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Procedia Engineering 199 (2017) 778–783

Procedia Engineering

www.elsevier.com/locate/procedia

X International Conference on Structural Dynamics, EURODYN 2017

Influence of the mechanics of escape on the instability of von Mises truss and its control

Diego Orlando[a]*, Paulo B. Gonçalves[b], Stefano Lenci[c], Giuseppe Rega[d]

[a]Department of Mechanics and Energy– FAT, University of State of Rio de Janeiro, UERJ, Resende, Brazil
[b]Department of Civil Engineerging, Catholic University,PUC-Rio, Rio de Janeiro, Brazil
[c]Department of Civil and Building Engineering, and Architecture, Polytechnic University of Marche, Ancona, Italy
[d]Department of Structural and Geotechnical Engineering, Sapienza University of Rome, Rome, Italy

Abstract

The elastic von Mises truss model is a prototype for bi-stable structures. It allows a deep understanding of the static and dynamic buckling of several planar and spatial truss systems and shallow lattice shell structures, including the geodesic dome, and has a theoretical and practical interest. This structure has a highly nonlinear response in the presence of static and dynamic loads. The geometric nonlinearity is particularly significant even at low load levels when this structure is shallow. This paper presents an exact nonlinear formulation, which is used to investigate the mechanics of erosion and escape from the safe pre-buckling well. As the static pre-load increases, the probability of escape increases in a nonlinear manner. Permanent and transient escapes, as well as the influence of random noise, on the dynamic buckling load are investigated. To increase the load carrying capacity, a method for controlling the global nonlinear dynamics for the elastic von Mises truss is employed. The control method consists of the (optimal) elimination of homoclinic intersection by properly adding superharmonic terms to a given harmonic excitation. Permanent and transient basins of attraction are obtained. The results highlight the complex nonlinear dynamics of this class of structures and the effectiveness of the control method in increasing the integrity of the basins of attraction of the system and its practical safety.

Keywords: Von Mises Truss, Instability, Transient Escape, Random Noise, Dynamical Integrity, Control.

1. Introduction

Shallow structures, such as shells, arches and reticulated structures, belong to a group of structures commonly used to span large spaces. Generally, they have a highly nonlinear behavior with increasing loss of effective stiffness in the presence of compressive stresses, which lead to limit point instability or unstable symmetric bifurcation along

* Corresponding author. Tel.: +55-21-99647-8627 fax: +0-000-000-0000 .
E-mail address: dgorlando@gmail.com

1877-7058
10.1016/j.proeng.2017.09.048

the nonlinear equilibrium path before reaching the first limit point [1]. This nonlinearity leads to multiple potential wells and a complex dynamic behavior and loss of stability under dynamic loads well below the static critical load, making, therefore, of vital importance the investigation of the nonlinear dynamic behavior of these structures in engineering and applied mechanics [2]. Among them are shallow pyramidal trusses, whose planar example is the well-known von Mises truss [3-5]. The static stability of this class of structures was studied by [1, 3, 4], among others. Recently, the deterministic nonlinear dynamics of these structural systems was studied by Orlando et al. [5], to identify the common types of instability and indicate the essential parameters for a safe design.

However, only a deterministic analysis is not enough and, according to Silva and Gonçalves [6], the influence of uncertainties and random noise on the evolution and stratification of the basins of attraction must also be addressed to identify a safe load level for design (see also [7]). Soliman and Thompson [8] studied the influence of random noise superimposed to a deterministic harmonic excitation on the evolution of the basin of attraction and quantified its effect in terms of a stochastic integrity measure. The influence of both random noise and system parameter uncertainties on the dynamic instability of structural systems liable to buckling was discussed in Gonçalves and Santee [9] and Silva and Gonçalves [6]. The influence of uncertainties and random noise is particularly important in the vibration control of dynamic systems with multiple coexisting attractors. For systems under coexisting attractors, also the influence of transient escape from a given potential well must be investigated [2, 10, 11]. In each case, the evolution of the basins of attraction as a function of a control parameter can be properly quantified by the dynamic integrity measures, as shown by Rega and Lenci [12]. The dynamic integrity of a desired solution can be however increased and escape delayed by the use of appropriate control techniques. In this paper, the effect of random noise and transient escape in the von Mises truss is analyzed, and a method for controlling global dynamics, recently developed and tested on various mechanical systems [12, 13], is applied.

2. Formulation

The nonlinear behavior and stability under static and dynamic loads of the von Mises shallow truss [4] is analyzed in this paper. The model (Fig. 1) is composed of two identical elastic bars of length L ($L^2 = H^2 + B^2$), axial stiffness EA and mass per unit length m. The shallow truss has a height H and span $2B$, with ($H/2B$)<<1.

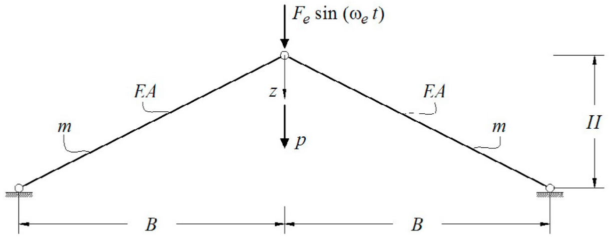

Fig. 1. Elastic von Mises truss model.

The potential energy of the truss under a static vertical load p is given by [9]

$$V = EA/4L^3\,[(H-z)^2 - H^2]^2 - pz \quad (1)$$

Adopting the nondimensional parameters $w = z/H$, $\alpha = B/H$ and $\lambda = p/EA$, the nonlinear equilibrium path is:

$$\lambda = w(2-w)(1-w)/\left(\sqrt{1+\alpha^2}\right)^3 \quad (2)$$

The nonlinear response is illustrated in Fig. 2(a) for $\alpha = B/H = 1/\tan(15°)$. The observed nonlinear behavior is typical of shallow structures. For a prescribed static load level p, between the two limit points, there are three equilibrium positions W_i (i=1,2,3) (two stable and one unstable). Fig. 2(b) shows the potential energy of the system under a load level equal to 30% of the snap-through load. Assuming as a reference a static equilibrium configuration w_i, the nondimensional potential energy $\overline{V}$ can be written as

$$\bar{V} = \frac{1}{2}w^2\left(1 - 3w_i + \frac{3}{2}w_i^2\right) - \frac{1}{2}w^3(1 - w_i) + \frac{1}{8}w^4 \tag{3}$$

For the loaded structure the first safe pre-buckling well is smaller than the second well associated with the buckled configuration. So, when considering a vertical harmonic excitation $F_e \sin(\omega_e t)$, preventing the escape from the pre-buckling well becomes an important design issue. To this aim, we intend to control the global bifurcation associated with the homoclinic orbit defining the safe potential well (one-side control) as suggested in [12], namely by adding optimal superharmonics to the basic harmonic excitation. The equation of motion of the pre-loaded structure under harmonic excitation is given by:

$$w_{,\tau\tau} + 2\xi w_{,\tau} + w\left(1 - 3w_i + \frac{3}{2}w_i^2\right) - \frac{3}{2}w^2(1 - w_i) + \frac{1}{2}w^3 =$$

$$F\,sin(\Omega\tau) + G(\tau; F, \Omega) + F\left[\sum_{j=2}^{n}\frac{F_j}{F}sin\left(j\Omega\tau + v_j\right)\right] \tag{4}$$

Here, $\tau = \omega_o t$, $c/m = 2\xi\omega_o$ and $\Omega = \omega_e/\omega_o$, where $\omega_o = \sqrt{2EAH^2/mL^3}$ is the natural frequency of the unloaded structure, ω_e is the excitation frequency, c is the damping, $G(\tau; F, \Omega)$ is the random noise, $F = F_e/mH\omega_o^2$ is the amplitude of the dynamic excitation, and F_j/F, v_j are the superharmonics control parameters.

The random term $G(\tau; F, \Omega)$ depends on the deterministic forcing magnitude F and frequency Ω. For the numerical calculations, the non-deterministic term in Eq. (4), $G(\tau; F, \Omega)$, is considered as a stationary and ergodic continuous stochastic process in time [6] and has a zero expected value; that is, $E[G(\tau; F, \Omega)] = 0$. It is assumed, without loss of generality, that the random term has the following spectral density function

$$\Phi_{GG}(\overline{\omega}) = \sigma_{GG}^2/2\omega_l \quad for \quad \Omega - \omega_l/2 < \overline{\omega} < \Omega + \omega_l/2 \tag{5}$$

where σ_{GG}^2 is the variance of the random force amplitude and ω_l is the frequency bandwidth of the excitation frequency. Additionally, the random force amplitude is assumed proportional to the deterministic one; i.e., $\sigma_{GG} = \delta F$, where δ is the standard deviation parameter. The numerical algorithms used here can be found in [6].

3. Nonlinear dynamics, system safety, and control

Figure 2(c) reports homoclinic orbits that delimit the two potential wells of the pre-loaded von Mises truss with a static load level equal to 30% of the snap-through load (stable pre-buckling position $w_i = 0.0636$). The safe pre-buckling potential well of the von Mises shallow truss is bounded by the left homoclinic orbit. The corresponding safe basin decreases with the static load level and vanishes at the limit point. Thus, as the static load increases, increases the probability of escape from the pre-buckling well due to dynamic perturbations of the reference static solution. A discussion of the overall dynamical scenario for the uncontrolled case is reported in [5]. Figure 2(d), where the escape boundary in the main resonance region for gradually applied load is depicted together with the loci of the relevant bifurcations, summarizes the dynamic behavior of the von Mises shallow truss. The homoclinic bifurcation occurs for load levels much lower than the escape loads of the slowly evolving system. However, at this point begin the unwanted dynamic phenomena that entail erosion of the safe basin and, after a sequence of dynamical events, finally lead to escape.

Under dynamic loads, the von Mises shallow truss may fail at load levels well below the theoretical limit point load. The escape may be permanent or transient and reduces the safety and the dynamic integrity of the structure; this often occurs as a consequence of the erosion of the basins of attraction of the safe pre-buckling solutions. So, investigating the escape from the pre-buckling well becomes an important issue in design for these structural systems. Transient escape occurs when the structure escapes from the safe well during the initial transient response but returns to it during the steady-state response. Transients in nonlinear structural systems may be rather long and may lead to large cross-well motions and stress peaks, leading possibly to damage or even to ruin of the structure.

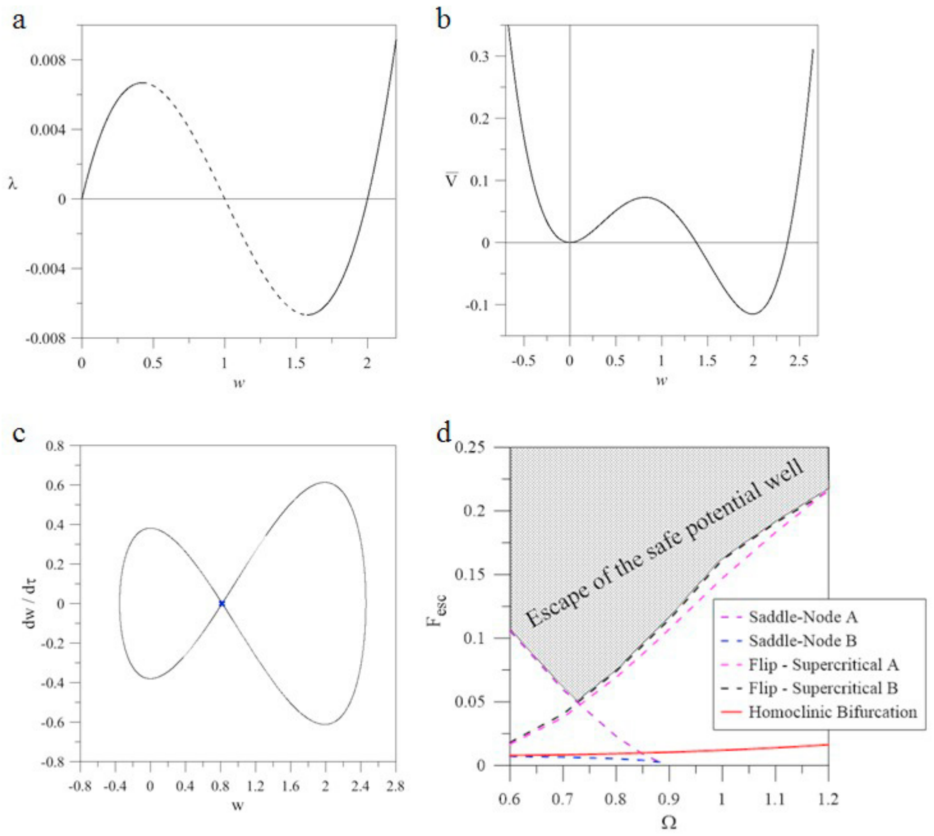

Fig. 2. (a) Nonlinear equilibrium path; (b) potential energy of the loaded truss; (c) homoclinic orbits that delimit the two potential wells of the pre-loaded von Mises truss; undamped system, $\xi = 0$; (d) behavior chart in the main resonance region for gradually applied load, $\xi = 0.01$.

An example of transient escape is shown in Fig. 3(a) for a suddenly applied load. The system response escapes from the first potential well during the transient response, but returns to this well after a rather long transient characterized by large amplitude cross-well motions. The red line represents the magnitude (coordinate) of the saddle point that delimits the potential well, see Fig. 2(c). Figure 3(b) shows the escape boundary of the suddenly loaded system, where the colors represent the load conditions (F and Ω) for which transient escape occurs within the indicated time interval and the white region corresponds to conditions that do never escape from the safe well. The dashed black line represents the escape when considering a gradually applied load (Fig. 2(d)). This chart shows how much the von Mises truss is sensitive to an abrupt loading, especially in the regions below the dashed black line.

The effect of a suddenly applied load and the transient escape is visualized more clearly through the basins of attraction. Figure 4 compares the basins of attraction obtained by considering either the permanent escape with (a) harmonic loading and (b) harmonic loading plus random noise or (c) the transient escape, for $F = 0.02$, $\xi = 0.01$, $\omega_l = 0.05$ and $\delta = 0.1$. The white region in Fig. 4(b) corresponds to initial conditions for which the final outcome (attractor) is uncertain due to the considered level of random noise for a sufficiently large number of simulations. The transient basin excludes all points outside the safe compact region around the attractor, which lead necessarily to transient oscillations within the second well or cross-well motions. As observed here, random noise has a marked influence on the basin topology, reducing its area, and the consideration of transient escape entails a further reduction. By building a sufficient number of basins of attractions it is possible to construct the erosion profiles [12], which report a measure of the dynamical integrity of the pre-buckling basin (herein characterizing the safety of the structure) versus the increasing excitation amplitude. Figure 5(a) shows the variation of the global integrity measure, GIM, which considers the entire safe area [12] as a function of the load magnitude in the deterministic and non-deterministic cases for a slowly increasing load and in the transient escape case under a suddenly applied load. The integrity measure is normalized with respect to its magnitude for $F = 0.0$. The random noise leads to an almost linear decrease in the safe area. Consideration of transient escape leads to a strong decrease of the GIM measure for any load level. In contrast, the profile (not reported) of the integrity factor IF, which measures the radius of the largest circle inscribed in the same area [12], is practically the same in the three cases. Indeed, the IF rules out from the integrity evaluation the fractal parts of the basins, which clearly do not contribute to the safety of the structure. The GIM integrity profiles are reported in Fig. 5(b) for the controlled and uncontrolled system. These numerical curves clearly show that the theoretical gain of about 40% (with one controlling superharmonic) and 60% (with two controlling superharmonics) in the homoclinic bifurcation threshold [13] also has a meaningful beneficial effect in terms of dynamical integrity, even in the case of a single controlling superharmonic. Indeed, in the controlled model, either the safe basin erosion starts later or it is even prevented, depending on the considered integrity measure, thus overall increasing the magnitude of the disturbances the structure can undergo without practically losing its stability and consequently its safety margin in a dynamic environment.

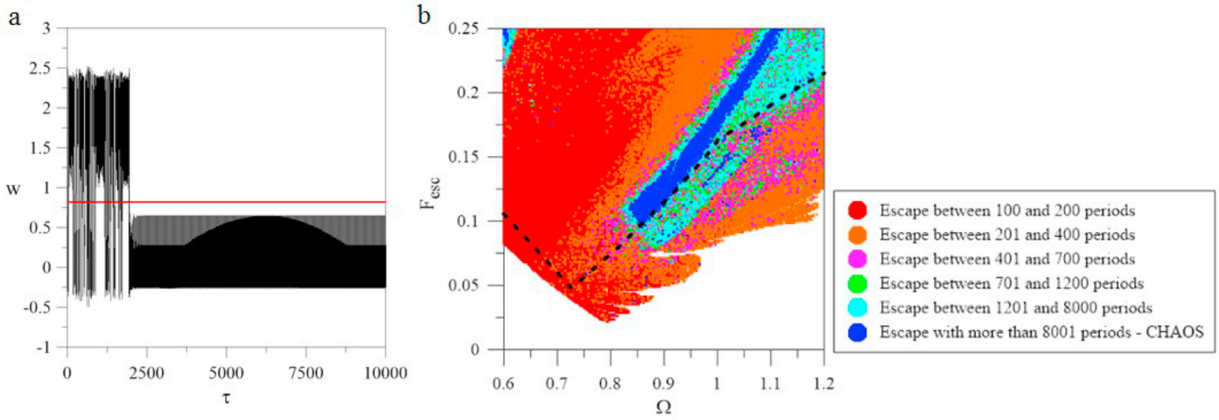

Fig. 3. (a) Time history, $F = 0.0825$, $\Omega = 0.902889$ and $\xi = 0.01$.The red line indicates the saddle point; (b) Escape boundary for the suddenly loaded structure, $\xi = 0.01$.

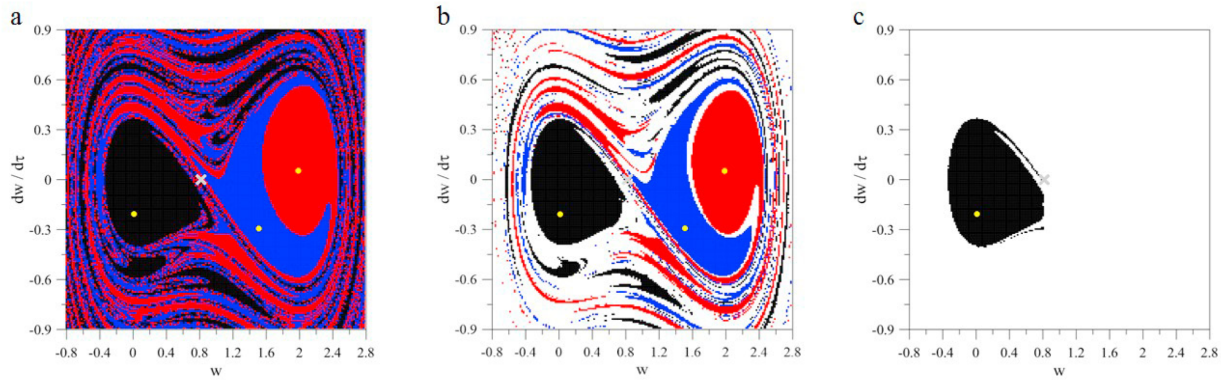

Fig. 4. Basins of attraction. (a) Harmonic load; (b) harmonic load plus random noise; (c) transient escape. $F = 0.02$, $\Omega = 0.902889$, $\xi = 0.01$, $\omega_l = 0.05$ and $\delta = 0.1$.

4. Summary

The mechanics of escape from a safe pre-buckling well in the shallow von Mises truss has been analyzed by considering a gradually or suddenly applied harmonic excitation, with also the possible addition of a random noise in the former case, and by evaluating their influence on the system safety, which is strongly affected by the transient escape entailed by sudden excitations. Dynamic integrity is evaluated by the global integrity measure, GIM, which considers the entire safe area, and the integrity factor, IF, which measures the radius of the largest circle inscribed in the same area. The results show that random noise and transient escape have a marked influence on the GIM, decreasing the basin area, but have small influence on the IF. In the case of a gradually applied harmonic excitation, a control method of the global dynamics is also applied, showing that the robustness of safe pre-buckling solutions in a dynamic environment, and thus the structure "practical" safety, are actually increased, even though the actual amount of the improvement depends on the adopted measure of dynamic integrity.

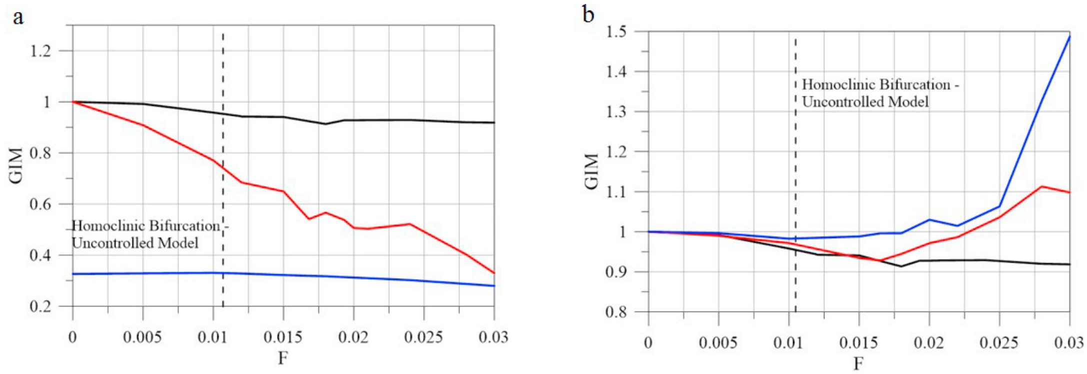

Fig. 5. Integrity profile GIM for increasing load. (a) Uncontrolled case. Black line: harmonic load, red line: harmonic load plus random noise ($\omega_l = 0.05$ and $\delta = 0.1$) and blue line: transient escape; (b) Black line: uncontrolled case, red line: controlled 1 (one superharmonic, $F_2 = 1.1605F$ and $\nu_2 = \pi$) and blue line: controlled 2 (two superharmonics, $F_2 = 2.5756F$, $\nu_2 = \pi$, $F_3 = 4.9331F$ and $\nu_3 = 0$); no random noise. $\Omega = 0.902889$ and $\xi = 0.01$.

Acknowledgements

The authors acknowledge the financial support of CAPES, CNPq and FAPERJ-CNE.

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