V \times V \to [0,1]$ are such that

$$\mu_2(v_i, v_j) \le \mu_1(v_i) \wedge \mu_1(v_j), \gamma_2(v_i, v_j) \le \gamma_1(v_i) \vee \gamma_1(v_j) \ \&$$

$$0 \le \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \le 1.$$

An arc (v_i, v_j) of an IFG G is called an strong arc if $\mu_2(v_i, v_j) = \mu_1(v_i) \wedge \mu_1(v_j)$, $\gamma_2(v_i, v_j) = \gamma_1(v_i) \vee \gamma_1(v_j)$.Let *G = (V,E)* be a IFG. Then the cardinality of G is defined to be

$$|G| = \left| \sum_{v_i \in V} [\frac{(1 + \mu_1(v_i) - \gamma_1(v_i))}{2}] \right| + \left| \sum_{v_i \in V} [\frac{(1 + \mu_2(v_i, v_j) - \gamma_1(v_i, v_j))}{2}] \right|$$

Let *G = (V,E)* be a IFG. The vertex cardinality of G is defined to be $|G| = \left| \sum_{v_i \in V} [\frac{(1+\mu_1(v_i) - \gamma_1(v_i))}{2}] \right|$ for all $v_i \in V, (i = 1,2,..n)$.

Let G = (V, E) be an IFG. An edge cardinality of G is defined to be $|G| = \left| \sum_{v_i \in V} [\frac{(1+\mu_2(v_i,v_j) - \gamma_1(v_i,v_j))}{2}] \right|$ for all $(v_i, v_j) \in E$

An IFG, G = (V, E) is said to be strong IFG if $\mu_{2ij} = \mu_1(v_i) \wedge \mu_1(v_j)$ and $\nu_{2ij} = \nu_1(v_i) \vee \nu_2(v_j)$ for all $(v_i, v_j) \in E$.

An IFG, G = (V, E) is said to be a complete-μ strong IFG if $\mu_{2ij} = \mu_1(v_i) \wedge \mu_1(v_j)$ and $\nu_{2ij} < \nu_1(v_i) \vee \nu_2(v_j)$ for all i and j.

An IFG, G = (V, E) is said to be a complete-ν strong IFG if $\mu_{2ij} = \mu_1(v_i) \wedge \mu_1(v_j)$ and $\nu_{2ij} = \nu_1(v_i) \vee \nu_2(v_j)$ for all i and j.

An IFG, G = (V, E) is said to be a complete IFG if $\mu_{2ij} = \mu_1(v_i) \wedge \mu_1(v_j)$ and $\nu_{2ij} = \nu_1(v_i) \vee \nu_2(v_j)$ for every $(v_i, v_j) \in E$.

A set D of V is said to be intuitionistic fuzzy dominating set of G if every $v \in V - D$ there exits $u \in D$ such that u dominates v.A fuzzy dominating set D of a fuzzy graph G is called minimal intuitionistic fuzzy dominating set of G, if every node $v \in D$, $D - \{v\}$ is not aintuitionistic fuzzy dominating set.The fuzzy dominating number $\gamma_f(G)$ of the fuzzy graph G is the minimum cardinality taken over all minimal intuitionistic fuzzy dominating set of G.

Let $G(\sigma, \mu)$ be a fuzzy graph. A set D is subset of V is said to be efficient dominating set of a intuitionistic fuzzy graph G if every $v \in V - D$ there is exactly one $u \in D$ dominates v i.e $N(u) \cap D = \{v\}$.

A efficient dominating set D of a intuitionistic fuzzy graph G is called minimal efficient dominating set of G, if every subset of d is not a efficientintuitionistic fuzzy dominating set. i.e every node $u \in D, D - \{u\}$, is not an efficient intuitionistic fuzzy dominating set.

The efficient intuitionistic fuzzy dominating number $\gamma_e(G)$ of the intuitionistic fuzzy graph G is the minimum cardinality taken over all minimal efficient intuitionistic fuzzy dominating set of G.

3. STRONG EFFICIENT DOMINATING IN FUZZY GRAPHS

In this section we define strong efficient dominating set and strong efficient dominating number in fuzzy graph and investigate some bounds of strong efficient dominating number

Definition 3.1: Let $G(\sigma, \mu)$ be a fuzzy graph. The neighbourhood of the vertex $u \in V$ is defined by set of all vertices $N(u) = \{v | \ \sigma(u) \wedge \sigma(v) = \mu(uv)\}$. A neighbourhood degree of the vertex

$u \in V$ is defined by $d_N(u) = \sum_{v \in N(u)} \sigma(v)$ and the effective neighbourhood degree of the vertex $u \in V$ is defined by $d_E(u) = \sum_{v \in N(u)} \mu(uv)$.

Definition 3.2: In fuzzy graph $G(\sigma, \mu)$ the vertex u strongly dominates v if (i) uv is a strong edge in $G(\sigma, \mu)$ (ii) $d_N(u) \geq d_N(v)$. The set $D \subseteq V$ is said to be a strong dominating set of $G(\sigma, \mu)$ such that every vertex $v \in V - D$ is strongly dominated by a vertex $u \in D$.

Definition 3.3: Let $G(\sigma, \mu)$ be a fuzzy graph. The subset $D \subset V$ is said to be an efficient dominating set of a fuzzy graph $G(\sigma, \mu)$ if every vertex $v \in V - D$ there exist a vertex $u \in D$ such that $N(u) \cap D = \{v\}$.

Definition 3.4: Let $G(\sigma, \mu)$ be afuzzy graph. The subset $D \subset V$ be strong dominating set of the fuzzy graph $G(\sigma, \mu)$. An strong dominating set D is said to be a strong efficient dominating set if if every vertex $v \in V - D$ there exist a vertex $u \in D$ such that $N(u) \cap D = \{v\}$.. The minimum cardinality among the minimal strong efficient dominating set is called a strong efficient domination number of $G(\sigma, \mu)$ and it is denoted by $\gamma_{SE}(G)$.

Definition 3.5: Let $G(\sigma, \mu)$ be a fuzzy graph. The subset $D \subset V$ be weak dominating set of the fuzzy graph $G(\sigma, \mu)$. A weak dominating set D is said to be a weakefficient dominating set if if every vertex $v \in V - D$ there exist a vertex $u \in D$ such that $N(u) \cap D = \{v\}$.. The minimum cardinality among the minimal weak efficient dominating set is called a weak efficient domination number of $G(\sigma, \mu)$ and it is denoted by $\gamma_{WE}(G)$.

Theorem 3.1: Let $G(\sigma, \mu)$ be a complete fuzzy graph, then the strong efficient dominating number $\gamma_{SE}(G) = |v|$, here u, v is the vertex having the maximum and minimum cardinality in $G(\sigma, \mu)$ respectively.Then

i) $\gamma_{SE}(G) = |v|$

ii) $\gamma_{WE}(G) = |u|$

Proof: Let $G(\sigma, \mu)$ be a complete fuzzy graph. This implies there is a strong edge between every pair of vertices.

(i) Assume the minimal efficient dominating set of $G(\sigma, \mu)$ isthe singleton set $D = \{v\}$, v is vertex having the maximum neighbourhood degree in the graph $G(\sigma, \mu)$ such that $d_N(v) \geq d_N(u)$. This implies v is vertex having the minimum cardinalityin $G(\sigma, \mu)$ .Thereforev strongly dominates every $x \in V - \{v\}$ is and $N(x) \cap D = \{v\}$. Hence D is a strong efficient dominating set of $G(\sigma,\mu)$. Therefore the minimum strong efficient domination number $\gamma'_{SE}(G)=|v|$.

(ii) Assume the minimal efficient dominating set of $G(\sigma,\mu)$ isthe singleton set $D=\{u\}$, u is vertex having the minimum neighbourhood degree in the graph $G(\sigma,\mu)$ such that $d_N(v)\geq d_N(u)$. This implies u is vertex having the maximum cardinality in $G(\sigma,\mu)$. Therefore u weakly dominates every $x\in V-\{u\}$ is and $N(x)\cap D=\{u\}$. Hence D is a weak efficient dominating set of $G(\sigma,\mu)$. Therefore the minimum weak efficient domination number $\gamma'_{SE}(G)=|u|$.

Example 3.1.

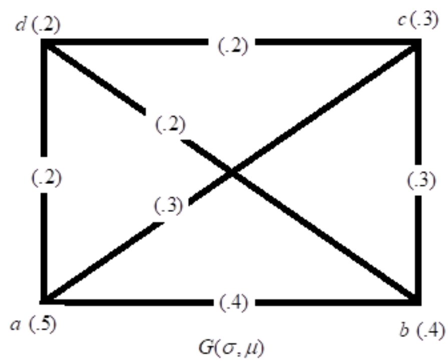


Figure 3.1

In the above complete fuzzy graph degree of the vertices are $d_N(a)=0.9, d_N(b)=1.0, d_N(c)=1.1,\ d_N(d)=1.2,\ and\ \ \Delta_N(G)=1.2,\ \ \delta_N(G)=0.9$. The minimum strong efficient number $\gamma_{SE}(G)=0.2$ and minimum weak efficient number $\gamma_{WE}(G)=0.5$.

Theorem 3.2: Let $G(\sigma,\mu)$ be a fuzzy graph .Then

$i)\ \gamma(G)\leq\gamma_{SE}(G)$

$ii)\ \gamma(G)\leq\gamma_{WE}(G)$

Proof: Let $G(\sigma,\mu)$ be a fuzzy graph and D, D_{SE}, D_{WE} is a $\gamma(G), \gamma_{SE}(G) and\ \gamma_{WE}(G)$ set respectively. Note that every strong efficient dominating set is a dominating set but not a minimal dominating set of $G(\sigma,\mu)$ .This implies $|D|\leq|D_{SE}|\Rightarrow\gamma(G)\leq\gamma_{SE}(G).$

Similarly every weak efficient dominating set is a dominating set but not a minimal dominating set of $G(\sigma,\mu)$ .This implies $|D|\leq|D_{WE}|\Rightarrow\gamma(G)\leq\gamma_{WE}(G).$

Theorem 3.3: Let $G(\sigma,\mu)$ be a fuzzy graph and $d_N(u)=\Delta_N(G)$ .Then $u\in\gamma_{SE}(G)$

Proof: Let $G(\sigma,\mu)$ be a fuzzy graph and $d_N(u)=\Delta_N(G)$. Every vertex in $v\in N(u)$ satisfies $d_N(u)\geq d_N(v)$. This implies every vertex $v\in N(u)$ strongly dominated by the vertex u. Suppose $u\notin\gamma_{SE}(G)$. There is vertex $v\in N(u)$ strongly dominates u. This is contradict to assumption $d_N(u)=\Delta_N(G)$. This implies our assumption is wrong. Hence $u\in\gamma_{SE}(G)$.

Theorem 3.4: Let $G(\sigma,\mu)$ be a fuzzy graph and $d_N(u)=\delta_N(G)$ .Then $u\in\gamma_{WE}(G)$

Proof: Let $G(\sigma,\mu)$ be a fuzzy graph and $d_N(u)=\Delta_N(G)$. Every vertex $v\in N(u)$ satisfies $d_N(u)\leq d_N(v)$. This implies every vertex $v\in N(u)$ strongly dominated by the vertex u. Suppose $u\notin\gamma_{WE}(G)$. There is a vertex $v\in N(u)$ weakly dominates u. This is contradict to assumption $d_N(u)=\delta_N(G)$. This implies our assumption is wrong. Hence $u\in\gamma_{WE}(G)$.

Example 3.2

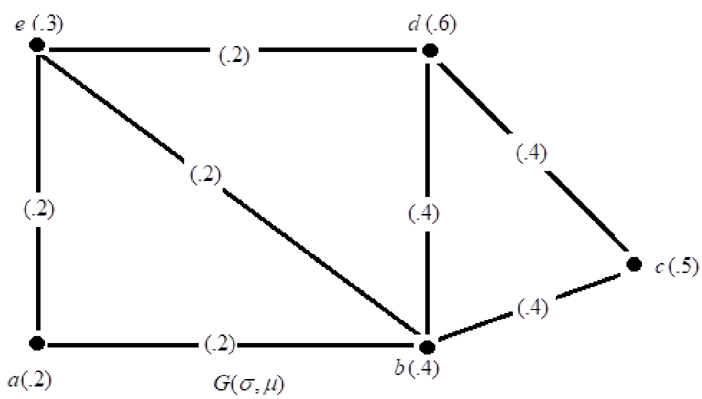


Figure 3.2

In the above fuzzy graph degree of the vertices are $d_N(a)=0.7, d_N(b)=1.3$,
$d_N(c)=0.4,\ d_N(d)=0.9,\ d_N(e)=0.2\ \ and\ \ \Delta_N(G)=1.3,\ \ \delta_N(G)=0.2$.
The minimal dominating set $D=\{a,b\}$, minimal strong efficient dominating set $D_{SE}=\{b,e\}$, minimal weak efficient dominating set $D_{WE}=\{a,c,d,e\}$, domination number $\gamma(G)=0.6$. strong efficient domination number is $\gamma_{SE}(G)=0.7$.and weak efficient domination number is $\gamma_{WE}(G)=1.6$.

Theorem 3.5: Let $G(\sigma,\mu)$ be a fuzzy graph and $d_N(u)=\Delta_N(G)$, $d_N(v)=\delta_N(G)$ then

(i). $\gamma_{SE}(G)\leq O(G)-\Delta_N(G)$

(ii). $\gamma_{WE}(G)\leq O(G)-\delta_N(G)$

Proof: Let $G(\sigma,\mu)$ fuzzy graph and $d_N(u)=\Delta_N(G)$, $d_N(v)=\delta_N(G)$.

(i) Assume D_{SE} be the strong efficient dominating setof $G(\sigma,\mu)$. By theorem 3.4, $u\in\gamma_{SE}(G)$,this implies

$$D_{SE}\subset V-N(u)$$
$$\Rightarrow|D_{SE}|<|V-d_N(u)|$$
$$\Rightarrow\gamma_{SE}(G)<O(G)-\Delta_N(G)$$

(ii) Assume D_{WE} be the weak efficient dominating set of $G(\sigma,\mu)$. By theorem 3.5, $v\in\gamma_{SE}(G)$, this implies

$$D_{WE}\subset V-N(v)$$
$$\Rightarrow|D_{WE}|<|V-d_N(u)|$$
$$\Rightarrow\gamma_{WE}(G)<O(G)-\delta_N(G)$$

Theorem 3.6. Let $G_1(\sigma_1,\mu_1)\,and\,G_2(\sigma_2,\mu_2)$ be a fuzzy graphs and the sets $D_1\,\&\,D_2$ are the strong efficient dominating set of $G_1(\sigma_1,\mu_1)\,and\,G_2(\sigma_2,\mu_2)$ respectively. Then

$$(i)\ \gamma_S(G_1+G_2)=|D_1|,if\ O(G_2)>O(G_1)$$
$$(ii)\ \gamma_S(G_1+G_2)=|D_1|,\ if\ O(G_1)>O(G_2)$$

Proof: Let $G_1(\sigma_1,\mu_1)\,and\,G_2(\sigma_2,\mu_2)$ be a fuzzy graphs and the sets $D_1\,\&\,D_2$ are the strong efficient dominating set of $G_1(\sigma_1,\mu_1)\,and\,G_2(\sigma_2,\mu_2)$ respectively. In (G_1+G_2) edges is of the form

$i)$ $uv\in G_1$

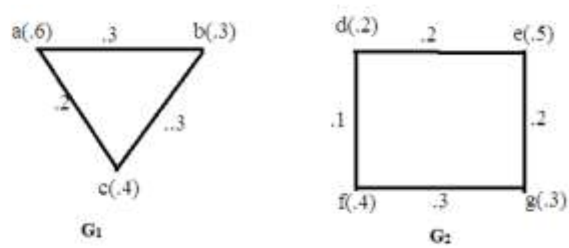
$ii)$ $uv\in G_2$

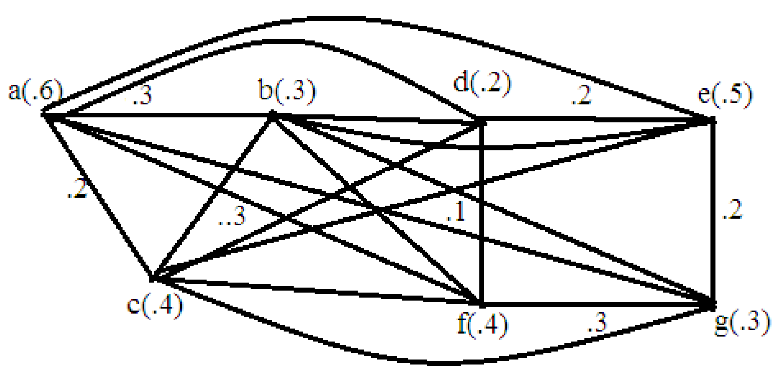
$iii)$ $uv\in G_1+G_2,\ if\ u\in G_1\ \&\ v\in G_2$

(i) If $O(G_2)>O(G_1)$, Now we prove D_1 is the strong dominating set of (G_1+G_2).Every vertex in G_1 strong efficiently dominated by the set D_1. Note that there is a strong edge between the vertices in vertices in $G_1(\sigma_1,\mu_1)\,and\,G_2(\sigma_2,\mu_2)$. This implies every vertices in $G_2(\sigma_2,\mu_2)$ is strongly dominated by the set D_1. Since every vertex $u\in G_1\ \&\ v\in G_2$ such that $d_N(v)>d_N(u)$. Therefore D_1 is the minimal strong dominating set of (G_1+G_2). This implies $\gamma_S(G_1+G_2)=|D_1|$.

(ii) If $O(G_1)>O(G_2)$, Now we prove D_2 is the strong dominating set of (G_1+G_2).Every vertex in G_2 are strong efficiently dominated by the set D_2. Note that there is a strong edge between the vertices in vertices in $G_1(\sigma_1,\mu_1)$ *and* $G_2(\sigma_2,\mu_2)$. This implies every vertices in $G_1(\sigma_1,\mu_1)$ is strongly dominated by the set D_2. Since every vertex $u \in G_1 \ \& v \in G_2$ such that $d_N(v) > d_N(u)$. Therefore D_2 is the minimal strong dominating set of (G_1+G_2). This implies $\gamma_S(G_1+G_2)=|D_2|$.

Example 3.3





G_1+G_2

Figure 3.3

In the figure 3.3, join of two fuzzy graphs $G_1(\sigma_1,\mu_1)$ *and* $G_2(\sigma_2,\mu_2)$ is given. The membership values of (G_1+G_2) is given below

Edges	Value	Edges	Value	Edges	Value	Edges	Value
(ab)	0.3	(fg)	0.3	(ag)	0.3	(cd)	0.2
(ac)	0.2	(df)	0.1	(bd)	0.2	(ce)	0.4
(bc)	0.3	(ad)	0.2	(be)	0.3	(cf)	0.4
(de)	0.2	(ae)	0.5	(bf)	0.3	(cg)	0.3
(eg)	0.2	(af)	0.4	(bg)	0.3	--	--

The strong efficient dominating set of $G_1(\sigma_1,\mu_1)$ *and* $G_2(\sigma_2,\mu_2)$ is $D_{SE}(G_1)=\{b\}$, $D_{SE}(G_2)=\{d,g\}$. In (G_1+G_2) the sets $D_{SE}(G_1)$,

$D_{SE}(G_2)$ are not a strong efficient dominating set . The order of the $G_1(\sigma_1,\mu_1)\, and\, G_2(\sigma_2,\mu_2)$ are $O(G_1)=1.3\ \&\, O(G_2)=1.4$ .the set $D_{SE}(G_1)=\{b\}$ is the minimal strong dominating set of (G_1+G_2).

4. STRONG EFFICIENT DOMINATION IN INTUITIONISTIC FUZZY GRAPHS

In this section we define strong efficient dominating set and strong efficient dominating number in intuitionisticfuzzy graph and investigate some bounds of strong efficient dominating number.

Definition 4.1: In fuzzy graph $G(\sigma,\mu)$ the vertex u strongly dominates v if (i) uv is a strong edge in $G(\sigma,\mu)$ (ii) $d_N(u)\geq d_N(v)$. The set $D\subseteq V$ is said to be a strong dominating set of $G(\sigma,\mu)$ such that every vertex $v\in V-D$ is strongly dominated by a vertex $u\in D$.

Definition 4.2: Let $G(\sigma,\mu)$ be a fuzzy graph. The subset $D\subset V$ is said to be an efficient dominating set of a fuzzy graph $G(\sigma,\mu)$ if every vertex $v\in V-D$ there exist a vertex $u\in D$ such that $N(u)\cap D=\{v\}$.

Definition 4.3: Let $G(\sigma,\mu)$ be a fuzzy graph. The subset $D\subset V$ be strong dominating set of the fuzzy graph $G(\sigma,\mu)$. An strong dominating set D is said to be a strong efficient dominating set if if every vertex $v\in V-D$ there exist a vertex $u\in D$ such that $N(u)\cap D=\{v\}$.. The minimum cardinality among the minimal strong efficient dominating set is called a strong efficient domination number of $G(\sigma,\mu)$ and it is denoted by $\gamma_{SE}(G)$.

Definition 4.4: Let $G(\sigma,\mu)$ be a fuzzy graph. The subset $D\subset V$ be weak dominating set of the fuzzy graph $G(\sigma,\mu)$. A weak dominating set D is said to be a weakefficient dominating set if if every vertex $v\in V-D$ there exist a vertex $u\in D$ such that $N(u)\cap D=\{v\}$.. The minimum cardinality among the minimal weak efficient dominating set is called a weak efficient domination number of $G(\sigma,\mu)$ and it is denoted by $\gamma_{WE}(G)$.

Theorem 4.1: Let $G(V,E)$ be a complete intuitionistic fuzzy graph, then the strong efficient dominating number $\gamma_{ISE}(G)=|v|$, here u, v is the vertex having the maximum and minimum cardinality in $G(V,E)$ respectively.Then

iii) $\gamma_{ISE}(G)=|v|$

iv) $\gamma_{IWE}(G)=|u|$

Proof: Let $G(\sigma,\mu)$ be a complete intuitionistic fuzzy graph. This implies there is a strong edge between every pair of vertices.

(i) Assume the minimal efficient dominating set of $G(V,E)$ isthe singleton set $D=\{v\}$, v is vertex having the maximum neighbourhood degree in the graph $G(\sigma,\mu)$ such that $d_N(v)\geq d_N(u)$. This implies v is vertex having the minimum cardinality in $G(V,E)$. Therefore v strongly dominates every $x\in V-\{v\}$ is and $N(x)\cap D=\{v\}$. Hence D is a strong efficient dominating set of $G(V,E)$. Therefore the minimum strong efficient domination number $\gamma'_{ISE}(G)=|v|$.

(ii) Assume the minimal efficient dominating set of $G(V,E)$ isthe singleton set $D=\{u\}$, u is vertex having the minimum neighbourhood degree in the graph $G(V,E)$ such that $d_N(v)\geq d_N(u)$. This implies u is vertex having the maximum cardinality in $G(V,E)$. Therefore u weakly dominates every $x\in V-\{u\}$ is and $N(x)\cap D=\{u\}$. Hence D is a weak efficient dominating set of $G(V,E)$. Therefore the minimum weak efficient domination number $\gamma'_{ISE}(G)=|u|$.

Example 4.1.

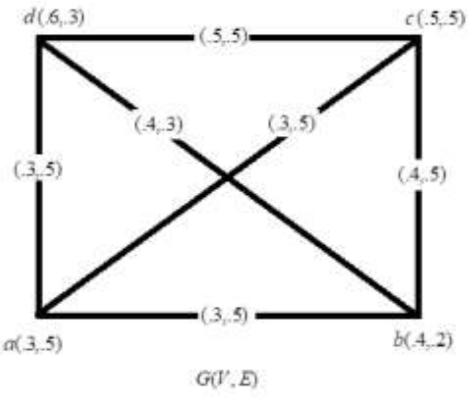


Figure 4.1

In the above complete fuzzy graph degree of the vertices are $d_N(a)=1.75, d_N(b)=1.55, d_N(c)=1.65,\ d_N(d)=1.5,\ and\ \Delta_N(G)=1.75,\ \delta_N(G)=1.5$. The minimum strong efficient number $\gamma_{ISE}(G)=0.4$ and minimum weak efficient number $\gamma_{WE}(G)=0.75$.

Theorem 4.2: Let $G(\sigma,\mu)$ be an intuitionistic fuzzy graph .Then

$i)\ \gamma_I(G)\leq\gamma_{ISE}(G)$

$ii)\ \gamma_I(G)\leq\gamma_{IWE}(G)$

Proof: Let $G(\sigma,\mu)$ be an intuitionistic fuzzy graph and D, D_{SE}, D_{WE} is a $\gamma(G),\gamma_{SE}(G) and\ \gamma_{WE}(G)$ set respectively. Note that every strong efficient dominating set is a dominating set but not a minimal dominating set of $G(\sigma,\mu)$ .This implies $|D|\leq|D_{SE}|\Rightarrow\gamma(G)\leq\gamma_{ISE}(G)$.

Similarly every weak efficient dominating set is a dominating set but not a minimal dominating set of $G(\sigma,\mu)$ .This implies $|D| \le |D_{WE}| \Rightarrow \gamma(G) \le \gamma_{IWE}(G).$

Theorem 4.3: Let $G(\sigma,\mu)$ be an intuitionistic fuzzy graph and $d_N(u) = \Delta_N(G)$ .Then $u \in \gamma_{ISE}(G).$

Proof: Let $G(\sigma,\mu)$ be anintuitionistic fuzzy graph and $d_N(u) = \Delta_N(G)$. Every vertex in $v \in N(u)$ satisfies $d_N(u) \ge d_N(v)$. This implies every vertex $v \in N(u)$ strongly dominated by the vertex u. Suppose $u \notin \gamma_{SE}(G)$. There is vertex $v \in N(u)$ strongly dominates u. This is contradict to assumption $d_N(u) = \Delta_N(G)$. This implies our assumption is wrong. Hence $u \in \gamma_{ISE}(G)$.

Theorem 4.4: Let $G(\sigma,\mu)$ be an intuitionistic fuzzy graph and $d_N(u) = \delta_N(G)$ .Then $u \in \gamma_{IWE}(G).$

Proof: Let $G(\sigma,\mu)$ be a fuzzy graph and $d_N(u) = \Delta_N(G)$. Every vertex $v \in N(u)$ satisfies $d_N(u) \le d_N(v)$. This implies every vertex $v \in N(u)$ strongly dominated by the vertex u. Suppose $u \notin \gamma_{WE}(G)$. There is a vertex $v \in N(u)$ weakly dominates u. This is contradict to assumption $d_N(u) = \delta_N(G)$. This implies our assumption is wrong. Hence $u \in \gamma_{WE}(G)$.

Example 4.2

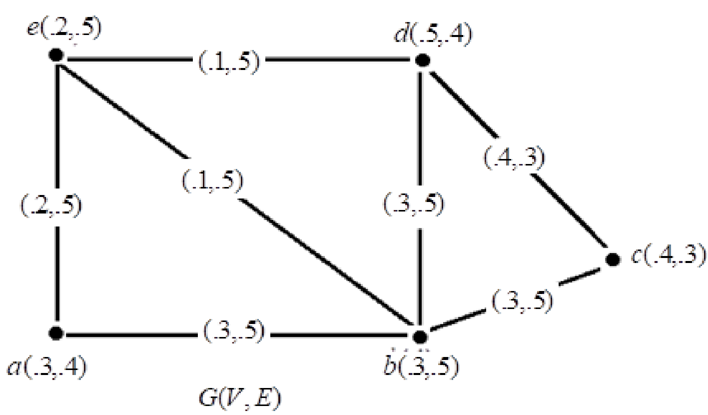


Figure 4.2

In the above fuzzy graph degree of the vertices are $d_N(a) = 0.75, d_N(b) = 1.55$
$d_N(c) = 0.35,\ d_N(d) = 0.35,\ d_N(e) = 0.4\ \ and\ \ \Delta_N(G) = 1.55,\ \ \delta_N(G) = 0.35$.
The minimal strong efficient dominating set $D_{SE} = \{b, e\}$, minimal weak

efficient dominating set $D_{WE} = \{a,c,d\}$, strong efficient domination number is $\gamma_{SE}(G) = 0.75$.and weak efficient domination number is $\gamma_{WE}(G) = 1.55.$

Theorem 4.5: Let $G(V,E)$ be an intuitionistic fuzzy graph and $d_N(u) = \Delta_N(G)$, $d_N(v) = \delta_N(G)$ then

(i). $\gamma_{ISE}(G) \le O(G) - \Delta_N(G)$

(ii). $\gamma_{IWE}(G) \le O(G) - \delta_N(G)$

Proof: Let $G(\sigma,\mu)$ be an intuitionistic fuzzy graph and $d_N(u) = \Delta_N(G)$, $d_N(v) = \delta_N(G)$.

(iii) Assume D_{SE} be the strong efficient dominating set of $G(\sigma,\mu)$. By theorem 4.3, $u \in \gamma_{ISE}(G)$, this implies

$D_{SE} \subset V - N(u)$

$\Rightarrow |D_{SE}| < |V - d_N(u)|$

$\Rightarrow \gamma_{ISE}(G) < O(G) - \Delta_N(G)$

(iv) Assume D_{WE} be the weak efficient dominating set of $G(\sigma,\mu)$. By theorem 4.4, $v \in \gamma_{ISE}(G)$, this implies

$D_{WE} \subset V - N(v)$

$\Rightarrow |D_{WE}| < |V - d_N(u)|$

$\Rightarrow \gamma_{IWE}(G) < O(G) - \delta_N(G)$

Theorem 4.6. Let $G_1(V_1,E_1)\, and\ G_2(V_2,E_2)$ be an intuitionistic fuzzy graphs and the sets $D_1 \& D_2$ are the strong efficient dominating set of $G_1(V_1,E_1)\, and\ G_2(V_2,E_2)$ respectively. Then

$(i)\ \gamma_S(G_1 + G_2) = |D_1|, if\ O(G_2) > O(G_1)$

$(ii)\ \gamma_S(G_1 + G_2) = |D_1|, if\ O(G_1) > O(G_2)$

Proof: Let $G_1(V_1,E_1)\, and\ G_2(V_2,E_2)$ be an intuitionistic fuzzy graphs and the sets $D_1 \& D_2$ are the strong efficient dominating set of $G_1(V_1,E_1)\, and\ G_2(V_2,E_2)$ respectively. In $(G_1 + G_2)$ edges is of the form

$i)\ \ uv \in G_1$

$ii)\ \ uv \in G_2$

$iii)\ uv \in G_1 + G_2, if\ u \in G_1 \,\&\, v \in G_2$

(iii) If $O(G_2) > O(G_1)$, Now we prove D_1 is the strong dominating set of $(G_1 + G_2)$.Every vertex in G_1 strong efficiently dominated by the set D_1. Note that there is a strong edge between the vertices in vertices in $G_1(\sigma_1,\mu_1)\, and\ G_2(\sigma_2,\mu_2)$. This implies every vertices in $G_2(\sigma_2, \mu_2)$ is strongly dominated by the set D_1. Since every vertex $u \in G_1$ & $v \in G_2$ such that $d_N(v) > d_N(u)$. Therefore D_1 is the minimal strong dominating set of $(G_1 + G_2)$. This implies $\gamma_S(G_1 + G_2) = |D_1|$.

(iv) If $O(G_1) > O(G_2)$, Now we prove D_2 is the strong dominating set of $(G_1 + G_2)$.Every vertex in G_2 are strong efficiently dominated by the set D_2 . Note that there is a strong edge between the vertices in vertices in $G_1(\sigma_1, \mu_1)\, and\, G_2(\sigma_2, \mu_2)$. This implies every vertices in $G_1(\sigma_1, \mu_1)$ is strongly dominated by the set D_2 . Since every vertex $u \in G_1$ &$v \in G_2$ such that $d_N(v) > d_N(u)$. Therefore D_2 is the minimal strong dominating set of $(G_1 + G_2)$. This implies $\gamma_S(G_1 + G_2) = |D_2|$.

Example 4.3:

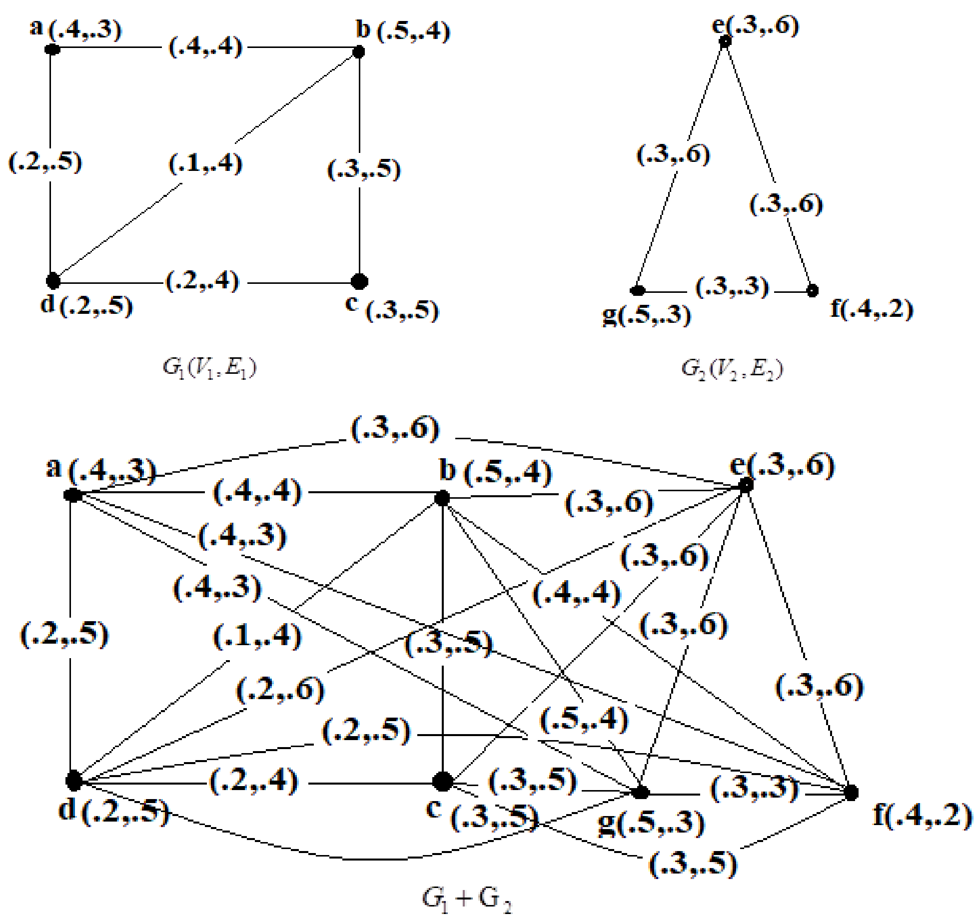


Figure 4.3

In the figure 3.3, join of two fuzzy graphs $G_1(V_1, E_1)\, and\, G_2(V_2, E_2)$ is given. In $G_1(V_1, E_1)\, and\, G_2(V_2, E_2)$ the degrees of the vertices are $d_N(a) = 0.9,\ d_N(b) = 0.95$ $d_N(c) = 0.55,\ d_N(d) = 0.55,\ d_N(e) = 1.2, d_N(f) = 0.35, d_N(g) = 0.35$. The strong efficient dominating set of $G_1(V_1, E_1)\, and\, G_2(V_2, E_2)$ is $D_{SE}(G_1) = \{a, b\}$, $D_{SE}(G_2) = \{e\}$. In $(G_1 + G_2)$ the sets $D_{SE}(G_1)$, $D_{SE}(G_2)$ are not a strong efficient dominating set . The order of the $G_1(\sigma_1, \mu_1)\, and\, G_2(\sigma_2, \mu_2)$ are $O(G_1) = 1.85\ \&\ O(G_2) = 1.55$. The set $D_{SE}(G_2) = \{e\}$ is the minimal strong dominating set of $(G_1 + G_2)$.

Conclusion

In this paper, the idea of strong efficient dominating set of fuzzy graphs and intuitionistic fuzzy graph is defined. Further we introduce the strong efficient dominating number of fuzzy graphs and an intuitionistic fuzzy graph is introduced. Finally investigate some bounds of the strong efficient dominating number in fuzzy graphs and intuitionistic fuzzy graph are derived.

References

1. Atanasson, Intuitionistic Fuzzy Sets: Theory and Applications, Physica-verlag, New York (1999).
2. Mordeson, J.N., and Nair, P.S., Fuzzy graphs and Fuzzy Hyper graphs, Physica-Verlag, Heidelberg, 1998, second edition, 20011.
3. Harary.F., Graph Theory, Addition Wesely, Third Printing, October 1972.
4. Tharmar , S., Veerasivaji, R and Senthil kumar, R. 2023, 'Theorytical Approach On Application Of Generalized Closed Sets Environmental Lifestyle Using r-neighbourhood Ideal spaces', 'E3S Web of conferences', 387, 04015, pp. 1-6.
5. Somasundaram,A.,Somasundaram,S.,1998, Domination in Fuzzy Graphs-I, Pattern Recognition Letters, 19, pp. 787–791.
6. Somasundaram, A., 2004, Domination in Fuzzy Graph-II, Journal of Fuzzy Mathematics.
7. R.Parvathi and G.Thamizhendhi, Domination in Intuitionistic Fuzzy Graphs, Fourteenth Int.Conf. On IFSs, Sofia 15-16 may 2010.
8. Harary, F 1973, Graph Theory, Reading, MA.
9. Mordeson, JN & Nair, PS 2000, 'Fuzzy Graphs and Fuzzy Hyper graphs', Physica-Verlag.
10. Vinothkumar**,** N &Geetharamani, G 2014, 'Edge domination in fuzzy graphs', Pensee Multi-Disciplinary Journal, vol. 76, no.1, pp. 61-70.
11. Vinothkumar, N &Geetharamani, G 2016, 'Vertex edge domination in operation of fuzzy graphs', 'International Journal of Advanced Engineering Technology', vol. 7, no. 2, pp. 401-405.

https://doi.org/10.1051/e3sconf/202339904026

12. Zimmerman, HJ 1996, Fuzzy Set Theory and its Applications, Allied Publishers.