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Energy Consumption Minimization Problem In A Railway Network

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Abstract

When train operations are perturbed, a new working timetable needs to be computed in real-time. In the literature, several algorithms have been proposed for optimizing this computation. This optimization usually does not consider energy consumption. However, minimizing energy consumption is a central issue both from the environmental and economic perspective. In this paper, we address the real-time problem of minimizing the energy consumption. The energy consumption depends on driving regimes used by the train drivers. Hence, we focus on the decision of the appropriate driving regimes throughout each train's travel along a given infrastructure. A model and solution approach for this problem are provided. We show a proof of concept on the applicability of this solution approach on a simple test case.

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Keywords: real-time optimization; energy consumption; railway network; mixed-integer optimization.

1. Introduction

In many countries, the trains traveling on the railway networks are not fully automated, but supervised by dispatchers. A dispatcher manually establishes routes and scheduling to perform regular traffic operations and minimize overall delays when perturbations occur. The real-time Railway Traffic Management Problem (rtRTMP) is the problem of automatically establishing the train routing and scheduling in real-time, minimizing a function of the delay propagation. The rtRTMP has been extensively investigated in the literature by many authors, e.g., D'Ariano et al. (2008), Corman et al. (2010), Mannino and Lamorgese (2015), Pellegrini et al. (2015) and Rodriguez (2007). The energy consumption is not considered in these works, although the green transportation has recently gained significant importance. Then some authors gave attention to the energy consumption in real-time railway traffic operation in the last years, e.g., Bai et al. (2014) and Song and Song (2016).

We define the Energy Consumption Minimization Problem (ECMP). When a train travels along a route, it must respect some constraints due to the infrastructure and the traffic condition. Its speed profile needs to be compatible

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with these constraints. Each train can change the driving regimes several times along its route to follow a feasible speed profile. We formulate the ECMP as the problem of finding the appropriate driving regime combination to ensure a feasible speed profile, optimal from the energy consumption perspective. The energy consumption is to be minimized in real-time, respecting the routing and the precedences fixed in a rtRTMP solution.

After an introduction on railway systems in Sec. 2, we describe both train motion equations and possible driving regimes in Sec. 3. Then, we introduce the problem and the algorithm used to solve it in Sec. 4 and Sec. 5. Last but not least, we present a detailed description of the model in Sec. 6 and report the obtained results on a simple test case in Sec. 7. Finally, we report the conclusion and the future works in Sec. 8.

2. Railway systems

Signaling systems in railway networks vary quite a lot from country to country. Signals ensure that the safety distance between trains is always respected. They delimit track segments called *block sections*, which can host one train at a time. The block sections can share some track portions and, in this case, they are called *incompatible*. Then each block section is incompatible with itself. Incompatible block sections cannot be concurrently occupied. The access to a block section is controlled by a signal. We assume that a signal has three aspects, precisely red, yellow or green (more complex signaling systems with more aspects exist). Red means that the subsequent block section is not available; yellow means that the subsequent block section is available, but the following one is not, and green means that the two subsequent block sections are available. Therefore, a train is allowed to enter the next block section at the planned speed only if the signal is green. A train must stop, if the signal is red and it must slow down to be able to stop by the next signal, if the signal is yellow. In general, if the signaling system has n aspects, at least $n-1$ available block sections follow a green signal. The train *route* is a sequence of block sections to be traversed by a train from its starting to its destination points. The train *schedule* is the sequence of entry times of the train itself in each block section it traverses.

A station is a location where trains may stop for specific activities or services, for example boarding and unloading of passengers. A station includes platforms where trains can stop for a waiting time called *dwell time*.

The initial timetable describes the movements of all trains running in a given time window by specifying for each train its schedule at a set of relevant points along its given route. Timetables are designed to satisfy railway traffic regulations. However, unexpected events occur during operations, which cause train delays with respect to the initial timetable. A *conflict* occurs when two trains, running at their planned speed, would require the same block section concurrently, implying that one of them needs to stop or slow down. The *running time* is the time needed to travel along a block section and the *clearing time* is the time needed to clear it. When the train enters a block section, the *route formation time* is needed. During this time, the train driver sees the signal's aspect and reacts to it, and the route that the train will follow is created. After the train exits a block section, the *route release time* is needed before another train may use a block section incompatible with it. During this time, the block section is brought back to the default situation. The route formation and release times may depend on specific block sections.

The rolling stock characteristics (maximum speed, mass, friction, acceleration, braking rates, ...) and the physical infrastructure characteristics (maximum allowed speed, signaling system in use, ...) combined with the possible driving regimes (acceleration, cruising, coasting, deceleration) imply the speed profiles that can be adopted by a train.

3. Train motion equations and driving regimes

The basic train motion equation describes the relation between the time and the speed of the train. Using the notation reported in 9, the train is modeled as a mass point, so the equation is

$$F_T - F_R(v) = m f_p a \tag{1}$$

where F_T and $F_R(v)$ are the tractive force and the resistance force in the railway network; v, m, a are, respectively, the train speed, mass and acceleration, while f_p is a mass factor considering the rotating parts of the train. The tractive force is the effort that the train makes to move. The resistance force is the sum of line, curve and vehicle resistance, L_R, C_R and M_R, respectively. Line resistance depends on the train mass and the slope angle α, $L_R = mg\sin(\alpha)$, where

$g = 9.81m/s^2$ is the gravity constant. It is usually approximated as $L_R = mg\tan(\alpha)$, because for small values of α, $\sin(\alpha) \approx \tan(\alpha)$. Curve resistance is due to curvature and mass and it is usually calculated by the Roeckl's formula

$$C_R = m\frac{6.3}{r_c - 55}\ \textit{if}\ r_c \geq 300m, \quad C_R = m\frac{4.91}{r_c - 30}\ \textit{if}\ r_c \leq 300m \tag{2}$$

where r_c is the curve radius. Finally, vehicle resistance combines both rolling resistance and air resistance and depends on the physical properties and on the current train speed. It is usually approximated by Davis's formula

$$M_R(v) = A + Bv + Cv^2 \tag{3}$$

where A, B and C are constants specific to the rolling stock.

The mechanical energy E is needed to move the train and it is $E = \int_0^t P(t)dt$ where $P(t) = F_T(t)v(t)$ is the mechanical power, depending on the adopted regime.

The theory of optimal control and the application of the Maximum Principle (see Howlett and Pudney (1995) for details) assure that there are four optimal driving regimes, which the train driver can follow. In particular, acceleration at full power, cruising at constant speed, coasting and deceleration. In this phase of the study, we omit, for the sake of simplicity, the possibility of power recovery, i.e., an efficient use of regenerative braking between near trains. The energy consumption evolves differently in the four driving regimes.

- The acceleration phase: the maximum power is given to the engine. $F_T = F_M(v)$, $P = F_M(v)v$, $a = \frac{F_M(v)-F_R(v)}{mf_p}$, where $F_M(v)$ is the maximum tractive effort that a train can make at speed v.
- The cruising phase: consists in maintaining the speed constant. Therefore, the acceleration is null and $F_T = F_R(v)$, $P = F_R(v)v$. The above formulas apply when the slope angle is large enough (greater than the threshold under which the descent may allow the train to accelerate without effort). Otherwise, the descent helps the train to move downwards. In this case, the train must brake to maintain the constant speed and the mechanical power is null. Precisely, the train must brake with deceleration $\frac{F_R(v)}{f_p m}$ and $F_T = 0$, $P = 0$.
- The coasting phase: the engine is stopped. The train moves by inertia, therefore the mechanical energy is null.
- The deceleration phase: allows the train to brake using either the maximum service or the sharp deceleration. Under normal circumstances, the maximum service deceleration b is recommended because it allows the train to brake in a comfort mode. The maximum sharp deceleration is reserved for an emergency stop. During this phase, the mechanical energy is null.

4. Problem

The ECMP is the real-time optimization problem of finding the driving regime combination for each train that minimizes the energy consumption, while respecting traffic management decisions. In the ECMP, a solution of the rtRTMP is considered and the train driving regime combination is optimized while satisfying it. Therefore, both the train routes and precedences are given.

We consider an infrastructure composed by block sections and signals with given speed limits and slopes, in which some trains travel on given routes. The infrastructure includes some stations that are represented by block sections where the train can stop. Each train travels along a route, that is, it passes through a sequence of block sections. T is the set of trains and for each train $i \in T$, B_i is the set of block sections that i must traverse to follow the route chosen by solving the rtRTMP. T' is the subset of T containing the trains that start from a station in the given infrastructure, so with initial speed 0. The complement $T \setminus T'$ is the set of trains arriving from a neighbor control area and entering at the planned speed. S_i is the subset of block sections associated with the stations where the train $i \in T$ has a scheduled stop.

When approaching a signal, each train may have to change the driving regime to respect a restrictive aspect. For ease of formulation, we assume that all signals' visibility distance is 0, that is, a train entering a block section with a yellow aspect will have to stop at the end of it. Once stopped, the driver sees the signal opening the next block section and may immediately accelerate if the signal already turned green or yellow. Each train can change the driving regime in predefined positions within the block sections belonging to its route. Let CO be the set of all acceptable combinations for all train in each block section.

The principal aim of this work is to define a Mixed-Integer Linear optimization Model to describe the ECMP. The continuous variables represent the times at which the trains enter the block sections along their route. The binary variables indicate the driving regimes combination adopted in each block section by each train. Other variables handle the possible stops in front of red signals. The Train Driving Regime Combinations Mixed-Integer Linear Problem algorithm (TDRC-MILP algorithm) used to solve the problem is described in Sec. 5. Then, a detailed description of the problem formulation is presented in Sec. 6.

5. TDRC-MILP algorithm

The TDRC-MILP algorithm is introduced to solve the problem. The algorithm is composed by two parts. Precisely, it begins by calculating the running time, the clearing time, the final speed and the energy consumption of a train in each block section of the given infrastructure. These *values* are pre-computed and then used in the real-time optimization. After the values pre-computation, the TDRC-MILP algorithm creates the model and solves the optimization problem in real-time.

5.1. Values pre-computation

The running time, the clearing time, the train final speed and the energy consumption depend on: the infrastructure, the train initial speed and the driving regime combination. The algorithm calculates these values for all trains in each block section of their route, starting from all possible initial speed and considering all the feasible driving regime combinations.

We assume that each train can change the driving regime in some predefined points of each block section. The algorithm splits each block section into n_s subsections in which the maximum speed is constant. In each of these subsections exactly one driving regime is adopted. For each subsection, if a constant acceleration would bring the train to exceed the maximum speed allowed, the values associated with the acceleration regime are actually computed considering a suitable combination of acceleration until the maximum speed allowed and cruising at this speed. Similarly, if a constant deceleration would bring the train to stop before the end of the subsection, the values associated with the deceleration regime are actually computed considering a suitable combination of cruising and deceleration to stop at the end of the subsection exactly.

In each subsection s of a block section $j \in B_i$, train i can start with an initial speed between 0 and the maximum allowed speed. Let V_{is} be the set of possible initial speeds of train i in the subsection s. Moreover, let V_{ij}^0 be the set of possible initial speeds of train i in the first subsection of j, then the possible initial speed in j. For all these initial speeds, the algorithm calculates the values associated with one driving regime between acceleration, cruising, coasting and deceleration. These values are calculated solving the basic motion equation (1) by means of a difference equation approach. To do so, s is divided in h steps of equal and sufficiently short length. After the values calculation in all subsections of block section $j \in B_i$, the algorithm combines them to obtain the values for j.

By doing so, the running time r_{ijkl}, the clearing time c_{ijkl}, the final speed v_{ijkl} and the energy consumption E_{ijkl} are calculated for each train $i \in T$, in each block section $j \in B_i$, considering each possible combination $l \in CO$ and each initial speed $k \in V_{ij}^0$.

6. Model

The model is a Mixed-Integer Linear Optimization Problem where the variables are:

- the continuous variables $t_{ij}, \forall i \in T, j \in B_i$. They define the time at which each train enters each block section of its route
- the binary variables $y_{ijkl}, \forall i \in T, j \in B_i, k \in V_{ij}^0, l \in CO$. They indicate the initial speed k and the combination l adopted in the block section j by each train i. Precisely,

$$y_{ijkl} = \begin{cases} 1 & \text{if } i \text{ enters } j \text{ with speed } k \text{ and traverses it using combination } l \\ 0 & \text{otherwise} \end{cases}$$

- the continuous variables $q_{ij}, \forall i \in T, j \in B_i$. They handle the possible train stop duration at the red signals
- the binary variables z_{ij}, $\forall i \in T, j \in B_i$. They indicate whether a train must stop at the red signal at the end of a block section (entering the block section with yellow signal). Precisely,

$$z_{ij} = \begin{cases} 1 & \text{if } i \text{ enters } j \text{ with a yellow signal} \\ 0 & \text{otherwise.} \end{cases}$$

The objective of our problem is the minimization of energy consumption. For each train, the energy consumption depends on the driving regime combination adopted in each block section, so the objective is:

$$\min \sum_{i \in T} \sum_{j \in B_i} \sum_{k \in V^0_{ij}} \sum_{l \in CO} E_{ijkl} y_{ijkl}. \tag{4}$$

Bound Constraints give the upper and the lower bound to all continuous variables.

$$LB_{ij} \leq t_{ij} \leq UB_{ij}, \quad \forall i \in T, j \in B_i \tag{5}$$

$$0 \leq q_{ij} \leq UB_{ij}, \quad \forall i \in T, j \in B_i. \tag{6}$$

LB_{ij} can be set as the time in which train i enters the infrastructure plus the minimum running time needed to cross all the block sections before j. UB_{ij} can be set as the maximum allowed train travel duration minus the minimum running time to cross all the block sections after j.

Binary Constraints ensure that y_{ijkl} and z_{ij} are binary variables.

$$y_{ijkl} \in \{0, 1\}, \forall i \in T, j \in B_i, k \in V^0_{ij}, l \in CO \tag{7}$$

$$z_{ij} \in \{0, 1\}, \forall i \in T, j \in B_i. \tag{8}$$

Route Constraints guarantee that each train follows exactly one combination of driving regimes in each block section, starting from any initial speed (con. (9)). They ensure that the final speed in each block section is equal to the initial speed of the following one (con. (10)). Moreover, they ensure that each train arrives at each block section after spending the necessary running time in the previous one (con. (11)). Constraints (11) are not applied to block sections in S_i where the train i has a scheduled stop. These block sections will be handled separately in the *Station Constraints*.

$$\sum_{k \in V^0_{ij}} \sum_{l \in CO} y_{ijkl} = 1 \quad \forall i \in T, j \in B_i. \tag{9}$$

$$\sum_{(k'l) \in Q_{(ij)_k}} y_{ijk'l} = \sum_{l \in CO} y_{ij'kl} \quad \forall i \in T, j \in B_i \setminus j^*_i, j' \textit{ follows } j, \forall k \in V^0_{ij'} \tag{10}$$

$$t_{ij'} - t_{ij} = \sum_{k \in V^0_{ij}} \sum_{l \in CO} r_{ijkl} y_{ijkl} + q_{ij}, \quad i \in T, j \in B_i \setminus j^*_i, j' \textit{ follows } j, j \notin S_i \tag{11}$$

$Q_{(ij)_k}$ is the set containing pairs $(k', l) \in (V^0_{ij}, CO)$ of train initial speeds and driving regime combinations that imply k as train final speed, for each train $i \in T$ and each block section $j \in B_i$. $j^*_i \in B_i$ is the last block section on the route of train i.

Station Constraints ensure that each train stops at least for the minimum dwell time in each station, where it has a scheduled stop (cons. (12), (13)). Moreover, the train must not leave the station before its planned departure time (con. (14)). Finally the train starts with initial speed 0 from a station in the given infrastructure (con. (15)).

$$t_{ij'} - t_{ij} = \sum_{k \in V^0_{ij}} \sum_{l \in CO} r_{ijkl} y_{ijkl} + d_{ij} + q_{ij}, \quad \forall i \in T, j \in S_i, j' \in B_i \setminus S_i, j' \textit{ follows } j \tag{12}$$

$$\sum_{(k,l)\in Q_{(ij)0}} y_{ijkl} = 1, \quad \forall i \in T, j \in S_i \tag{13}$$

$$t_{ij'} \geq w_{ij}, \quad \forall i \in T, j \in S_i, j' \in B_i, j' \text{ follows } j \tag{14}$$

$$\sum_{l\in CO} y_{ij_i^\circ 0l} = 1 \quad \forall i \in T' \tag{15}$$

d_{ij} is the dwell time for each train $i \in T$ and each block section $j \in S_i$, while w_{ij} is the originally scheduled time at which the train i should be entering $j \in S_i$. $j_i^\circ \in B_i$ is the first block section on the route of train i.

Precedence Constraints ensure that only one train at a time can traverse each block section. The precedence between trains are given. Let $P_{jj'}$ be the set that contains the ordered pairs of trains with respect to their precedence in incompatible block sections j and j'. Precisely, when train i must traverse block section j and i' must traverse j', if $(i', i) \in P_{jj'}$ then i' must release j' before i can enter j.

$$t_{ij} - t_{i'j} \geq \sum_{k\in V_{ij}^0} \sum_{l\in CO} g_{i'jkl} y_{i'jkl}, \quad \forall (i', i) \in P_{jj'}, j \in B_i, j' \in B_{i'}, j' \text{ incompatible with } j. \tag{16}$$

$g_{i'jkl} = r_{i'jkl} + c_{i'jkl} + f_j + r_j$, where $c_{i'jkl}$ is the clearing time and f_j and r_j are, respectively, the route formation and release times of j.

Signal Constraints ensure that a train stops with red signal at the end of a block section when traffic imposes it. This is the case when it has to give precedence to one or more other trains. In particular, the signal will be red if there is at least one of these trains which is traversing or is still to traverse a block section that is incompatible with the one opened by the signal itself (cons. (17),(18)). Precisely, we consider train i and block section j''. The latter is followed by j on the route of train i. Block section j' belongs to the route of train i' and it is incompatible with j. Train i' has the precedence in j' with respect to i in j, that is, $(i', i) \in P_{jj'}$. If i is traversing j'' before i' releases j', then i will have to stop at the end of j'' (red signal) to guarantee safety. It will wait in front of the signal for $q_{ij''}$ time units ($q_{ij''} > 0$), because the signals' visibility distance is 0. To let the train be ready to face a red signal, the signaling system sets the signal opening j'' to be yellow when the train crosses it.

$$Mz_{ij''} + t_{ij''} \geq t_{i'j'} + \sum_{k\in V_{ij'}^0} \sum_{l\in CO} g_{i'j'kl} y_{i'j'kl}, \quad j'', j \in B_i, j \text{ follows } j'', j' \in B_{i'}, j' \text{ incompatible with } j. \tag{17}$$

$$\sum_{k\in Q_{(ij)0}} y_{ij''kl} \geq z_{ij''}, \forall i \in T, j'' \in B_i \tag{18}$$

$M >> 0$ is a big constant and its minimum value is the distance between the time needed train i' to exit from j' and the time needed train i to enter j''. Constraints (17) assure that

$$\exists i' \in T, (i', i) \in P_{jj'} : t_{ij''} \leq t_{i'j'} + \sum_{k\in V_{ij'}^0} \sum_{l\in CO} g_{i'j'kl} y_{i'j'kl} \Rightarrow z_{ij''} = 1.$$

That is, if some train i' exists which is traversing a block section j' while train i is about to enter block section j coming from block section j'', j' is incompatible with j and i' has precedence over i, then the variable $z_{ij''}$ is set to 1. If $z_{ij''} = 1$, that is, train i is traversing block section j'', while i' is traversing block section j', then the constraints (18) assure that i will stop at the end of j''. Otherwise, the train stop is not compulsory.

Duration Time Constraints ensure that q_{ij} is 0 when a train does not stop at a red signal. Then

$$q_{ij} \leq UB_{ij} z_{ij}, \forall i \in T, j \in B_i. \tag{19}$$

Delay Constraints ensure that the trains delay is less than a maximum allowed delay D.

$$\sum_{j\in S_i} (t_{ij} - w_{ij}) + (t_{ij_i^*} - t_{ij_i^*}^*) \leq D, \forall i \in T. \tag{20}$$

w_{ij} and $t_{ij_i^*}^*$ are the originally scheduled time at which train i should be entering block section $j \in S_i$ and last block section j_i^* of its route, respectively.

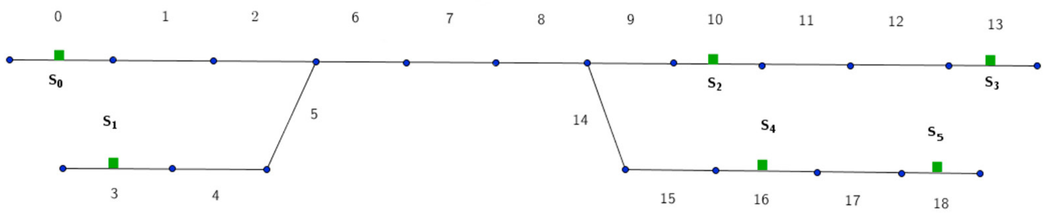

Fig. 1. The infrastructure used to test the model.

7. Proof of concept

7.1. Implementation detail

The TDRC-MILP algorithm is implemented in Java using IntelliJIDEA that is an integrated development environment (IDE) for developing computer software. It is integrated with IBM ILOG CPLEX Optimization Studio academy integer linear solver. Precisely, after the initial values pre-computation, the optimization problem is solved with $CPLEX\_Studio126$. We solve the test case reported in the following subsection.

7.2. Test case

We consider a simple infrastructure in Fig. 1 without slope and curvature. The infrastructure has 19 block sections, in which block sections 0, 3, 10, 13, 16, 18 are stations S_0, S_1, S_2, S_3, S_4 and S_5, respectively. Each block section is long $3000m$ with route formation and release time of $3s$ and maximum speed limit of $33.33m/s$ ($120km/h$). Two trains i_0 and i_1 with equal rolling stock characteristics run the infrastructure. Precisely, the train mass, mass factor, length and maximum speed are, respectively, $156.75t$, 1.05, $72.30m$, $44.44m/s$. The parameters in the Davis's Formula are, respectively, $A = 1307N$, $B = 32.04Ns/m$ and $C = 5.68Ns^2/m^2$, the maximum tractive force is reported in the Table 1 and the maximum service braking is $1.1m/s^2$. The train routes are, respectively,

- i_0 : 0, 1, 2, 6, 7, 8, 9, 10, 11, 12, 13
- i_1 : 3, 4, 5, 6, 7, 8, 14, 15, 16, 17, 18.

i_0 starts from S_0 and it must stop at S_2 and S_3 for $20s$. Train i_1 starts from S_1 and it must stop at S_4 and S_5 for the same dwell time. We assume that the trains depart from a station when they enter the block section following it. Hence the trains actual depart corresponds to the time at which they enter block sections 1 and 3, respectively. Both trains must travel through block sections 6 , 7 and 8. We suppose that train i_1 has the precedence.

We assume $n_s = 4$, so each block section is divided into 4 subsections and the driving regime combinations are composed by 4 regimes in each block section. The subsections have the same length ($h = 750m$) and each subsection is divided into steps of length $1m$.

Table 1. The train maximum tractive force.

v (m/s)	Maximum Tractive Force (N)
$0 \leq v < 16.46$	184050
$16.46 \leq v < 29.34$	$10918643800/v$
$29.34 \leq v < 44.44$	$1151206560000/v^2$

Both trains start at 8:00 from the first block section of their route. i_0 is scheduled to enter station S_2 at 8:09 and S_3 at 8:15:30. i_1 is scheduled to enter S_4 at 8:09 and S_3 at 8:10:30. Supposing that the trains start at 0, shifting back all times, we assume the big constant $M = 3600$. The maximum allowed delay D is $30s$, the lower and the upper bound LB_{ij}, UB_{ij} are, respectively, $0s$ and $1800s$ for all block sections. Precisely, the upper bound is twice the time calculated supposing that trains travel at constant maximum speed $33.33m/s$.

7.3. Results

The TDRC-MILP algorithm finds the result in $8s$, precisely $4s$ are required for the initial values's computation and the remaining $4s$ to create and solve the linear model by means of $CPLEX\_Studio126$.

The optimal value for energy consumption is $77.75MJ$. The optimal schedule for i_0 and i_1 is reported in Table 2.

Table 2. The trains optimal schedule.

i_0	Block Section	Optimal Schedule	i_1	Block Section	Optimal schedule
	1	8:00		4	8:00
	2	8:01:26		5	8:01:24
	6	8:04:57		6	8:02:33
	7	8:06:24		7	8:03:41
	8	8:07:36		8	8:04:49
	9	8:08:49		14	8:05:57
	10	8:10:03		15	8:07:06
	11	8:12:13		16	8:08:14
	12	8:13:39		17	8:10:20
	13	8:14:56		18	8:11:45

The algorithm finds that $z_{02} = 1$ and $q_{02} = 1m42s$, so train i_0 stops with red signal at the end of block section 2 for $1m42s$ to give precedence to train i_1. Moreover, the algorithm finds that $z_{010} = 1$ and $z_{116} = 1$. In fact, trains i_0 and i_1 stop at stations 10, 16 for dwell time, respectively. Therefore, constraints (13) imply the feasibility of constraints (18) when $z_{010} = 1$ and $z_{116} = 1$. Finally, the initial, the final speed and the driving regime combination for each train in each block section are reported in Table 3.

Table 3. The train initial and final speed, and the driving regime combination. ACC, CRU, COA and DEC stand for acceleration, cruising, coasting and deceleration regime, respectively.

i_0	Block Section	Initial Speed	Final Speed	Regime 1	Regime 2	Regime 3	Regime 4
	1	$0m/s$	$31m/s$	ACC	COA	COA	COA
	2	$31m/s$	$0m/s$	ACC	COA	COA	DEC
	6	$0m/s$	$31m/s$	ACC	COA	CRU	COA
	7	$31m/s$	$31m/s$	CRU	CRU	CRU	COA
	8	$31m/s$	$31m/s$	CRU	CRU	CRU	COA
	9	$31m/s$	$29m/s$	CRU	COA	COA	COA
	10	$29m/s$	$0m/s$	ACC	COA	COA	DEC
	11	$0m/s$	$31m/s$	ACC	COA	COA	COA
	12	$31m/s$	$28m/s$	COA	COA	COA	COA
	13	$28m/s$	$0m/s$	ACC	COA	COA	DEC
i_1	Block Section	Initial Speed	Final Speed	Regime 1	Regime 2	Regime 3	Regime 4
	4	$0m/s$	$33m/s$	ACC	COA	ACC	DEC
	5	$33m/s$	$33m/s$	CRU	CRU	CRU	COA
	6	$33m/s$	$33m/s$	CRU	CRU	CRU	DEC
	7	$33m/s$	$33m/s$	CRU	CRU	CRU	COA
	8	$33m/s$	$33m/s$	CRU	CRU	CRU	COA
	14	$33m/s$	$33m/s$	CRU	CRU	CRU	DEC
	15	$33m/s$	$32m/s$	CRU	CRU	COA	COA
	16	$32m/s$	$0m/s$	ACC	CRU	COA	DEC
	17	$0m/s$	$32m/s$	ACC	CRU	COA	COA
	18	$32m/s$	$0m/s$	ACC	COA	COA	DEC

8. Conclusion

The real-time optimization problem of minimizing the energy consumption in the railway networks is an interesting and complex open problem. We have designed TDRC-MILP: an algorithm which finds the optimal solution to the problem when the routing and the train precedences are given; it is based on the solution of a MILP formulation.

As a proof of concept, we have tested it on a simple test case and we have obtained encouraging results. Future works will be devoted to test the TDRC-MILP algorithm on more complicated test cases where the slope, the curvature and the speed limits change on the track segments. Finally, we will test it on a real infrastructure.

9. Notation

Tables 4, 5, 6 contain the symbols used in paper. Precisely, Table 4 contains the symbols introduced in Sec. 3. Table 5 and Table 6 contain the variables of the model and the symbols used to define the problem, the TDRC-MILP algorithm and the model, respectively, introduced in Sec. 4, 5 and 6.

Table 4. Notation introduced in Train Motion Equations and Driving Regimes (Sec. 3).

Symbol	Meaning	Unit
v	Train velocity	m/s
m	Train mass	kg
f_p	Train mass factor	
a	Train acceleration	m/s^2
F_T	Train tractive force	N
$F_R(v)$	Train resistance, function of v	N
$P(t)$	Mechanical power at instant t	W
$F_M(v)$	Train maximum tractive effort, function of v	N
$F_R(v)$	Train resistance, function of v	N
$L_R(v)$	Line resistance, function of v	N
$C_R(v)$	Curve resistance, function of v	N
$M_R(v)$	Vehicle resistance, function of v	N
A	First parameter in Davis's formula	N
B	Second parameter in Davis's formula	Ns/m
C	Third parameter in Davis's formula	Ns^2/m
α	Slope angle	rad
b	Maximum Service Braking	m^2
g	Gravity Constant	m/s^2
r_c	Curve Radius	m
E	Mechanical Energy	J

Table 5. The variables of the model.

Variable	Meaning
q_{ij}	Duration of stop of train i at red signal in block section j, $\forall i \in T, j \in B_i$
t_{ij}	Time at which Train i enters Block Section j
y_{ijkl}	Binary variable to handle the driving regime combination
z_{ij}	Binary variable to handle the stop at red signals

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Table 6. Notation introduced to describe the ECMP, the TDRC-MILP algorithm and the model (Sec. 4, 5 and 6).

Symbol	Meaning
T	Set of trains that travel along the Infrastructure
T'	Subset of trains that start from a station withing the considered infrastructure
B	Set of block sections
i	A train
j	A block section
k	A train initial speed
l	A train driving regime combination
B_i	Sequence of block sections along the route of train i, $\forall i \in T$
S_i	Set of stations where train i must stop, $\forall i \in T$
CO	Set of allowed combinations
s	A subsection in a block section
n_s	Number of subsections in a block section
V_{is}	Possible initial speeds for train i in subsection s
V^0_{ij}	Possible initial speeds for train i in the block section $j \in B_i$ in its first subsection
h	Number of steps in a subsection
r_{ijkl}	Running time
c_{ijkl}	Clearing time
v_{ijkl}	Train final speed
E_{ijkl}	Energy consumption
j°_i	First block section in $B_i, \forall i \in T$
j^*_i	Last block section in $B_i, \forall i \in T$
$Q_{(ij)_k}$	Set containing pairs $(k', l), k' \in V^0_{ij}, l \in CO$ that imply $k \in V^0_{ij}$ as train final speed
r_j	Release time for block section $j \in B$
f_j	Route formation time for block section $j \in B$
q_{ij}	Duration of stop of train i at red signal in block section j, $\forall i \in T, j \in B_i$
d_{ij}	Dwell time for train $i \in T$ in j, $\forall j \in S_i$
t_{ij}	Time at which train i enters block section j
y_{ijkl}	Binary variable to handle the driving regime combination
z_{ij}	Binary variable to handle the stop at red signals
LB_{ij}	Lower bound of time variables for train $i, \forall i \in T$ in block section j
UB_{ij}	Upper bound of time variables for train $i, \forall i \in T$ in block section j
g_{ijkl}	Constraints' coefficient given by $r_{i'jkl} + c_{i'jkl} + f_j + r_j, \forall i \in T, j \in B_i, k \in V_{ij}, l \in CO$
D	Maximum delay allowed
M	Big Constant >> 0
w_{ij}	Originally scheduled exit time of train i from block section $j \in S_i$
$t^*_{ij^*_i}$	Originally scheduled exit time of train i from last block section $j^*_i \in B_i$
$P_{jj'}$	Set containing the ordered pairs $(i, i'), i, i' \in T$, such that i' traverses $j' \in B_i$ before i traverses $j \in B_i$ with j and j' incompatible block sections

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