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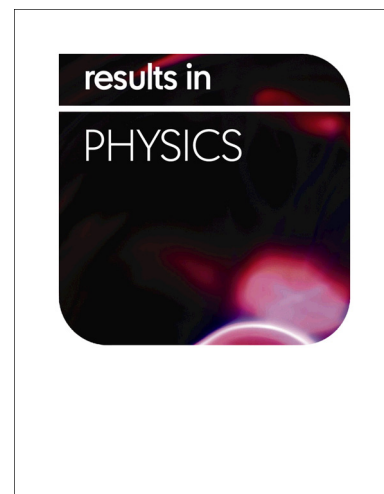
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Fisher information and quantum state estimation of two-coupled atoms in presence of two external magnetic fields

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Abstract: We consider a quantum system of an atom-atom interaction in the presence of two external magnetic and classical fields in x , and z directions. The dynamical behavior of single atom quantum quantifiers such as the atomic Wehrl entropy, atomic Fisher information and quantum entropy are investigated. The quantum entanglement degradation in regard to the atomic state estimation are discussed through the evolution of the atomic Fisher information flow, and the effects of the external classical fields and types of time dependent coupling between the two atoms are examined. We explore the relationship between different statistical quantities in the absence and presence of classical field during the time evolution. The results show that the creation and manipulation of entanglement by external fields greatly benefit within the suitable choice of the kind of time dependent coupling among the two atoms and external.

Keywords: Atomic Wehrl entropy, Atomic Fisher information, Atom-atom interaction; single atom entanglement; time dependent coupling.

1. Introduction

FI (Fisher information) was firstly introduced by Fisher as the main quantity in estimation theory [1]. The members of probability distributions can be detected by FI. In this regard, FI can perform optimal tasks in detecting the optimal measurement of the considered quantum systems at the point of view of the theory of quantum estimation. The parameter estimation problem being the more significant is the main support of various science and technology branches. The phase estimation plays the essential role of state generation, decoherence and loss [2-7]. The lower bound of QFI (Quantum Fisher Information) can be obtained from the quantum version of the Cramér-Rao (CR) inequality [8]. Interestingly, the mathematical treatments of the lower bound in physical models has been clarified [9,10], in this way the optimal resource for the phase estimation has been discussed [11,12].

In recent years, many devices such as such, as beam splitter [13-15], cavity QED [16], nanoresonators [17] and NMR systems [18] have been realized and proposed experiments for generating quantum entanglement. So the investigation of nonlocal correlation outdated its context of bipartite and multipartite pure quantum states, considering the case of mixed states as well as investigates the entanglement in large scale many particle quantum systems. However, a complete investigation of entanglement in complex systems is still absent until present. The quantification of this kind correlation in both discrete and continuous variables in the framework of theoretical approaches [19-21]. A measure of entanglement must be invariant under local operations and classical communication. The understanding derivable measures of quantum entanglement is account bearing in mind when searching to detect entanglement in large dimensional complex systems such as is considered in biological systems. Until present, the quantum concepts of nonlocal correlation were

related to entangled states [22]. The separation between classical and quantum correlations in composite systems has attracted recently much attention in different branches of physics. It is shown that the shared the total correlation in composite system can be presented by quantum mutual information [23,24], but an important question is how to determine the amount of classical and quantum correlations from the total correlation.

As known that the entanglement between two partite quantum systems can be measured by some statistical quantities such as the entanglement of formation (EOF) or the von Neumann entropy (VNE). All the entanglement measures must be invariant under LOCC (Local Operation Classical Communication). On the other hand the entanglement of complex physical system cannot be measure by the standard measure such as the VNE. So, different attempts have been done to discovers a new quantifier of entanglement of a compost quantum system. In this regard the atomic Wehrl entropy (AWE) and the atomic Fisher information (AFI) have been used to quantify the entanglement and compared by the standard VNE measure [25]. It has been shown that a clear relationship between the behavior of AFI and AWE during the time evolution has observed. Although the dynamical properties of the flow of AFI has not been studied. The follow of the statistical quantities or information quantifier depends of the first derivative. In this regard the flow of Fisher information can be used to detect the Markovian and non Markovian dynamics of a two-level atom system driven by a phase noise laser [26]. Also, it was used to characterize the damping effect and unveil a fundamental connection between non-Markovian behavior and dynamics of system–environment correlations under phase noise laser [27]. The purpose of this paper is to examine the effect of the time dependent coupling and classical fields on the dynamical properties of the VNE, the AWE and the AFI. Moreover the flow of AFI will be used to detect the entanglement between the two coupled atoms as alternative quantifier of the VNE. Also, exploring the link between the statistical quantities during the time evolution.

2. Model and dynamics

Recently, much effort has been given from many authors to develop studies of this kind of interaction, see for example Refs [28,29,30]. This in fact encouraged us to develop some previous investigations. The atom-atom interaction is one of the cornerstones in quantum optics and quantum information theory. The Hamiltonian model describes the interaction between TCAs (two coupled atoms) and external classical fields has the form

$$\frac{\hat{H}(t)}{\hbar} = \sum_{j=1}^2 \left\{ \frac{\bar{\omega}_j(t)}{4} \hat{R}_z^{(j)} + \bar{\lambda}_j(t) (\hat{R}_+^{(j)} + \hat{R}_-^{(j)}) \right\} + \lambda f(t) \sum_{\ell=x,y,z} \hat{R}_\ell^{(1)} \hat{R}_\ell^{(2)} \quad (1)$$

where $f(t)$ is the time-dependent coupling between the TCAs and $\bar{\lambda}_j(t)$, is the time-dependent coupling of the two external classical fields (i.e. $j=1,2$). Also, the external magnetic field frequencies are $\bar{\omega}_j(t)$, $j=1,2$. Furthermore, $\hat{R}_\pm^{(j)}$ and $\hat{R}_z^{(j)}$ are playing the usual role of the Pauli operators which have the properties

$$\left[\hat{R}_+^{(j)}, \hat{R}_-^{(i)} \right] = 2\hat{R}_z^{(j)} \delta_{ij}, \quad \left[\hat{R}_z^{(i)}, \hat{R}_\pm^{(j)} \right] = \pm 2\hat{R}_\pm^{(j)} \delta_{ij}. \quad (2)$$

The coupling $\bar{\lambda}_j$ can be removed by using the transformation

$$\left. \begin{aligned} \hat{R}_z^{(j)} &= \hat{S}_z^{(j)} \cos 2\varphi_j - \hat{S}_x^{(j)} \sin 2\varphi_j \\ \hat{R}_x^{(j)} &= \hat{S}_x^{(j)} \cos 2\varphi_j + \hat{S}_z^{(j)} \sin 2\varphi_j \end{aligned} \right\} \quad (3)$$

where $\tan 2\varphi_j = 2\bar{\lambda}_j / \bar{\omega}_j$, and $R_y^{(j)} = S_y^{(j)}$, under the condition $\bar{\lambda}_1 / \bar{\omega}_1 = \bar{\lambda}_2 / \bar{\omega}_2$. Also, let us assume that $\bar{\omega}_j = \omega_j f(t)$, and $\bar{\lambda}_j = \lambda_j f(t)$ then $\Omega_j = \sqrt{4\lambda_j^2 + \omega_j^2}$.

Now, the time dependent Hamiltonian which describes an atom-atom interaction in terms of the frequencies $\bar{\Omega}_j$ that depends explicitly on the classical field coupling $\bar{\lambda}_j$ is given by

$$\hat{H}(t) = \lambda f(t) \left\{ \sum_{j=1}^2 \frac{\Omega_j}{2} \hat{R}_z^{(j)} + \sum_{\ell=x,y,z} \hat{R}_\ell^{(1)} \hat{R}_\ell^{(2)} \right\}. \quad (4)$$

The equations of motion according to the time dependent Hamiltonian (4) are given by

$$\begin{aligned} \frac{d\hat{S}_z^{(1)}}{dt} &= i\lambda f(t) (\hat{S}_+^{(1)} \hat{S}_-^{(2)} - \hat{S}_-^{(1)} \hat{S}_+^{(2)}), \\ \frac{d\hat{S}_+^{(1)}}{dt} &= \frac{i\lambda f(t)}{2} (2\Omega_1 \hat{S}_+^{(1)} - \hat{S}_z^{(1)} \hat{S}_+^{(2)} + \hat{S}_z^{(2)} \hat{S}_+^{(1)}), \\ \frac{d\hat{S}_-^{(1)}}{dt} &= -\frac{i\lambda f(t)}{2} (2\Omega_1 \hat{S}_+^{(1)} - \hat{S}_z^{(1)} \hat{S}_+^{(2)} + \hat{S}_z^{(2)} \hat{S}_+^{(1)}), \end{aligned} \quad (5)$$

where the operators in Eq. (5) are satisfy the following communication relations

$$[\hat{S}_+^{(j)}, \hat{S}_-^{(j)}] = \hat{S}_z^{(j)} \delta_{ij}, \quad [\hat{S}_z^{(i)}, \hat{S}_\pm^{(j)}] = \pm 2\hat{S}_\pm^{(j)} \delta_{ij}, \quad [\hat{S}_+^{(i)}, \hat{S}_+^{(i)}] = I \quad (\hat{S}_\pm^{(j)})^2 = 0, \quad (\hat{S}_z^{(j)})^2 = I.$$

The second qubit equations can also be obtained by replacing the superscript in the above equations index (1) by index (2). From equations (5) it is easy to see that $\hat{S}_z^{(1)} + \hat{S}_z^{(2)} = \hat{c}$, is constant of motion. The constant of motion enable us to find the solution of equation (5). Upon above results the wave function corresponding to the transferred Hamiltonian (4) can be obtained as

$$|\bar{\psi}_{AB}(t)\rangle = \exp\left[-i \int_0^t H(\tau) d\tau\right] |\bar{\psi}_{AB}(0)\rangle \quad (6)$$

where $|\bar{\psi}_{AB}(0)\rangle = \cos(\varepsilon)|eg\rangle + \sin(\varepsilon)\exp(i\varphi)|ge\rangle$, with ε is the atomic state position and φ is the relative phase between the atomic states. Eq.(6) will be changed under the transformation (3). Using the inverse of the transformation (3), we can obtain the TCAs density matrix $\hat{\rho}_{AB}(t) = |\bar{\psi}_{AB}(t)\rangle\langle\bar{\psi}_{AB}(t)|$, therefore the proposed quantum quantifies or the statistical quantities will be defined and obtained in the next section.

3. Atomic Fisher information and phase space entropy

Here we use the Q -function which will be the basis for calculating the atomic Wehrl entropy. This quasiprobability distribution is defined as [31,32]

$$Q_A(\Theta, \Phi, t) = \frac{1}{2\pi} \langle \Theta, \Phi | \hat{\rho}_{AB}(t) | \Theta, \Phi \rangle, \quad (7)$$

where $|\Theta, \Phi\rangle$, is the atomic coherent state based on the atomic phase space parameters Θ, Φ as

$$|\Theta, \Phi\rangle = \sum_{m=0}^{2\ell} \sqrt{\binom{2\ell}{m}} (\cos(\Theta/2))^m (\sin(\Theta/2)e^{-i\Phi})^{2\ell-m} |m\rangle, \quad (8)$$

for the TCAs then the spin $\ell = 3/2$, one can recast equation (7) in the case of single two-level atom (i.e. $\ell = 1/2$) in the following form:

$$Q_A(\Theta, \Phi, t) = \frac{1}{2\pi} \left\{ \cos(\Theta/2) |1\rangle + \sin(\Theta/2) e^{-i\Phi} |0\rangle \right\}, \quad (9)$$

where $|1\rangle$ ($|0\rangle$) is the upper (lower) level of the two-level atom.

In a parallel definition for the field Wehrl entropy the atomic Wehrl entropy (AWE) has defined as [31,32]

$$S_{AW}(t) = - \int_0^{2\pi} \int_0^\pi Q_A(\Theta, \Phi, t) \ln(Q_A(\Theta, \Phi, t)) \sin \Theta d\Theta d\Phi \quad (10)$$

The AFI has defined from analog to the FI of the as a promised measure in the atomic phase space of the quantum system and depends on the atomic state estimation [25]

$$I_{AF}(t) = \sum_{j=1}^2 \int_0^{2\pi} \int_0^\pi \left\{ \sqrt{Q_A(\Theta, \Phi, t)} \sigma_{g_j}(t) \frac{\partial \ln Q_A(\Theta, \Phi, t)}{\partial g_j} \right\}^2 \sin \Theta d\Theta d\Phi, \quad (11)$$

where $\sigma_\Theta^2, \sigma_\Phi^2$ are the variances for the atomic phase space variables Θ, Φ given by $\sigma_{g_j}(t) = \int_0^{2\pi} \int_0^\pi Q_A(\Theta, \Phi, t) g_j \sin \Theta d\Theta d\Phi$, with $(g_1, g_2) = (\Theta, \Phi)$ respectively. The flow of the AFI is defined as $\frac{\partial}{\partial t} I_{AF}(t)$.

To measure the amount of single atom entanglement [33,34], can be quantified by the standard VNE measure. Let the bipartite system denoted by ρ_{AB} , the VNE is given by [35,36]

$$S_V = -\text{Tr}\{\rho_A \ln(\rho_A)\}, \quad (12)$$

where $\rho_A = \text{Tr}_B(\rho_{AB})$, is the reduced density for the atom A which is given by

$$\rho_A = \text{Tr}_B(\rho_{AB}) = \begin{bmatrix} \rho_{11} + \rho_{22} & \rho_{13} + \rho_{24} \\ \bar{\rho}_{13} + \bar{\rho}_{24} & \rho_{33} + \rho_{44} \end{bmatrix}. \quad (13)$$

Therefore, the VNE in terms of the eigenvalues of the atomic density matrix ρ_A [39]

$$S_V = -\eta_1 \ln \eta_1 - \eta_2 \ln \eta_2, \quad (14)$$

where S_V varies from zero value for the separable state to $\ln(2)$, for a maximally entangled state.

4. Numerical Results and Discussion

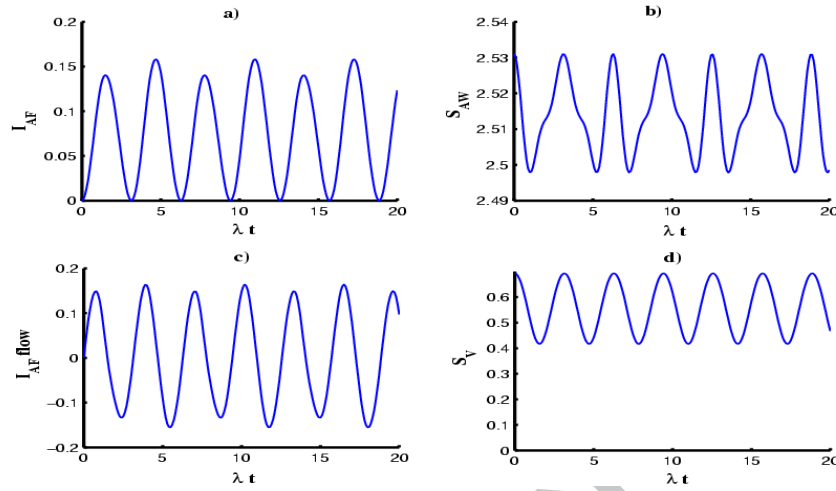


Fig. 1: The dynamics of the single atom: (a) the AFI donated by $I_{AF}(t)$, (b) the AWE $S_{AW}(t)$, (c) the $I_{AF}(t)$ flow, and (d) the entanglement measured by the von Neumann entropy $S_V(t)$. The initial atomic state parameter $\varepsilon = \pi/6$, and $\varphi = \pi/4$. The magnetic fields effect is ignored (i.e. $\bar{\lambda}_j/\bar{\omega}_j = 0$ for $j = 1, 2$) and the constant coupling between the TCAs (i.e. $f(t) = 1$).

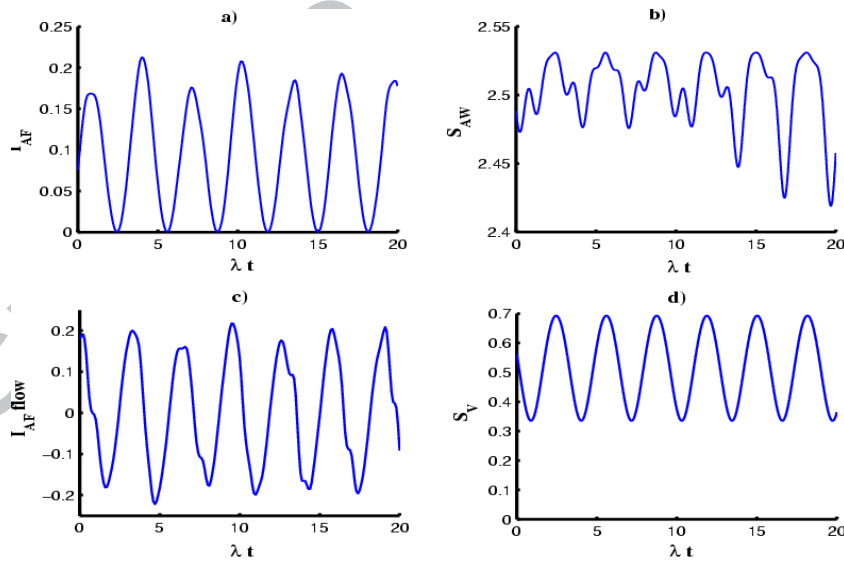


Fig. 2: The same as figure 1 but the influence of the magnetic fields is considered (i.e. $\bar{\lambda}_j/\bar{\omega}_j = 1$).

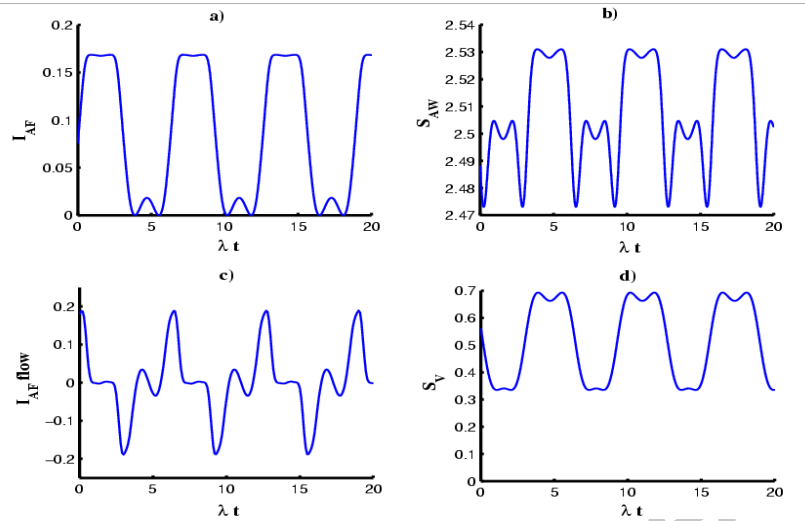


Fig. 3: Effect of the time-dependent coupling between the TCAs (i.e. $f(t) = \cos(t)$) on the dynamical behavior of the single atom: (a) the AFI $I_{AF}(t)$, (b) the AWE S_{AW} , (c) the AFI flow and (d) the von Neumann entropy $S_V(t)$. The initial atomic state parameter $\varepsilon = \pi/6$, and $\varphi = \pi/4$. The magnetic fields effect is considered (i.e. $\bar{\lambda}_j/\bar{\omega}_j = 1$ for $j = 1, 2$).

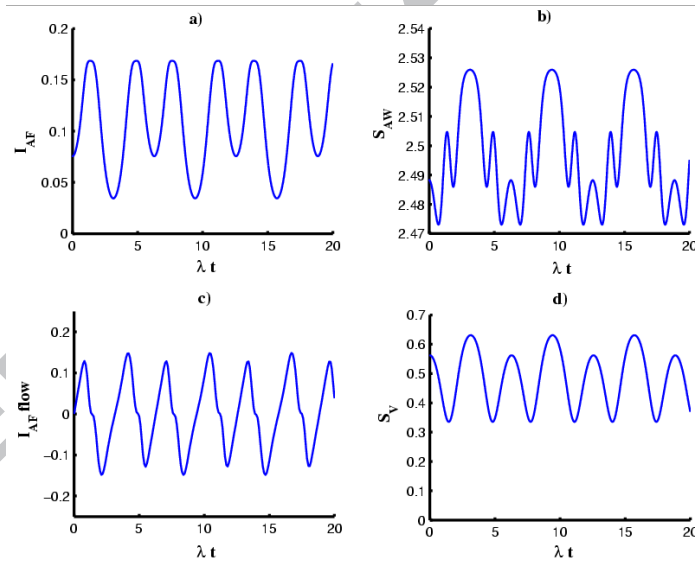


Fig. 4: The same as figure 3 but the time-dependent coupling between the TCAs is $f(t) = \sin(t)$.

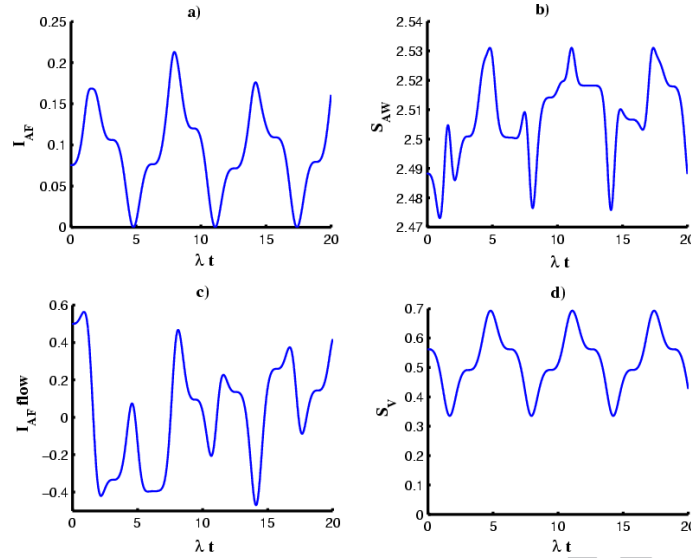


Fig. 5: The same as figure 3 but the time-dependent coupling between the TCAs is $f(t) = \sin^2(t)$.

In Figs. 1–5, we present our main results by exhibiting the influence of the physical parameter s on the AFI, the AFI flow, the AWE and the von Neumann entropy. A reasonable comparison between the results enables us to understand the contribution of the external magnetic fields and time-dependent coupling between the TCAs on the evolution of the different physical quantities.

In Fig. 1, the respective variations of the S_V , S_{AW} , I_{AF} , and I_{AF} flow are plotted against the scaled time λt in the absence of the external magnetic fields (i.e. $\bar{\lambda}_j/\bar{\omega}_j = 0$) and constant coupling between the TCAs. As seen that when we set the parameter $\bar{\lambda}_j/\bar{\omega}_j$ equals to zero value the AFI has zero value corresponding to the maximum entanglement between the TCAs at the periodic time $\lambda t = m\pi$ where $m = 0, 1, 2, \dots$ while the AFI tends to its maximum value which corresponding to the disentanglement between the TCAs at the half of periodic time (i.e. $\lambda t = (m + 1/2)\pi$). Also, the regular fluctuations over the moving envelope in which the fluctuations are with similar amplitudes. In addition, the AWE also has a regular oscillation same as the behavior of the AFI where the function of AWE fluctuates between the maximum values and minimum values in the time interval consideration as observed in Fig. (1b).

To understand the system better we reset in figure 2 the parameter $\bar{\lambda}_j/\bar{\omega}_j$ to nonzero value to check the effect of the external magnetic fields within the case of constant coupling between the TCAs (i.e. $f(t) = 1$). The external magnetic fields has a clear effect on the dynamical properties of the AWE compared with the other quantities. It is noticed that after adding the external magnetic fields the function of AFI oscillates between a maximum and minimum values with a periodic and regular behavior during the time evolution. Also, we see that the maximum values of AFI are increased after

the external magnetic fields taken into account (see Fig. (2a)). On the other hand, we see that the external magnetic fields have more effect on the dynamics of AWE in the beginning time where the amplitude of oscillations are reduced compared with the previous case. Over a short time, the amplitude of the oscillation is increased and the effectiveness of the external magnetic field decreases as observed in Fig. (2b). The AFI flow has a positive value in the case of increasing the AFI and negative value in the case of decreasing of AFI. Interestingly, it found that the monotonic behavior between AFI flow and entanglement measured by the von Neumann entropy. This results clarified that the single atom entanglement can be detected by the AFI flow of the system under consideration (see figure 2).

Now we come to discuss the effect of the type of time dependent coupling between the TCAs which presented in figures 3,4,5. In figure 3 we have presented the effect of the time dependent coupling with the cosine shape function case (i.e. $f(t) = \cos(t)$). It can be seen that the statistical quantities have a more regular and periodic during the time evolution. Also, it depicts an opposite behavior between the AFI and the VNE. As seen from the comparison between the temporal behavior of the AFI (a) and the VNE (d) are approximately connected by the relation $S_V(t) \approx 0.34 + I_{AF}(t)$. Then S_V and S_{AW} , have a similar behavior at the time points $\lambda t = (2m + 3/2)\pi$, $m = 0,1,2,\dots$ and for the time $\lambda t = (2m + 1/2)\pi$ both of them have an opposite behavior.

To visualize the effect of changing the shape function of the time dependent coupling from the cosine case to sine case we have in figure 4 the variation of quantum quantities as a function of the scaled time λt . It is regarded that the nonlocal correlation decreases during the time dynamics. Also, the AWE has a more meaningful in the quantifying the amount of single atom entanglement under this type of time dependent coupling.

Now we focus the light of a realistic and experimental situation in the interaction between the TCAs in the presence of two external magnetic fields. In this kind of coupling the alignment or orientation of the molecular or atomic dipole moment using laser pulse and motion of the atom through the cavity [33,37,38]. This case of time dependent coupling has been considered in different investigations. The main results that the entanglement is enhanced for this kind of time dependent coupling. The behavior of the statistical quantities being a quasi periodic and regular. The maximum entanglement corresponds to the maximum value of the AWE and minimum value of the AFI (see figure 5).

5. Conclusions

To summarize, we have presented the model of an atom-atom interaction in the presence of an external magnetic field in x and z directions by exploiting different cases of time dependent coupling. We have investigated in detail the variation of single atom entanglement, atomic Wehrl entropy and Fisher information. The atomic Fisher information and its flow have used to quantify the entanglement and dynamical properties of the system. We have shown that the different quantifiers are very sensitive to the type time dependent coupling between the two coupled atoms. Also, the entanglement can be indicated by the flow of atomic Fisher information or the atomic Wehrl entropy. In addition the control and manipulation of these quantities greatly benefit in the presence of an external magnetic field and the kind of coupling among the two atoms. Furthermore, we have shown that all the statistical quantities has a monotonic or opposite monotonic behavior which depends of the type of time dependent coupling between the two atoms. It is expected that our theoretical results can provide an important reference to implement different tasks of quantum optics and information.

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