



Exploring noise effects in chaotic optical networks



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ARTICLE INFO

Article history:

Received 10 April 2017

Accepted 8 May 2017

Available online 12 May 2017

Keywords:

Coherence resonance

Stochastic resonance

Control

Noise

ABSTRACT

We report the experimental evidence coherence and stochastic resonance in a dynamics of fast chaotic spiking of a semiconductor laser with optical feedback using an external nonwhite noise in the pumping current. We characterize both coherence and stochastic resonance in the time and frequency domain. We show that the regularity of the chaotic pulses in the intensity of laser diod increases when adding noise and it is optimal for some intermediate value of the noise intensity. We find that the power spectrum of the signal develops a peak at a finite frequency at intermediate values of the noise. The results show that noise may help in extracting the periodic signal without synchronization in chaotic communication. Then we reported the effect of external noise numerically on a single system by using bifurcation diagram. Finally, we considered Chaos synchronization in a network of 2^8 distinct chaotic systems with independent initial conditions when a normal Gaussian noise is added. The transition between non-synchronization to synchronization states using a suitable spatio-temporal representation has been reported. The role of coherence has also been considered.

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Introduction

The significant issue from a chaotic dynamics, there is not clear inquiries on how the dynamics could be controlled by an external perturbation. Several techniques have been proposed to control the dynamics of different systems, noise is one of these techniques.

Noise has always been realized as a source of perturbation or disorder for dynamical systems, but in nonlinear systems, an adequate amount of noise causes a more ordered behavior. After the investigation by Benzi [1] which showed a resonance of dynamical system when subjected to both periodic forcing and random perturbation, the phenomenon attracted a lot of interest in neural sciences [2–4], electronic circuits [5], and optical systems [6–16]. The first experimental observation of coherence resonance in an optical system made of a semiconductor laser with external optical feedback done by Giudici et al. [10].

The two examples of noise-induced ordering in nonlinear dynamical systems are stochastic resonance (SR) and coherence resonance (CR). Coherence Resonance refers to coherent motion

stimulated by noise on the intrinsic dynamics of the regime without the presence of an external periodic forcing [14]. SR was defined as an improvement of the consistency of a system output for certain range of noise amplitudes when the system is driven by a weak periodic signal. This phenomenon has been studied in different kinds of nonlinear systems Lindner et al. [17], in excitable [6,11,15], bistable [5,18,19], and nonlinear systems [20]. CR has been studied theoretically [6,21] and experimentally [10,22] in excitable physical systems, which are characterized by their response to external perturbations, and in a bistable system with delay [23].

Synchronization phenomena are widely present in physics, chemistry, biology, social science and many other fields and have attracted much attention for years. The increasing interest in chaotic synchronization is motivated by its potential applications [24]. In particular, synchronization of chaotic oscillations in coupled nonlinear systems is an important issue since its prediction by Pecora and Carroll in 1990 [25]. One of the most surprising results of the last few decades in the field of the nonlinear dynamics is that a dynamical system and its copies can be synchronized with each other when they are linked by the common excitation only [26]. Optical chaos can be used to hide information so it can be used in privacy and security of optical communication [27,28]. It

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requires synchronization between emitter and receiver lasers to encode and decode information.

Noise plays a constructive role in enhancing synchronization [15,29]. Noise-enhanced synchronization may have significant implications in various fields. For example, the circumstance where different systems are not coupled or only weakly coupled but subjected to an ordinary random forcing is of high relevance in life sciences, especially in neuroscience or ecology [29]. Relevant studies on the noise-induced synchronization of limit-cycle oscillators were carried out by Termae and Tanaka and by Goldobin and

Pikovsky, who analytically showed, by using the phase reduction method that two identical limit-cycle oscillators driven by typical weak Gaussian white noise always synchronize with each other [30]. The equal standard deviations of the injected noise to the uncoupled chaotic systems, colored noise gives rise to a higher synchronization degrees than white noise [31]. On the other hand, Wang et al. showed white noise plays a better role in enhancing synchronization than colored noise [32].

The goal of this paper is to demonstrate the effects of noise correlation time on the high chaotic rate optical system, to be specific, a semiconductor laser with ac-coupled optical feedback [33]. On the other hand, we studied the synchronization in an optical network induced by the Gaussian white noise of zero means. For Coherence resonance we add to the dc pump current controllable ac-coupled noise. In the second section, we illustrate the SR in the system, besides the noise, a weak periodic modulation is added to pumping current. By changing the noise level for fixed modulation frequency or, changing the modulation frequency at a particular noise level. In all cases, the bias current is set constant to be sure that the system has high chaotic intensity.

To obtain a quantitative measurement for CR and SR, we present signal to noise ratio (SNR) indicator of the power spectrum for increasing values of the noise variance σ , measured in dBm/Hz. This magnitude is defined as $SNR = 10\log(\frac{P_s}{P_n})$, where P_s is the value of the power spectrum of the experimental signal and P_n is the comparing value of the power spectrum for the extrapolated noise background, i.e. in the absence of external perturbation.

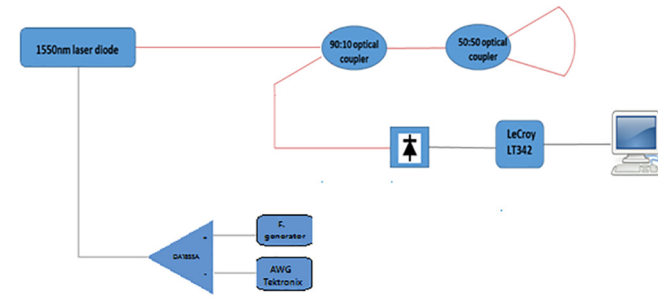


Fig. 1. Experimental setup for coherence and stochastic resonance measurements.

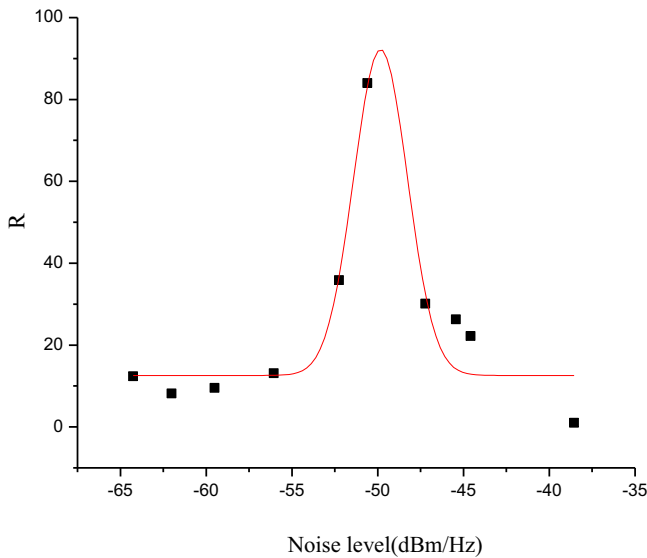


Fig. 2. The coefficient of variation R as a function of noise intensity.

The experimental setup

We demonstrate these effect of noise in a single-mode semiconductor laser experimentally. The Experimental setup is shown in Fig. 1, it consists of a fiber-coupled source laser, is conveniently package a pigtailed Fabry-Perot laser diod and current controller into a single bench unit of type S1FC1550 with a wavelength of 1550 nm from Thorlabs. The light passed through two directional couplers with the splitting ratios of 90:10 and 50:50. To generate ac-optical feedback direction toward the cavity of the semiconductor laser, we connect the two output ports of Y coupler. The laser bias current is always 19 mA. A high-speed InGaAs photodetector (model 1811-125 MHz) from new focus, typical bandwidth is 125 MHz with a current gain of 40 V/mA converts 90% of the detected light. The optical output detected by the photodetector is observed with a LeCroyWaverunner LT342 500 MHz digital storage oscilloscope connected to a personal computer running Windows, enables to exchange data with a variety of Windows applications such as OriginLab software.

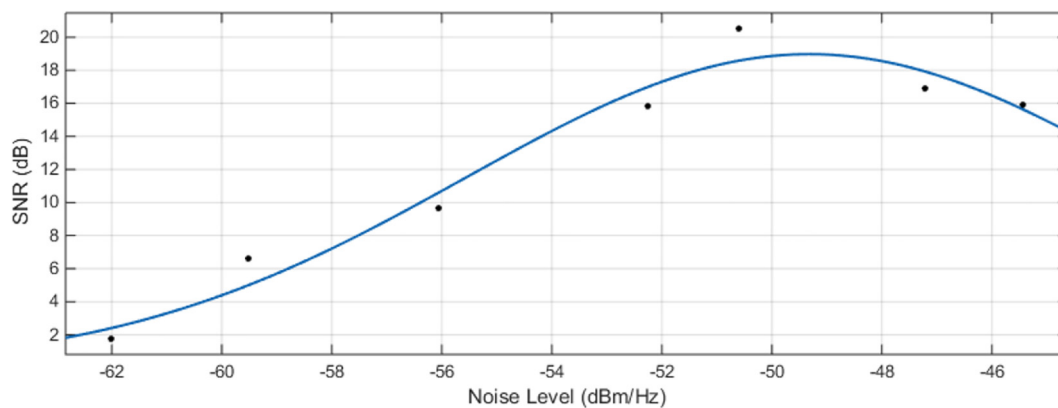


Fig. 3. CR signal to noise ratio (SNR).

To investigate the role of external perturbation a Gaussian noise generator has been inserted into the feedback loop by Tektronix AWG420, 200MS/s Arbitrary Waveform Generator whose amplitude is a controllable parameter.

As an external perturbation, we add a periodic forcing to the driving current of SL. As a result, the pump parameter P_0 is modulated by an external periodic signal with amplitude A and frequency f_e [14], as $p(t) = p_0[1 + A\sin(2\pi f_e t)]$.

Results

Coherence resonance

In the absence of noise, the laser output intensity is low, when the noise is introduced to the system we observe the random distribution of spiking packets in time. When the noise level is increased the spiking packets rate is increased as well until the

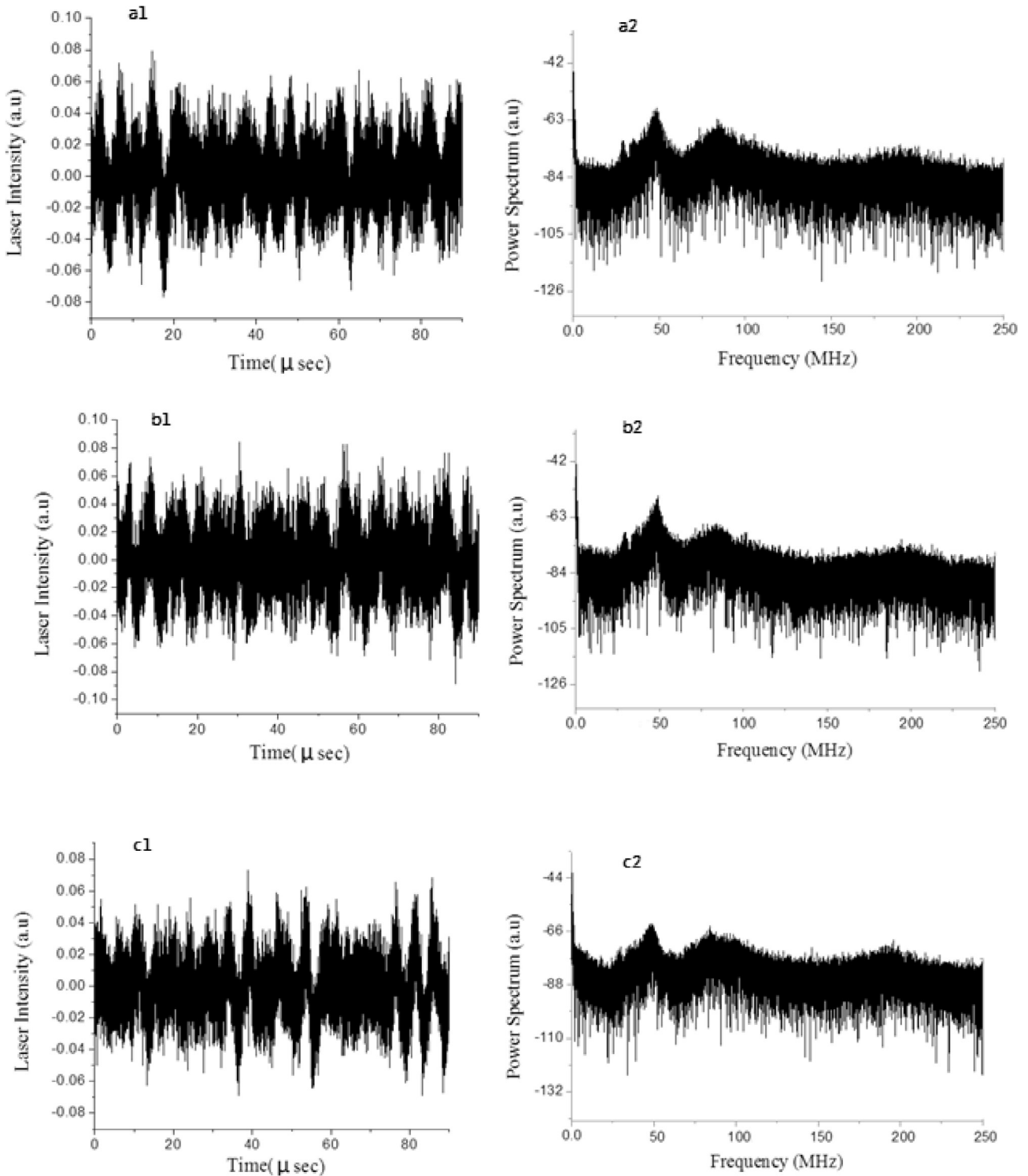


Fig. 4. Time series evolution and corresponding FFT characteristic at optimum noise value (a) 200 kHz, (b) 48 MHz (c) for 83 MHz.

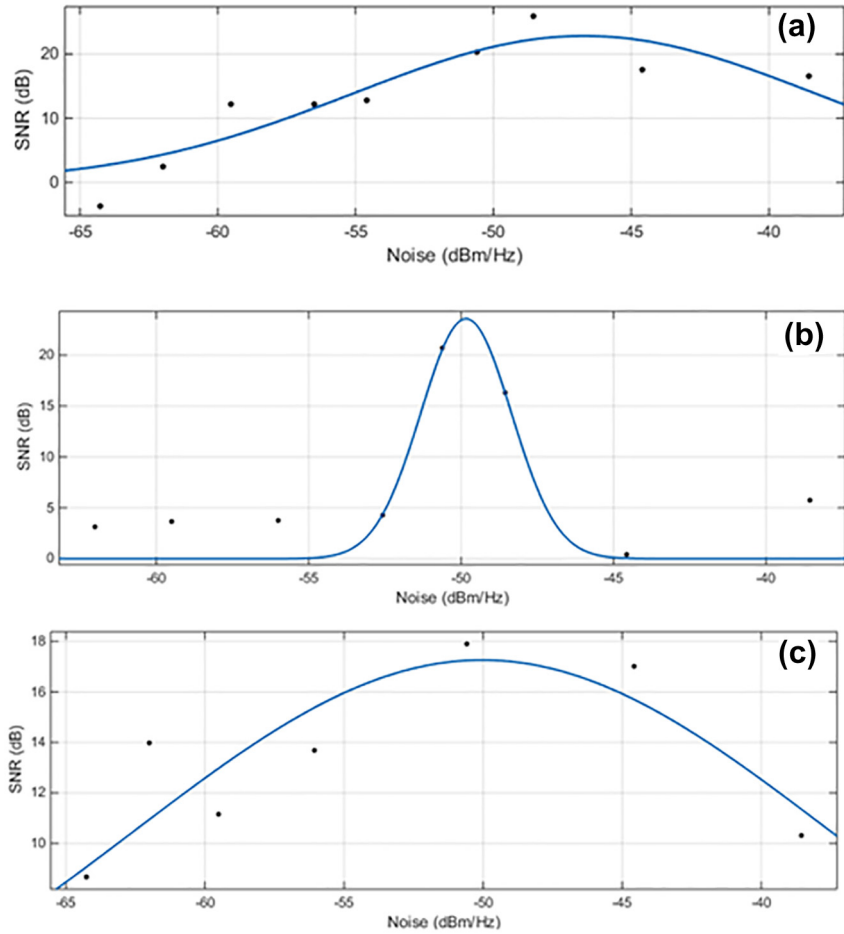


Fig. 5. SR signal to noise ratio (SNR) (a) 200 kHz. (b) 48 MHz. (c) 83 MHz.

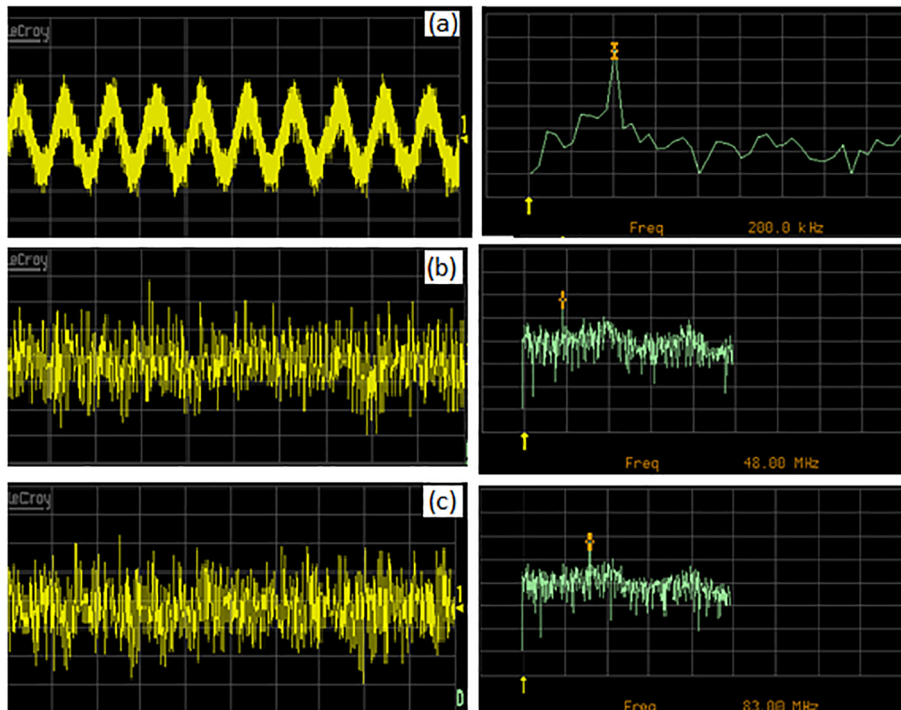


Fig. 6. Time evolution and corresponding FFT characteristic at low noise level -64.26 dBm/Hz for optimum frequency values (a) 200 kHz (b) 48 MHz (c) 83 MHz.

signal becomes more periodic, for higher noise the signal becomes irregular completely.

We study the coefficient of noise variation, which is given as the ratio of the standard deviation of interspike intervals from the corresponding mean value, Fig. 2 at optimal noise intensity -50.6 dBm/Hz, R has a maximum value with the quality coefficient of fit being $R^2 = 0.9197$, indicating a quite good fit to the data which is considered as a key indication for CR. Fig. 3 shows the another quantitative indicator, signal to noise ratio which is also -50.6 dBm/Hz.

Stochastic resonance

The laser has been directly modulated by an external periodic signal and external noise generation (the experimental setup Fig. 1). Weak sinusoidal signal (200 kHz, 48 MHz, and 83 MHz frequency) with amplitude of 200microvolt was embedded to the feedback loop. The power spectra for different noise levels were analyzed. Although the added frequency is embedded, a sharp peak at the FFT was observed. The power spectra of the laser output signals for three values of the embedded frequencies were reported in Fig. 4.

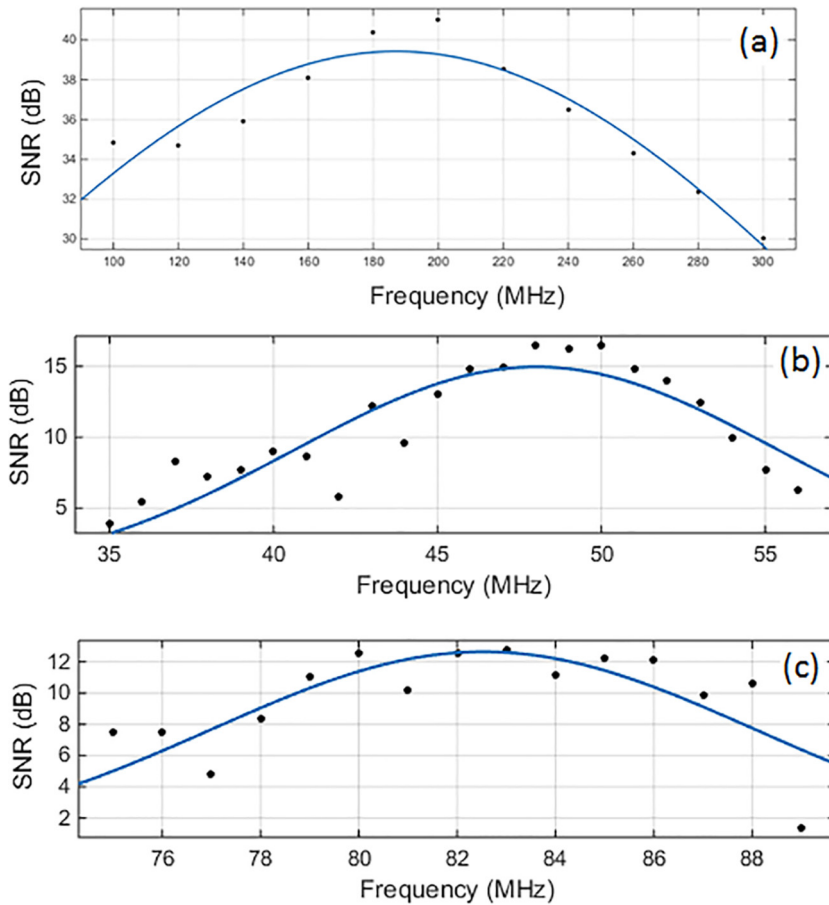


Fig. 7. Stochastic resonance (SNR) at fixed noise level as a function of applied frequencies for optimal frequency (a) 200 kHz (b) 48 MHz (c) 83 MHz.

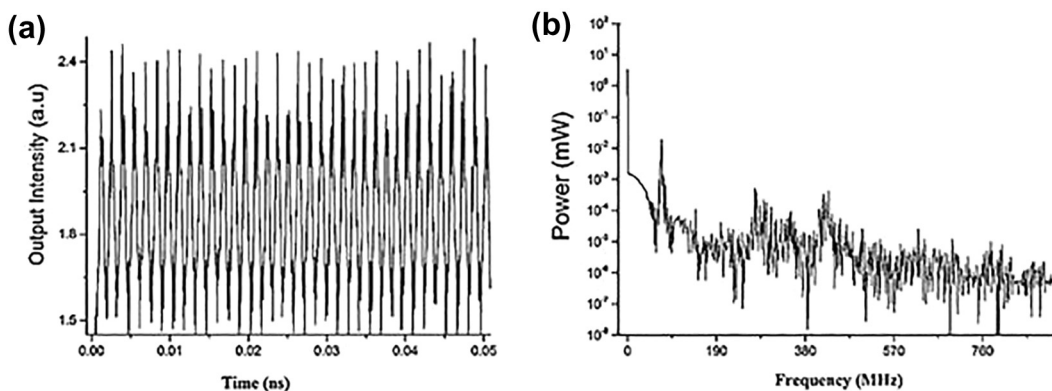


Fig. 8. (a) The time series for dynamical model when $X = 0.5$ mV, (b) The FFT for dynamical model.

To obtain a quantitative measure, we introduce the signal to noise ratio (SNR) of the power spectrum for each value of the embedded frequency Fig. 5. The optimal noise intensity is -50.6 dBm/Hz for embedded driving frequencies 48 MHz, and 83 MHz, and -48.56 dBm/Hz for 200 kHz embedded frequency.

The same sequence was observed by fixing the noise level at -64 dBm/Hz and changing the embedded driving frequencies with an amplitude of 200 microvolt. The small noise level was chosen according to [11]. The power spectra of the laser output signals were reported in Fig. 6, the small level of noise enhanced the three frequencies 200 kHz, 48 MHz, and 83 MHz as shown in Fig. 7 by the quantitative value of SNR.

The dynamical model

A system of semiconductor lasers with optical feedback is modeled by delay differential equations. The Semiconductor laser is classified into class B. Therefore, the polarization term is adiabatically eliminated, and the effect is simply replaced by the linear relation between the field and the polarization. The population inversion for semiconductor lasers is replaced by the carrier density N produced by electron-hole recombination. The photon number (which is equivalent to the absolute square of the field amplitude) and the carrier density are frequently used as the variables of the rate equations for semiconductor lasers [27]. The dynamic of the single mode semiconductor lasers can be described by Lang-Kobayashi equations [34]. The rate equation for the evolutions of the complex amplitude electric field $E(t)$ and the carrier number $N(t)$ read as [35]:

$$\frac{dE(t)}{dt} = \frac{1}{2}(1 + i\alpha)[G(t) - \gamma]E(t) + KE(t - \tau_f) \exp(-\omega\tau_f) + \sqrt{2\beta N(t)}X \tag{1}$$

$$\frac{dN(t)}{dt} = \frac{I}{q} - \gamma_c N(t) - G(t)|E(t)|^2 \tag{2}$$

$G(t)$ is the optical gain that is defined by:

$$G(t) = \frac{g[N(t) - N_0]}{1 + s|E(t)|^2}$$

where I is the injection current, $|E(t)|^2$ is the number of photons inside the cavity.

In the simulation, a set of typical semiconductor laser parameters is considered with a line width enhancement factor $\alpha = 1.5$,

transparency carrier number $N_0 = 1.5 \times 10^8$, differential gain coefficient $g = 1.5 \times 10^4$, gain saturation coefficient $s = 5 \times 10^{-7}$, photon decay rate $\gamma = 500 \text{ ns}^{-1}$, carrier decay rate $\gamma_c = 0.5 \text{ ns}^{-1}$, spontaneous emission rate $\beta = 1.6 \times 10^{-6} \text{ ns}^{-1}$, delay time $\tau_f = 10 \text{ ns}$, feedback strength $K = 20 \text{ ns}^{-1}$, Injection strength $\sigma = 20 \text{ ns}^{-1}$. The spontaneous emission processes are considered by introducing independent Gaussian noise sources X with zero mean and unity variance into the rate equation.

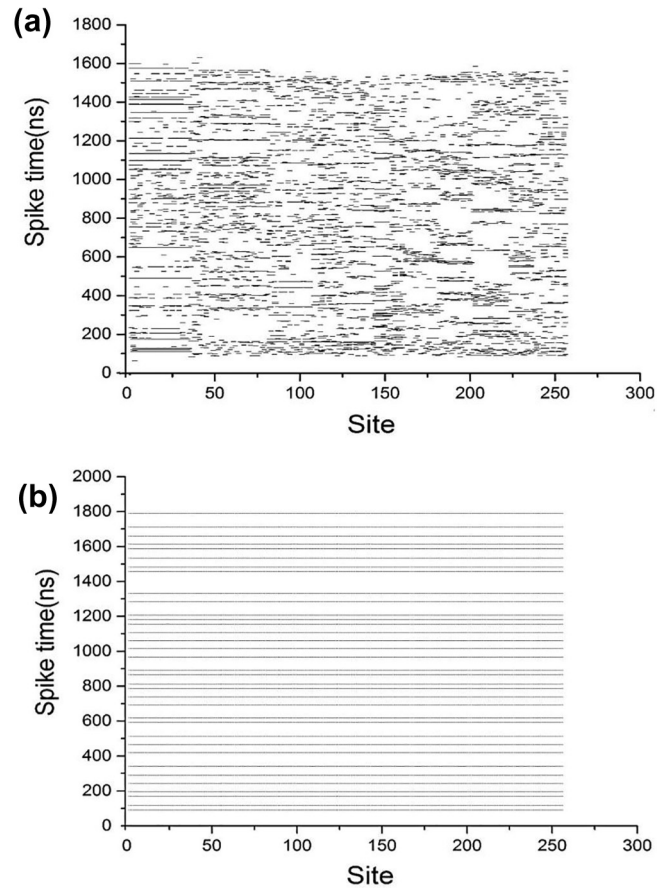


Fig. 10. Spatio-Temporal distribution for (a) non-synchronization, (b) synchronization condition.

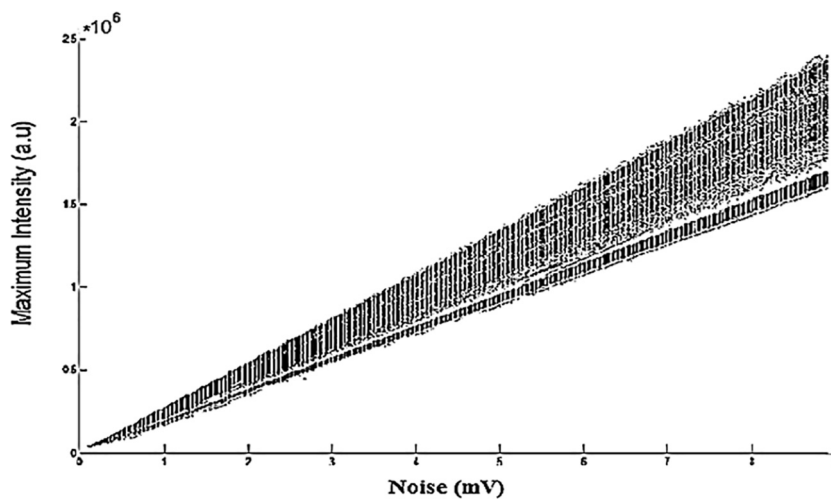


Fig. 9. The bifurcation diagram, feedback strength as a control parameter.

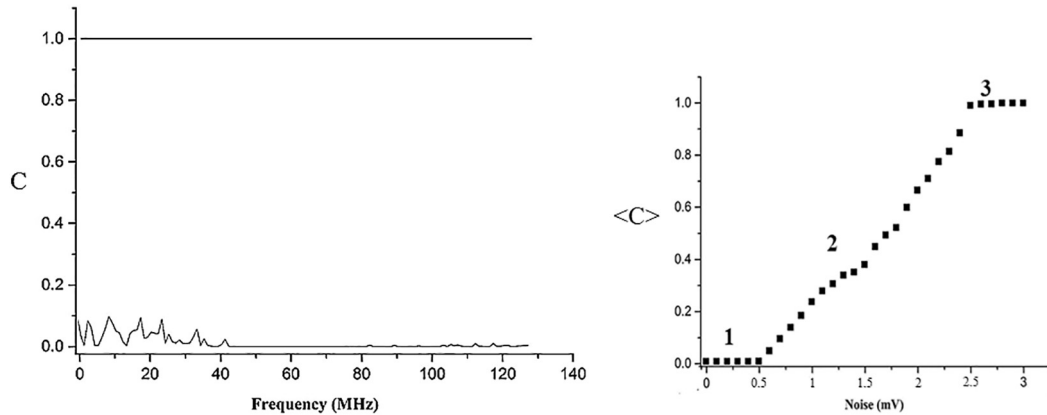


Fig. 11. The mean coherence value (C) as a function of noise level.

Influences of noise on chaos

The effect of noise in dynamical systems has been concerned, the modeling of this behavior is satisfied by programming the theoretical model of the nonlinear system where the bias current and feedback strength fixed at 0.5 mA, 20 ns⁻¹, respectively and (X) is varied. When X = 0.2 mV, the laser output becomes unstable and exhibits a periodic oscillation. By increasing the noise level

X = 0.5 mV, the system turns from periodic state to chaotic state as shown in Fig. 8a, the decay in FFT diagram indicates that the oscillation is chaotic running (Fig. 8b). Fig. 9 shows the bifurcation diagram provides a full characterization of the response of our system with Gaussian white noise intensity. At noise equal to 0, the output form of the semiconductor laser is steady. When the noise is increased to 0.2 mV, a fast scale dynamics behave like a period doubling. As X is increased to 0.5 mV, the dynamics of the oscillator becomes chaotic with small amplitude as shown in the bifurcation diagram. After these values, the chaotic dynamic of the system increases gradually with the increasing of noise parameter.

Optical networks

In the following, Synchronization phenomena in such optical network have been characterized and reported. Numerical results show that the synchronization of 2⁸ chaotic semiconductor lasers is possible using Gaussian noise to couple the system.

We characterize the degree of order in the system by mean of the spatiotemporal distribution and the coherence (C) To describe the abrupt change quantitatively. In this part, Fig. 10 shows the Space-Time representation of semiconductor lasers intensities. When the coupling is zero, the time of peaks for each oscillator are different from the other as illustrated in Fig. 10a, when the coupling is 2.8, the birth of peaks for fixed time difference is observed due to the time correlation between spikes at adjacent sites as shown in Fig. 10b.

The synchronization and non-synchronization for the oscillator could also be described by calculating the mean of the coherence value in both non-synchronization and synchronization conditions.

Coherence takes values of one in the case of the perfect linear interdependence between processes x any y and values close to zero in the absence of any interaction at frequency ω .At a given frequency, if the Coherence equal to 1, the two signals are considered to correspond to each other perfectly at that frequency and if the Coherence is zero the two signals are unrelated at that frequency.

The Coherence is a function of frequency that measures the degree of linear dependence of two signals by testing whether they

contain similar frequency components. The equation of Coherence is:

$$C_{xy}(f) = \frac{|P_{xy}(f)|^2}{P_{xx}(f)P_{yy}(f)} \tag{2}$$

where C_{xy}(f) is the Coherence function, P_{xy}(f) is the Fourier transformation of the cross-covariance function of process x and y, P_{xx}(f) P_{yy}(f) is the Fourier transformation of the auto-covariance function of process x and y, respectively. The cross-spectrum P_{xy}(f) is normalized by the auto-spectrum P_{xx}(f)P_{yy}(f) lead to coherence. Fig. 11a shows the dependence of the synchronization quality and the chaotic complexities on the mean complex value, when there is no coupling between oscillators X = 0, the Coherence is close to zero and it equal to 1 at synchronization condition x = 2.8. Fig. 11b represents the mean value of Coherence as a function of noise level, it shows three regions; the first region represents the non-synchronization condition at zero or small value of noise, the second region represents the transition from non-synchronization condition to synchronization condition with increasing of noise, and the last region represents the synchronization condition at noise level equal to 2.8 mV.

Conclusions

We have shown the existence CR and SR in high chaotic spiking rate with optical feedback.

The system response to the external noise and exhibits a maximal coherence and stochastic resonance for the optimal value of noise level and it considered as a critical parameter that controls the dynamics of the system. The results show that noise may help in extracting the periodic signal without synchronization in chaotic communication.

The synchronization in the chaotic optical network is possible when adding a typical Gaussian noise to the optical network that works with different and independent initial condition and leads to perfect synchronization as a consequence of a noise induced change in time scale.

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