Available online at www.sciencedirect.com

ScienceDirect

Procedia Engineering 10 (2011) 2098–2103

Engineering Procedia

ICM11

Discontinuous FE approach and lattice models to describe cracking behaviour in fibre-reinforced brittle materials

Roberto Brighenti[a]*, Andrea Carpinteri[a], Andrea Spagnoli[a], Daniela Scorza[a]

[a] *Department of Civil-Environmental Engineering and Architecture, University of Parma, Parma 43100, Italy*

Abstract

The numerical evaluation of the mechanical behaviour of quasi-brittle materials like rocks, concrete, ceramics etc. is difficult from the computational point of view due to complex highly nonlinear phenomena occurring in the fracture failure mechanism, and the material softening behaviour due to the cracking process usually leads to a strong mesh dependence. The use of reinforcing fibres is an efficient and economic method to enhance the mechanical behavior of such materials. For a quasi-brittle multiphase material, such as a fibre-reinforced concrete (FRC), several phenomena must be considered in the computational simulation, such as matrix cracking, fibre bridging effects, fibre debonding, fibre breaking and so on. Continuum mechanics approaches as well as micromechanical ones have been developed for the computational solution of such problems. In the present paper, the mechanical behaviour of fibre-reinforced materials is analysed by adopting both a new discontinuous-like FE approach and a lattice model. The main phenomena involved, such as crack formation and propagation, crack fibre bridging, fibre debonding, fibre breaking, etc., are taken into account and examined using the above two models. The basic assumptions and theoretical background of such approaches are outlined and, finally, some experimental data related to notched plain or fibre-reinforced concrete specimens under mixed-mode monotonic loading are analysed.

Keywords: Quasi-brittle materials; fracture; strain localization; fibre; lattice model; discontinuous FEM.

1. Introduction

The numerical evaluation of the mechanical behaviour of quasi-brittle materials like rocks, concrete, ceramics etc. is difficult due to complex phenomena occurring in the fracture failure mechanism. The softening mechanical characteristics of such a class of materials involve a highly nonlinear behaviour

* Corresponding author. Tel.: +39 0521 905910; fax: +39 0521 905924.
E-mail address: brigh@unipr.it.

1877-7058 © 2011 Published by Elsevier Ltd. Open access under CC BY-NC-ND license.
Selection and peer-review under responsibility of ICM11 doi:10.1016/j.proeng.2011.04.347

which can be responsible of non-uniqueness of the solution. Due to the cracking process, which can be considered as a strain localization phenomenon, a strong mesh dependence typically arises for these materials. To overcome the above drawbacks of quasi-brittle materials, the use of reinforcing fibres can be an efficient and economic method to enhance their mechanical behaviour. For a quasi-brittle multiphase material, such as a fibre-reinforced concrete (FRC), several phenomena must be considered in the computational simulation, such as fibre bridging in the cracked zone, fibre debonding, fibre breaking and so on. In order to properly evaluate the mechanical behaviour of quasi-brittle materials, some continuum mechanics approaches have been developed such as remeshing strategies [1], strain softening in the context of plasticity [2], meshless methods [3], X-FEM approach [4], discontinuous finite elements [5-8] and so on. By observing the micro-mechanics of rupture in quasi-brittle materials, a physically-based micromechanical approach can be adopted for simulations. Lattice models belong to such a class of methods, and have been successfully applied to analyse fracture processes [9-11]. Even heterogeneous materials and multi-phase composites (such as fibre-reinforced materials) can successfully be modelled.

In the present paper, the mechanical behaviour of fibre-reinforced materials is analysed by adopting both a new discontinuous-like FE approach [12] and a triangular lattice model [13] developed for two-dimensional problems. The main phenomena involved, such as crack formation and propagation, crack fibre bridging, fibre debonding, fibre breaking, etc. are taken into account and examined using the above models. The basic assumptions and theoretical background of such approaches are outlined and, finally, experimental data related to notched plain or fibre-reinforced concrete specimens under mixed-mode monotonic loading are analysed. The results provided by the two approaches are compared and discussed.

2. A continuous approach to fracture: the discontinuous-like FE model

From a mathematical point of view, when a discontinuity of the displacement field occurs along a straight line S crossing a finite element (Fig. 1a), the displacement field $\mathbf{u}(\mathbf{x})$ can be written as follows [7, 8]:

$$\mathbf{u}(\mathbf{x}) = \mathbf{N}(\mathbf{x}) \cdot \boldsymbol{\delta} + \left[H(\mathbf{x}) - \mathbf{N}^{+}(\mathbf{x}) \right] \cdot \mathbf{w}_n \tag{1}$$

where the interpolated FE displacement field $\mathbf{u}(\mathbf{x})$ in the element is the sum of the continuous displacement field $\mathbf{N}(\mathbf{x}) \cdot \boldsymbol{\delta}$ (where $\mathbf{N}(\mathbf{x})$ is the classical FE shape function matrix, and $\boldsymbol{\delta}$ is the nodal displacement vector), and of the discontinuous part $\left[H(\mathbf{x}) - \mathbf{N}^{+}(\mathbf{x}) \right] \cdot \mathbf{w}_n$ (where $H(\mathbf{x})$ is the Heaviside jump function, $H(\mathbf{x}) = 0$ if $\mathbf{x} \in \Omega_e^{-}$, $H(\mathbf{x}) = 1$ if $\mathbf{x} \in \Omega_e^{+}$, Fig. 1). The displacement jump vector can be written as $\mathbf{w}_C = \mathbf{u}_C + \mathbf{v}_C$, where $\mathbf{u}_C$ and $\mathbf{v}_C$ are the displacement jumps normal and parallel to the crack line, respectively (Fig. 1b). By using the nodal counterparts, the above relationship can be written as $\mathbf{w}_n = \mathbf{u}_n + \mathbf{v}_n$, where $\mathbf{u}_n$ and $\mathbf{v}_n$ are the nodal displacement jumps normal and parallel to the crack, respectively. The small strain field and the bounded part of the linear elastic stress field can be written:

$$\boldsymbol{\varepsilon}(\mathbf{x}) = \underbrace{\mathbf{B}(\mathbf{x})\boldsymbol{\delta} - \left[\mathbf{B}^{+}(\mathbf{x}) \otimes \mathbf{w}_n \right]^s}_{\boldsymbol{\varepsilon}^b(\mathbf{x})} + \underbrace{\delta_s \mathbf{w}(\mathbf{x})}_{\boldsymbol{\varepsilon}^u(\mathbf{x})} \qquad \boldsymbol{\sigma}(\mathbf{x}) = \mathbf{C} : \boldsymbol{\varepsilon}^b(\mathbf{x}) = \mathbf{C} : \left[\mathbf{B}(\mathbf{x})\boldsymbol{\delta} - \left[\underbrace{\mathbf{B}^{+}(\mathbf{x})}_{\sum_{i \in \Omega_e^+} B_i(\mathbf{x})} \otimes \mathbf{w}_n \right]^s \right] \tag{2}$$

where $\mathbf{B}(\mathbf{x})$ is the compatibility matrix and δ_s is the Dirac delta function in S. Further, $\boldsymbol{\varepsilon}^b(\mathbf{x})$ and $\boldsymbol{\varepsilon}^u(\mathbf{x})$ are the bounded part and unbounded part of the strain tensor, respectively. Outside a narrow region across the discontinuity line S, the material can be assumed to remain continuous (i.e. in Ω_e^{-} and Ω_e^{+}, Fig. 1b), so that the strains and the stresses are bounded in $\Omega \setminus S$. Once the nodal crack jump displacement vector $\mathbf{w}_n$ is known through the material's cohesive law, the effective displacement field

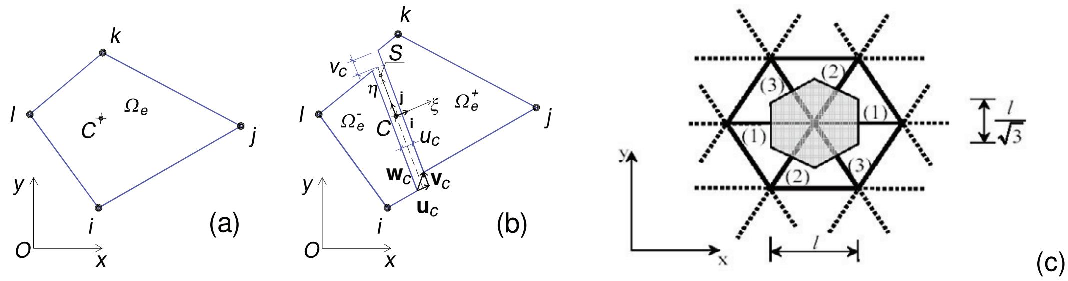

Fig. 1. Embedded discontinuity of the displacement field in a finite element: uncracked (a) and cracked 4-noded finite element (b). The unit cell of a regular triangular lattice (c).

can be found. As a matter of fact, it is well-known that, in the so-called process zone around the crack tip, we can assume the existence of a bridging and/or of a friction stress, which is usually well described by a decreasing function of the jump displacement $\mathbf{w}$. The following relationships for the bridging normal and shearing stresses are used [12]:

$$\sigma_c(u_c) = f_t \cdot e^{\frac{2f_t(u_0-u_c)}{2G_f - f_t \cdot u_0}} \quad \text{with} \quad G_f = \int_{v_0}^{+\infty} f_t \cdot e^{\frac{2f_t(u_0-u_c)}{2G_f - f_t \cdot u_0}} du_c \quad \text{and} \quad \tau_c(u_c) = \begin{cases} \sigma_c(u_c) \cdot \upsilon \left(1 - \left(\dfrac{u_c}{2 \cdot r_c}\right)^m\right) & \text{if } 0 < u_c < 2 \cdot r_c \\ 0 & \text{if } u_c > 2 \cdot r_c \end{cases} \tag{3}$$

where f_t is the maximum tensile strength of the material, u_0 is the lower crack opening limit at which the bridging process starts, G_f is the fracture energy (energy for unit surface crack which governs the softening behaviour), equal to the area under the curve $\sigma_c(u_c)$, and υ is the friction coefficient.

The fracture of the material can be treated in a way similar to that typical of the classical elastic-plastic FE formulation for non-linear materials, i.e. by introducing an ad-hoc FE stress field relaxation in order to represent the effective mechanical effects of the crack (stress bridging) on the damaged material. This approach allows us to keep the continuity of the effective displacement field inside each finite element, [12]. The FE equilibrium equations are fulfilled iteratively by driving the unbalanced nodal force vector $\mathbf{f}_{e,u}^{(i)}$ to very small values. At the load step *i*, the unbalanced nodal force vector can be written as follows:

$$\mathbf{f}_{e,u}^{(i)} = \mathbf{f}_{e,ext}^{(i)} - \int_{\Omega_e} \mathbf{B}^t \cdot \boldsymbol{\sigma}_{rel}(\mathbf{w}_c)\, d\Omega \tag{4}$$

where $\mathbf{f}_{e,ext}^{(i)}$ is the external nodal force vector. Further, $\boldsymbol{\sigma}_{rel}(\mathbf{w}_c)$ is the stress tensor evaluated in the finite element fulfilling the crack bridging stress-strain relationship, and $\mathbf{w}_C = \mathbf{u}_C + \mathbf{v}_C$ is the relative displacement vector across the crack. By taking into account the relaxed stress field [12], the incremental variational formulation of the problem leads to:

$$\mathbf{K}d\boldsymbol{\delta} - (\mathbf{K}_n + \mathbf{K}_s)d\boldsymbol{\delta} = \int_{\Omega} \mathbf{N}^t\, d\mathbf{b} d\Omega + \int_{\Gamma_t} \mathbf{N}^t\, d\mathbf{t} d\Gamma = d\mathbf{f}_1 \quad \text{with} \tag{5}$$

$$\mathbf{K}_n = \int_{\Omega} \mathbf{B}^t \mathbf{C}\, \mathbf{B} \cdot \mathbf{N}(\mathbf{x}) [A(\boldsymbol{\delta}, \sigma_c(u_c)) \cdot \mathbf{D}] d\Omega, \quad \mathbf{K}_s = \int_{\Omega} \mathbf{B}^t \mathbf{C}\, \mathbf{B} \cdot \mathbf{N}(\mathbf{x}) [B(\boldsymbol{\delta}, \tau_c(u_c)) \cdot \mathbf{E}] d\Omega$$

where $\boldsymbol{\delta}$ is the nodal displacement vector, $A(\boldsymbol{\delta}, \sigma_c)$ and $B(\boldsymbol{\delta}, \sigma_c)$ are matrices depending on the crack face displacement jump, and $\overline{\mathbf{K}} = [\mathbf{K} - (\mathbf{K}_n + \mathbf{K}_s)]$ assumes the role of the condensed stiffness matrix of the cracked FE [12].

The mechanical behaviour of FRC can be modeled by assuming that the fibre volume fraction is constant and that the fibres are homogeneously distributed in every directions so that a reference elementary volume (REV) can be used to represent the macroscopically homogeneous composite. The equivalent elastic tensor of the homogenised material $\mathbf{C}_{eq}$ can be written as follows [14]:

$$\mathbf{C}_{eq} = \frac{1}{V} \cdot \int_V \left\{ \kappa(\mathbf{x}) \cdot \mathbf{C}_m + \sum_{p=1}^{q} \chi_p(\mathbf{x}) \cdot E_f \cdot [\mathbf{Q} \otimes \mathbf{Q}] \right\} dV = \mu \cdot \mathbf{C}_m + \eta \, E_f \cdot \int_V \mathbf{Q} \otimes \mathbf{Q} \, dV \tag{6}$$

where $\mathbf{Q} = \mathbf{i} \otimes \mathbf{i}$ ($\mathbf{i}$ being the generic fibre direction versor). The matrix and fibres volume fractions are $\mu = V_m/V$, $\eta = V_f/V$ and the fibre elastic modulus is E_f. To take into account for the fibre debonding, a sliding scalar function s can be introduced to assess the matrix-fibre strain jump, $[\![\varepsilon_{f-m}]\!] = \varepsilon_f^m \cdot (1 - s(\varepsilon_f^m))$ where ε_f^m is the matrix strain measured in the fibre direction. The equivalent elastic tensor (Eq. (6)) can be updated as follows:

$$\mathbf{C}_{eq} = \mu \cdot \mathbf{C}_m + \eta_p \cdot E_f \cdot \left[s(\varepsilon_f^m) + \varepsilon_f^m \cdot \frac{ds(\varepsilon_f^m)}{d\varepsilon_f^m} \right]_p \cdot \int_V \mathbf{Q} \otimes \mathbf{Q} dV \tag{7}$$

In order to take into account the fibre failure, the initial fibre length ($2L_f$) can be simply reduced (to represent the fibre fragmentation) once the maximum tensile strength is reached.

3. A discrete approach to fracture: the lattice model

A two-dimensional lattice is adopted to discretize the continuum model of the material. In such a lattice model, a regular triangular geometry (i.e. with hexagonal unit cells) constituted by truss elements is employed. The length l of the truss elements dictates the level of the discretization. The Young modulus of the truss elements in the lattice model determines the stiffness of the continuum discretized through the lattice. The relationship between the Young modulus of the truss ($\overline{E}$) and that of the continuum (E) is given by [9]:

$$\overline{E} = E\sqrt{3} l / 2A \tag{8}$$

where A is the cross-sectional area of the truss elements. The adopted lattice of truss elements enforces a Poisson ratio value of the continuum equal to 1/3 [9, 10].

In the continuum model, the tensile behaviour of a fibre-reinforced composite can be described according to the cohesive crack approach. Hence, the stress-strain curve is determined through three constituting laws: the constitutive law of solid concrete (bulk material); the crack bridging law of plain concrete; the crack bridging law related to fibres. The resulting stress-strain curve is characterized by a perfectly elastic behaviour in compression; the tensile behaviour is elastic up to a first cracking stress, and a linear piecewise postcracking curve with softening branches follows. With reference to the lattice model, the stress-strain curve in the continuum combining the above three constituting laws can be re-written by using an appropriate continuum-to-lattice stress transformation rule and by smearing the strain components in the continuum along the truss length l [13].

4. Numerical applications

The (mixed mode) fracture behaviour of a four-point shear loaded single-edge notched beam is herein examined. Such a beam configuration has been used by several authors as a benchmark test for numerical analyses [7, 8]. The geometrical parameters of the structure and of the FE and lattice discretisations are displayed in Fig.2. A beam thickness equal to 0.1 m is adopted and a plane stress condition is assumed in the beam. The mechanical parameters of the beam material are the following: Young modulus $E = 35\,\text{GPa}$, ultimate tensile strength $f_t = 2.8\,\text{MPa}$, fracture energy $G_f = 100\,\text{N/m}$. Both plain and fibre-reinforced cases are examined. In particular, two FRCs are adopted: one with PVA fibres ($\eta_f = 5\%$, $E_f = 60\,\text{GPa}$, $2L_f = 6\text{mm}$, fibres diameter $\phi_f = 14\mu\text{m}$, $\tau_{au} = 3\,\text{MPa}$, $\tau_{fu} = 0.1\tau_{au}$), and the second one with steel fibres ($\eta_f = 5\%$, $E_f = 200\,\text{GPa}$, $2L_f = 24\text{mm}$, $\phi_f = 0.5\text{mm}$, $\tau_{au} = 1.5\,\text{MPa}$, $\tau_{fu} = 0.1\tau_{au}$). In the present analyses, the aspect ratios of fibres are assumed as such as to exclude fibre tensile failure (only fibre debonding is herein considered). A FE model with an irregular mesh (non structured mesh), 436 four-node bilinear elements and 479 nodes (Fig. 2a), and a lattice model with l = 0.5 mm (Fig. 2b) are adopted. In order to reduce bias of the crack trajectory, the triangular lattice is made irregular by randomly perturbing the nodal coordinates. In more detail, each node of the lattice is shifted by a random amount, following a uniform probability distribution, within a square box with size a (a value of $a = 0.25l$ is arbitrarily adopted herein). The analysis is performed under displacement control by imposing a progressive vertical displacement at the two bottom loaded points (Fig. 2a), and keeping the ratio between the displacements of the above two points equal to that obtained from a load-controlled linear-elastic analysis with P and $P/10$, respectively.

The vertical applied load P against the crack mouth sliding displacement d (CMSD, [7, 8]) is graphically represented in Fig. 3a together with literature results [7, 8]. All the results are in satisfactorily agreement. It can be noted as the lattice model slightly overestimates the initial stiffness of the structure while the peak load is well evaluated for the plain concrete beam. Some differences can be appreciated in the softening branch of the numerical FEM curves for the case of plain concrete. Moreover, it can be observed that both the models being presented well describe the increase of load-carrying capacity and of the ductility in the material due to the presence of fibres, showing a better behaviour in the case of PVA fibres with respect to steel ones. Finally, the crack patterns obtained from the two models are displayed in Figs 3b-e. For plain concrete and the same load level, a more extended and diffused fracture zone for is provided by the FEM model, while the lattice approach provides a narrow fracture zone.

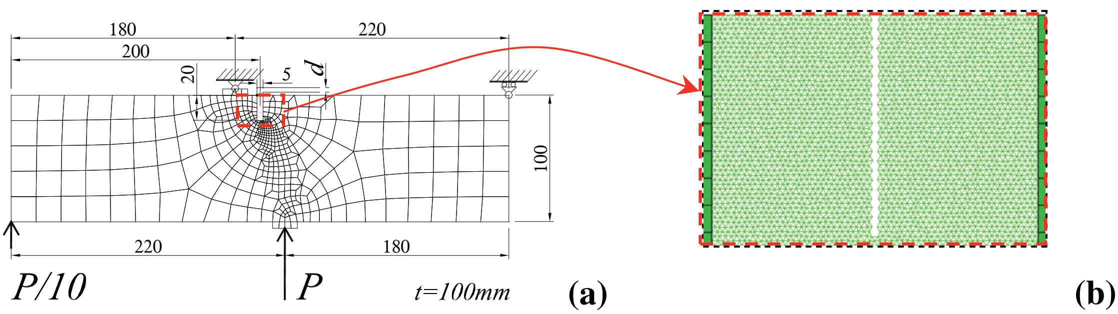

Fig. 2. Single-edge notched beam under four point shear: FE discretisation with 436 four-noded bilinear elements and 479 nodes (a). Detail of lattice discretization with truss length l = 500 μm (b).

5. Conclusions

The crack formation and propagation in plain and fibre-reinforced quasi-brittle materials have been analysed by using a discontinuous-like FE approach and a microstructural lattice model. The main theoretical aspects of the two approaches have been outlined, and experimental results related to plain and

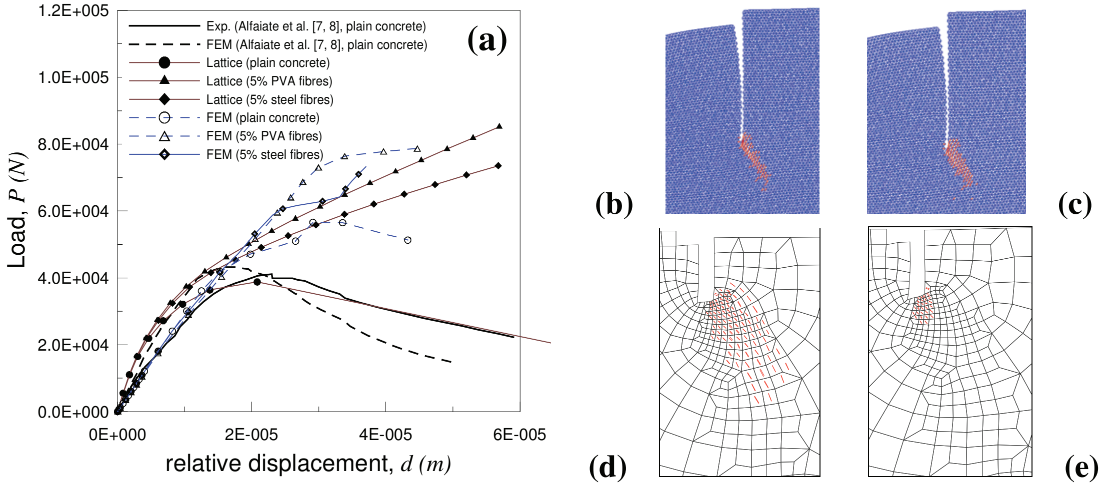

Fig. 3. Vertical bottom applied load vs crack mouth sliding displacement *d* the for plain and fibre-reinforced materials (a). Crack patterns obtained through the lattice (plain (b), with PVA Fibres (c)) and FEM (plain (d),with PVA Fibres (e)) at $P \cong 16KN$.

fibre-reinforced concrete specimens under mixed mode of fracture have been examined. The comparison of the simulation results shows that the lattice model allows a very detailed description of the fracture pattern, whereas the discontinuous FE approach gives mainly global information, in terms of both crack path and stress-strain response curve also for fibre-reinforced brittle materials. Nevertheless, the FE approach is computationally convenient and is a useful tool for studying problems which do not require a detailed description of the fracture process.

References

[1] Rashid MM. The arbitrary local mesh refinement method: an alternative to remeshing for crack propagation analysis. *Comput Meth Appl Mech Engng* 1998; **154**: 133–150.

[2] de Borst R, Remmers JJC, Needleman A, Abellan MA. Discrete vs smeared crack models for concrete fracture: bridging the gap. *Int J Numer Anal Meth Geomech* 2004; **28**: 583–607.

[3] Belytschko T, Krongauz Y, Organ D, Fleming M. Krysl P. Meshless methods: an overview and recent developments. *Comput Meth Appl Mech Engng* 1996; **139**: 3–47.

[4] Dolbow J, Möes N, Belytschko T. An extended finite element method for modelling crack growth with frictional contact. *Comput Meth Appl Mech Engng* 2001; **190:** 6825–6846.

[5] Belytschko T, Fish J, Engelmann BE. A finite element with embedded localization zones. *Comput Meth Appl Mech Engng* 1988; **70**: 59–89.

[6] Oliver J. On the discrete constitutive models induced by strong discontinuity kinematics and continuum constitutive equations. *Int J Sol Struct* 2000; **37**: 7207–7229.

[7] Alfaiate J, Wells GN, Sluys LJ. On the use of embedded discontinuity elements with crack path continuity for mode-I and mixed-mode fracture. *Engng Fract. Mechs* 2002; **69**: 661–686.

[8] Alfaiate J, Simone A, Sluys LJ. Non-homogeneous displacement jumps in strong embedded discontinuities. *Int J Sol Struct.* 2003; **40**: 5799–5817.

[9] Schlangen E, van Mier JGM. Simple lattice model for numerical-simulation of fracture of concrete. *Mat. and Struct.* 1992; **25**: 534-542.

[10] Schlangen E, Garboczi EJ. Fracture simulations of concrete using lattice models. *Engng Fract. Mechs*, 1997; **57**: 319-332.

[11] Guo L.-P., Carpinteri A., Roncella R., Spagnoli A., Sun W., Vantadori S., Fatigue damage of high performance concrete through a 2D mesoscopic lattice model. *Comput Mat Sci* 2009; **44**: 1098-1106.

[12] Carpinteri A., Brighenti R., A new continuum FE approach for fracture mechanics discontinuous problems. *Comput Mat Sci* 2009; **45**: 367-377.

[13] Spagnoli A. A micromechanical lattice model to describe the fracture behaviour of engineered cementitious composites *Comput Mat Sci* 2009; **46**: 7-14.

[14] Brighenti R. A mechanical model for fibre reinforced composite materials with elasto-plastic matrix and interface debonding. *Comput Mat Sci* 2004; **29**: 475-493.