Static vs Dynamic SAGAs*

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SAGAs calculi (or simply SAGAs) have been proposed by Bruni et al. as a model for long-running transactions. The approach therein can be considered static, while a dynamic approach has been proposed by Lanese and Zavattaro. In this paper we first extend both static SAGAs (in the centralized interruption policy) and dynamic SAGAs to deal with nesting, then we compare the two approaches.

1 Introduction

Computing systems are becoming more and more complex, composed by a huge number of components interacting in different ways. Also, interactions are frequently loosely-coupled, in the sense that each component has scarce information on its communication partners, which may be unreliable (e.g., they may disconnect, or may not follow the expected protocol). Communication may be unreliable too, for instance in the case of wireless networks. Nevertheless, applications are expected to provide reliable services to their users. For these reasons, a main concern is the management of unexpected events.

In the case of loosely-coupled distributed systems (e.g., for web services), unexpected events are managed according to the *long-running transaction* approach. A long-running transaction is a computation that either *commits* (i.e., succeeds), or it *aborts* and is *compensated*. Compensating a (long-running) transaction means executing a sequence of actions that revert the effect of the actions that lead to abortion, so as to reach a consistent state. This is a relaxation of the properties of ACID transactions from database theory [12], based on the fact that in the systems we are interested in rollback cannot always be perfect (e.g., one can not undo the sending of an e-mail, and if one tries to undo an airplane reservation, (s)he may have to pay some penalty).

Recently, many proposals of formal models to reason about properties of long-running transactions, and about systems exploiting them, have been put forward. We concentrate on process calculi, since they are a good tool to experiment with different primitives and compare their relative merits and drawbacks. Later on, the results of these experiments can drive the design of real languages. Process calculi approaches to long-running transactions divide in two main categories: *interaction-based calculi* and *flow composition approaches*. Interaction-based calculi are obtained by extending name passing calculi with dedicated primitives, and one of their main concerns is the interplay between communication and transactions. We recall among them the π t-calculus [1], c-join [5], web π [15], dc π [18], the ATc calculus [2] and SOCK [11]. Flow composition approaches instead deal with the composition of atomic activities, studying how to derive compensations for complex activities from compensations of basic ones. We recall for instance SAGAs [10], StAC [8], cCSP [7] and the SAGAs calculi [6]. Some of the primitives for long-running transactions have been introduced in real languages such as WS-BPEL [17] and Jolie [16]. Long-running transactions have also been analyzed in a choreographic setting in [9]. However, only a

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few works [3, 14, 13] until now have tried to clarify the relationships between the different approaches. We want to go forward in the understanding of those relationships.

As shown in [13], a main distinction to be done is between *static compensations*, where essentially the order of execution of compensations depends on the syntactic structure of the program, and *dynamic* compensations, where it depends on the order of execution of the activities at runtime and cannot be determined statically. The analysis in [13] has been carried on in the case of interaction-based calculi. However compensations are also heavily studied in the framework of flow composition languages. The only dynamic approach to compensations in the framework of flow composition languages we are aware of is the one of dynamic SAGAs [14]. There however only non-nested SAGAs have been defined, and they have been contrasted with the dynamic interaction-based calculus SOCK [11], but not with the classic static SAGAs calculi [6]. Here we want to carry on this last comparison. More precisely, since different flavors of static SAGAs calculi exist, we contrast them with the *centralized interruption* policy defined in [3]. Here "interruption" means that, if there are many concurrent flows of computation and one of them aborts, the other ones are stopped. "Centralized" instead means that the compensation procedure of parallel flows of computation is started only after all forward flows have been stopped. We have chosen this approach since it is the one that more closely matches the behavior of systems programmed in WS-BPEL or Jolie. Actually, also this flavor of SAGAs has been defined only in the non-nested case, while we are interested in the nested case too. In fact, nested SAGAs are fundamental to model complex flows of activities. Thus the contributions of this paper are:

- a description of the semantics of nested SAGAs, both under the static centralized interruption approach (Section 2) and the dynamic approach (Section 3); both the extensions are non trivial, as we will show when we present their semantics;
- a comparison between the two semantics (Section 4), showing that the computations allowed by the dynamic semantics are a strict subset of the ones allowed by the static semantics, and this is due to the fact that the dynamic semantics is more strict on the possible orders of execution of compensations of parallel activities; this comparison has also been used as a sanity check for the formalization of the two semantics.

2 Static SAGAs

SAGAs calculi [6] (SAGAs from now on) are calculi for defining compensating processes. A process is composed of *activities*, ranged over by A, B, \ldots , and each activity may have its own compensating activity. Processes form long-running transactions, called *sagas* in this context. A saga either *commits* (i.e., succeeds), or it *aborts* and is *compensated*. Abortion of the compensation causes a *failure*, which is recognized as a catastrophic event and terminates the whole process. We are interested in nested sagas, thus sagas are processes too.

Definition 1 (Sagas). Saga processes are defined by the following grammar: $P ::= \mathbf{0} | A | A \div B | P; P | P|P | \{[P]\}$

Processes can be the empty activity **0**, an activity *A* without a specified compensation, or an activity $A \div B$ specifying *B* as compensation for *A*. Processes can be composed in sequence (P;P) or in parallel (P|P). Sagas can be nested, thus a saga $\{[P]\}$ is a process too. In the following we will disregard activities *A*, since they can be considered as a particular instance of compensable activities $A \div B$ where B = 0.

The idea underlying the static SAGA semantics is that compensations of sequential activities are executed in reverse order, while compensations of parallel activities are executed in parallel. In particular,

(ZERO) (S-ACT) $\Gamma \vdash \langle 0, \beta \rangle \xrightarrow{0} \langle \Box, \beta \rangle$ $A \mapsto \boxdot, \Gamma \vdash \langle A \div B, \beta \rangle \xrightarrow{A} (\boxdot, B; \beta)$ (S-STEP) (F-ACT) $\frac{\Gamma \vdash \langle P, \beta \rangle \xrightarrow{\alpha}_{s} \langle \boxdot, \beta'' \rangle \quad \Gamma \vdash \langle Q, \beta'' \rangle \xrightarrow{\alpha'}_{s} \langle \boxdot, \beta' \rangle}{\Gamma \vdash \langle P; Q, \beta \rangle \xrightarrow{\alpha; \alpha'}_{s} \langle \boxdot, \beta' \rangle}$ $A \mapsto \boxtimes, \Gamma \vdash \langle A \div B, \beta \rangle \xrightarrow{0} \langle \boxtimes, \beta \rangle$ (A-STEP) $\frac{\Gamma \vdash \langle P, \beta \rangle \xrightarrow{\alpha}_{s} \langle \Box, \beta' \rangle \quad \Box \neq \boxdot}{\Gamma \vdash \langle P; Q, \beta \rangle \xrightarrow{\alpha}_{s} \langle \Box, \beta' \rangle}$ (S-PAR $\frac{\Gamma \vdash \langle P, 0 \rangle \xrightarrow{\alpha_1}_{s} \langle \Box_1, \beta_1 \rangle \quad \Gamma \vdash \langle Q, 0 \rangle \xrightarrow{\alpha_2}_{s} \langle \Box_2, \beta_2 \rangle \quad \Box_1, \Box_2 \in \{ \boxdot, \boxtimes, \overline{\boxtimes} \}}{\Gamma \vdash \langle P | Q, \beta \rangle \xrightarrow{\alpha_1 | \alpha_2}_{s} \langle \Box_1 \land \Box_2, (\beta_1 | \beta_2); \beta \rangle}$ (F-PAR) $\underline{\Gamma \vdash \langle P, 0 \rangle} \xrightarrow{\alpha_1} s \langle \Box_1, \beta_1 \rangle \quad \Gamma \vdash \langle Q, 0 \rangle \xrightarrow{\alpha_2} s \langle \Box_2, \beta_2 \rangle \quad \Box_2 \in \{ \mathbb{R}, \overline{\mathbb{R}}, \overline{\mathbb{H}} \}$ $\Gamma \vdash \langle P | Q, \beta \rangle \xrightarrow{\alpha_1 | \alpha_2}_s \langle \Box_1 \land \Box_2, 0 \rangle$ (FORCED-ABT) $\Gamma \vdash \langle P, \beta \rangle \xrightarrow{0}_{c} \langle \overline{\boxtimes}, \beta \rangle$ $\Gamma \vdash \langle P, \beta \rangle \xrightarrow{0} \langle \overline{|*|}, 0 \rangle$ (SUB-ABT) (SUB-CMT) $\frac{\Gamma \vdash \langle P, 0 \rangle \xrightarrow{\alpha}_{s} \langle \boxtimes, \beta' \rangle \quad \Gamma \vdash \langle \beta', 0 \rangle \xrightarrow{\beta'}_{s} \langle \boxdot, 0 \rangle}{\Gamma \vdash \langle \{[P]\}, \beta \rangle \xrightarrow{\alpha; \beta'}_{s} \langle \boxdot, \beta \rangle}$ $\frac{\Gamma \vdash \langle P, 0 \rangle \xrightarrow{\alpha}_{s} \langle \boxdot, \beta' \rangle}{\Gamma \vdash \langle \{\![P]\!\}, \beta \rangle \xrightarrow{\alpha}_{s} \langle \boxdot, \beta'; \beta \rangle}$ (SUB-FAIL-2) (SUB-FAIL-1) $\frac{\Gamma \vdash \langle P, 0 \rangle \xrightarrow{\alpha}_{s} \langle \Box, 0 \rangle \quad \Box \in \{\mathbb{K}, \overline{\mathbb{K}}, \overline{\mathbb{H}}\}}{\Gamma \vdash \langle \{[P]\}, \beta \rangle \xrightarrow{\alpha}_{s} \langle \Box, 0 \rangle} \qquad \frac{\Gamma \vdash \langle P, 0 \rangle \xrightarrow{\alpha}_{s} \langle \boxtimes, \beta' \rangle \quad \Gamma \vdash \langle \beta', 0 \rangle \xrightarrow{\beta''}_{s} \langle \boxtimes, 0 \rangle}{\Gamma \vdash \langle \{[P]\}, \beta \rangle \xrightarrow{\alpha; \beta''}_{s} \langle \Box, 0 \rangle}$ (SUB-FORCED-1) $\frac{\Gamma \vdash \langle P, 0 \rangle \xrightarrow{\alpha}_{s} \langle \boxtimes, \beta' \rangle \quad \Gamma \vdash \langle \beta', 0 \rangle \xrightarrow{\beta''}_{s} \langle \overline{\boxtimes}, 0 \rangle}{\Gamma \vdash \langle \{ [P] \}, \beta \rangle \xrightarrow{\alpha; \beta''}_{s} \langle \overline{\boxtimes}, 0 \rangle}$ (SUB-FORCED-2) $\frac{\Gamma \vdash \langle P, 0 \rangle \xrightarrow{\alpha}_{s} \langle \overline{\boxtimes}, \beta' \rangle \quad \Gamma \vdash \langle \beta', 0 \rangle \xrightarrow{\beta''}_{s} \langle \Box_{1}, 0 \rangle}{\Gamma \vdash \langle \{\![P]\!], \beta \rangle \xrightarrow{\alpha;\beta''}_{s} \langle \Box_{2}, 0 \rangle} \quad \Box_{2} = \left\{ \begin{array}{cc} \overline{\boxtimes} & \text{if } \Box_{1} = \boxdot \\ \overline{\boxplus} & \text{if } \Box_{1} \in \{\boxtimes, \divideontimes\} \end{array} \right.$

Table 1: Static semantics of nested SAGAs.

the possible orders of execution for compensation activities can be determined statically, looking at the structure of the process. However, as shown by [3], different design choices concerning the behavior of parallel activities are possible. As already said, we consider the semantics with *interruption of parallel activities and centralized compensations* proposed in [3]. According to this semantics parallel activities are stopped when one of them aborts, while in the semantics without interruption they are run to the end (and then compensated). Also, compensations are handled in a centralized way (but for subtransactions), while in the semantics with distributed compensations each flow is responsible for executing its own compensations. The semantics presented in [3] however does not consider nested sagas, while we consider them important both from a theoretical and a practical point of view. From a practical point of view, nesting is fundamental to model large systems in a compositional way. From a theoretical point of view nesting raises interesting questions on the interplay between the behavior of a saga and of its subsagas. For this reason we extend the semantics to deal with nested SAGAs, taking inspiration from the one in [6], which has however distributed compensations.

Definition 2 (Static semantics of SAGAs). The static semantics \rightarrow_s of SAGAs is the LTS defined in Table 1 (we assume symmetric rules for parallel composition).

A saga may commit, abort or fail, denoted respectively by \Box , \boxtimes and B. Also, a saga may acknowledge an external abortion or failure, and these two possibilities are denoted by $\overleftarrow{\boxtimes}$ and $\overleftarrow{\textcircled{B}}$ respectively. Finally, a saga may answer an external abortion with a failure (when an external abort causes it to compensate a subsaga, and the compensation fails), denoted as $\overleftarrow{\boxplus}$. Note that this situation never occurs without nesting. In fact, under the centralized compensation approach, only subsagas are compensated locally, while other processes are compensated at the top level.

The behavior of a saga is determined by the behavior of its constituent activities, which is specified by an environment Γ mapping each activity to either \Box or \boxtimes . The semantics of SAGAs is given as a relation $\Gamma \vdash \langle P, \beta \rangle \xrightarrow{\alpha} \langle \Box, \beta' \rangle$, defined in the big-step style. Here label α is the observation, showing the successfully executed activities. Observations are obtained by combining activities in sequence and in parallel. If one considers an interleaving setting, label A|B can be considered as a shortcut for the two possible sequences, A;B and B;A. We consider observations up to the following axioms: $0; \alpha = \alpha$, $\alpha; 0 = \alpha, 0 | \alpha = \alpha, \alpha | 0 = \alpha$. Also, β is the compensation stored for execution at the beginning of the computation and β' the final stored compensation. Finally, \Box ranges over $\{ \Box, \boxtimes, \boxtimes, \overline{\boxtimes}, \overline{\boxplus}, \overline{\boxplus} \}$, the possible outcomes of the saga.

The first three rules execute the empty activity and basic activities. Note that rule (F-ACT) does not execute the compensation (differently from the rules for distributed compensations in the literature), since this will be executed in a centralized way (see rule (SUB-ABT)). Rule (S-STEP) deals with sequential composition when the first part of the computation succeeds. Rule (A-STEP) deals with all the other cases. Rules (S-PAR) and (F-PAR) concern parallel composition. The operator \land in these rules is the symmetric closure of the one defined in the table below:

\wedge	·	\boxtimes	\boxtimes	*	*	$\overline{\boxplus}$
•	•					
\boxtimes	\boxtimes	_				
\boxtimes	\boxtimes	\boxtimes	\boxtimes			
*	*	_	_	_		
*	*	_	_	*	*	
\blacksquare	\blacksquare	*	_	—	\blacksquare	_

The two rules differ since in the first case the compensation is stored, in the second one it is discarded (since a failure is involved). Rules (FORCED-ABT) and (FORCED-FAIL) show that a process can be

stopped either by an external abort or by an external failure. In the second case the compensation is discarded. Rule (SUB-CMT) allows a saga to commit upon commit of its internal computation. Rule (SUB-ABT) instead allows a saga to commit upon abort of the internal computation and successful compensation. Rule (SUB-FAIL-1) propagates to a saga a catastrophic outcome of its internal computation. Rule (SUB-FAIL-2) establishes failure for an aborted saga whose compensation aborts too. Rule (SUB-FORCED-1) allows an external failure to interrupt a compensating saga. Finally, rule (SUB-FORCED-2) deals with external requests of abortion for sagas. The saga is interrupted and compensated. If the compensation is not successful then a failure is propagated to the outer level. Note that it is not possible to make a saga abort while it is executing its own compensation: the execution of a compensation is protected from further aborts. Compensations of sagas are executed locally, in parallel with the normal flow of external activities, and before starting the compensations of external sagas containing them.

We show now a few derivable transitions to clarify the semantics.

Example 1. Consider a ship for transporting goods. Assume that two different kinds of goods, *A* and *B*, have to be loaded, and the order is not relevant. Also, *A* is not necessary, while *B* is. After loading the ship can leave. This can be modeled using a SAGA process $P = (\{[loadA \div unloadA]\}|loadB \div unloadB); leave$. Assume that all the activities but *leave* succeed. We can derive as a big-step:

 $\Gamma \vdash \langle P, 0 \rangle \xrightarrow{loadA \mid loadB}_{s} \langle \boxtimes, unloadA \mid unloadB \rangle$

thus the process has aborted. If we put the whole process inside a saga then compensation is actually executed and the saga succeeds:

 $\Gamma \vdash \langle \{[P]\}, 0 \rangle \xrightarrow{(loadA|loadB); (unloadA|unloadB)}_{s} \langle \boxdot, 0 \rangle$

Assume now that all the activities but *loadB* and *unloadA* succeed. If the failure of *loadB* occurs before the execution of *loadA* then we have the transition: $\Gamma \vdash \langle P, 0 \rangle \xrightarrow{0}_{S} \langle \boxtimes, 0 \rangle$

derived using rule (S-PAR) where the left component performs $\overline{\boxtimes}$ (derived using rule (SUB-FORCED-2) with left premise $\Gamma \vdash \langle loadA \div unloadA, 0 \rangle \xrightarrow{0}_{s} \langle \overline{\boxtimes}, 0 \rangle$) and the right one performs \boxtimes .

If the failure of *loadB* occurs after the execution of *loadA* we have instead the transition: $\Gamma \vdash \langle P, 0 \rangle \xrightarrow{loadA}_{s} \langle \mathbb{H}, 0 \rangle$

derived using rule (S-PAR) where the left component performs $\overline{\boxplus}$ (derived using rule (SUB-FORCED-2) with left premise $\Gamma \vdash \langle loadA \div unloadA, 0 \rangle \xrightarrow{loadA}_{s} \langle \overline{\boxtimes}, unloadA \rangle$) and the right one performs \boxtimes .

Having $\overline{\boxtimes}$ or $\overline{\mathbb{R}}$ instead of $\overline{\mathbb{H}}$ (which is novel of this semantics) would not faithfully model the intuition. In fact, in the first case the result of the transition would be \boxtimes instead of \mathbb{R} , while in the second case the transition would not be derivable at all, since $\boxtimes \wedge \overline{\mathbb{R}}$ is undefined (otherwise an abort could make a transaction fail, even if compensations were successful).

Example 2. We consider here a modification of the example above, so to clarify another aspect of the semantics. We consider a SAGA process $P' = \{[loadA1 \div unloadA1; loadA2 \div unloadA2]\} | (loadB1 \div unloadB1; loadB2 \div unloadB2)$, where each load activity has been split in two subactivities. Assume that activity *loadB2* aborts, while all the other activities succeed. On the right-hand side we have a transition:

 $\Gamma \vdash \langle loadB1 \div unloadB1; loadB2 \div unloadB2, 0 \rangle \xrightarrow{loadB1}_{s} \langle \boxtimes, unloadB1 \rangle$

This interacts with a left transition of the form:

 $\Gamma \vdash \langle \{ [loadA1 \div unloadA1; loadA2 \div unloadA2] \}, 0 \rangle \xrightarrow{loadA1; unloadA1} _{s} \langle 0, \overline{\boxtimes} \rangle$

Thus the label of the whole transition is (loadA1; unloadA1)|loadB1. In particular, the compensation of the left branch, unloadA1, can be executed before loadB1, i.e. before the fault actually happens. This can be justified by considering an asynchronous scenario, where the observer receives events from different

parallel processes out-of-order. The same problem occurs with the distributed semantics [3], and is due to the fact that sagas are compensated locally. We will see that the dynamic semantics solves this problem. An approach for solving the problem also in the static scenario can be found in [4].

3 Dynamic SAGAs

Dynamic SAGAs have been proposed in [14], in the non-nested case, to reduce the degree of nondeterminism in saga execution. In static SAGAs, in fact, the compensation of $A_1 \% B_1 | A_2 \% B_2$ is $B_1 | B_2$. Thus both the orders $B_1; B_2$ and $B_2; B_1$ are allowed, independently of the order in which activities A_1 and A_2 have been executed. Dynamic SAGAs specify instead that compensations of parallel activities are executed in reverse order: if the execution of normal activities has been $A_1; A_2$ then the execution of compensations is $B_2; B_1$. Thus the order of execution of compensations depends on runtime information on the execution of the basic activities. While the semantics of static SAGAs is normally given in the big-step style, the semantics of dynamic SAGAs is given in the small-step one. A more detailed motivation for this will be given in the next section.

Static and dynamic SAGAs have the same syntax, differing only in the semantics. However, to define the semantics of dynamic SAGAs, we find it convenient to exploit an extended syntax:

 $P ::= \dots \mid \{[P,P]\} \mid \llbracket P \rrbracket_{\boxdot} \mid \llbracket P \rrbracket_{\boxtimes}$

Here $\{[P,\beta]\}$ is a running saga, where *P* is the body and β a stored compensation (syntactically, a process obtained as sequential composition of basic activities). From now on, $\{[P]\}$ stands for $\{[P,0]\}$. Also, $[\![P]\!]_{\Box}$ and $[\![P]\!]_{\boxtimes}$ are executing compensations. Notation $[\![P]\!]_{\Box}$ ranges over both of them. Compensations should be executed in a protected way since we do not want further abortions to stop them. Similar solutions are exploited for instance in SOCK [11], WS-BPEL [17] and others. The difference between $[\![P]\!]_{\Box}$ and $[\![P]\!]_{\boxtimes}$ is that $[\![P]\!]_{\Box}$ has been triggered by the transition itself and commits if *P* commits, while $[\![P]\!]_{\boxtimes}$ has been activated by an external abort, thus it has to re-raise the abort if *P* commits.

We need to extract the protected compensations from a process (to actually protect them, cfr. rule (A-PAR-D)), and we will use to this end the function $extr(\bullet)$ defined below. Similar functions are used, e.g., in SOCK [11] and dc π [18].

$$extr(0) = 0$$

$$extr(A \div B) = 0$$

$$extr(P;Q) = extr(P)$$

$$extr(P|Q) = extr(P)|extr(Q)$$

$$extr(\{[P,\beta]\}) = extr(P);\beta$$

$$extr([[P]]_{\square}) = P$$

Finally, we assume a predicate null(P), which holds if P has no behavior:

The dynamic semantics of SAGAs that we present extends the one in [14] to deal with nested sagas. The extension is non trivial: for instance for the non-nested case neither the function $extr(\bullet)$ nor the auxiliary runtime syntax were needed.

Definition 3 (Dynamic semantics of SAGAs). The dynamic semantics \rightarrow_d of SAGAs is the LTS defined in Table 2 (we assume symmetric rules for parallel composition).

Basic steps are as for the standard semantics. Rules for composition operators allow deriving both intermediate steps $\Gamma \vdash \langle P, \beta \rangle \xrightarrow{a}_{d} \langle P', \beta' \rangle$ and final steps $\Gamma \vdash \langle P, \beta \rangle \xrightarrow{a}_{d} \langle \Box, \beta' \rangle$ (here \Box ranges over $\{ \odot, \boxtimes, \mathbb{R} \}$ and a is an activity name). Also \dagger is a possible label, denoting an abortion which is delayed to wait for termination of running compensation activities. Rules (STEP-D), (K-STEP-D), (S-STEP-D), (A-STEP-D) and (F-STEP-D) deal with the possible evolutions of sequential composition. Rules (PAR-D), (S-PAR-D) and (F-PAR-D) concern normal computation, commit and failure of one branch of parallel composition, respectively. Rule (A-PAR-D) deals with abortion of one branch. If the other branch includes some running compensations, then abortion is delayed and compensation execution is completed first. Running compensations are extracted by function extr(P) and thus preserved, while other parallel activities are discarded. Delayed abortion is propagated using label †. Label † is propagated by rule (K-PAR-D), extracting running compensations from parallel processes. When all the compensations have been completed (rule (A-PAR-FIN-D)) abortion is raised again. Rules (SAGA-D), (S-SAGA-D) and (F-SAGA-D) deal with normal computation, commit and failure of the internal computation in a saga, respectively. Rule (K-SAGA-D) stops the propagation of label †. Rule (A-SAGA-D) deals with abortion of the internal computation of a saga: the stored compensation is executed in a protected way. The behavior of protection (of the two kinds) is defined by rules (PROT-D) for normal steps, rule (K-PROT-D) for delayed abortion (actually, this can happen only for $[P]_{\boxtimes}$) and rule (A-PROT-D) for abortion (producing a failure). The two kinds of protection differ in case of commit of the internal computation: $[P]_{\Box}$ commits (rule (S-PROT-D)) while $\llbracket P \rrbracket_{\boxtimes}$ aborts (rule (S-KILLED-D)), re-raising the delayed abortion.

We show a few computations as examples.

Example 3. Let us consider the saga process defined in Example 1. Remember that $P = (\{[loadA \div unloadA]\}|loadB \div unloadB); leave$. Assume that all the activities but *leave* succeed. We can derive, e.g., the following computation:

$$\begin{array}{ll} \Gamma \vdash \langle P, 0 \rangle & \xrightarrow{loadA}_{d} & \langle (loadB \div unloadB); leave, unloadA \rangle \\ & \xrightarrow{loadB}_{d} & \langle leave, unloadB; unloadA \rangle \\ & \xrightarrow{0}_{d} & \langle \boxtimes, unloadB; unloadA \rangle \end{array}$$

thus the process has aborted. If we put the whole process inside a saga then compensation is actually executed and the saga succeeds:

$$\begin{split} \Gamma \vdash \langle \{[P]\}, 0 \rangle & \xrightarrow{loadA}_{d} \xrightarrow{loadB}_{d} & \langle \{[leave, unloadB; unloadA]\}, 0 \rangle \\ & \xrightarrow{0}_{d} & \langle [\![unloadB; unloadA]\!]_{\odot}, 0 \rangle \\ & \xrightarrow{unloadB}_{d} & \langle [\![unloadA]\!]_{\odot}, 0 \rangle \\ & \xrightarrow{unloadA}_{d} & \langle [\![unloadA]\!]_{\odot}, 0 \rangle \end{split}$$

We consider here another possible computation, so to clarify one of the most tricky cases of the dynamic semantics. Assume that instead of activity *loadA* with compensation *unloadA* we have a sequential composition of two activities, *loadA*1 with compensation *unloadA*1 and *loadA*2 with compensation

(ZERO-D)	(S-ACT-D)
$\Gamma dash \langle 0, oldsymbol{eta} angle \stackrel{0}{ o}_d \langle oldsymbol{arphi}, oldsymbol{eta} angle$	$A\mapsto \boxdot, \Gamma$
(F-ACT-D)	(STEP-D)
$A \mapsto \boxtimes, \Gamma \vdash \langle A \div B, \beta \rangle \xrightarrow{0}_{d} \langle \boxtimes, \beta \rangle$	$\Gamma \vdash \langle I$
	$\Gamma \vdash \langle P; Q \rangle$
(K-STEP-D)	(S-STEP-D)
$\frac{\Gamma \vdash \langle P, \beta \rangle \xrightarrow{\tau}_{d} \langle P', \beta' \rangle}{\tau}$	$\Gamma \vdash \langle P, \rangle$
$\Gamma \vdash \langle P; Q, \beta \rangle \xrightarrow{\dagger}_{d} \langle P', \beta' \rangle$	$\Gamma \vdash \langle P; Q \rangle$
$(A-STEP-D)$ $\Gamma \vdash \langle \boldsymbol{D}, \boldsymbol{Q} \rangle \stackrel{0}{\longrightarrow} \langle \boldsymbol{\nabla} \boldsymbol{Q}, \boldsymbol{Q} \rangle$	(F-STEP-D)
$\frac{\Gamma \vdash \langle P, \beta \rangle \xrightarrow{0}_{d} \langle \boxtimes, \beta' \rangle}{0}$	$\Gamma \vdash \langle P,$
$\Gamma \vdash \langle P; Q, \beta \rangle \stackrel{0}{\rightarrow}_{d} \langle \boxtimes, \beta' angle$	$\Gamma \vdash \langle P; Q \rangle$ (K-PAR-D)
$ \begin{array}{c} {}^{(\text{PAR-D)}} \\ \Gamma \vdash \langle P, \beta \rangle \xrightarrow{a}_{d} \langle P', \beta' \rangle \end{array} $	(к-рак-d)
$\frac{\Gamma \vdash \langle P Q,\beta \rangle \stackrel{a}{\rightarrow}_{d} \langle P' Q,\beta' \rangle}{\Gamma \vdash \langle P Q,\beta \rangle \stackrel{a}{\rightarrow}_{d} \langle P' Q,\beta' \rangle}$	
$1 \vdash \langle \boldsymbol{r} \boldsymbol{\mathcal{Q}}, \boldsymbol{p} \rangle \rightarrow_{d} \langle \boldsymbol{r} \boldsymbol{\mathcal{Q}}, \boldsymbol{p} \rangle$ (S-PAR-D)	$\Gamma \vdash \langle P Q$ (A-PAR-D)
$\Gamma \vdash \langle P, \beta \rangle \xrightarrow{a}_{d} \langle \boxdot, \beta' \rangle$	$\Gamma \vdash \langle P, \beta$
$\frac{(P) P - a}{\Gamma \vdash \langle P Q, \beta \rangle} \xrightarrow{a}_{d} \langle Q, \beta' \rangle$	\) I ²
(A-PAR-FIN-D)	(F-PAR-D)
$\Gamma \vdash \langle P, \beta \rangle \xrightarrow{0}_{d} \langle \boxtimes, \beta' \rangle extr(Q) = \beta'' null(\beta'')$	$\Gamma \vdash \langle P, \rangle$
$\Gamma \vdash \langle P O, \boldsymbol{\beta} \rangle \xrightarrow{0}_{d} \langle \boldsymbol{\boxtimes}, \boldsymbol{\beta}' \rangle$	$\Gamma \vdash \langle P Q$
(SAGA-D)	(K-SAGA-D
$\Gamma dash \langle P, oldsymbol{eta} angle \stackrel{a}{ ightarrow_d} \langle P', oldsymbol{eta}' angle$	Г
$\overline{\Gamma \vdash \langle \{ [P, \beta] \}, \beta'' \rangle} \xrightarrow{a}_{d} \langle \{ [P', \beta'] \}, \beta'' \rangle$	$\Gamma \vdash \langle \{ [P,$
(S-SAGA-D)	(A-SAGA-E
$\Gamma \vdash \langle P, \boldsymbol{\beta} \rangle \xrightarrow{a}_{d} \langle \boxdot, \boldsymbol{\beta}' \rangle$	Γ ⊢
$\Gamma \vdash \langle \{ [P, oldsymbol{eta}] \}, oldsymbol{eta}'' angle \stackrel{a}{ ightarrow_d} \langle \boxdot, oldsymbol{eta}'; oldsymbol{eta}'' angle$	$\Gamma \vdash \langle \{ [P,$
(F-SAGA-D)	(PROT-D)
$\Gamma dash \langle P, oldsymbol{eta} angle \stackrel{0}{ o}_d \langle \mathbb{B}, 0 angle$	$\Gamma \vdash \langle$
$\Gamma dash \langle \{ [P, oldsymbol{eta}] \}, oldsymbol{eta}'' angle rac{0}{ odeta_d} ig \langle {f eta}, 0 angle$	$\Gamma \vdash \langle \llbracket P \rrbracket$
(K-PROT-D)	(S-PROT-D)
$\Gammadash \langle P,oldsymbol{eta} angle \stackrel{ au}{ ightarrow}_d \langle P',oldsymbol{eta}' angle$	$\Gamma \vdash \langle P,$
$\Gamma \vdash \langle \llbracket P \rrbracket_{\boxtimes}, oldsymbol{eta} angle \stackrel{\dagger}{ o}_d \langle \llbracket P' \rrbracket_{\boxtimes}, oldsymbol{eta}' angle$	$\Gamma \vdash \langle \llbracket P \rrbracket$
(S-KILLED-D)	(A-PROT-D
$\frac{\Gamma \vdash \langle P, \beta \rangle}{\overset{a}{\longrightarrow}_{d} \langle \boxdot, \beta' \rangle}$	$\Gamma \vdash \langle P, \rangle$
$\Gamma \vdash \langle \llbracket P \rrbracket_{\boxtimes}, \beta \rangle \xrightarrow{a}_{d} \langle \boxtimes, \beta' \rangle$	$\Gamma \vdash \langle \llbracket P \rrbracket$

$$\begin{array}{l} (\text{S-ACT-D}) \\ A \mapsto \boxdot, \Gamma \vdash \langle A \div B, \beta \rangle \xrightarrow{A}_{d} \langle \boxdot, B; \beta \rangle \\ (\text{STEP-D}) \\ \hline \Gamma \vdash \langle P, \beta \rangle \xrightarrow{a}_{d} \langle P', \beta' \rangle \\ \hline \Gamma \vdash \langle P; Q, \beta \rangle \xrightarrow{a}_{d} \langle \Box, \beta' \rangle \\ \hline \Gamma \vdash \langle P; Q, \beta \rangle \xrightarrow{a}_{d} \langle \Box, \beta' \rangle \\ \hline \Gamma \vdash \langle P, \beta \rangle \xrightarrow{a}_{d} \langle \Box, \beta' \rangle \\ \hline \Gamma \vdash \langle P; Q, \beta \rangle \xrightarrow{a}_{d} \langle \Box, \beta \rangle \\ \hline \Gamma \vdash \langle P, \beta \rangle \xrightarrow{a}_{d} \langle \Box, \beta \rangle \\ \hline \Gamma \vdash \langle P, \beta \rangle \xrightarrow{a}_{d} \langle B, 0 \rangle \\ \hline \Gamma \vdash \langle P, \beta \rangle \xrightarrow{a}_{d} \langle B, 0 \rangle \\ \hline \Gamma \vdash \langle P, \beta \rangle \xrightarrow{a}_{d} \langle P' | [[extr(Q)]]_{\boxtimes}, \beta' \rangle \\ \hline \Gamma \vdash \langle P, \beta \rangle \xrightarrow{a}_{d} \langle P' | [[extr(Q)]]_{\boxtimes}, \beta' \rangle \\ \hline \Gamma \vdash \langle P, \beta \rangle \xrightarrow{a}_{d} \langle B, 0 \rangle \\ \hline \Gamma \vdash \langle P, \beta \rangle \xrightarrow{a}_{d} \langle B, 0 \rangle \\ \hline \Gamma \vdash \langle P, \beta \rangle \xrightarrow{a}_{d} \langle B, 0 \rangle \\ \hline \Gamma \vdash \langle P, \beta \rangle \xrightarrow{a}_{d} \langle B, 0 \rangle \\ \hline \Gamma \vdash \langle P | Q, \beta \rangle \xrightarrow{a}_{d} \langle B, 0 \rangle \\ \hline \Gamma \vdash \langle P | Q, \beta \rangle \xrightarrow{a}_{d} \langle B, 0 \rangle \\ \hline \Gamma \vdash \langle P, \beta \rangle \xrightarrow{a}_{d} \langle B, 0 \rangle \\ \hline \Gamma \vdash \langle \{[P, \beta]\}, \beta'' \rangle \xrightarrow{0}_{d} \langle \langle B, 0 \rangle \\ \hline \Gamma \vdash \langle \{[P, \beta]\}, \beta'' \rangle \xrightarrow{0}_{d} \langle \langle B, \beta' \rangle \\ \hline \Gamma \vdash \langle \{[P, \beta]\}, \beta'' \rangle \xrightarrow{0}_{d} \langle \langle B' |_{\Box}, \beta'' \rangle \\ \hline \Gamma \vdash \langle [P]_{\Box}, \beta \rangle \xrightarrow{a}_{d} \langle P', \beta' \rangle \\ \hline \Gamma \vdash \langle [P]_{\Box}, \beta \rangle \xrightarrow{a}_{d} \langle [P']_{\Box}, \beta' \rangle \\ \hline \Gamma \vdash \langle P, \beta \rangle \xrightarrow{a}_{d} \langle \Box, \beta' \rangle \\ \hline \Gamma \vdash \langle P, \beta \rangle \xrightarrow{a}_{d} \langle \Box, \beta' \rangle \\ \hline \Gamma \vdash \langle P, \beta \rangle \xrightarrow{a}_{d} \langle \Box, \beta' \rangle \\ \hline \Gamma \vdash \langle P, \beta \rangle \xrightarrow{a}_{d} \langle \Box, \beta' \rangle \\ \hline \Gamma \vdash \langle P, \beta \rangle \xrightarrow{a}_{d} \langle \Box, \beta' \rangle \\ \hline \Gamma \vdash \langle [P]_{\Box}, \beta \rangle \xrightarrow{a}_{d} \langle \Box, \beta' \rangle \\ \hline \Gamma \vdash \langle [P]_{\Box}, \beta \rangle \xrightarrow{a}_{d} \langle \Box, \beta' \rangle \\ \hline \Gamma \vdash \langle [P]_{\Box}, \beta \rangle \xrightarrow{a}_{d} \langle \Box, \beta' \rangle \\ \hline \Gamma \vdash \langle [P]_{\Box}, \beta \rangle \xrightarrow{a}_{d} \langle \Box, \beta' \rangle \\ \hline \Gamma \vdash \langle [P]_{\Box}, \beta \rangle \xrightarrow{a}_{d} \langle \Box, \beta' \rangle \\ \hline \Gamma \vdash \langle [P]_{\Box}, \beta \rangle \xrightarrow{a}_{d} \langle \Box, \beta' \rangle \\ \hline \Gamma \vdash \langle [P]_{\Box}, \beta \rangle \xrightarrow{a}_{d} \langle \Xi, \beta' \rangle \\ \hline \Gamma \vdash \langle [P]_{\Box}, \beta \rangle \xrightarrow{a}_{d} \langle \Xi, \beta' \rangle \\ \hline \Gamma \vdash \langle [P]_{\Box}, \beta \rangle \xrightarrow{a}_{d} \langle \Xi, \beta' \rangle \\ \hline \Gamma \vdash \langle [P]_{\Box}, \beta \rangle \xrightarrow{a}_{d} \langle \Xi, \beta' \rangle \\ \hline \Gamma \vdash \langle [P]_{\Box}, \beta \rangle \xrightarrow{a}_{d} \langle \Xi, \beta' \rangle \\ \hline \Gamma \vdash \langle [P]_{\Box}, \beta \rangle \xrightarrow{a}_{d} \langle \Xi, \beta' \rangle \\ \hline \Gamma \vdash \langle [P]_{\Box}, \beta \rangle \xrightarrow{a}_{d} \langle \Xi, \beta' \rangle \\ \hline \Gamma \vdash \langle [P]_{\Box}, \beta \rangle \xrightarrow{a}_{d} \langle \Xi, \beta' \rangle \\ \hline \Gamma \vdash \langle [P]_{\Box}, \beta \rangle \xrightarrow{a}_{d} \langle \Xi, \beta' \rangle \\ \hline \Gamma \vdash \langle [P]_{\Box}, \beta \rangle \xrightarrow{a}_{d} \langle \Xi, \beta' \rangle \\ \hline \Gamma \vdash \langle [P]_{\Box}, \beta \rangle \xrightarrow{a}_{d} \langle \Xi, \beta' \rangle \\ \hline \Gamma \vdash \langle [P]_{\Box}, \beta \rangle \xrightarrow{a}_{d} \langle \Xi, \beta' \rangle \\ \hline \Gamma \vdash \langle [P]_{\Box}, \beta \rangle \xrightarrow{a}_{d} \langle \Xi, \beta' \rangle \\ \hline \Gamma \vdash \langle [P]_{\Box}, \beta \rangle \xrightarrow{a}_{d} \langle \Xi, \beta' \rangle \\ \hline \Gamma \vdash \langle [P]_{\Box}, \beta \rangle \xrightarrow{a}_{d} \langle \Xi, \beta' \rangle \\ \hline \Gamma$$

Table 2: Dynamic semantics of nested SAGAs.

unloadA2. Assume also that activity loadB aborts, while the other activities succeed.

$$\begin{split} \Gamma \vdash \langle P'', 0 \rangle & \xrightarrow{loadA1}_{d} & \langle (\{[loadA2, unloadA1]\} | loadB \div unloadB); leave, 0 \rangle \\ & \xrightarrow{\dagger}_{d} & \langle [\![unloadA1]\!]_{\boxtimes}, 0 \rangle \\ & \xrightarrow{unloadA1}_{d} & \langle \boxtimes, 0 \rangle \end{split}$$

Here *loadB* aborts when the parallel saga is still running, thus abortion is postponed. After compensation has been performed, the abortion is raised again.

Example 4. We show now the behavior of the saga in Example 2 when executed under the dynamic semantics.

$$\Gamma \vdash \langle P', 0 \rangle \xrightarrow{loadA1}_{d} \quad \langle \{ [loadA2 \div unloadA2, unloadA1] \} | loadB1 \div unloadB1; loadB2 \div unloadB2, 0 \rangle = 0$$

Here unloadA1 is not enabled, and it becomes enabled only after loadB1 is observed and loadB2 is executed, triggering the abort:

$$\begin{split} \Gamma \vdash \langle P', 0 \rangle & \xrightarrow{loadA1}_{d} & \langle \{ [loadA2 \div unloadA2, unloadA1] \} | loadB1 \div unloadB1; loadB2 \div unloadB2, 0 \rangle \\ & \xrightarrow{loadB1}_{d} & \langle \{ [loadA2 \div unloadA2, unloadA1] \} | loadB2 \div unloadB2, unloadB1 \rangle \\ & \xrightarrow{\dagger}_{d} & \langle [unloadA1] _{\boxtimes}, unloadB1 \rangle \\ & \xrightarrow{unloadA1}_{d} & \langle \boxtimes, unloadB1 \rangle \end{split}$$

4 Static vs Dynamic SAGAs

In this section we compare the static and dynamic semantics of nested SAGAs. In particular, we show that each computation obtained from the dynamic semantics is compatible with a big-step of the static semantics. We show also that the static semantics allows for more nondeterminism in the order of execution of activities, i.e. it allows for some computations not valid according to the dynamic semantics.

Labels of a big-step correspond to sets of computations of small-steps (assuming an interleaving interpretation for parallel composition). We write $\stackrel{\alpha}{\Longrightarrow}$ with $\alpha = a_1; \ldots; a_n$ to denote $\stackrel{a_1}{\longrightarrow}_d \cdots \stackrel{a_n}{\longrightarrow}_d$. We remove from α both 0 and \dagger . However, big-step labels may also include parallel composition operators, thus to perform the comparison we have to introduce the concept of linearization. We consider the set of linearizations $\ln(\alpha)$ of a big-step label α , which is defined by structural induction on α :

$$\begin{array}{lll} \ln(A) &=& \{A\} \\ \ln(\alpha; \alpha') &=& \{\gamma; \gamma' \mid \gamma \in \ln(\alpha) \land \gamma' \in \ln(\alpha')\} \\ \ln(\alpha \mid \alpha') &=& \bigcup_{\gamma \in \ln(\alpha) \land \gamma' \in \ln(\alpha')} \gamma \mid\mid \gamma' \end{array}$$

where ||| is defined as follows:

$$\begin{array}{rcl} 0 \parallel \gamma &=& \{\gamma\} \\ \gamma \parallel 0 &=& \{\gamma\} \\ A; \gamma \parallel A'; \gamma' &=& \{A; \gamma'' \mid \gamma'' \in \gamma \parallel A'; \gamma'\} \cup \{A'; \gamma'' \mid \gamma'' \in A; \gamma \parallel \gamma'\} \end{array}$$

In words, ||| computes the set of all possible interleavings of the sequences of actions in its two arguments.

Summarizing, a big-step label α corresponds to the set of small-step computations with labels in $lin(\alpha)$.

Next lemma discusses the properties of †-labeled transitions.

Lemma 1. If $\Gamma \vdash \langle P, \beta \rangle \stackrel{\dagger}{\Longrightarrow} \langle P', \beta' \rangle$ then P' is a parallel composition of terms of the form $\llbracket P_i \rrbracket_{\boxtimes}$.

We can now prove our main theorems, relating the behavior of static and dynamic semantics for SAGAs. We have two theorems, one for each direction.

Theorem 1. If $\Gamma \vdash \langle P, \beta \rangle \xrightarrow{\gamma} \langle \Box, \beta' \rangle$ with $\Box \in \{ \boxdot, \boxtimes, \circledast \}$ then there is a big-step $\Gamma \vdash \langle P, \beta \rangle \xrightarrow{\alpha}_{s} \langle \Box, \beta'' \rangle$ with $\gamma \in lin(\alpha)$ and $\beta' \in lin(\beta'')$.

Proof. The proof is by structural induction on *P*. Actually for the induction we need a stronger hypothesis, requiring also that:

- if $\Gamma \vdash \langle P, \beta \rangle \xrightarrow{\gamma} \langle P', \beta' \rangle$ and $\Gamma \vdash \langle extr(P'), 0 \rangle \xrightarrow{\gamma'} \langle \Box, 0 \rangle$, then there is a big-step $\Gamma \vdash \langle P, \beta \rangle \xrightarrow{\alpha; \alpha'} \langle \overline{\Box}, \beta'' \rangle$ with $\gamma \in \operatorname{lin}(\alpha), \gamma' \in \operatorname{lin}(\alpha')$ and $\beta' \in \operatorname{lin}(\beta'')$;
- if $\Gamma \vdash \langle P, \beta \rangle \xrightarrow{\gamma} \langle P', \beta' \rangle$ and $\Gamma \vdash \langle extr(P'), 0 \rangle \xrightarrow{\gamma'} \langle \Box, 0 \rangle$ with $\Box \in \{ \boxtimes, \mathbb{R} \}$, then there is a big-step $\Gamma \vdash \langle P, \beta \rangle \xrightarrow{\alpha; \alpha'} _{s} \langle \overline{\boxplus}, 0 \rangle$ with $\gamma \in lin(\alpha)$ and $\gamma' \in lin(\alpha')$;
- if $\Gamma \vdash \langle P, \beta \rangle \xrightarrow{\gamma} \langle P', \beta' \rangle$, then there is a big-step $\Gamma \vdash \langle P, \beta \rangle \xrightarrow{\alpha} \langle \overline{\mathbb{H}}, 0 \rangle$ with $\gamma \in \operatorname{lin}(\alpha)$. We have the following cases:
- P = 0: the only non trivial computation is $\Gamma \vdash \langle 0, \beta \rangle \xrightarrow[]{0}{d} \langle \boxdot, \beta \rangle$. The big-step $\Gamma \vdash \langle 0, \beta \rangle \xrightarrow[]{0}{d} \langle \boxdot, \beta \rangle$ derived from rule (ZERO) satisfies the thesis. As far as the empty computation is concerned the two big-steps $\Gamma \vdash \langle 0, \beta \rangle \xrightarrow[]{0}{d} \langle \boxdot, \beta \rangle$ and $\Gamma \vdash \langle 0, \beta \rangle \xrightarrow[]{0}{d} \langle \boxdot, \beta \rangle$, derived from rules (FORCED-ABT) and (FORCED-FAIL) respectively, satisfy the thesis.
- $P = A \div B$: we have a case analysis according to $\Gamma(A)$. If $\Gamma(A) = \Box$ then the only non trivial computation is $\Gamma \vdash \langle A \div B, \beta \rangle \xrightarrow{A}_d \langle \Box, B; \beta \rangle$, and we have a corresponding big-step, derived from rule (S-ACT). Similarly for the case $\Gamma(A) = \boxtimes$, using rule (F-ACT). The empty computations can be matched as before.
- $P = P_1; P_2$: assume that there is a computation $\Gamma \vdash \langle P_1; P_2, \beta \rangle \xrightarrow{\gamma} \langle \Box, \beta' \rangle$ with $\Box \in \{ \boxdot, \boxtimes, \boxtimes \}$. We have to consider the three cases $\Box = \boxdot, \Box = \boxtimes$ and $\Box = \boxtimes$.
 - $\Box = \boxdot: \text{ let us consider the first part of the computation. The only possibility is to have the first zero or more steps derived using as last rule (STEP-D) followed by one step derived using as last rule (S-STEP-D). By concatenating the premises we have a computation <math>\Gamma \vdash \langle P_1, \beta \rangle \xrightarrow{\gamma'} \langle \boxdot, \beta'' \rangle$. By inductive hypothesis we have a big-step $\Gamma \vdash \langle P_1, \beta \rangle \xrightarrow{\alpha'}_{s} \langle \boxdot, \delta'' \rangle$ with $\gamma' \in \ln(\alpha')$ and $\beta'' \in \ln(\delta'')$. Also, using the last part of the computation we have a big-step $\Gamma \vdash \langle P_2, \beta'' \rangle \xrightarrow{\alpha''}_{s} \langle \boxdot, \delta' \rangle$ with $\gamma' \in \ln(\alpha'')$, $\beta' \in \ln(\delta')$ and $\gamma = \gamma'; \gamma''$. The thesis follows by rule (S-STEP). It is not possible to have the first zero or more steps derived using as last rule (STEP-D) followed by one step derived using as last rule (K-STEP-D) since this computation can not succeed (see Lemma 1).
 - $\Box = \boxtimes$: we have a few possibilities here, according to which is the first rule applied different from (STEP-D). If it is rule (S-STEP-D) then the thesis follows by inductive hypothesis applying rule (S-STEP). If it is rule (A-STEP-D) then the thesis follows by inductive hypothesis applying rule (A-STEP). If it is rule (K-STEP-D) then the thesis follows by inductive hypothesis, again applying rule (A-STEP).

 $\Box = \mathbb{R}$: similar to the one above.

As far as computations leading to processes are concerned, a similar reasoning can be done. The thesis follows from rule (A-STEP) if the computation is only composed by steps from rule (STEP-D). If the computation includes also a step from rule (S-STEP-D) then the thesis follows from rule (S-STEP). If the computation includes also a step from rule (K-STEP-D) then the thesis follows from rule (A-STEP).

- $P = P_1 | P_2$: assume that there is a computation $\Gamma \vdash \langle P_1 | P_2, \beta \rangle \xrightarrow{\gamma} \langle \Box, \beta' \rangle$ with $\Box \in \{ \boxdot, \boxtimes, \mathbb{R} \}$. We have to consider the three cases $\Box = \boxdot, \Box = \boxtimes$ and $\Box = \mathbb{R}$.
 - $\Box = \boxdot: \text{ let us consider the first part of the computation. The only possibility is to have the first zero or more steps derived using as last rule (PAR-D) followed by one step derived using as last rule (S-PAR-D). Assume for simplicity that (S-PAR-D) eliminates the first component of the parallel composition. By concatenating the premises of those transitions that concern the first component we have a computation <math>\Gamma \vdash \langle P_1, 0 \rangle \xrightarrow{\gamma'} \langle \boxdot, \beta'' \rangle$. By inductive hypothesis we have a big-step $\Gamma \vdash \langle P_1, 0 \rangle \xrightarrow{\alpha'}_{s} \langle \boxdot, \delta'' \rangle$ with $\gamma' \in \text{lin}(\alpha')$ and $\beta'' \in \text{lin}(\delta'')$. Also, using the premises of the transitions involving the second component and the last part of the computation we have a big-step $\Gamma \vdash \langle P_2, \beta \rangle \xrightarrow{\alpha''}_{s} \langle \boxdot, \delta''' \rangle$ with $\gamma'' \in \text{lin}(\alpha'')$ and $\beta''' \in \text{lin}(\delta''')$. Also, γ is an interleaving of γ' and γ'' and β' is obtained by prefixing β with an interleaving of β'' and β''' . The thesis follows by rule (S-PAR) (since $\boxdot \land \boxdot = \boxdot$). It is not possible to have the first zero or more steps derived using as last rule (STEP-D) followed by one step derived using as last rule (K-STEP-D) since this computation can not succeed (see Lemma 1).
 - $\Box = \boxtimes :$ we have a few possibilities here, according to which is the first rule applied different from (PAR-D). If it is (S-PAR-D) then the thesis follows by inductive hypothesis applying rule (S-PAR) (since $\Box \land \boxtimes = \boxtimes$). If it is (A-PAR-D) then from the premises concerning *P* we can derive an abortion for *P*. From *Q* we can derive a computation leading to *Q'* and an abortion for $[[extr(Q')]]_{\boxtimes}$, i.e. a commit for extr(Q'). Thus we can derive a big-step leading to $\overline{\boxtimes}$ for *Q*. The thesis follows from rule (S-PAR) (since $\boxtimes \land \overline{\boxtimes} = \boxtimes$). The case of rule (K-PAR-D) is similar, with the abortion coming after all compensations have been consumed (i.e., when rule (A-PAR-FIN-D) is triggered).
 - $\Box = \textcircled{B}: we have a few possibilities here, according to which is the first rule applied different from (PAR-D). If it is (S-PAR-D) then the thesis follows by inductive hypothesis applying rule (F-PAR) (since <math>\Box \land \textcircled{B} = \textcircled{B}$). If it is (A-PAR-D) then from the premises concerning *P* we can derive an abortion for *P*. From *Q* we can derive a computation leading to *Q'* and a failure for $\llbracket extr(Q') \rrbracket_{\boxtimes}$, i.e. an abort or a failure for extr(Q'). Thus we can derive a big-step leading to $\overline{\boxplus}$ for *Q*. The thesis follows from rule (F-PAR) (since $\boxtimes \land \overline{\boxplus} = \textcircled{B}$). The case of rule (K-PAR-D) is similar. The case of rule (F-PAR-D) follows from rule (F-PAR), since $\textcircled{B} \land \overline{\textcircled{B}} = \textcircled{B}$.

As far as computations leading to processes are concerned, a similar reasoning can be done. One has to distinguish interrupt from abort and interrupt from failure. In the first case the thesis follows by inductive hypothesis applying rule (S-PAR) (with $\overline{\boxtimes} \land \overline{\boxtimes} = \overline{\boxtimes}$) if both the compensations succeed. If at least one of the compensations fails then the other one is interrupted by a failure and the thesis follows by inductive hypothesis applying rule (F-PAR) (with $\overline{\boxplus} \land \overline{\boxtimes} = \overline{\boxplus}$). If interruption is from a failure, then the thesis follows from rule (F-PAR) (with $\overline{\boxplus} \land \overline{\boxtimes} = \overline{\boxplus}$).

 $P = \{[P_1]\}: \text{ assume that there is a computation } \Gamma \vdash \langle \{[P_1]\}, \beta \rangle \xrightarrow{\gamma} \langle \Box, \beta' \rangle \text{ with } \Box \in \{ \boxdot, \boxtimes, \circledast \}. \text{ We have to consider the three cases } \Box = \boxdot, \Box = \boxtimes \text{ and } \Box = \circledast.$

- $\Box = \boxdot$ we have two possibilities here. The first one is to have the first zero or more steps derived using as last rule (SAGA-D) followed by one step derived using as last rule (S-SAGA-D). By concatenating the premises of those transitions we have a computation $\Gamma \vdash \langle P_1, 0 \rangle \xrightarrow{\gamma} \langle \boxdot, \beta' \rangle$. By inductive hypothesis we have a big-step $\Gamma \vdash \langle P_1, 0 \rangle \xrightarrow{\alpha}_s \langle \boxdot, \delta' \rangle$ with $\gamma \in \text{lin}(\alpha)$ and $\beta' \in \text{lin}(\delta')$. The thesis follows by rule (SUB-CMT). The second case exploits rule (SAGA-D) at the beginning (or possibly (K-SAGA-D)), then one transition from rule (A-SAGA-D), then some transitions from rule (PROT-D) and finally one transition from rule (S-PROT-D). Here by considering the premises of the first part of the computation we have a computation $\Gamma \vdash \langle P_1, 0 \rangle \xrightarrow{\gamma'} \langle \boxdot, \beta'' \rangle$. From the second part we get a computation $\Gamma \vdash \langle \beta'', 0 \rangle \xrightarrow{\gamma''} \langle \boxdot, 0 \rangle$. The thesis follows from the inductive hypothesis by applying rule (SUB-ABT).
- $\Box = \boxtimes$: there is no possibility for a saga computation to lead to an abort, thus this case can never happen.
- $\Box = \textcircledleteroise is the end of th$

As far as computations leading to processes are concerned, a similar reasoning can be done. If the computation is only composed by applications of rules (SAGA-D) or (K-SAGA-D) then the thesis follows by inductive hypothesis from rule (SUB-FAIL-1) or from rule (SUB-FORCED-2). If there is an application of rule (A-SAGA-D) then the thesis follows from rule (SUB-FORCED-1).

The following theorem considers the other direction.

Theorem 2. If $\Gamma \vdash \langle P, \beta \rangle \xrightarrow{\alpha}_{s} \langle \Box, \beta'' \rangle$ with $\Box \in \{ \boxdot, \boxtimes, \mathbb{R} \}$ then there are $\gamma \in \operatorname{lin}(\alpha)$ and $\beta' \in \operatorname{lin}(\beta'')$ such that $\Gamma \vdash \langle P, \beta \rangle \xrightarrow{\gamma} \langle \Box, \beta' \rangle$

Proof. The proof requires a case analysis similar to the one of Theorem 1. We omit the details. \Box

Example 5. Consider the ship example and its two executions at the beginning of Example 1 (with static semantics) and Example 3 (with dynamic semantics). It is easy to see for instance that $loadA; loadB \in lin(loadA|loadB)$ and $unloadB; unloadA \in lin(unloadA|unloadB)$. Thus the dynamic computation is compatible with the static big-step, for a suitable choice of the interleaving of parallel activities.

Notably, it is not possible to prove Theorem 2 by requiring a computation *for each* possible linearization of α . In fact the chosen linearization depends on the runtime execution of activities. For instance, in Example 3 it is not possible to have as observation *loadA*;*loadB* and as compensation *unloadA*;*unloadB*. Also, abortion of nested sagas is managed in a different way, as shown in the example below.

Example 6. Consider the saga in Example 2. According to the static semantics it has a big-step with label (loadA1; unloadA1)|loadB1. In particular, loadA1; unloadA1; loadB1 is a possible linearization of (loadA1; unloadA1)|loadB1. However, as shown in Example 4, there is no dynamic computation with this behavior. There is however a dynamic computation compatible with the big-step, as shown in Example 4, considering the linearization loadA1; loadB1; unloadA1.

These are the main differences between static and dynamic SAGAs: among all the computations compatible with the static semantics, only some are acceptable for the dynamic semantics, and whether a computation is acceptable or not depends on the relation between the order of execution of activities and of their compensations and on the interplay between nesting and parallel composition.

This also explains why we used a small-step semantics for dynamic SAGAs (while classic semantics for static SAGAs is big-step): it is difficult to catch the dependencies between the order of execution of activities and the order of execution of compensations using big-step semantics. For instance, a rule such as (S-PAR) in Table 1 tailored for dynamic SAGAs should require that the interleaving chosen for activities in $\alpha_1 | \alpha_2$ is the same as the one chosen for their compensations in $\beta_1 | \beta_2$, and one would need to track the correspondence between activities and their compensations.

Summarizing, the theorems above provide the following insights:

- the static and the dynamic semantics are strongly related, in the sense that for each static big-step there is (at least) one dynamic computation compatible with it, and for each dynamic computation there is a compatible big-step;
- the two semantics are not equivalent, since not all the computations compatible with a big-step are valid dynamic computations;
- in the dynamic semantics the order of execution of compensations depends on the order of execution of basic activities, while this is not the case for the static semantics;
- in the dynamic semantics compensations of subtransactions can be observed only after the abort itself has been observed.

5 Conclusion

We have presented two semantics for nested SAGAs, a static semantics following the centralized interruption policy and a dynamic semantics. The two semantics are a non trivial step forward w.r.t. the non-nested case presented in the literature (cfr. [3] and [14]). Even the static semantics is quite different from the (static) semantics of nested SAGAs with distributed interruption presented in [6]. The main difference relies in the different ways compensations have to be managed in the two cases. Actually, we think that the semantics with distributed interruption is realistic only in asynchronous settings since, as said in [3], it "includes a "guessing mechanism" that allows branches on the forward flow to compensate even before an activity aborts". This forbids for instance an encoding into a lower-level framework such as the one in [14]. A similar behavior occurs also in our static semantics, but only in the case of nested transactions.

As far as future work is concerned, many different proposals of primitives and models for longrunning transactions have been put forward in the last years, yet a coherent picture of the field is still far. Understanding the relationships between the different formalisms is fundamental so to understand which of them are best suited to be introduced in real languages. Even restricting our attention to SAGAs, the interplay between dynamicity and the different approaches presented in [3] has to be fully analyzed. Also, we are currently working [4] on a static semantics for SAGAs which is distributed but does not require the guessing mechanism described above.

6 Bibliography

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