

# Buckling of imperfect thin cylindrical shell under lateral pressure

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## Analysis and modelling

### ABSTRACT

**Purpose:** This paper investigates buckling behaviour of imperfect thin cylindrical shell with analytical and experimental assessment methods, in the context of NPP applications as, for instance, the IRIS LWR integrated Steam Generator (SG) tubes.

**Design/methodology/approach:** In this paper, thin shell, homogeneous and isotropic material, also tube geometric imperfections as eccentricity/ovality/welding are assumed to investigate the effects of latter on the limit pressure load in conditions for which, at present, a complete theoretical analysis was not found in literature. At Pisa University a research activity is being carried out on the buckling of thin walled metal specimen, with a test equipment (and the necessary data acquisition facility), suitable for carrying out test series on this issue, as well as numerical models implemented on the MARC FEM code, were set up.

**Findings:** The experiments were conducted on test specimens of the same material (AISI 316) tube with and without longitudinal welding. The comparison between numerical and experimental results, for the same geometry and loading conditions, shows a good agreement between the elastic-plastic finite-element predictions and the experimental data.

**Research limitations/implications:** The presented research results may be considered preliminary in the sense that it would be important to enlarge the statistical base of the results themselves, even if they are yet certainly meaningful to highlight the real problem, considering the relatively large variability of the geometrical imperfections also in high quality production tubes.

**Originality/value:** From the point of view of the practical implication, besides the addressed problem general interest in industrial plant technology, it is worth to stress that straight and curved axis tubes are foreseen specifically in innovative nuclear reactors SG design

**Keywords:** Numerical techniques

## 1. Introduction

The stability of circular cylindrical shells under uniform lateral pressure has been widely investigated. The behaviour of cylindrical shells under external pressure is very sensitive to geometric imperfections. There have been many theoretical studies investigating the strength of cylinders with specific imperfection forms, and it is well established that axisymmetric imperfections cause the greatest reductions in strength (Koiter 1963; Yamaki 1984).

Many researchers have studied buckling of circular cylindrical shell under external pressure and more accurate solutions of the present problem were obtained for short cylindrical shells and for anisotropic shells, respectively. However, actual calculations seem to be confined mainly to some special ranges of the shells geometries, boundary and loading conditions [1]. Moreover, the literature concerning unequal wall transition joints in thin shell is limited. When thin shells were subjected to external pressure, the collapse was initiated by yielding, which was often the dominant factor, but the interaction with the instability is meaningful. In

fact, the presence of imperfections reduces the load bearing capacity by an amount of engineering significance; so the classical elastic solution, like Timoshenko and Gere approach [2], appear to be not adequate. The major factors that affected the collapse pressure of pipes were the diameter-to-thickness ratio  $D/t$ , the Young's modulus and yield stress of the material in the circumferential direction, and initial imperfections in the form of ovality and wall thickness variations [3-4], as for eccentricity and presence of welding joint. The current paper examined the buckling issue of a thin circular cylindrical shell in the dimension range of possible interest for nuclear the steam generators (SG) of type foresee in the IRIS Reactor. IRIS is currently under way of preliminary design; it was an integral pressurized water reactor. Its reactor vessel houses not only the nuclear core, but also all the major reactor coolant system components including pumps, steam generators, pressurizer, control rod drive mechanisms and neutron reflector. The IRIS integral vessel is larger than a traditional PWR pressure vessel, but the size of the IRIS containment results to be smaller than that of corresponding loop reactors, resulting in a significant reduction in the overall size of the reactor plant [5]. The IRIS SGs were once through, with the primary fluid outside the tubes. In general buckling analysis is used to predict failure of long pipelines, subjected to external over-pressure. A selected configuration was used to perform the different analysis significant for the specified field of interests (numerical as were as experimental). The present study might also serve as a base for other loading models, which include different boundary and geometrical conditions. Buckling phenomenon occurs when most of the strain energy, which is stored as membrane energy, can be converted to the bending energy requiring large deflections. Nonlinear buckling pressures can be evaluated using a nonlinear stress analysis by observing the first change in the slope (i.e., stiffness of the structure) in the load–deflection curve [6-7]. In the nonlinear buckling analysis; however, an initial imperfection, either in terms of the geometry or load, is necessary to trigger the buckling phenomenon. The load carrying capacity of shell subjected to an external pressure could be characterised by the yield load. The first yield load was not considered as a failure pressure, because it was no equivalent to an immediate loss of stability.

## 2. Numerical analysis

The presence of imperfections, such as the ovality, the eccentricity or the variation of the thickness, due to presence of longitudinal welding, make elastic analysis inadequate to the purpose of determining the critical load, so a nonlinear analysis was required. In this paper of nonlinear analysis was performed using a numerical analysis, in which either load or displacement was used as a control parameter. This paper would address in the first case, the effect of imperfections, such as ovality or eccentricity, on buckling load, and in the second one the effect of a welded joint presence, which involves unequal wall thickness transition, on the strength of structure. Some different elastic-plastic buckling analyses were carried out by means of the finite element program MSC.MARC to evaluate the failure behaviour of the transition joint and the effects of geometry. The assumption of perfect plasticity permitted a better assessment of the effects of circumferential instability. The tube was a long cylindrical shell

of length  $L$ , uniform thickness  $t$ , diameter  $D$ ; the two ends were assumed as fixed hedges, under simultaneous action of external lateral pressure as sketched in Figure 1. The material was as previously said, AISI 316, in all applications (with or without longitudinal welding), homogeneous and isotropic with Young's modulus  $E = 200$  GPa and Poisson's ratio  $\nu = 0.3$ . The length was large enough in respect to the tube diameter to allow to disregard the end conditions. It was assumed that all variables involved in the analysis were constant along the tube length. This assumption is valid until localized buckle occurs, for long tubes, in which the effect of end constraints can be neglected.

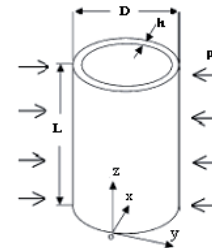


Fig. 1. Schematic thin shell under lateral pressure

In such cases, the models were realized with solid 3-D finite elements. The cylindrical surface was modeled by use of 20-node brick elements, because shell elements were not adequate to compute the contribution of radial compression. The adopted mesh involved 60 elements over the circumferential section, and 5 elements on the thickness; moreover, the mesh in the case of transition-welded joint was more refined in the welded area. In the numerical analyses a non-linear approach was adopted. Weld-induced residual stresses are usually discontinuous at the weld in a transition joint and can be significant in pipe-line [8]. However, appropriate simulation has indicated that the differences of the failure pressure at plastic collapse were not noticeable with or without weld residual stress, thus this local effect, in the present analysis, was not considered. The imperfections considered were of 20% of tube thickness for the eccentricity; 2.5% of nominal diameter for the ovality, while a thickness reduction of 12.5% (from the certified tolerance) was considered in the weld joint. In the hypotheses of cylindrical shell in a bendingless state, before buckling, and of homogeneous external pressure, the critical load for long thin shell was performed adopting the equation:

$$\frac{D}{h} \nabla^8 \omega + Ek_x^2 \frac{\partial^4 \omega}{\partial y^4} + 2Ek_x k_y \frac{\partial^4 \omega}{\partial x^2 \partial y^2} + Ek_y^2 \frac{\partial^4 \omega}{\partial x^4} - \sigma_x^{(0)} \nabla^4 \left( \frac{\partial^2 \omega}{\partial x^2} \right) - 2\sigma_{xy}^{(0)} \nabla^4 \left( \frac{\partial^2 \omega}{\partial x \partial y} \right) - \sigma_y^{(0)} \nabla^4 \left( \frac{\partial^2 \omega}{\partial y^2} \right) = 0 \quad (1)$$

where  $\sigma_x^{(0)}$ ,  $\sigma_{xy}^{(0)}$ ,  $\sigma_y^{(0)}$  are the initial membrane stresses;  $\omega$  is the buckling deflection in the  $z$  direction;  $E$  is Young's modulus,  $\nu$  is Poisson's ratio;  $h$  and  $L$  are the thickness and the length of the shell;  $D = Eh^3/[12(1-\nu^2)]$  is the bending stiffness;  $k_x$ ,  $k_y$  were the curvatures in  $x$ ,  $y$  direction [9]. The boundary and load conditions were the same (real) ones, adopted for the experimental buckling tests on the AISI 316 tube. Rigorous tolerance criteria for equilibrium iterations were applied and the size of increment was automatically determined according to the number of iterations in the previous increments. Mesh sizing used has demonstrated that is suitable to assure the convergence of results and to accurately predict collapse behaviour. The deepening of the initial buckle was

either a result of a local buckling process, characterized by a local instability, and/or of a progressive stress-deflection dependence. However, the reaching of a certain depth of the initial on going buckle can lead to the formation of another adjacent buckle, if the material local resistance is larger than the local deformation one. To quantify the effect of the imperfection parameters and the influence of the geometry on the load bearing behaviour, shells of a wide range of diameters and thickness were investigated. The influence of mentioned imperfections on the buckling loads is indicated for a wide range of geometries in Table 1. The number of waves around the circumference decreases as the length increases and takes the minimum value of two only for very long tubes ( $L/R \geq 50$ ). The results, listed in the mentioned table, show the differences between the collapse loads for perfect shell and for imperfect ones. It is evident that the capacity of bearing load decreases in presence of important imperfections. The eccentricity and the ovality have shown the most detrimental effect on tube collapse, while the longitudinal welding seems to induce a stiffening effect along the welded joint. The numerical results point out that load level collapse is strictly dependent from the initial imperfections, so the level of the first instability load initially falls with the increasing imperfection amplitudes.

Table 1. Buckling loads for perfect and imperfect shells

D (mm)	t (mm)	Buckling Load (Pa)			
		Perfect shell	Eccentricity	Ovality	Weld joint
15	1.2	3.117E+7	3.053E+7	2.840E+7	3.210E+7
16	1.2	3.370E+7	3.316E+7	3.277E+7	3.373E+7
17	1.5	4.307E+7	4.230E+7	4.102E+7	4.418E+7
19	1.2	4.127E+7	4.074E+7	3.958E+7	4.215E+7
20	1.5	5.015E+7	4.620E+7	4.249E+7	5.255E+7
20	2	5.691E+7	5.248E+7	5.375E+7	6.507E+7
30	1.2	6.182E+7	5.714E+7	5.227E+7	6.514E+7

### 3. Experimental device

The experiments were conducted on commercially available stainless steel AISI 316 tube specimens with nominal diameters of 20 mm and nominal thickness of 1.5 mm. For a better explanation, type A and B, in Table 2, indicate specimens with and without welding joint. The tests specimens were cut, from the own certified piece, to the corresponding length of 828 mm.

Firstly, the diameter (D) has been measured at twelve equidistant points along the meridional length. This type of measure has been repeated along two equally spaced points on the same circumference, for type A as well as for those B. Measured values of external diameter are in agreement with the nominal diameter for each test specimen. In fact, the small variations registered on the test specimens allowed to evaluate that the real mean ovality is about the 0.3 %, while the thickness mean variation measured around the transition joint is about the 10%. A schematic diagram of the hydraulic pressure device is represented in Figure 2.

The test specimen was confined between two rigid cylindrical sliding base supports, manufactured in the laboratory of Pisa University. These supports maintain the tube in the vertical position, so any rotations or preloaded states on the specimens, during the stages of the assembling, are prevented.

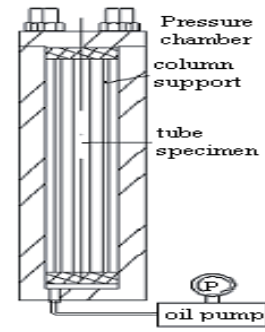


Fig. 2. Buckling test machine

Buckling tests adopted the same boundary conditions, end restraints, geometric properties and load acting on the tube, used in all the numerical simulations. The required load and the strain relationship are recorded for each test and used later to check the numerical buckling results. Data acquisition system was based on strain gauge instrumentation, which allows recording the shell deformation. A piezoelectric pressure transducer was used to measure the pressure in the test chamber, filled with oil, in which the test specimen and its supports were positioned. Previously the data acquisition procedure was tested to verify the accuracy and reliability of the measures. Some recorded relations of pressure-strain circumferential in the upper part of diagrams and the axial in the bottom, for the AISI 316 specimen types A and B, are shown in Figures 3 and 4.

In the A tests, the effect of the thickness mismatch had produced an increasing as for the failure pressure (over 50 MPa) as for the hoop stress in the tube. The influence of welding is shown in the strain shapes of Figure 3. In the B tests, the tubes were buckled to a final value of about 40 MPa pressure. In these graphs buckling phenomenon corresponds to the non-linear part observed, more evident for B tests.

### 4. Comparison between buckling pressure loads

All FEM cases discussed above, for clamped end and external pressure loading, have shown numerical results that indicate a similar behaviour and deformation mechanism. However, some significant differences are also observed for the B specimens: the radial expansions of tube and fitting are small, so the rotation and the bending stress near the weld are small. In the Table 2 the main buckling tests results are indicated.

Table 2. Experimental pressure load

Specimen Test N°	Buckling Pressure (Pa)	
	type A (welded tubes)	type B
1	4.48E+7	3.72E+7
2	4.74E+7	3.80E+7
3	4.81E+7	3.90E+7
4	4.74E+7	3.71E+7
5	4.95E+7	3.63E+7
6	4.57E+7	3.71E+7

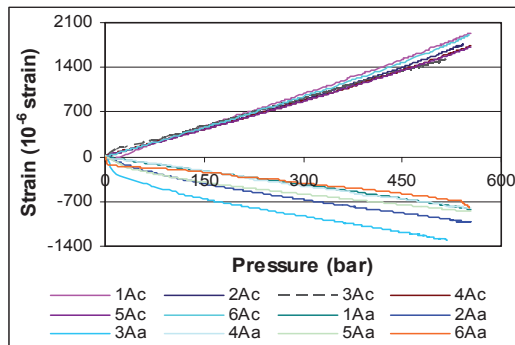


Fig. 3. Diagrams of strain versus pressure for A tests

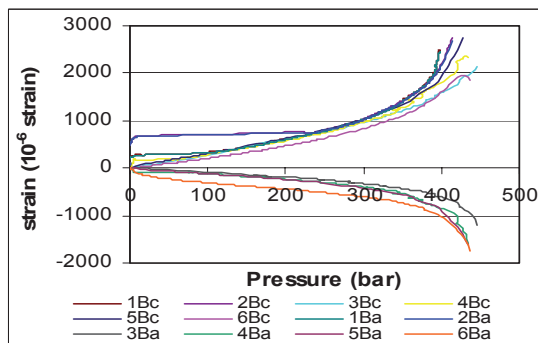


Fig. 4. Diagrams of strain versus pressure for B tests

A good agreement, instead, has been obtained between the predicted numerical and experimental buckling pressure values in the first test. Experimental values of collapse, for specimens B, are shown about 37–42 MPa. The reason of this discrepancy is probably due to the other imperfections, which could not be detected on the tube specimens. The presence of the assumed geometrical imperfections on the shell produces a reduction of 15% on buckling pressure respect to that of perfect shell. Experimental values of collapse, for specimens A, have shown that the geometry discontinuities at the weld were insignificant in the plastic collapse analysis of transition joints.

Schematic picture of collapsed tube, visible in Figures 5 (a) and (b), shows the buckling modes obtained, for the specimen A and B. In the latter case, it is shown that buckling mode will always follow the lowest mode without additional constraints.

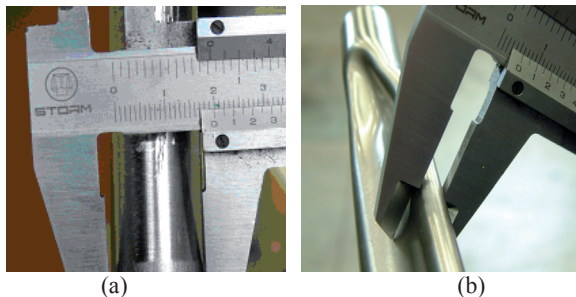


Fig. 5. Schematic A and B tubes deformation after buckling

## 5. Conclusions

Noted that the tubes of an integrated PWR steam generator operate at a differential pressure of about 10 MPa, it can be shown that the tube can bear fairly high external pressure levels. The present work has investigated the plastic collapse failure behaviour and considered the effect of thickness mismatch in the weld joint, of ovality and eccentricity. The pressure-strain relationship used in the mentioned analyses and showed in the above graphs, is crucial to correctly predict the tubes collapse behaviour. The preliminary results show that the numerical approach, implemented with FEM codes, as MSC.MARC, seems able to achieve a good evaluation of the critical load, as a good agreement was obtained between test results and numerical predictions. Moreover, it has been shown by means of extensive analyses, that this method can accurately predict the collapse behaviour of thin tubes, both in linear analyses and under the combined effects of material non-linearity and geometrical imperfections, respectively. Numerical analysis has also shown that the thickness mismatch and the strength of welded joint are parameters that control the buckling of welded thin shell. Most of the analytical approaches proved to be suitable to foresee the collapse loads of the investigated thin tubes, in absence of geometrical imperfections (ovality, eccentricity, welding), even if with discrepancies larger than the ones characteristic of the numerical approach.

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