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Research Letter

Optimal Filtering in Pilot-Aided Carrier Recovery

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The paper deals with carrier recovery based on pilot symbols in single-carrier systems. Wiener's method is used to determine the optimal unconstrained filter in estimation of phase noise assuming that a sequence of equally spaced pilot symbols is available. Our analysis allows to capture two effects that are not considered in the existing literature: the impact of aliasing due to sampling of the phase noise sequence at the pilot rate and the cyclostationary nature of the estimate hence of its performance. Experimental results are derived also for the case, where the filter is constrained to the cascade of two moving averages. These results show that, in the considered example, the mean-square phase error of the constrained filter is within 0.35 dB from the MSE of the optimal filter.

1. Introduction

The performance of single-carrier microwave radio systems operating at high spectral efficiency is often limited by the phase noise introduced by local oscillators that generate the sinusoid used for up/down-conversion. As the number of constellation points grows, the S-curve of the conventional decision-directed phase detector used in carrier recovery based on phase-locked loop becomes narrow and narrow. As a consequence, the receiver becomes less and less able to recover the phase noise that affects the incoming carrier, up to the appearance of the phenomenon of cycle slip, that definitively degrades performance. To overcome this limit, pilot symbols can be periodically introduced in the data sequence to improve the robustness of the receiver in estimating the carrier phase, as proposed in [1] for synchronous transmission systems. In [2] the influence of the pilot pattern in burst transmission is investigated. Pilot symbols have proven their usefulness also in iterative joint decoding and synchronization [3, 4].

In the present paper, we consider the optimization of the filter that produces the phase estimate at symbol rate from the noisy observation of the phase noise at pilot rate. One crucial point in pilot-aided carrier recovery is the design of the pilot rate. The pilot rate should be kept as small as possible to maintain low the overhead due to the pilot symbols. However, a too small pilot rate induces substantial aliasing in the sequence of sampled phase noise, compromising the performance of carrier recovery. The novelty of the present contribution is the consideration of aliasing in the design of the carrier recovery mechanism and in the evaluation of its performance.

The paper is organized as follows. In Section 2 the system model is introduced, and the performance of the system is defined. In Section 3 the optimal pilot filter is derived by optimizing the performance criterion introduced in the previous section, and the performance is analyzed. In Section 4 simulation results are presented together with the results coming from the analysis for the case where the phase noise is modelled as random phase walk. In Section 5 conclusion is drawn.

2. System Model and Problem Statement

Consider a passband signal of power P embedded in white noise of two-sided power spectral density $N_0/2$. We assume that the frequency of the carrier of the passband signal is known and therefore that the signal can be down-converted around $f = 0$. We also assume that perfect symbol timing is available and that the received signal is free of intersymbol interference. Let

$$x(k) = a(k)e^{j\theta(k)} + w(k) \tag{1}$$

be the kth element of the sequence obtained after sampling at symbol frequency. In the above equation, $\{a(k)\}$ is the data sequence that includes payload symbols and pilot symbols, j is the complex unit, $\{\theta(k)\}$ is the random sequence of samples of the phase of the free-running oscillator, and $\{w(k)\}$ is the complex envelope of the sequence of samples of white channel noise. Independency between data, phase noise, and channel noise is assumed. We assume that the baseband signal is scaled in such a way that the data sequence $\{a(i)\}$ has zero mean and unit variance, and that the power of the zero-mean complex white noise $\{w(k)\}$ is

$$E\left\{|w(k)|^2\right\} = \frac{1}{\text{SNR}}, \tag{2}$$

where

$$\text{SNR} = \frac{PT}{N_0} \tag{3}$$

is the channel signal-to-noise ratio, with T^{-1} being the symbol rate. The random sequence $\{\theta(k)\}$ is a real-valued stationary random sequence of phase noise characterized by the power spectral density $\Psi(f)$. Suppose that the pilot symbols have unit amplitude, and that the pilot rate is $(MT)^{-1}$, meaning that one pilot symbol is inserted after $M-1$ payload symbols. Without loosing generality, we assume that the ith pilot symbol occurs at time iM.

Using the pilot symbols the receiver produces the zero-padded sequence

$$\left\{e^{j\theta(kM)} + w'(kM)\right\}, \tag{4}$$

where $w'(kM)$ is complex AWGN statistically equivalent to $w(kM)$. The pilot filter is optimized by assuming that its input is the phase sequence

$$y(kM) = \theta(kM) + n(kM), \tag{5}$$

where $n(kM)$ is real AWGN with zero mean and power

$$E\{n^2(kM)\} = \frac{1}{2\cdot\text{SNR}}. \tag{6}$$

The estimate of the phase sequence produced by the pilot filter is

$$\hat{\theta}(m) = \sum_i y(iM)h(m-iM), \tag{7}$$

where $\{h(i)\}$ is the impulse response of the pilot filter, that is hereafter assumed to be noncausal and of unconstrained duration. The criterion that we adopt to design the pilot filter is the minimization of the mean-square (phase) error (MSE):

$$\text{MSE}_m = E\left\{\left(\theta(m) - \hat{\theta}(m)\right)^2\right\}. \tag{8}$$

Note that the sequence $\hat{\theta}(m)$ is cyclostationary with period M, therefore

$$\begin{aligned} &E\left\{\left(\theta(m) - \hat{\theta}(m)\right)^2\right\} \\ &\quad = E\left\{\left(\theta(m+lM) - \hat{\theta}(m+lM)\right)^2\right\}, \quad \forall l. \end{aligned} \tag{9}$$

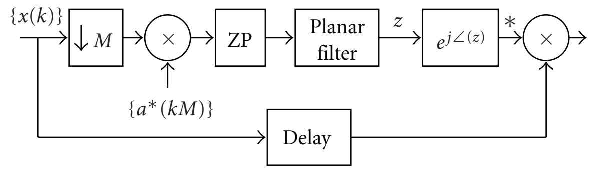

Figure 1: Block diagram of carrier recovery. The block ZP produces a zero-padded sequence at symbol rate by introducing $M-1$ zeros between two successive samples at pilot rate. After the ZP block the system works at symbol rate.

In view of the above periodicity, in what follows we will restrict our attention to

$$m = 0, 1, \ldots, M-1. \tag{10}$$

Also, in what follows we will use index m to indicate the M phases of the cycle, and, anywhere m occurs, it is understood that that equation should be considered for the values of m of (10).

A remark about the implementation of the pilot filter is in order. A common approach in feedforward synchronization is that of *planar* filtering. In planar filtering, the complex envelope (4) is filtered in place of its phase. The angle $\hat{\theta}(t)$ is then extracted from the filtered complex envelope. An example of this approach can be found in the paper by A. J. Viterbi and A. M. Viterbi [5]. One prefers not to extract the phase from the unfiltered complex envelope because extraction is subject to errors in phase unwrapping, with the errors being caused by the large power of the unfiltered noise that affects the unfiltered complex envelope. It should be observed that, when the complex envelope (4) is narrowband frequency modulation plus noise, filtering the complex envelope by a given transfer function and then extracting the angle produce virtually the same result as filtering the unwrapped phase through the same transfer function. In other words, when (4) is narrowband frequency modulation plus noise, the optimized phase transfer function can be used for the transfer function of the filter that receives at its input the complex envelope, virtually without loosing optimality. This observation is motivated by the classical result, that can be found in Middleton [6], that narrowband frequency modulation approximates the cascade of integration in time domain of the frequency modulating signal and amplitude modulation. Hence narrowband frequency modulation can be treated as amplitude modulation, with the modulating signal being $\theta(t)$. The block diagram of the system based on planar filtering is shown in Figure 1.

3. Optimal Pilot Filter

Given a discrete-time sequence $\{s(k)\}$ whose spectrum is

$$S(f) = \sum_k s(k)e^{-j2\pi kfT}, \tag{11}$$

define the spectra of the down-sampled sequences over M phases

$$S_m(f) = \sum_k s(m + kM)e^{-j2\pi(m+kM)fT}. \quad (12)$$

The following properties of $S_m(f)$ will be used in what follows.

(i) $S_m(f - n(MT)^{-1})$ is determined from $S_m(f)$ as

$$S_m\left(f - \frac{n}{MT}\right) = S_m(f)e^{j(2\pi nm/M)}. \quad (13)$$

(ii) From (13) we see that $|S_m(f)|^2$ is periodic with period $(MT)^{-1}$,

$$|S_m(f)|^2 = \left|S_m\left(f - \frac{n}{MT}\right)\right|^2, \quad \forall n. \quad (14)$$

(iii) $S_m(f)$ can be obtained by folding $S(f)$ as

$$S_m(f) = \frac{1}{M}\sum_{i=0}^{M-1} S\left(f - \frac{i}{MT}\right)e^{-j(2\pi mi/M)}. \quad (15)$$

(iv) For $m = 0$, from (15) one gets

$$\int_0^{T^{-1}} S(f)df = M\int_0^{(MT)^{-1}} S_0(f)df. \quad (16)$$

Let

$$H_m(f) = \sum_k h(m + kM)e^{-j2\pi(m+kM)fT}, \quad (17)$$

and let

$$H(f) = \sum_{i=0}^{M-1} H_i(f) \quad (18)$$

indicate the frequency response of the pilot filter. Since each of the M phases of the down-sampled sequence is a stationary sequence, one can apply the classical result for the stationary MSE (see, e.g., [7]) to each of the M phases

$$\begin{aligned}\mathrm{MSE}_m &= T\int_0^{T^{-1}} \Psi(f)|1 - H_m(f)|^2 df \\ &\quad + T\int_0^{T^{-1}} \frac{|H_m(f)|^2}{2\cdot \mathrm{SNR}}df.\end{aligned} \quad (19)$$

The first term in the right-hand side of the above equation is due to the high-frequency components of phase noise that are rejected by the filter, hence that are not recovered by the estimate. The second term is due to the white additive noise that passes through the filter. Filter design should optimize the compromise between the two terms. Specifically, large bandwidth is desired in order the keep small the first term, while narrow bandwidth is desired in order to keep small the second term.

The complex exponentials appearing in (15) form an orthogonal basis

$$\begin{aligned}\int_0^{T^{-1}} \Psi(f)H_m^*(f)df &= \int_0^{T^{-1}} \Psi_m(f)H_m^*(f)df \\ &= M\int_0^{(MT)^{-1}} \Psi_m(f)H_m^*(f)df.\end{aligned} \quad (20)$$

Using the above equality together with (16) and (14) the MSE (19) is written as

$$\begin{aligned}\mathrm{MSE}_m = MT\int_0^{(MT)^{-1}} &\Bigg(\Psi_0(f)\left(1 + |H_m(f)|^2\right) \\ &+ -2\Re\{\Psi_m(f)H_m^*(f)\} \\ &+\frac{|H_m(f)|^2}{2\cdot \mathrm{SNR}}\Bigg)df.\end{aligned} \quad (21)$$

Setting to zero the derivative of (21) with respect to $H_m(f)$ in the range $0 \le f < (MT)^{-1}$ one gets the solution

$$H_m(f) = \frac{\Psi_m(f)}{\Psi_0(f) + (2\cdot \mathrm{SNR})^{-1}}, \quad 0 \le f < (MT)^{-1}. \quad (22)$$

By using (13) we see that the restriction can be removed, getting

$$H_m(f) = \frac{\Psi_m(f)}{\Psi_0(f) + (2\cdot \mathrm{SNR})^{-1}}. \quad (23)$$

Substituting the optimal filter into (21), the mth MSE is easily seen to be

$$\mathrm{MSE}_m = MT\int_0^{(MT)^{-1}} S_m(f)df, \quad (24)$$

where

$$S_m(f) = \Psi_0(f)\left(1 - \frac{|\Psi_m(f)|^2}{|\Psi_0(f)|^2 + (2\cdot \mathrm{SNR})^{-1}\Psi_0(f)}\right). \quad (25)$$

4. Numerical Results

The model of phase noise considered in this section is the popular *random phase walk*

$$\theta(k) = \theta(k - 1) + \gamma(k), \quad (26)$$

where $\{\gamma(k)\}$ is white noise with zero mean and variance σ^2. The power spectral density of phase noise is

$$\Psi(f) = \frac{\sigma^2}{(1 - e^{j2\pi fT})(1 - e^{-j2\pi fT})}. \quad (27)$$

To derive specific results, we put $\sigma = 0.3^\circ$, that is, $\sigma = 5.23 \cdot 10^{-3}$ radians. This is the experimental setting used in [4] to characterize carrier recovery in a system operating at high spectral efficiency, hence at high channel SNR.

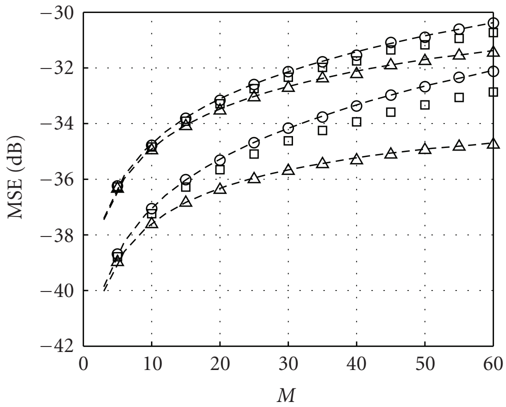

FIGURE 2: MSE versus M for SNR $= 25$ dB (upper family of curves) and SNR $= 30$ dB. Dashed lines: computed minimum and maximum MSE_m. Squares: simulation of the mean MSE. Circles: simulation of the maximum MSE_m. Triangles: simulation of the minimum MSE_m.

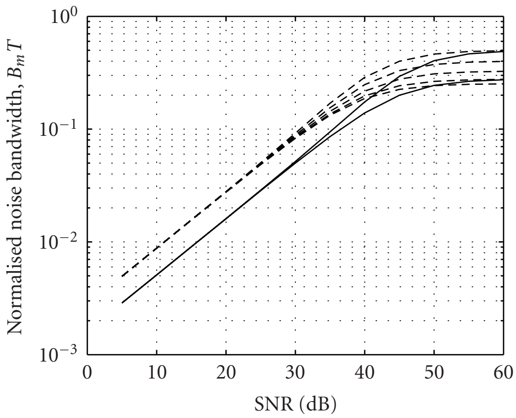

FIGURE 3: Normalized noise bandwidth of the M subfilters versus SNR for two values of M. Solid lines: $M = 3$. Dashed lines: $M = 9$.

Figure 2 shows two families of curves corresponding to two values of SNR. For each family, in the figure are shown the computed and the simulated minimum and maximum MSE and the simulated mean value of MSE versus pilot spacing M. The mean value of MSE is defined as

$$\frac{1}{M}\sum_{m=0}^{M-1}\mathrm{MSE}_m. \tag{28}$$

The gap between the maxima and the minima of the MSE is due to the cyclostationary nature of the sequence $\{\widehat{\theta}(k)\}$, which is obtained by filtering a zero-padded sequence. The minimum inside the period M occurs for $m = 0$, which indicates the estimate of phase noise at the time instant where the pilot symbol occurs. The maximum occurs for

$$m = \left\lfloor \frac{M}{2} \right\rfloor, \tag{29}$$

where $\lfloor x \rfloor$ indicates the integer part of x. The above m is the maximum time distance between the estimate of phase noise

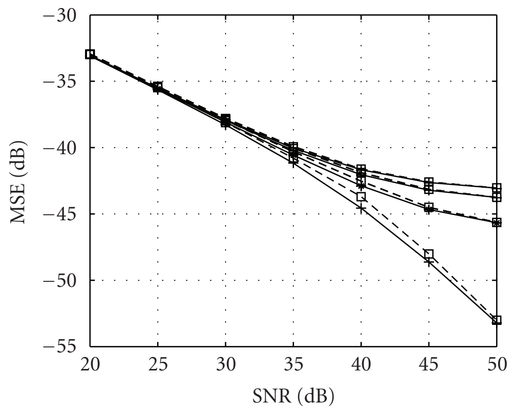

FIGURE 4: MSE versus SNR. Solid line: computed optimal filter. Dashed line: computed suboptimal filter. Crosses: simulation, optimal filter. Squares: simulation, suboptimal filter.

and the closer pilot symbol. As expected, the gap grows with the pilot spacing M. Also, the gap grows with the SNR, since the larger is the SNR, the larger is the influence of phase noise on system performance compared to the influence of the channel SNR.

Figure 3 shows the normalized noise bandwidth of the mth subfilter, defined as

$$B_m T = MT\int_0^{1/2MT} |H_m(f)|^2 df, \tag{30}$$

versus SNR, for $M = 3$ and $M = 9$. The noise bandwidth increases with SNR and with the pilot spacing M. The increase of the noise bandwidth with SNR is due to the optimization of the compromise between the two terms appearing in the right side of (19). Specifically, at high SNR the compromise between the two mentioned terms is optimized by a filter with large bandwidth. The increase of the noise bandwidth with M is explained by noting that the spectral effect of the time spacing M between to successive pilot symbols is that of folding M times the spectrum of channel noise. Since the noise bandwidth is a measure of the channel noise that passes through the filter, one expects that, due to aliasing, the noise bandwidth increases with M. Also, from the figure one appreciates that the difference between subfilter bandwidths becomes nonnegligible for $B_mT > 10^{-1}$, independently of M and SNR.

In Figure 4 it is shown the family of curves $\{\mathrm{MSE}_m\}$, with $M = 7$, versus channel SNR. For comparison it is also shown $\{\mathrm{MSE}_m\}$ for a suboptimal FIR pilot filter obtained by cascading two moving averages, each one of duration NM samples. The impulse response of the suboptimal filter has the following triangular shape:

$$h(k) = \begin{cases} \dfrac{1}{MN^2}(MN - |k|), & |k| < NM, \\ 0, & |k| \geq NM. \end{cases} \tag{31}$$

Figure 5 shows the impulse response of the optimal pilot filter for $M = 7$ and SNR $= 35$ dB along with the impulse response of the best suboptimal FIR pilot filter.

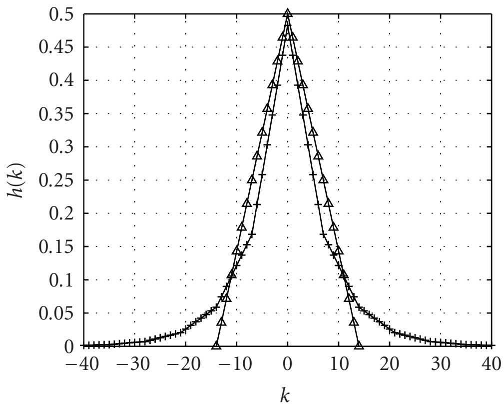

FIGURE 5: Impulse response of the planar filter for $M = 7$, SNR = 35 dB. Crosses: optimal filter. Triangles: suboptimal filter ($N = 2$).

The parameter N used to derive the results of Figure 4 is optimized by computing (19) for several values of N and for all the values of m between 0 and $M - 1$, then by taking the N that optimizes the mean MSE. Disregarding the case $m = 0$, that corresponds to estimating the phase noise at the time instant where the pilot symbol occurs, the performance loss of the suboptimal filter is within 0.35 dB ($m = 1$, SNR = 40 dB) from the performance of the optimal filter.

5. Conclusion

The performance of pilot-aided carrier recovery has been analyzed, and the optimal planar filter has been derived taking aliasing into account. The main contribution of the paper is that of having captured the important role of aliasing in the design of pilot rate. Compared to the previous literature [1], our analysis also puts light on the cyclostationary nature of the recovered phase, which becomes substantial when aliasing is nonnegligible, that is, at high SNR and for low pilot rate. Simulation results, obtained for the case where phase noise is modelled as random phase walk, confirm the theory. Also, the results presented in the paper show that the performance of a suboptimal filter obtained by cascading two moving averages is close to that of the optimal filter in cases of practical interest.

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