Quantum Uncertainty in Weakly Non-Ideal Astrophysical Plasma

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Galitskii and Yakimets showed that in dense or low temperature plasma, due to quantum uncertainty effect, the particle distribution function over momenta acquires a power-like tail even under conditions of thermodynamic equilibrium. We show that in weakly non-ideal plasmas, like the solar interior, both non-extensivity and quantum uncertainty should be taken into account to derive equilibrium ion distribution functions and to estimate nuclear reaction rates and solar neutrino fluxes. The order of magnitude of the deviation from the standard Maxwell-Boltzmann distribution can be derived microscopically by considering the presence of random electrical microfield in the stellar plasma. We show that such a nonextensive statistical effect can be very relevant in many nuclear astrophysical problems.

1 Introduction

Recent progress in statistical mechanics indicates the Tsallis non-extensive thermostatistics as the natural generalization of the standard classical and quantum statistics when memory effects and long-range forces are not negligible [1, 2]. The treatment in the frame of the extensive description is based on short-range interaction, on neglecting surface effects and on fluctuations of thermodynamic quantities relatively too small to be observed.

The solar core plasma represents a system of particles that, for many reasons, has an equilibrium distribution deviating very slightly from the Maxwell-Boltzmann (MB) distribution. In fact, the mean Coulomb energy is not much smaller than the thermal kinetic energy; the Debye length is of the order of the inter-ionic distance a_i; it is not possible to clearly separate collective and individual degrees of freedom; the presence of the scales of energies of the same size produces deviations from the standard statistics [4].

In literature, a plasma is characterized by the value of the plasma parameter Γ

$$\Gamma = \frac{(Ze)^2}{a\,kT} \ , \qquad (1)$$

where $a = n^{-1/3}$ is of the order of the interparticle average distance (n is the average density). The plasma parameter is a measure of the ratio of the mean (Coulomb) potential energy and the mean kinetic (thermal) energy.

Depending on the value of the plasma parameter, we can distinguish three regimes that are characterized by different effective interactions and require different theoretical approaches.

- $\Gamma \ll 1$. The plasma is described by the Debye-Hückel mean-field theory as a dilute weakly interacting gas. The screening Debye length

$$R_D = \sqrt{\frac{kT}{4\pi e^2 \sum_i Z_i^2 n_i}} \ , \qquad (2)$$

  is much greater than the average interparticle distance a, hence there is a large number of particles in the Debye sphere ($N_D \equiv (4\pi/3)R_D^3$). Collective degrees of freedom are present (plasma waves), but they are weakly coupled to the individual degrees of freedom (ions and electrons) and, therefore, do not affect their distribution. Binary collisions through screened forces produce the standard velocity distribution.

- $\Gamma \approx 0.1$. The mean Coulomb energy potential is not much smaller of the thermal kinetic energy and the screening length $R_D \approx a$. It is not possible to clearly separate individual and collective degrees of freedom. The presence of at least two different scales of energies of the same rough size produces deviations from the standard statistics which describe the system in terms of a single scale, kT.

- $\Gamma > 1$. This is a high-density/low-temperature plasma where the Coulomb interaction and quantum effects start to dominate and determine the structure of the system.

In the region of plasma parameter $\Gamma = Z^2e^2/(a_ikT) \approx 0.1$, to which the solar core plasma belongs, the Debye character of the screening is maintained but strong interactions at distances of the order of $n^{-1/3}$ (n is the particle density) become the main interactions while involving multiple particles. The reaction time to build up screening after a hard collision is the inverse of the plasma frequency; the collision time is comparable to it. Therefore many collisions are necessary before particles loose memory of the initial state and the scattering process can not be considered Markovian, non-extensive effects should be included [3]. Tsallis non-extensive thermostatistics is actually a well-known tool successfully applied to many different physical problems and can account quite naturally of all the above mentioned features of the solar core plasma by means of the entropic parameter q [1, 4].

Recently, in Ref.s [5, 6] the Green's function technique has been used to examine tunneling reaction probability and the rôle of the many-body effects (or quantum uncertainty effect, following Galitskii and Yakimets [7]) on the rates of nuclear reactions in the laboratory fusion plasma and in the weakly non-ideal sun core. The implications of such effects in the evaluation of the solar neutrino fluxes are discussed in Ref. [6] (for an updated reading on the solar neutrino problem one can refer to Ref.s [8-10] and to the recent Ref. [11] where neutrino decay solution is reexamined in the light of the SuperKamiokande data).

In this paper we find a microscopic interpretation of the non-extensive parameter q in the solar core (in other words, the entity of deviation of the Maxwellian distribution of the ionic equilibrium distribution we use in this work) in terms of the plasma parameter and the ion correlation parameter. After that, we study both the non-extensive (q) and the quantum uncertainty (Q) effects [6, 7], in the calculation of the solar reaction rates and in the solar neutrino fluxes. The two effects should be simultaneously active if the collision frequency is comparable with the plasma frequency; this requirement is fulfilled in the solar core. Finally, we discuss many astrophysical open problems that can be solved assuming that the systems be described by generalized nonextensive statistical distributions.

2 Electrical microfields and power law distribution in the solar core

In this section we want to investigate about a microscopic justification of a metastable power-law stationary distribution inside a stellar core. At this scope, let us start by observing that the time-spatial fluctuations in the particles positions produce specific fluctuations of the microscopic electric field (with energy density of the order of 10^{-16} MeV/fm^3) in a given point of the plasma. The rates are changed respect to the standard ones because of the presence of microfield distributions (with energy density of the order of 10^{-16} MeV/fm^3) which modifies, under particular conditions, the ion distribution tail. The influence of microfields on the rates of nuclear tunneling reactions has been widely studied since the early work of Holtsmark [12] and later, for instance, in the Ref.s [13-16].

In the solar core, the effects of random electric microfields are of crucial importance. These microfields have in general long-time and long-range correlations, can generate anomalous diffusion and may be decomposed in three main components: 1) A slow varying component due to plasma oscillations. The particles see this component as an almost constant external mean field over several collisions. 2) A fast random component related to the diffusive cross section ($\sigma_d \approx 1/v$); this component does not affect the distribution that remains Maxwellian. 3) The third component is related to a short-range two-body strong Coulomb effective interaction. As we will see later, this is the component that alters the distribution.

Let us derive the ion distribution function to be used to calculate tunneling reactions rates in solar plasma, when an electric microfield distribution is present. The equilibrium distribution we are deriving differs from the MB distribution, if particular conditions are fulfilled. In fact, the presence of the electric microfield average energy density, $\langle E^2\rangle$, modifies the stationary solution of the Fokker-Planck equation and the ion equilibrium distribution can be written as [17, 18]

$$f(v) = C\exp\left\{-\int_0^v \frac{mv\,dv}{kT+\frac{2}{3}\frac{e^2\langle E^2\rangle}{xm\nu^2}}\right\}\,, \tag{3}$$

where ν is the total collision frequency, x the elastic energy-transfer coefficient between two particles of the plasma, $x = 2m_1m_2/(m_1+m_2)^2$, m the reduced mass, T the temperature, C the normalization constant.
Defining a critical field $E_c = \nu\sqrt{3xmkT/2e^2}$, we can see from Eq.(3) that if $E \ll E_c$ the distribution is Maxwellian whatever be the value of the frequency ν; if $E \gg E_c$ and ν is not a constant but depends on v, the distribution is a Druyvesteyn like distribution [17]. In the solar core being E not too larger than E_c, the distribution of Eq.(3) differs slightly from the Maxwellian but such small deviation is quite important in the evaluation of the nuclear rates.
The condition that ν be a function of velocity v is verified by the fact that the elastic collision cross section is $\sigma = \sigma_d + \sigma_0$, where $\sigma_d \propto 1/v$ is the elastic diffusion cross section and σ_0 is the enforced Coulomb cross section. The total frequency $\nu = \langle\sigma v n\rangle$ satisfies the relation $\nu^2 = \nu_d^2 + \nu_0^2$ without interference terms.
The expression of σ_0 is due to Ichimaru [19] which developed a strict enforcement of the Wigner-Seitz ion sphere model yielding the elastic cross section $\sigma_0 = 2\pi(\alpha a)^2$ where a is the inter-particle distance, α is a one dimensional parameter related to the probability that the nearest neighbor be at a distance R and therefore related to the pair-correlation function $g(R,t)$.

The explicit expression of the equilibrium distribution (3) for the solar interior can be written as a function of the kinetic energy ϵ_p

$$f(\epsilon_p) = N\exp\left[-\varphi\frac{\epsilon_p}{kT} - \delta\left(\frac{\epsilon_p}{kT}\right)^2\right]\,, \tag{4}$$

where N is the normalization constant and

$$\varphi = \frac{\hat\varphi}{1+\hat\varphi}\,, \quad \hat\varphi = \frac{9}{2}\,x\,\frac{n^2(kT)^2}{Z^2e^2\langle E^2\rangle}\langle\sigma_d^2\rangle\,, \tag{5}$$

$$\delta = \left(\frac{3\langle\sigma_d^2\rangle}{\sigma_0^2} + \frac{1}{\hat\delta}\right)^{-1}\,, \quad \hat\delta = \hat\varphi\frac{\sigma_0^2}{3\langle\sigma_d^2\rangle}\,. \tag{6}$$

In solar interior $\varphi \approx 1$ ($\hat{\varphi} \gg 1$), therefore the equilibrium distribution function containing random microfields with collision frequency depending on the velocity, is given by the Maxwellian distribution times the factor $\exp[-\delta(\epsilon_p/kT)^2]$. It represents, if we recall δ as $\delta = (1-q)/2$, the approximation of the Tsallis distribution when $q \approx 1$ [1]. It is remarkable that such distribution has been postulated ad hoc by Clayton more than twenty years ago in the solution of the solar neutrino problem [20].

By means of Eq.s(4)-(6) we have established a strict connection between the entity of the deviation from the MB in the solar interior, the δ (q) parameter, and the elastic diffusion and Coulomb cross sections. Expliciting such cross sections in terms of the Ichimaru parameter α and the plasma parameter Γ, after straightforward calculations, we obtain that, in the small correction limit relevant to the solar core ($\varphi \approx 1$), the δ parameter can be written as

$$|\delta| \approx \frac{\sigma_0^2}{3\langle\sigma_d^2\rangle} = 12\,\alpha^4\,\Gamma^2 \ll 1\,. \tag{7}$$

Eq.(7) is a crucial result of this paper because it establishes that the presence of electric microfields implies a deviation from the MB distribution, the entity depending on the value of the plasma parameter and the collision frequency by means of the α parameter. As we will see, very small deviations from the Maxwellian tail can produce strong deviations of the reaction rates from their standard values.

Let us remark that we do not discuss the electron screening effect [21]. We can realize that at the temperature, elemental densities and plasma parameter of the sun core, the rates of the reactions could be modified of only few per cent and even less by the screening factor f_0. Its value can be taken equal to unity because its effect on the rates is negligible if compared to the depletions (or enhancements) of the rates due to other effects (electromagnetic fluctuations in a plasma) responsible of the equilibrium distribution function we are considering and evaluating in this work.

3 The uncertainty quantum effect

In a weakly non-ideal plasma, like the solar core, the high value of the collision frequency $\nu =< n v \sigma >$ leads to another interesting effect: the distribution function over momenta can acquire a power-like tail, while the distribution over energy remains Maxwellian. This quantum tail has an influence on the nuclear rates, but how to calculate it is not clear in the sense that the averaging can be made over momentum or over energy.

One can assume the following procedure within a semi classical approach: in the reaction cross section $\sigma(E)$ substitute the energy with $\epsilon_p = p^2/2m$ and average over momentum distribution rather than energy distribution. This procedure is supported by a rigorous treatment based on Green functions technique, recently developed by Savchenko [5]. In a medium the momentum or kinetic energy distribution has relevance to the probability of the barrier penetration rather than energy distribution.

Tunneling particles undergo simultaneous collisions with other particles of the plasma maintained in thermodynamical equilibrium. In free gas approximation, momentum p and energy E of a colliding particle are independent variables not connected by the usual dispersion relation $\delta(E - p^2/2m)$. However, for weakly non-ideal interacting particles, the following relation, obtained in the Born approximation collisions, should be more appropriate:

$$\delta_\gamma(E,p) = \frac{1}{\pi}\,\frac{\gamma(E,p)}{(E-\epsilon_p-\Delta(E,p))^2+\gamma(E,p)^2}\,, \tag{8}$$

where $\gamma \approx h\nu$ and $\Delta \approx -\Gamma kT/2$.

Following a rigorous treatment we should use the non-extensive approach to the Green's function formalism, as discussed by Rajagopal, Mendel and Lenzi [22], and explicitly introduce a generalized spectral function, in the framework of the Tsallis statistics and then deduce the momentum distribution as an energy integral over Green functions. In fact, nonlinearity of the tunneling problem makes the rate very sensible to the behavior of Green function at high momenta. In the framework of extensive statistics, reaction rates are faster by many orders of magnitude compared to their values calculated with the spectral function $\delta(\epsilon_p - p^2/2m)$, if instead the δ_γ of Eq.(8) is used [5, 6]. In quasi-particle approximation real and imaginary parts of the self-energy have a very important rôle.

In this paper, for the sake of simplicity and for a clear understanding of the physical meanings, we use a less technical approach, whose validity in the solar core and other similar plasmas is well verified when $q \approx 1$. We follow the approach of Galitskii and Yakimets [7], generalized by Savchenko and collaborators [5, 6] and calculate the distribution function over momenta as an integration of Tsallis energy distribution and the undeformed spectral density (8). Performing this calculation, we obtain [18]

$$f(p) = f_c(p) + C\,\frac{h\nu\,kT}{\epsilon_p^2}\,\exp\left[-\frac{1-q}{2}\left(\frac{\epsilon_p}{kT}\right)^2\right]\,, \tag{9}$$

where $f_c(p)$ is the Tsallis distribution for $q \approx 1$ and C is a normalization constant. This last expression can be easily used for numerical calculations. We note that the second term in the r.h.s. of Eq.(9) represents the correction to the high momentum tail due to the quantum many-body effects.

4 Signals in astrophysical problems

Let us illustrate few problems where we can find signals of the presence of deviations from the MB distribution. Their solutions can be achieved by means of modified (or generalized) rates calculated by means of deformed distributions. Among the others, we quote A) Solar neutrino fluxes; B) Temperature dependence of modified CNO nuclear reaction rates and resonant fusion reactions. C) Jupiter energy production; D) Atomic radiative processes in electron nuclear plasmas; E) Abundance of Lithium.

For brevity we will discuss here the first two points only. A detailed discussion of the other problems can be found in Ref.s [18, 23].

4.1 Nuclear reaction rates with non-extensive and uncertainty quantum corrections

We know that a solution which coincides with a non-extensive distribution can be obtained only if the electric microfield E is greater than a certain critical value E_c, and if the frequency of collision depends on v. This condition is verified if the Coulomb collisional cross section, which describes the strong part of the collision, is a constant: $\sigma_0 = 2\pi(\alpha a)^2$ and is due to the random contribution of the electric microfield. The above assumptions do not modify the treatment of Galitskii and Yakimets (they consider the Coulomb interaction, but the random part of it is not taken into account), rather are responsible of some changements in the final rates.

The non-extensive distribution function, given by Eq.(4), modifies the reaction rate R_q and can be written as

$$R_q = R_M \, e^{-\delta\gamma_{ij}} \,, \tag{10}$$

where R_M are the Maxwellian rates, $\delta = (1-q)/2$, $\gamma_{ij} = (\epsilon_0^{ij}/kT)^2$, $\epsilon_0^{ij} = 5.64(Z_i Z_j \, A_i A_j/(A_i + A_j)\, T_c/T)^{1/3}$, is the most effective energy and $T_c = 1.36$ keV is the temperature at the center of the Sun.

If we consider only the quantum effect Q, the correction r_{ij} to the Maxwellian rate ($R_Q = R_M r_{ij}$) can be written as [5]

$$r_{ij} = \frac{3^{19/2}}{8\pi^{3/2}} \sum_b \frac{h\nu}{kT} \left(\frac{m_{\rm coll}}{m_r}\right)^{7/2} \frac{e^{\tau_{ij}}}{\tau_{ij}^8} \,, \tag{11}$$

$$m_{coll} = \frac{m_r m_b}{m_r + m_b} \,, \quad m_r = \frac{m_i m_j}{m_i + m_j} \,, \tag{12}$$

$$\tau_{ij} = 3\,(\frac{\pi}{2})^{2/3} \left(100 Z_i^2 Z_j^2 \frac{A_i A_j}{A_i + A_j} T_{\rm keV}^{-1}\right)^{1/3} \,, \tag{13}$$

where m_b is the mass of the background particles colliding with tunneling particles.

Including the Tsallis non-extensive effects, the total rate (i.e. including both the q and Q effects) is

$$R = R_q + R_Q e^{-\delta\gamma_{ij}^*} = R_M(e^{-\delta\gamma_{ij}} + r_{ij} e^{-\delta\gamma_{ij}^*}) \,, \tag{14}$$

where $\gamma^* = (\epsilon_Q/kT)^2$ and ϵ_Q is the effective energy of the quantum correction. Considering the appropriate expression of the collision frequency ν in terms of σ_0 and σ_d, we can verify that approximately holds the numerical relation: $\epsilon_Q \approx 3\epsilon_0$. Although the values r_{ij} calculated in Ref.[6] are of order $10^8 \div 10^9$, the factor $e^{-\delta\gamma^*}$ has a large exponent and can strongly suppress the enhancement given by r_{ij}. In fact, the quantum uncertainty corrections calculated on the basis of Tsallis distribution $r_{ij} e^{-\delta\gamma^*}$ are of the same order of $e^{-\delta\gamma}$ and even less (if $\delta \approx 10^{-2} \div 10^{-3}$, as we can deduce from Eq.(7)). Solar conditions admit small non-extensive statistics effects in the equilibrium distribution, showing very small deviations in the Maxwellian tail. By using a numerical code based on a complete evolutionary stellar model, it has been verified that the consequence of this statement are compatible to the experimental results on neutrino fluxes [24].

Let us now define the following quantities:

$$A = \frac{\Phi(Be^7)/\Phi^M(Be^7)}{\Phi(B)/\Phi^M(B)} \,, \quad B = \frac{\Phi(B)}{\Phi^M(B)} \,,$$
$$C = \frac{\Phi(Be^7)}{\Phi^M(Be^7)} \,,$$

and $k_{e7} = z k_{17}^M$, $k_{e,7} = y\, k_{17}$, where k_{ij} and k_{ij}^M are respectively the modified and Maxwellian rates; the value of the constant $z = 227$ can be found, for instance, in Ref.[25].
We can take the usual time-dependent equations of the rates with the solar luminosity constraint (as reported for instance in Ref.[6]) to derive the steady state solutions for the elemental densities. Using the expressions of the rates of Eq.(14) with quantum uncertainty and non-extensive corrections, the following set of equations can be derived:

$$A = \frac{C}{B} = \frac{e^{\delta_{17}\gamma_{17}^*}}{r_{17}} e^{-\delta_{17}\gamma_{17}} \ll r_{17} e^{-\delta_{17}\gamma_{17}^*} \,, \quad \frac{y}{z}\frac{1}{A} = 1 \,,$$
$$\frac{n_{Be^7}}{n_{Be^7}^M} = \frac{n_3}{n_3^M}(e^{-\delta_{34}\gamma_{34}} + r_{34} e^{-\delta_{34}\gamma_{34}^*}) \times$$
$$\frac{1}{[1 + (e^{-\delta_{17}\gamma_{17}} + e^{-\delta_{17}\gamma_{17}^*})/(2z)]} \,.$$

A reasonable evaluation of α gives: $\alpha = 0.55$, with $\Gamma \sim 0.1$ we obtain $q = 0.990$ ($\delta = 0.005$) for all components (see the Conclusions for a comment on the uncertainty of these values).

If we assume $n_3/n_3^M \simeq 3 \cdot 10^{-3}$ (let us remark that density is not changed alone, but all the tunneling rates are changed consistently solving the set of the mentioned equations) we obtain

$$\frac{\Phi(Be^7)}{\Phi^M(Be^7)} = \frac{1}{50} \,, \quad \frac{\Phi(B)}{\Phi^M(B)} \approx \frac{1}{2} \,,$$
$$\text{Gallium} = 81 \text{ SNU} \,, \quad \text{Chlorine} = 2.8 \text{ SNU} \,,$$

$\Phi(pp)$ and luminosity are practically unchanged respect to the SSM values.

The CNO rates that are strongly enhanced by the quantum uncertainty effect are remarkably reduced by the factor $e^{-\delta\gamma^*}$. We have introduced the assumption that n_3 is of the order of about 10^{-3} reduced respect SSM and consistent with the set of time-dependent equations of the rates. This value is within the constraints actually imposed by helioseismology because in the region $r/R_{sun} < 0.2$ the value of n_3 can be submitted to a large variability [26].

4.2 Temperature dependence of modified CNO nuclear reaction rates

The temperature dependence of CNO cycles nuclear rates is strongly affected by the presence of nonextensive effects in Sun like stars evolving towards white dwarfs ($10^7 \div 10^8$ K). Small deviations ($q = 0.991$) from MB distribution strongly increase the rates and can explain the presence of heavier elements (Fe, Mg) in final composition of white dwarfs, consistently with recent limit of the fraction of energy the Sun produces via the CNO fusion cycle (neutrino constraints). We obtain that [27] i) the luminosity yield of the pp chain is slightly affected by the deformed statistics, with respect to the luminosity yield of the CNO cycle; ii) the nonextensive CNO correction ranges from 37% to more than 53%; iii) above $T \approx 2\cdot 10^7$ K, the luminosity is mainly due to the CNO cycle only, thus confirming that CNO cycle always plays a crucial role in the stellar evolution, when the star grows hotter toward the white dwarf stage. Our results are reported in Fig. 1 and Fig. 2. In Fig. 1, we plot the dimensionless luminosity over temperature, for the *pp* chain and the CNO cycle. In Fig. 2, we report the dimensionless equilibrium concentrations of CNO nuclei over temperature.

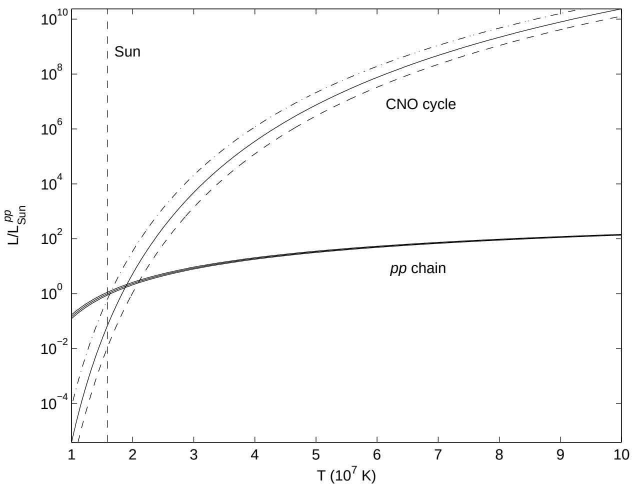

Figure 1. Log-linear plot of dimensionless luminosity over temperature, for the *pp* chain and the CNO cycle. Dashed line, $\delta = +0.0045$, $q = 0.991$; dash-dotted line, $\delta = -0.0045$, $q = 1.009$. The vertical line shows the Sun's temperature. All curves are normalized with respect to the *pp* luminosity inside the Sun.

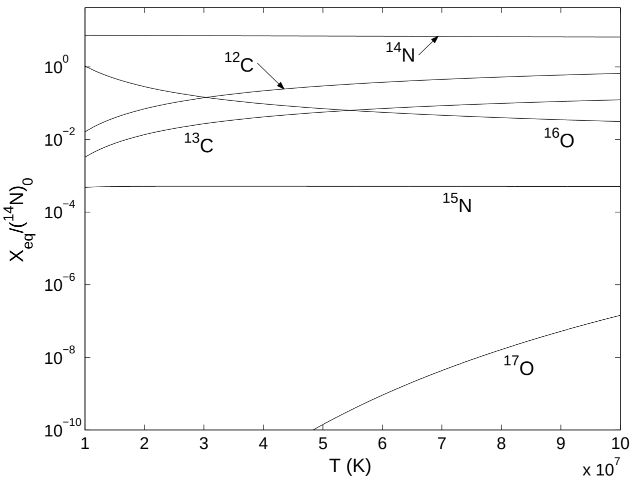

Figure 2. Log-linear plot of dimensionless equilibrium concentrations of CNO nuclei over temperature. Classical statistics has been used. All curves are normalized with respect to the initial density $(^{14}N)_0$ inside the Sun.

For $\delta < 0$ (i.e. $q > 1$), we obtain a remarkable increase of the CNO reaction rates with a more relevant contribution to the star luminosity with respect to the one obtained in the classical picture. Such a modification is consistent with the recent solar neutrino constraints that fix the deformation parameter to the value $|\delta| < 0.0045$. The modified CNO reaction rates thus imply a faster evolution of stellar nuclear plasma towards heavier elements (like Mg and Fe) at high temperature. Such a behavior can be very relevant to understand the nature of matter in white dwarfs stars. In fact, a comparison of current observed mass-radius determinations with the theoretical curves seems to confirm that the composition of most white dwarfs is dominated by medium weight elements (carbon and oxygen). However, a small minority of white dwarfs do have relatively small radii indicating the presence of iron cores, which presents an intriguing puzzle from the point of view of stellar evolution. A slight non-extensivity of the system could explain the presence of heavy elements in the final composition of white dwarf's core.

4.3 Resonant reaction rates in astrophysical plasma

Cussons, Langanke and Liolios [28] proposed, on the basis of experimental measurements at energy $E \sim 2.4\,\mathrm{MeV}$, that the resonant behavior of the stellar $^{12}\mathrm{C}+^{12}\mathrm{C}$ fusion cross section could continue down to the astrophysical energy range.

The reduction of the resonant rate due to resonant screening correction amounts to 11 orders of magnitude at the resonant energy of $400\,\mathrm{keV}$, with important implications for hydrostatic burning in carbon white dwarfs.

We have analytically derived two first-order formulae that can be used to express the non-extensive reaction rate as a product of the classical reaction rate times a suitable corrective factor for both narrow and wide resonances.

Concerning the fusion reactions between two medium-weighted nuclei, for example the $^{12}\mathrm{C} + {}^{12}\mathrm{C}$ reaction, our non-extensive factor, which can be formally defined as follows [29]

$$f_{NE} = 1 + \frac{15}{4}\delta - \left(\frac{E_R}{\mathrm{k_B T}}\right)^2 \delta \,,$$

gives rise to further correction beside the screening and the potential resonant screening

$$F = f_{NE} \cdot f_S \cdot f_{RS} \,, \tag{15}$$

where f_S and f_{RS} account for the Debye-Hückel screening and the resonant screening effect respectively.

We have applied our results to a physical model describing a carbon white dwarf's plasma, with a temperature of $T = 8 \cdot 10^8\,\mathrm{K}$ and a mass density of $\rho = 2 \cdot 10^9\,\mathrm{g/cm^3}$ (the plasma parameter is, correspondingly, $\Gamma \simeq 5.6$). Furthermore, we have set a deformation parameter $|\delta| = 10^{-3}$, regardless of its sign, and we have kept the energy of the possible resonance, E_R, as a free parameter. In Fig. 3 we plot our estimation of the effective total factor F as a function of the resonance energy E_R.

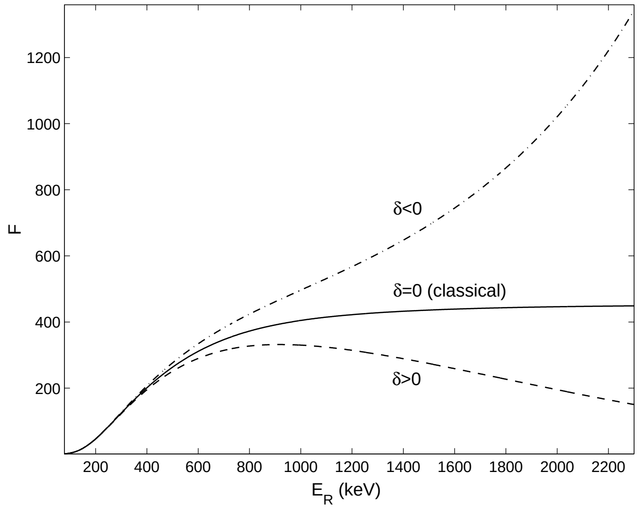

Figure 3. Linear plot of the effective factor F, defined in Eq.(15), against the resonance energy E_R. The dash-dotted (upper) line refers to super-extensivity, the dashed (lower) line to sub-extensivity, while the solid (middle) line describes the classical (MB) result.

All the plasma enhancements due to the presence of long-range many-body nuclear correlations and memory effects, that can be described within the non-extensive statistics by means of the entropic parameter $q > 1 (\delta < 0)$, are in the direction of still more increasing the effective factor F of nuclear rates of hydrostatic burning and white dwarfs environment.

5 Conclusions

In a weakly non-ideal plasma, like the solar interior, ions and electrons behave like quasi-particles and, due to the quantum uncertainty, momentum and energy of colliding particles are independent variables. This many-body effect is responsible of an additional power-law tail to the Maxwellian momentum equilibrium distribution, as discussed by Galitskii and Yakimets [7], while the energy particle distribution remains Maxwellian. The central and more important quantity that enters into the quantum correction of the distribution is the elastic collision frequency which, in these weakly non-ideal plasmas, has a value of the same order as the collective plasma frequency. Being such a value of collision frequency in systems of particles with long-range interactions related also to memory effects with long-time tails, the equilibrium non-extensive statistics appears to be the most suitable statistics to use and it can take into account, in a very natural way, both quantum and non-extensive effects.

Savchenko [5] has shown that the rigorous procedure of averaging of tunneling probability, based on Green function technique, gives results nearly coincident with the results obtained by the simple averaging. Therefore, we can show that, by using the modified distribution function, Eq.(4), (which corresponds to the equilibrium Tsallis distribution when deviations from standard statistics are small) instead of the MB one, the strong increase of the nuclear reaction rates caused by quantum uncertainty effects alone is greatly (or fully) reduced. We can confirm the validity of the Tsallis statistics to describe weakly non-ideal plasmas such as the solar core and of its use to calculate solar neutrino fluxes towards a closer agreement to the experimental measurements than standard calculations. In fact, the calculated fluxes are in good agreement with the experimental results using the reaction rates of Eq.(14) with quantum uncertainty and non-extensive corrections and a ^3He density (in the core) 10^{-3} times the value usually taken in SSM. This figure is consistent with helioseismology constraints. Finally, we have justified the entropic parameter q (δ) of the non-extensive statistics because we have related this quantity to the electric microfields distribution in the plasma (or explicitly, to the plasma parameter Γ and to α, the ion correlation function). Therefore the quantity q cannot be considered a free parameter. However, the evaluation of q (or δ) is not absent of some uncertainty because the quantity α (δ depends on the fourth power of α) is still to be deeply analyzed and evaluated, in spite of the many efforts spent in the past on space-time correlation functions of ions in stellar plasma.

Deviations from momentum Maxwellian distribution are quite small for the astrophysical systems, like the Sun, and the main properties of the stars do not change at all for these deviations. Nevertheless, very slight deviations can sensibly affect the evaluation of nuclear fusion rates and could be useful in solving the problem, for instance, of lithium abundance in the universe, among many other applications.

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