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Limits and potentials of Mixed Integer Linear Programming methods for optimization of polygeneration energy systems

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Abstract

The simultaneous production of different energy vectors from hybrid polygeneration plants is a promising way to increase energy efficiency and facilitate the development of distributed energy systems. The inherent complexity of polygeneration energy systems makes their economic, environmental and energy performance highly dependent on system synthesis, equipment selection and capacity, and operational strategy. Mixed Integer Linear Programming (MILP) is the state of the art approach to tackle the optimization problem of polygeneration systems. The guarantee of finding global optimality in linear problems and the effectiveness of available commercial solvers make MILP very attractive and widely used in optimization problems of polygeneration systems. Nevertheless, several drawbacks affect the MILP formulation, such as: the impossibility of taking into account nonlinear effects; the necessity of considering all the time periods at once; the risk of high-dimensionality of the problem. To tackle these limitations, several techniques have been developed, such as: piecewise linearization methods; rolling horizon approaches; dimensionality reduction by means of energy demands clustering algorithms. In this paper, limits and potentials of MILP methods for the optimization problem of polygeneration energy systems are reviewed and discussed.

Keywords: Mixed Integer Linear Programming (MILP); Optimization; Polygeneration energy systems; Cogeneration, CCHP.

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1. Introduction

Cogeneration, also known as Combined Heat and Power (CHP), is the simultaneous production of electric energy and useful heat. Cogeneration systems can also include absorption chillers, which use heat recovered from power units to produce cooling; those systems are usually referred to as trigeneration or Combined Cooling Heat and Power (CCHP) systems. More generally, an energy supply system which delivers simultaneously more than one form of energy to the final user is defined as a polygeneration energy system [1]. Polygeneration systems can use multiple energy sources (renewable and non-renewable), and provide multiple energy services (heating, cooling, electricity) and other products (water, hydrogen, etc.) [2]. Possible configurations of tri and polygeneration systems have been comprehensively presented in [3], and a thorough review of trigeneration technologies can be found in [4].

Compared to conventional separate production, polygeneration systems can achieve significantly higher energy efficiencies, and are considered a viable solution to promote the development of distributed energy systems. Basically, the high energy efficiency comes from utilizing otherwise wasted energy in the process.

Nevertheless, the inherent complexity of polygeneration energy systems makes their economic, environmental and energy performance highly dependent on system synthesis, equipment selection and capacity, and operational strategy. Therefore, the thermodynamic modelling and the optimization problem formulation of a polygeneration system are crucial tasks, which require suitable optimization techniques integrated with the thermodynamic model [5].

Several optimization techniques have been adopted over the years to tackle the optimization problem of polygeneration systems [6]. The operations research state-of-the-art approach consists in linearizing the original non-linear problem to obtain a Mixed Integer Linear Problem (MILP) [7]. This is due to the advantages offered by the linearity of the problem and to the effectiveness of available commercial MILP solvers. Nevertheless, several drawbacks affect the MILP formulation, such as: the impossibility of taking into account nonlinear effects; the necessity of considering all the time periods at once; the risk of high-dimensionality of the problem. To tackle these limitations, several techniques have been developed, such as: piecewise linearization methods; rolling horizon approaches; dimensionality reduction by means of energy demands clustering algorithms. In this paper, limits and potentials of MILP methods for the optimization problem of polygeneration energy systems are reviewed and discussed.

The remainder of the paper is organized as follows. In Section 2 the Mixed Integer Linear Programming approach for polygeneration energy systems is presented. Limits affecting the MILP formulation and available techniques able to tackle those limitations are discussed in Section 3. Eventually, the last section contains concluding remarks.

2. Mixed Integer Linear Programming methods for polygeneration systems

Fig. 1 shows a schematic configuration of a typical polygeneration system that produces electricity, heat and cooling. A typical polygeneration system consists of a power generation unit (e.g., an internal combustion engine), a heat recovery system, and a thermally activated cooling technology (e.g., an absorption chiller). Other subunits may be included to increase the operational flexibility of the system, such as: auxiliary boilers, electric or fuel driven chillers and heat pumps, and thermal energy storages.

As schematically summarized in Fig. 2, the overall problem of optimization of a polygeneration system can be decomposed into three levels: synthesis, design, and operation [8]. At each level is associated a subproblem [9]. The synthesis subproblem aims at identifying the components to be installed or excluded in the plant, as well as the optimal layout of the plant itself and the energy distribution network. The inclusion/exclusion of a unit can be represented by a binary variable [10]. The design subproblem deals with the optimal sizing/capacity problem of the components. Finally, the optimal operation subproblem aims at determining the operating condition of each unit in each timestep. In this case, the use of binary variables allows to apply a minimum operation constraint to the units, operating in on/off mode. The most common approach in literature is to optimize the operational strategy on hourly basis. The three subproblems should be solved jointly, since they are intrinsically linked. Of course, in case the design is already given, the optimization problem is reduced to determining the optimal operational strategy of the plant.

Due to non-linear constraints corresponding to the performance curves of generation units (typically nonlinear with respect to load, temperature, and size), the resulting problem is generally a Mixed Integer Non-Linear Problem

(MINLP), which is an optimization problem with continuous and discrete variables and nonlinear functions in the objective function and/or the constraints. A recent survey of MINLP solution methods can be found in [11]. However, the MINLP problem is extremely challenging because of nonlinearity, potential non-convexity, and the large number of variables [7]. Tackling MINLP problems, even with cutting-edge algorithms, is extremely challenging. For this reason, Mixed Integer Linear Programming (MILP) is the most common approach to tackle the optimization problem of polygeneration systems.

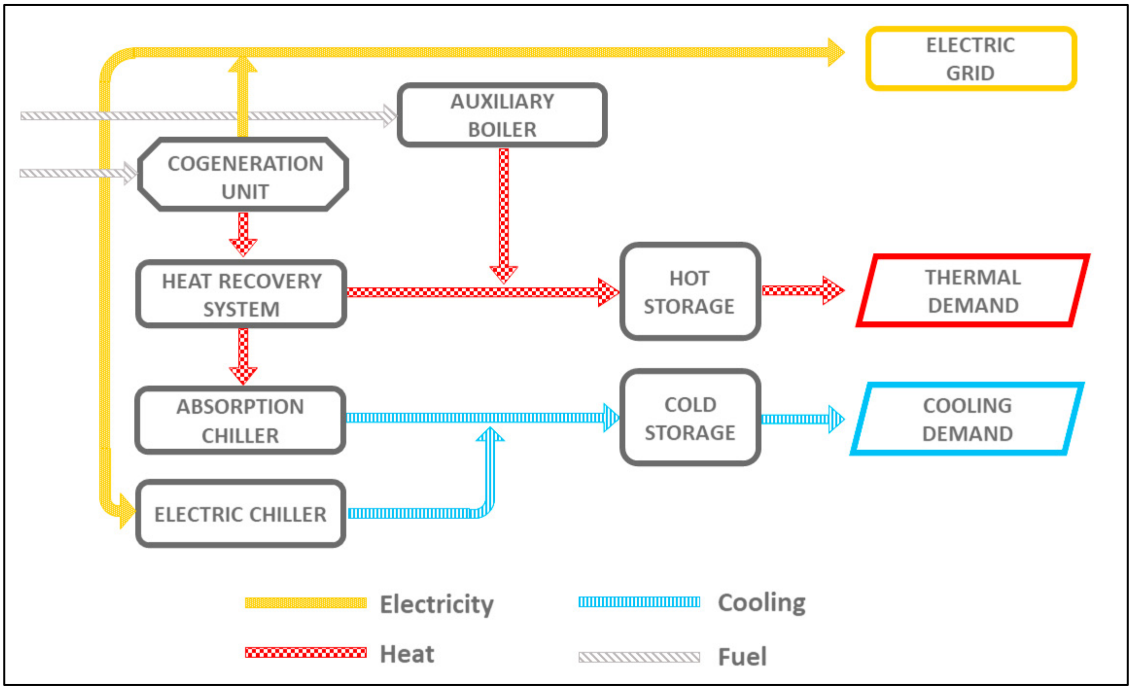

Fig. 1. Schematic representation of a typical polygeneration energy system

The formulation of a MILP problem is usually as follows

$$\underset{x}{\text{minimize}}\, f^t x \tag{1}$$

subject to

$$\begin{cases} x(intcon)\ are\ integers \\ A \cdot x \leq b \\ A_{eq} \cdot x = b_{eq} \\ lb \leq x \leq ub \end{cases} \tag{2}$$

where $f, x, intcon, b, b_{eq}, lb$ and ub are vectors, and A and A_{eq} are matrices.

Compared to MINLP, the mixed-integer linear formulation has two main advantages [7]. First, the convergence of the solution is guaranteed. In fact, the solution is globally optimal, without the risk of finding a local optimum, thanks to the convexity of linear problems. Only when constraints contradict between them (the feasible region is empty) or when the problem is unbounded in the direction of the objective function, no optimal solution can be found [12]. In addition, the solution accuracy is indicated by the branch and bound gap [13]. Another significant advantage is that MILP problems can be tackled by extremely fast and effective commercially available solvers (e.g., CPLEX [14], Gurobi [15], Xpress [16], MATLAB [17]).

Many example of MILP models of polygeneration systems can be found in both recent and long-standing literature. For instance, Arcuri et al. [18] presented a MILP model for the determination of the design and the running conditions of a trigeneration plant for a hospital complex. Bischi et al. [19] proposed an optimization model for determining the operating schedule that minimizes the operating and maintenance costs of a cogeneration system, with a given design. Carvalho el al. [20] found the optimal synthesis of a trigeneration system subject to environmental constraints by means of MILP technique. Ren et al. [21] developed a MILP model for the integrated plan and evaluation of distributed energy systems, minimizing the overall energy cost by selecting the units to install and determining their operating schedules. Ameri et al. [22] proposed a MILP model for optimizing design and operation of trigeneration systems connected to district heating and cooling networks.

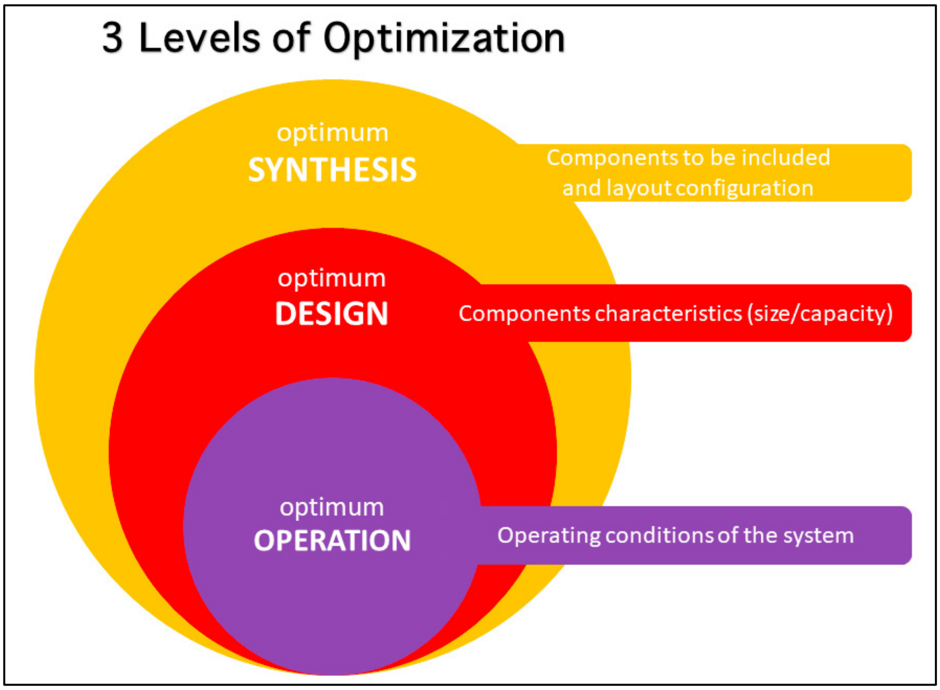

Fig. 2. Three levels of optimization of polygeneration energy systems

3. Mixed Integer Linear Programming limits and supporting techniques

On the other hand, several limitations affect MILP methods. First, non-linear effects obviously cannot be taken into account. In particular, when dealing with the optimal design problem, the efficiency of the units must be kept constant. Therefore, several effects cannot be considered such as: the variation of the nominal efficiency of the components in relation to their size, the variation of the component unitary cost in relation to their size, and part-load effects on nominal efficiency [23]. This problem can be tackled by a decomposition strategy, based either on an iterative procedure [23] or on a multi-stage algorithm [7].

Instead, when dealing with the optimal scheduling problem, a linear relation between the efficiency of the components and their load factor can be easily considered. Nevertheless, real performance curves are usually non-linear, and a further expedient must be adopted, namely piece-wise linearization. For each unit, a piecewise linear approximation, with an appropriate number of intervals, can be selected. Since ambient temperature may affect unit performance, the range as well as the shape of the performance curves can vary with temperature [19]. For instance, Zhou et al. [24] presented a model that approximates the performance curves of an internal combustion engine, a boiler, and an absorption chiller with piecewise linear functions. Dvorák and Havel [25], instead, approximated the extraction-condensing turbine and steam generator efficiency with a piecewise linear function of the fuel mass flow rate. It should also be noted that, while the piecewise linear approximation of one-degree of freedom performance curves is relatively straightforward, piecewise linearization of two-degree of freedom units can be more challenging [19], and dedicated approaches can be found in literature (e.g., [26]).

Moreover, when the capacity of the thermal energy storage is a decision variable of the problem, the temperature of the storage cannot be directly tracked, since it would result in an endogenous problem which cannot be solved within the MILP formulation [27]. For this reason, only the energy stored in each time-step can be tracked, by considering a linear energy balance, and heat losses are accounted on the basis of the amount of energy stored in the thermal energy storage. Nevertheless, the accuracy of the linear model can be satisfactorily increased by considering an additional parameter accounting for static heat losses [28].

Another limit afflicting the MILP formulation, is the need to consider the whole time horizon at once, when dealing with the synthesis and/or design problem. In fact, the synthesis and design problem, and the scheduling problem must be tackled simultaneously. This results in a very large number of variables and constraints, thus making the problem very challenging from the computational point of view. To tackle this issue, several approaches have been proposed.

One kind of approach is based on decomposition methods. To name a few, Yokoyama et al. [29] applied the block angular structure to derive a time-effective suboptimal solution to tackle the optimal design of energy supply systems. Rubio-Maya et al. [30] proposed a two-step procedure for the optimization of a polygeneration unit. The first step consists on the selection and sizing of the units based on monthly average requirements, while the second step deals with hourly data and analysis. Elsido et al. [7], instead, presented a two-stage algorithm: at the upper level, synthesis and design are optimized by means of evolutionary algorithms, while, at the lower level, the operational problem is dealt by means of a MILP method.

A further approach, which can be used in conjunction with the former, is the dimensionality reduction by selecting a series of typical or representative days, instead of the full year [31]. The simplest method to obtain typical periods is to average hourly data over a period (e.g., a month or a season). This approach is broadly used in literature (e.g., [32,33]), because of its simplicity and straightforwardness (the annual sums of energy demands is preserved). Nevertheless, it has some drawbacks: the peak demands are filtered and the correlation between different demand types may be lost. For these reasons, state-of-the-art approach for the selection of typical periods is based on clustering algorithms. These algorithms aim at preserving the most important characteristics, such as the peak demands, the demand duration curves, and the temporal inter-relationship between different demand types [34]. They are usually based on k-means [35] or k-medoids [34] clustering techniques.

An additional method to tackle the high-dimensionality problem is the rolling-horizon technique. It consists of solving MILP problems for short periods (or horizons, e.g., one week), then implementing the solution for one or a few timesteps, and then solving the problem for the following period and so on, until the whole time period has been explored. Rolling-horizon methods are mostly adopted for optimal scheduling problems and they cannot be coupled with design problems while maintaining a MILP formulation, unless a multi-stage formulation is involved (e.g., [36]). The minimum length of the rolling-horizons, so that the solution is not different from the whole-period-at-once solution, depends on the size of the energy storages. In fact, energy storages make the problem “dynamic”, while, when no storage is included in the system, the overall optimum coincides with the sum of optimums of every single timestep. In that case, the problem is “static” because the physical system has no memory of the previous timesteps (as in [37,38]).

Examples of rolling-horizon approaches in literature are as follows. Bischi et al. [39] proposed a rolling-horizon MILP algorithm for optimizing the scheduling of a cogeneration system, taking into account also yearly fiscal incentive. Silvente et al. [40] presented a MILP approach based on a combination of rolling-horizon and stochastic programming formulations. Ommen et al. [41] used the principle of rolling horizon in daily energy planning optimization. Marquant et al. [42] demonstrated the possibility to improve by 15 to 100 times the computational time required to solve energy optimisation problems without affecting the quality of the results, by means of a rolling-horizon technique.

4. Conclusions

Polygeneration energy systems are a promising way to increase energy efficiency and promote the development of distributed energy systems. The performance of polygeneration systems is highly dependent on their layout, units size and capacity, and operational strategy. For these reasons, the optimization problem of polygeneration systems is a topic of current interest. Methodologies based on Mixed Integer Linear Programming represent the state-of-the-art approach to tackle the three-level optimization problem of polygeneration energy systems. In this paper, the MILP formulation has been presented and a concise but meaningful review of related literature has been carried out. Advantages of the MILP approach, such as the guarantee of global optimality and the availability of efficient commercial solvers, as well as limitations afflicting the mixed-integer linear formulation have been discussed. Moreover, several techniques able to tackle those limitations have been presented. Decomposition strategies and piece-wise linearization can deal with non-linear behavior of the energy systems. The high-dimensionality of the MILP problem, mainly due to the necessity of considering the whole time period at once, can be tackled by decomposition methods, clustering algorithms for the selection of typical periods, and rolling-horizon approaches.

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