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On the design, elastic modeling and experimental characterization of novel tensegrity units

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Received 13 July 2018
Accepted 23 July 2018

Abstract

Purpose – This study aims to focus on a short review on recent results dealing with the mechanical modelling and experimental characterization of a novel class of tensegrity structures, named class $\theta = 1$ tensegrity prisms. The examined structures exhibit six bars connected by two disjoint sets of strings.

Design/methodology/approach – First, the self-equilibrium problem of tensegrity $\theta = 1$ prisms is numerically investigated for varying values of two aspect parameters and, next, their prestress stability is studied. The mechanical behavior of the examined structures in the large displacements regime under uniform compression loading is also numerically computed through a path-following procedure. Finally, the predicted constitutive response is validated through experimental tests.

Findings – The presented results highlight that the examined structures exhibit a large number of infinitesimal mechanisms from the freestanding configuration, and reveal that they exhibit tunable elastic response switching from stiffening to softening.

Originality/value – This multi-faceted elastic response is in agreement with previous literature results on the elastic response of minimal tensegrity prism, and suggests that such units can be usefully used as non-linear springs in next-generation tensegrity metamaterials.

Keywords Class theta tensegrity prism, Lattice metamaterials, Form-finding, Prestress stability, Path following, Large displacements

Paper type Review paper

1. Introduction

Over the past decade, the research area of tensegrity metamaterials has drawn significant interest due to their unconventional behaviors which are often impossible to be achieved with natural materials (Skelton and De Oliveira, 2010; Fraternali *et al.*, 2012; Amendola *et al.*, 2015; Skelton, 2002; Skelton and de Oliveira, 2010a; Skelton and de Oliveira, 2010b; Carpentieri *et al.*, 2017; Skelton *et al.*, 2014; Carpentieri *et al.*, 2015). Their elastic response, for example, may range from stiffening (specific to crystalline solids) to softening response (specific to foams),

The authors acknowledge financial support from the Italian Ministry of Education, University and Research (MIUR) under the "Departments of Excellence" grant L.232/2016.

depending on the architecture of the unit cell, the load arrangements and prestress control as opposed to their chemical composition (Fraternali *et al.*, 2014; Amendola *et al.*, 2014; Donahue *et al.*, 2014; Fraternali and Amendola, 2017; Mascolo *et al.*, 2018). The present work presents numerical and experimental investigation on the mechanical characterization of a special class of tensegrity metamaterials (Mascolo *et al.*, 2018; Modano *et al.*, 2018; Bieniek *et al.*, 2017; Bieniek, 2017a) obtained by the superimposition of two minimal tensegrity prisms (cf. Section 2). The examined structures are formed by discontinuous sets of tensile elements, which connect two sets of three compressed members. We have analysed two different physical models that correspond to thick and slender aspect ratios. First of all, we studied the existence of freestanding placements in absence of external loads, i.e. self-equilibrium configurations (Tilbert and Pellegrino, 2011) of the class $\theta = 1$ tensegrity prisms by varying values of two aspect parameters: the relative twisting angle between the terminal bases and the inclination above the horizontal plan of the internal strings (cf. Section 2.3). Local solutions of the self-equilibrium problem are numerically derived through Newton–Raphson method. Next, we solved the kinematic problem searching for possible infinitesimal mechanisms of the analysed physical model of the structure. The prestress stability of the structure is related to the positive definiteness of geometric stiffness matrix in correspondence to each possible mechanism (cf. Section 2.4). The following investigation of the elastic response of class $\theta = 1$ tensegrity prisms in large displacements regime under a uniform uniaxial loading took place. The incremental equilibrium problem is numerically solved through a path-following approach (cf. Section 2.5). Finally, an experimental validation of a thick and a slender model under quasi-static axial compression loading has been presented in Section 3. In the same section, the numerical results are compared with the experimental ones and subsequently discussed. Concluding Remarks section points out that the experimental results are in close agreement with numerical solutions, and highlights the multi-faceted elastic response of class $\theta = 1$ tensegrity prisms that can switch from softening to stiffening. Such peculiar response can be usefully exploited for the design and construction of novel tensegrity meta-materials (Fraternali *et al.*, 2014; Amendola *et al.*, 2018; Spadoni and Daraio, 2010; Theocharis *et al.*, 2013; Herbold and Nesterenko, 2013).

2. Materials and methods

2.1 Physical models

This study considered two different physical models of class $\theta = 1$ tensegrity prisms, which correspond to the value of the design variables given in Figure 1(b) and (c). The difference lies in their shape: the first one has a thick aspect ratio (i.e. it has approximately equal values of the lengths of horizontal and vertical cables) and the other one has a slender aspect ratio (i.e. it has lengths of vertical cables that is double of the horizontal ones). According to the labelling and nomenclature introduced in Figure 1(a) and in the works of Mascolo *et al.* (2018) and Modano *et al.* (2018), these two prisms consist of 12 nodes, 6 compressible members (i.e. bars) with length b, two sets of 3 horizontal tensile members (i.e. cables) with length l, 3 cross cables with length v and 6 inner cables with length c.

2.2 Experimental setup

We performed quasi-static compression tests on two prism samples (S1 and S2) at the Strength's partner laboratory (Geo Consult s.r.l., Avellino, Italy) with a displacement control rate of 4 mm/min. The tested samples were mounted on a Matest® electromechanical testing machine equipped with 50 kN load cell, ensuring interposing a rotating lubricated base among the test sample and the machine base.

The test specimens were manufactured using M4 threated bars made of zinc plated grade 4.8 steel (DIN 976-1) with a nominal cross-sectional area of 8.78 mm^2 and 203.53 GPa of Young modulus; Spectra® cables with 0.76 mm and 0.36 diameter in samples S1 and S2, respectively, 30.00 GPa of Young Modulus and 2 GPa yield stress. The samples assembled through the procedure illustrated in Modano *et al.*'s study (2018) are shown in Figure 1(b) and (c).

2.3 A numerical approach to the form-finding problem

For the purpose of studying the mechanics of the examined units, we first run a search for freestanding configurations. They are the placements in which the structure is in equilibrium under the action of zero external forces and under self-equilibrated internal forces induced by cables pre-stretch. We studied the self-equilibrium problem varying the value of two aspect parameters α and β. The latter represent the twisting angle between the top and bottom bases and the slope of the internal strings relative to the horizontal plane [Figure 1(a)], respectively. For this purpose, we introduced the Gramian **G** (Gentle, 2017) of the equilibrium matrix **A** (Mascolo *et al.*, 2018):

$$G = A^T A \tag{1}$$

and searched for local minimum of the function:

$$f(\alpha, \beta) = \det(G) \tag{2}$$

where det($\boldsymbol{G}$) means the determinant of the Gramian matrix. By imposing the minimum condition and using Newton–Raphson iterations, we obtained some zero points corresponding to degenerate configurations ($\alpha = 0°$ and $\beta = 90°$) of no practical relevance, and some other zero points corresponding to non-degenerate freestanding configurations, which are the values that we are looking for. By assuming the values of the design variables of the analysed systems (S1 and S2), we obtained $\alpha = 24° \ 27'$ and $\beta = 16° \ 03'$ for unit S1 and $\alpha = 24° \ 27'$ and $\beta = 29° \ 32'$ for unit S2.

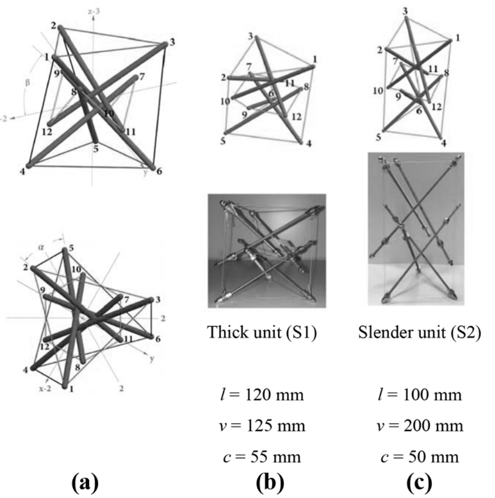

Figure 1.
(a) Reference configuration of a class $\theta = 1$ tensegrity prisms: 3D (top) and plan (bottom) views; (b and c) graphical (top) and physical (bottom) models of the analysed prisms

2.4 Prestress stability

This section presents the study of the possible infinitesimal mechanisms exhibited by the analysed structures. Such mechanisms are infinitesimal displacements from the freestanding configuration that do not induce elongations in all members. They are the elements of the null space of the transpose of the equilibrium matrix A (i.e. compatibility matrix A^{T}).

Computing the null space of A^{T}, we detected a large number of infinitesimal mechanisms for each of the examined structures. Excluding rigid body motion, they are equal to ten which highlights an interesting kinematic behavior. Furthermore, it can be easily established that these infinitesimal mechanisms can be suitably stabilized by applying a state of internal prestress. According to the nomenclature given in the works by Mascolo *et al.* (2018), Modano *et al.* (2018), Micheletti (2013) and Fraternali *et al.* (2015b), the prestress stability of the analysed structures can, indeed, be easily confirmed by checking positive definiteness of the quadratic form:

$$Q_m = M^T K_G M \tag{3}$$

of its geometric stiffness matrix K_{G} relative to the mechanisms M.

In both of the cases under examination (S1 and S2), the stability condition $Q_m > 0$ is verified, and then we can conclude that their freestanding configurations are prestress stable.

2.5. Elastic response under large displacements

In this section, the elastic response of the examined structures under a uniform uniaxial compression loading condition was investigated through a path-following algorithm. Such an algorithm provides solutions to the equilibrium problem in the large displacement regime, by adding a constraint to a Newton–Raphson linearized form of the incremental equilibrium problem (full details can be found in Mascolo *et al's* [2018] study). The imposed constraint equation enforces displacement-control loading.

3. Results and discussion

3.1 Numerical results

Figures 7 and 9 of (Mascolo *et al.*, 2018) illustrate the numerical predictions of the force (F) vs vertical displacements (w) responses of the tensegrity units S1 and S2 for different value of prestrain p_O. The hollow circle mark (°) denotes that the vertical strings get slack, the black circle mark (•) denotes the local Eulerian buckling of the bars and the breaking point of the F-w curves denotes the point at which the bars were close to touch each other. It should be underlined that the post-local buckling paths are just theoretical, as the numerical procedure presented in Section 2.5 does not model the post-buckling and post-yielding behavior of the analysed prisms.

For low values of the prestrain ($p_0 = 0.01$ per cent and $p_0 = 0.1$ per cent), both the systems S1 and S2 exhibit a stiffening branch near the origin, which corresponds to an increasing slope of the F-w branch, followed by a force-softening path. In unit S1, when the force reaches the maximum value, the tangent modulus rapidly decreases as the deformation increases with an unstable, descending path which leads the structure to a snap-buckling collapse or global buckling. As the prestrain gets bigger ($p_0 = 1$ per cent) we observe that the stiffness brunch tends to disappear; this is most apparent in the thick unit S1. It may also be noted that in the slender prism S2, the F-w curves corresponding to different values of the prestrain p_O are more close to each other.

Moreover, the axial displacements that the structure is able to reach are larger than the ones reached by unit S1. Furthermore, in the mechanical response of the slender unit S2, the locking phase (i.e. bars touching each other) anticipates the global snapping of the structure. Finally, it is worth noting that, for both the analysed systems, the local buckling of the bars took place long before the global collapse.

3.2 Experimental validation

Figures 12 and 13 (Mascolo *et al.*, 2018) of shows a comparison between observed and predicted force vs displacement curves of the analysed tensegrity prisms S1 and S2. For low values of the prestrain ($p_0 = 0.01$ per cent and $p_0 = 0.1$ per cent), the experiment revealed a satisfactory agreement with the predicted numerical response, confirming the tunable stiffening and softening behavior of the samples. In both samples, bars touched each other before the global buckling.

Finally, it should be emphasized that the breaking point of the experimental F-w curves anticipates the theoretical ones because of non-negligible size of the bars.

4. Concluding remarks

We have reviewed literature results on the statics, kinematics and elastic response of novel tensegrity units, named class $\theta = 1$ prisms. Such results included a numerical approach to the form-finding problem of the examined structures by varying two aspect parameters. The examined approach can be easily generalized to arbitrary tensegrity structures. The infinitesimal mechanisms of class $\theta = 1$ prisms have also been computed, obtaining that the freestanding configuration has static indeterminacy equal to one and kinematic indeterminacy equal to then. Such a configuration can be stabilized through a suitable set of self-equilibrated internal forces. The nonlinear elastic response of class $\theta = 1$ tensegrity prisms under compressive loading was also reviewed, through numerical and experimental results, by discovering an unconventional elastic behavior switching from stiffening to softening for increasing values of the internal prestress and the applied compression load. The multi-faceted elastic response of class $\theta = 1$ tensegrity prisms suggests that such units can be used in tensegrity metamaterials to be experimented as waveguides, impact protection systems, vibration isolation devices and sound proof materials (Mascolo *et al.*, 2018; Modano *et al.*, 2018).

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