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Entanglement and photon statistics of two dipole–dipole coupled superconducting qubits with Kerr-like nonlinearities

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ABSTRACT

The engineering of Kerr and time-dependent coupling interactions is of great attention for treating quantum information in quantum systems and for investigating the collective behavior of large numbers of interacting particles in a cavity-qubit network. In this manuscript, we investigate the time evolution of the entanglement and some nonclassical properties of two superconducting qubits interacting with a single-mode field in the presence of a Kerr-like medium and dipole–dipole interaction without and with time-dependent coupling effect. We show that a slight alteration in the interaction, detuning, and Kerr parameters might cause a change in the entanglement of subsystem states during the evolution. By taking into account the influence of the different physical parameters, we show the statistical distributions produced in the photons of the single mode field through the calculation of the Mandel's parameter. Finally, we find that the time-dependent Mandel's parameter not only provide the statistical properties of the field, but also include the information of quantum entanglement for the subsystem states.

1. Introduction

Superconducting (SC) circuits, in the concept of Josephson junctions (JJs), are eventuality good prospects to execute quantum bits (qubits), examine the physical properties of the charge qubits, and provide a convenient controlling on qubits in diverse domains of quantum information communication and processing (QICP) [1–12]. The majority of effective JJ-qubit considers a teeny junction that relates n excess Cooper-pair charges with capacitance and JJ-coupling energy to a SC electrode and a charging energy of an electron. Essentially, when we consider the case of a qubit only 2-charge states will be participating with $n = 0, 1$. Whereas all the other states containing greater energy are negligent. Therefore, in a Cooper box system, a SC-qubit reflects as a physical quantum system of 2-charge states [13], known as an artificial system, and that the energy differences between the 2-states are manipulated by the normalized gate charge. An essential intriguing suggestion is a possible implementation of several domains of QICP within the framework of circuits in the cavity QED [14]. The empirical results of the quantum properties of such system having already mentioned, such as the phenomenon of the coherent oscillations among degenerate states [15] and of the charge states-superposition [16,17]. Recently, a necessary advance for realizing the quantum regime is achieved. Moreover, it is found that a SC-qubit interacts with microwave photons [18] where lately it has been treated and examined the coupling properties among quantum circuits and photons within the context of atomic cavity QED [19].

In the theory of quantum mechanics, entangled states have been defined for which the state of each sub-system can be described out of its relation to each other and the total knowledge of a quantum state of a composite system can be determined [20,21]. It is widely known that the entanglement displays one of the most favorable phenomena in QICP, that can be is considered as the nonlocal correlations among physical quantum systems [22,23]. In this regard, the study of the entanglement phenomenon is introduced as a special kind of quantum correlations that leads to develop diverse physical problems [24,25]. Lately, the development of the field of QICP has provided an enhancement in the literature of this phenomenon [26–40]. The importance of the entanglement in various applications has given rise to examination of high-dimensional systems and exploring the particular importance of this kind of correlations in many-body systems [41]. The preparation of the entanglement is presently considered as one of the aims in the empirical feasibility or implementation.

The analysis of the interaction among SC-qubit-field systems relates at the core of quantum technologies. The SC-qubit interacted with a quantized field within an optical cavity is considered as one of the unpretentious examples which can be theoretical characterized through the Jaynes Cummings model under the approximation of the rotating wave [42,43]. Various works have been introduced to generalize this

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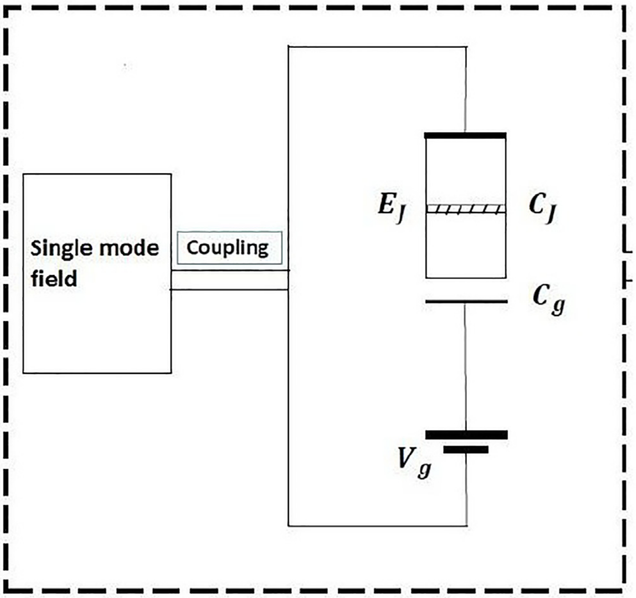

Fig. 1. Diagram of a superconducting-qubit (SC) electronic circuit coupled to a quantized electromagnetic field. Branches represent a Josephson junction (E_J), Capacitances (C_J and C_g), and voltage source V_g.

model [44,45]. In this context, the time-dependent coupling strength is a substantial feature for various interesting phenomena in quantum optics and can be utilized to optimize several physical quantum properties. Circuit quantum electrodynamics is considered as the directing quantum technologies for developing and investigating applications of diverse tasks of QICP. The possibility of rapid controlling the characteristic parameters of the interaction among SC-qubit systems and electromagnetic fields allows for the empirical achievement of quantum optical systems in formerly inaccessible regimes, permitting the reconnaissance and understanding the behavior of a large class of quantum systems [46,47].

We extend the usual model of SC-qubit systems by considering the dipole interaction and both the nonlinear coupling between the SC-qubits and the single mode field and a supplemental Kerr-like term. Here, the medium is characterized by a nonlinear harmonic oscillator with the term $(\hat{a}^{\dagger})^2\hat{a}^2$. Any real problem with nonlinear interaction can be described by this generalized model [48–51]. This allows us to execute the counterpart in the quantum mechanical for nonlinear optical phenomena such as Kerr nonlinearity, parametric interaction, and different kinds of nonlinear wave mixings. It has been shown that the systems involving discussed nonlinearities can lead to so-called photon/phonon blockade effect [52–55], such systems are called as nonlinear quantum scissors. In general, models involving Kerr-type nonlinearities can lead to the generation of the maximally entangled states. For instance, we can mention W states [56–58]. Moreover, beside quantum entanglement, quantum steering effects can also appear in various Kerr-type models [59–61]. Here, we will study qualitatively the time evolution of the entanglement and some nonclassical properties of two SC-qubits coupled to a single-mode field by considering a nonlinear Kerr-like medium and dipole–dipole interaction without and with time-dependent coupling effect. We will show that a slight change in the interaction, detuning, and Kerr medium parameters yield to a considerable amount of entanglement among the two SC-qubits and field (SC-

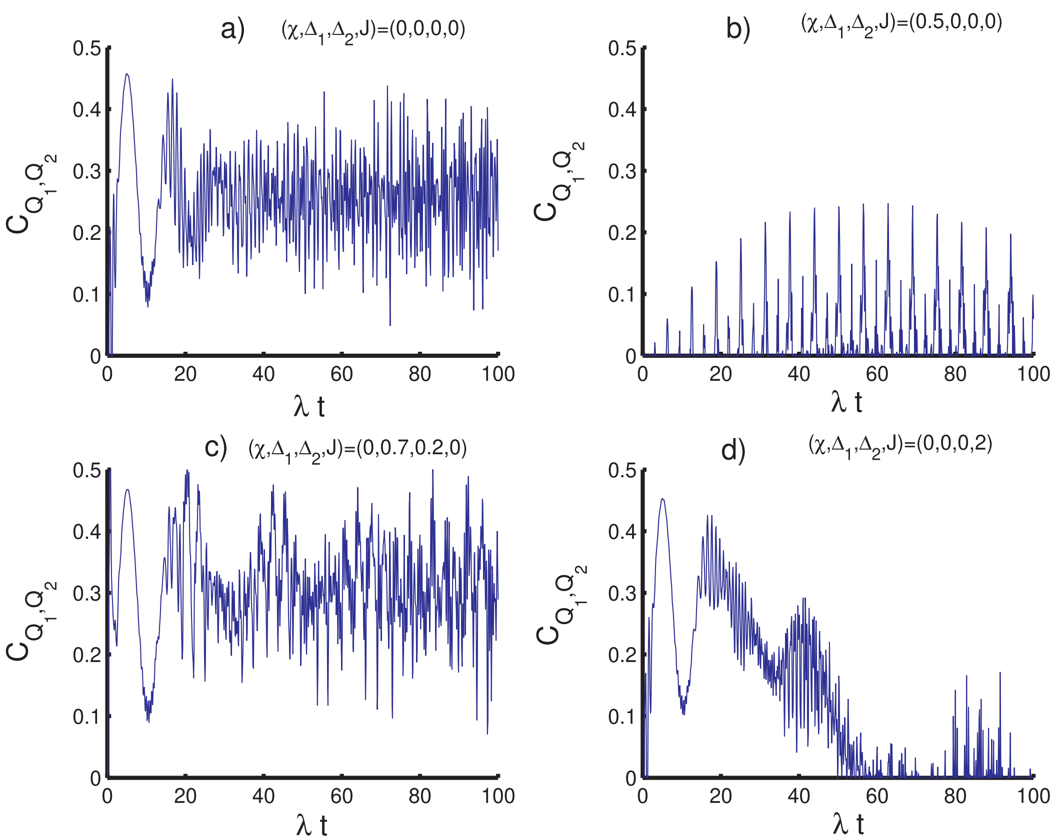

Fig. 2. SC-qubit–SC-qubit concurrence against the scaled time when the single mode is initially in the Glauber coherent state with $\beta = \sqrt{10}$, in the absence SC-qubit motion effect $\lambda(t) = \lambda$. Fig. (a) is obtained in the absence of detuning, Kerr, and dipole–dipole coupling i.e. $(\chi, \Delta_1, \Delta_2, J) = (0, 0, 0, 0)$. Figs. (b), (c), and (d) are obtained for different values of the Kerr, detuning and dipole terms as: Fig. (b) is for $(\chi, \Delta_1, \Delta_2, J) = (0.5, 0, 0, 0)$, Fig. (c) is for $(\chi, \Delta_1, \Delta_2, J) = (0, 0.7, 0.2, 0)$, and Fig. (d) is for $(\chi, \Delta_1, \Delta_2, J) = (0, 0, 0, 2)$.

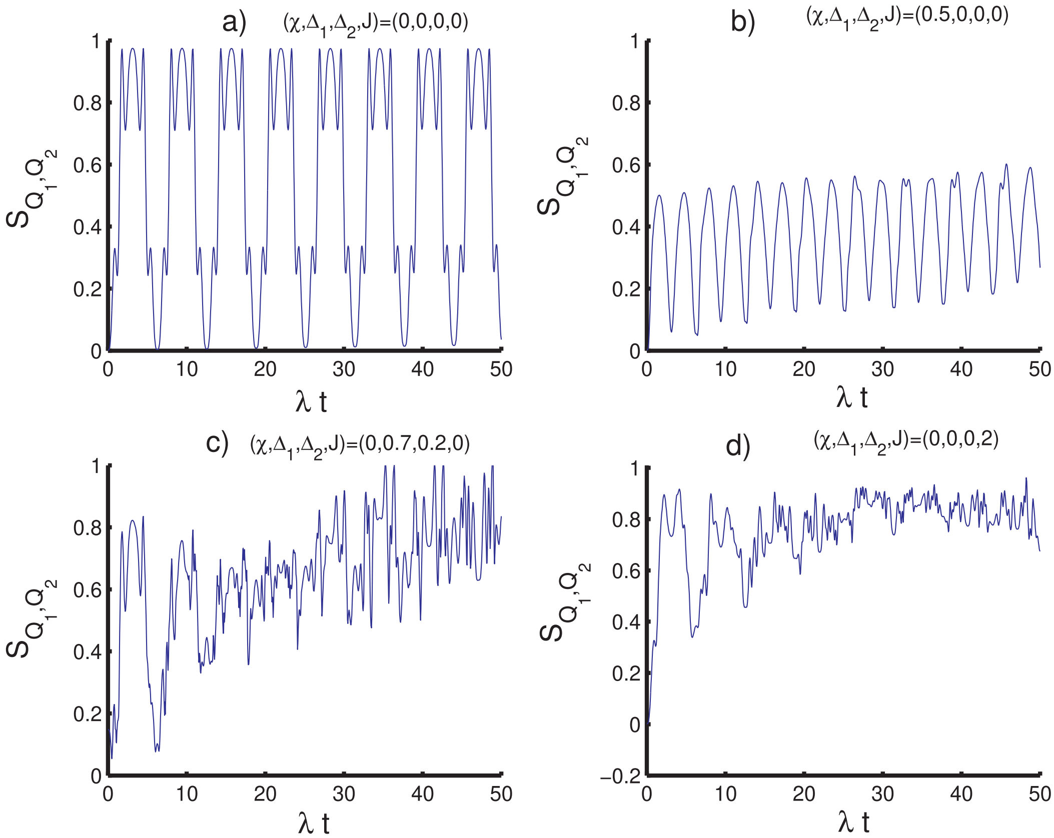

Fig. 3. The same as Fig.1 under the effect of SC-qubits motion for $\lambda(t) = \lambda \sin(t)$.

qubit–SC-qubit subsystem and two SC-qubits-field subsystem) during the evolution. By choosing suitable initial physical parameters, we discuss the time dependence of the statistical distributions produced in the photons for the quantized field through the calculation of the Mandel's parameter.

The paper is structured as follows. In Sec. II, we present the physical model and describe the different physical quantifiers. In Sec. III, we show the numerical results by examining the entanglement among the states of subsystems and statistical properties of the field with the help of the dynamical behavior of the Mandel's parameters. Finally, some conclusions are given in IV.

2. Model and dynamics

Let us start from one superconducting qubit (SC-qubit). The Hamiltonian of a SC-qubit coupled to a resonator in nonlinear single mode field is given by

$$H = \hbar_r\omega_r\hat{a}^+\hat{a} + \frac{\hbar E_J}{2}\hat{\sigma}_z - \frac{eC_g}{C_t}\sqrt{\frac{\hbar\omega_r}{LC}}(\hat{a}^+ + \hat{a})\hat{\sigma}_x, \tag{1}$$

where $C_t = C_g + C_J$ denotes the total box capacitance with C_g presents the gate capacitance to the island, $\hat{\sigma}_x$ and $\hat{\sigma}_z$ are, respectively, the Pauli operators for the x and z-components, $\omega_r = 1/\sqrt{LC}$ presents the frequency at the resonance, $\hat{a}$ and $\hat{a}^+$ are the annihilation and creation operators of the quantized field, respectively (See Fig. 1). By taking into account the rotating wave approximation the Hamiltonian reads

$$H = \hbar_r\omega_r\hat{a}^+\hat{a} + \frac{\hbar\omega_s}{2}\hat{\sigma}_z + \lambda(t)(\hat{\sigma}_+\hat{a} + \hat{a}^+\hat{\sigma}_-). \tag{2}$$

Here, we introduce a nonlinear model which is constructed from the standard model by considering the interaction of a SC-qubit with a quantized field and taking into account the influence of a Kerr-like medium. The nonlinear Hamiltonian is given by:

$$H_{1N} = \hbar\Omega_r\hat{n} + \hbar\chi\hat{n}^k + \frac{1}{2}\hbar\omega_s\hat{\sigma}_z + \hbar\lambda(t)[\hat{\sigma}_+\hat{a}\hat{N} + \sigma_+\hat{N}\hat{a}^+], \tag{3}$$

where χ is called the anharmonicity parameter with $0 \leqslant \chi \ll \omega_r$, k is an integer number with $k \geqslant 1$, and $\Omega_r = \omega_r - \chi$. The parameter χ describes the optical properties of the Kerr medium and the operator $\hat{N}$ is given as

$$\hat{N} = \left(1 - \frac{\chi}{\omega_r}(1 - \hat{n}^{k-1})\right)^{\frac{1}{2}}. \tag{4}$$

In the $\chi = 0$ or $k = 1$ limit, we recover the linear (or standard) model. For $k = 2$ the Hamiltonian is the Kerr Hamiltonian given by

$$H_{1N} = \hbar\Omega_r\hat{n} + \hbar\chi\hat{n}(\hat{n} - 1) + \frac{1}{2}\hbar\omega_s\hat{\sigma}_z + \hbar\lambda(t)(\hat{\sigma}_+\hat{a} + \sigma_+\hat{a}^+), \tag{5}$$

When the refractive index of the medium varies with the intensity of the field, then medium exhibits the Kerr effect, and we have the phenomenon of nonlinear optics.

In this paper, we consider two coupling SC-qubits that are coupled to a single mode field with the effective Hamiltonian:

$$H_{2N} = \hbar\omega_r\hat{n} + \hbar\chi\hat{n}(\hat{n} - 1) + \frac{1}{2}\hbar\omega_{s_1}\sigma^z_{s_1} + \frac{1}{2}\hbar\omega_{s_1}\sigma^z_{s_1} \tag{6}$$

$$+ \hbar\sum_{i=1,2}\lambda_i(t)(\sigma^+_{s_i}\hat{a} + \sigma^-_{s_i}\hat{a}^+) + \hbar J(\sigma^-_{s_1}\sigma^+_{s_2} + \sigma^-_{s_2}\sigma^+_{s_1}).$$

where $\sigma^z_{s_1}$ and $\sigma^z_{s_2}$ are Pauli matrices of two SC-qubits S_1 and S_2, $\sigma^\pm_{s_1}$ and $\sigma^\pm_{s_2}$ are raising and lowering operators, ω_{s_1} and ω_{s_2} are Rabi frequencies, and $\lambda_1(t)$ and $\lambda_2(t)$ are coupling strengths between the field (of the frequency ω_r) and the SC-qubits S_1 and S_2, respectively, with the detunings $\Delta_1 = \omega_r - \omega_{s_1}$ and $\Delta_2 = \omega_r - \omega_{s_2}$. J is an interacting strength between the two SC-qubits. We consider that the single mode field is

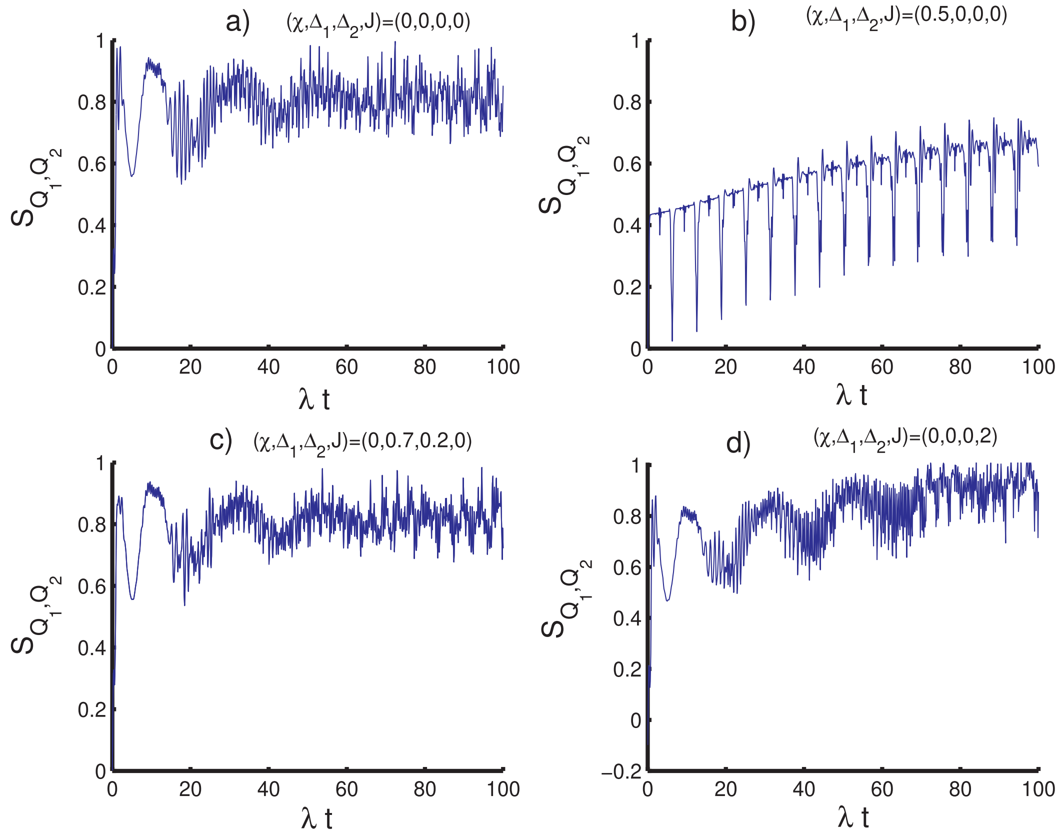

Fig. 4. SC-qubits-field entanglement against the scaled time when the single mode is initially in the Glauber coherent state with $\beta = \sqrt{10}$, in the absence SC-qubit motion effect $\lambda(t) = \lambda$. Fig. (a) is obtained in the absence of detuning, Kerr, and dipole–dipole coupling i.e. $(\chi, \Delta_1, \Delta_2, J) = (0, 0, 0, 0)$. Figs. (b), (c), and (d) are obtained for different values of the Kerr, detuning and dipole terms as: Fig. (b) is for $(\chi, \Delta_1, \Delta_2, J) = (0.5, 0, 0, 0)$, Fig. (c) is for $(\chi, \Delta_1, \Delta_2, J) = (0, 0.7, 0.2, 0)$, and Fig. (d) is for $(\chi, \Delta_1, \Delta_2, J) = (0, 0, 0, 2)$.

initially described by a Glauber state, $|\beta\rangle = \exp(-|\beta|^2/2)\sum_{n=0}^{\infty}(\beta^n)/\sqrt{n!}\,|n\rangle$, and that the two SC-qubit system are initially defined in the state $|\phi\rangle = |e_1e_2\rangle$.

The equation of motion for the considered system state is given by

$$i\hbar\frac{\partial}{\partial t}\left|\Psi(t)\right\rangle = H_{2N}\left|\Psi(t)\right\rangle. \tag{7}$$

The quantum state of the two SC-qubits and the single mode field is written as a function of the states $|e_1e_2, n\rangle$, $|e_1g_2, n+1\rangle$, $|g_1e_2, n+1\rangle$, and $|g_1g_2, n+2\rangle$ where $|g_i\rangle$ (resp. $|e_i\rangle$) defines the ground (excited) state of the i^{th}SC-qubit. At the instant t, the quantum state of the whole system (two SC-qubits + field) reads

$$\left|\Psi(t)\right\rangle = \sum_{n=0}^{\infty} C_{e_1e_2,n}(t)\left|e_1e_2, n\right\rangle + C_{e_1g_2,n+1}(t)\left|e_1g_2, n+1\right\rangle + C_{g_1e_2,n+1}(t)\left|g_1e_2, n+1\right\rangle + C_{g_1g_2,n+2}(t)\left|g_1g_2, n+2\right\rangle \tag{8}$$

The wavefunction amplitudes satisfy the coupled differential equations:

$$i\dot{C}_{e_1e_2,n} = \left[n\omega_r + n\left(n-1\right)\chi + \left(\frac{\omega_{s_1}+\omega_{s_2}}{2}\right)\right]C_{e_1e_2,n} + \lambda_1\sqrt{n+1}\,C_{g_1e_2,n+1} + \lambda_2\sqrt{n+1}\,C_{e_1g_2,n+1} \tag{9}$$

$$i\dot{C}_{e_1g_2,n+1} = \left[\left(n+1\right)\omega_r + n\left(n+1\right)\chi + \left(\frac{\omega_{s_1}-\omega_{s_2}}{2}\right)\right]C_{e_1g_2,n+1} + \lambda_1\sqrt{n+2}\,C_{g_1g_2,n+2} + \lambda_2\sqrt{n+1}\,C_{e_1e_2,n} + JC_{g_1e_2,n+1} \tag{10}$$

$$i\dot{C}_{g_1e_2,n+1} = \left[\left(n+1\right)\omega_r + n\left(n+1\right)\chi - \left(\frac{\omega_{s_1}-\omega_{s_2}}{2}\right)\right]C_{g_1e_2,n+1} + \lambda_1\sqrt{n+1}\,C_{e_1e_2,n} + \lambda_2\sqrt{n+2}\,C_{g_1g_2,n+2} + JC_{e_1g_2,n+1} \tag{11}$$

$$i\dot{C}_{g_1g_2,n+2} = \left[\left(n+2\right)\omega_r + \left(n+1\right)\left(n+2\right)\chi - \left(\frac{\omega_{s_1}+\omega_{s_2}}{2}\right)\right]C_{g_1g_2,n+2} + \lambda_1\sqrt{n+2}\,C_{e_1g_2,n+1} + \lambda_2\sqrt{n+2}\,C_{g_1e_2,n+1}, \tag{12}$$

where the values of $C_{e_1e_2,n}$, $C_{e_1g_2,n+1}$, $C_{g_1e_2,n+1}$ and $C_{g_1g_2,n+2}$ are reliant on the choice of the initial conditions. For the case where the state of the two SC-qubit system is the state $|e_1e_2\rangle$, we have $C_{e_1e_2,n}(0) = C_n(0)$ and $C_{e_1g_2,n+1}(0) = C_{g_1e_2,n+1}(0) = C_{g_1g_2,n+2}(0) = 0$ with $C_n(0) = \exp(-|\beta|^2/2)\beta^n/\sqrt{n!}$.

After getting the two SC-qubits mixed state density operator, $\hat{\rho}_{2SF}(t)$, we will consider the Wootters' [62] concurrence as a measure of entanglement, which can be adopted as an exactly calculable quantity to measure the degree of entanglement of an arbitrary two SC-qubit state. It can be defined as

$$C_{2S} = 2\max\{0, \mu_1 - \mu_2 - \mu_3 - \mu_4\} \tag{13}$$

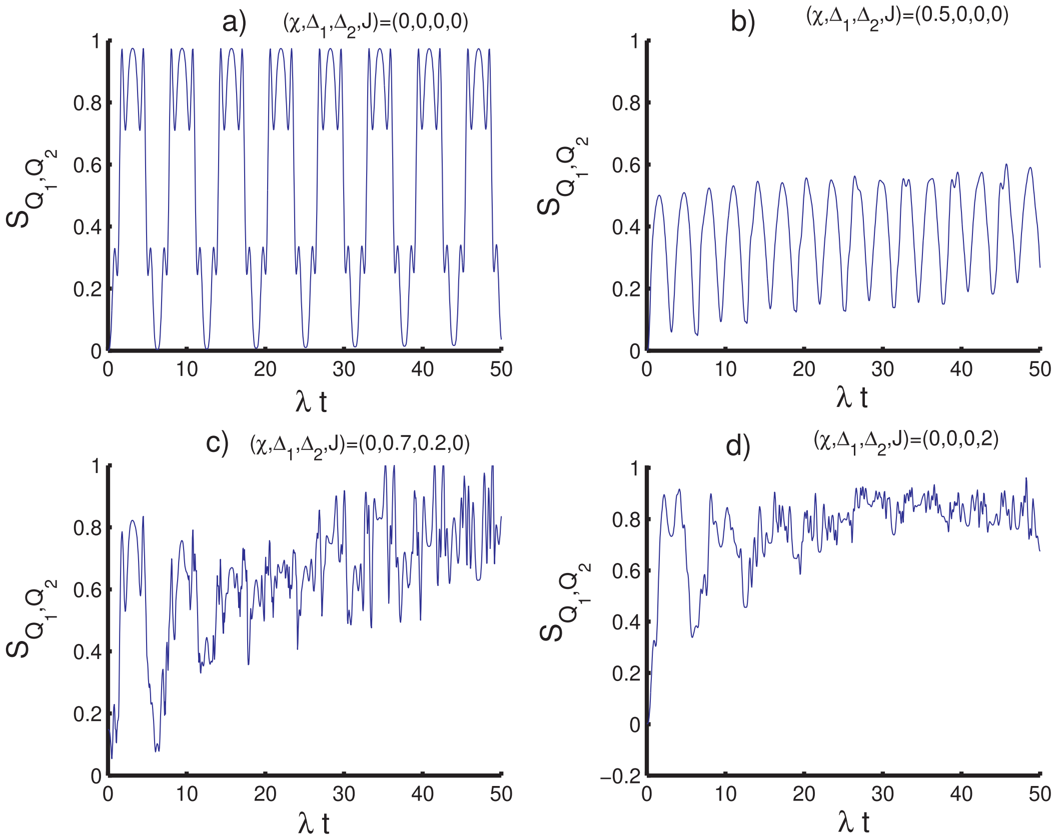

Fig. 5. The same as Fig.3 under the effect of SC-qubits motion for $\lambda(t) = \lambda\sin(t)$.

where μ_i (i = 1, 2, 3, 4) define the square roots of the eigenvalues of the operator $\rho_{2S}(\sigma_{s1}^y \otimes \sigma_{s2}^y)\rho_{2S}^*(\sigma_{s1}^y \otimes \sigma_{s2}^y)$ in decreasing order. The concurrence varies from $C_{2S} = 0$ (disentangled state) to $C_{2S} = 1$ (maximally entangled state).

To quantify the degree of the entanglement between the SC-qubits and the single mode field, we consider the von Neumann entropy, which is supposed as a measure of the purity and the disorder for the quantum states. In our situation, we examine the purity resulting from the quantum state defined in Eq. (8), $\hat{\rho}_{2S-F}(t) = |\psi(t)\rangle_{2S-F}\ {}_{2S-F}\langle\psi(t)|$, by performing a trace over one of the subsystems, and we have

$$S_V = S_{2S}(t) = S_F(t). \tag{14}$$

We simply evaluate the amount of the entropy of one subsystem to achieve the dynamics of the entanglement for the quantum state. The von Neumann entropy will vary from $S_V = 0$ (factorizable pure state) to $S_V = 1$ (maximally entangled state).

The Mandel's parameter serves as a physical indicator to study the distribution of the photon statistics for the quantized field during the interaction. It is defined by [63]

$$Q_P(t) = \frac{\langle(\Delta\hat{n})^2\rangle - \langle\hat{n}\rangle}{\langle\hat{N}\rangle}, \tag{15}$$

Here $\langle\hat{n}\rangle$defines the mean photon number and $\Delta\hat{n}$ presents the dispersion. Through the Mandel's parameter, we can determine whether the distribution of the photons is sub-Poissonian (classical case), $Q_P < 0$, Poissonian (semi-classical states), $Q_P = 0$, or super-Poissonian, $Q_P > 0$.

3. Results and discussion

In the present section, we show the time-evolution of the entanglement and statistical properties considering the nonlocal correlation of the system of two dipole–dipole coupled SC-qubits when they simultaneously interact with quantized field and the photon statistics of the single mode field under the effect of the Kerr medium and dipole–dipole strength in the absence and presence of time-dependent coupling effect. We discuss the time dependency coupling strength on these physical quantities, modeled by the coupling function $\lambda_1(t) = \lambda_2(t) = \lambda(t) = \lambda\sin t$. In comparison with the aforementioned papers, our present work from the phenomenological viewpoint might be more practical to explain and understand some empirical observations on the nonclassical properties and entanglement with realistic phenomenon providing more hints for future investigation on the quantum technology topics.

In Figs. 2 and 3 we plot the SC-qubit–SC-qubit concurrence as a function of λt in the cases of $\lambda(t) = \lambda$ and $\lambda(t) \neq \lambda$ with respect to different values of the model parameters. Fig. 2 displays the impact of the Kerr medium parameter, detuning, and dipole interaction on the evolution of the entanglement for the two SC-qubit state in the absence of the time-coupling effect. We find that the behaviour of the concurrence during the dynamics is oversensitive to the values of the model parameters. Generally, the concurrence shows a random behaviour with extremely rapidly oscillations. Interestingly, we note that the presence of Kerr medium and dipole–dipole interaction have a destroy effect on the amount of entanglement exhibiting sudden and sudden birth phenomena. The entanglement sudden death refers to the fact that even a very weakly environment effect can degrade the specifically quantum portion of the correlation to zero in just a finite time rather than by successive halves. In fact, in the previous studies, the couplings with the environment have been considered as the critical point of disentanglement and entanglement sudden death [64,65]. The dynamics of entanglement and the phenomenon of entanglement sudden death [64] are discussed in bipartite systems, measured by Wootters Concurrence. Our calculation shows that for the considered initial

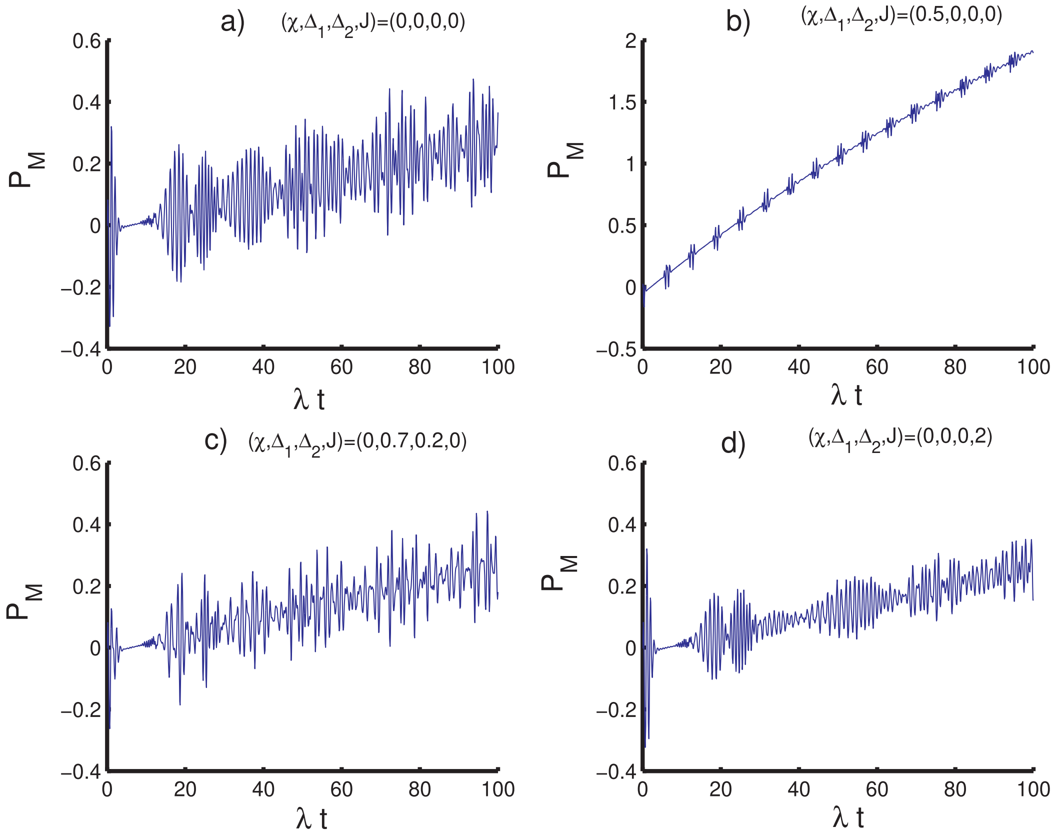

Fig. 6. The Mandel's parameter against the scaled time when the single mode is initially in the Glauber coherent state with $\beta = \sqrt{10}$, in the absence SC-qubit motion effect $\lambda(t) = \lambda$. Fig. (a) is obtained in the absence of detuning, Kerr, and dipole–dipole coupling i.e. $(\chi, \Delta_1, \Delta_2, J) = (0, 0, 0, 0)$. Figs. (b), (c), and (d) are obtained for different values of the Kerr, detuning and dipole terms as: Fig. (b) is for $(\chi, \Delta_1, \Delta_2, J) = (0.5, 0, 0, 0)$, Fig. (c) is for $(\chi, \Delta_1, \Delta_2, J) = (0, 0.7, 0.2, 0)$, and Fig. (d) is for $(\chi, \Delta_1, \Delta_2, J) = (0, 0, 0, 2)$.

condition, the entanglement disappeared in a finite time and then revived after a dark period due to the interaction between the SC-qubits and with the field. We note also that the existence of the detuning effect does not certainty affect the dynamical behavior of the entanglement. In fact, when the detuning parameter is large enough, the two SC-qubits tend to exchange energy with each other instead of with the single mode field, which acts as a medium. On the other hand, we find that the existence of the time-dependent coupling strength yields to a decrease in the oscillations of the concurrence and decrease the amount of entanglement during the time-evolution (see Fig. 3). We can see that the SC-qubit–SC-qubit concurrence is a periodic function in the case of $\lambda(t) \neq \lambda$, where the SC-qubits become periodically entangled and separable. Such as a SC-qubit–SC-qubit entanglement occurs via the single-mode field due to the periodic emissions and absorptions of photons by SC-qubits.

We also consider the plots for the SC-qubits-field entropy as a function of time. Figs. 4 and 5 display the von Neumann entropy of the two SC-qubits-field state versus the dimensionless time λt for $\lambda(t) = \lambda$ and $\lambda(t) \neq \lambda$, respectively, according to different initial values of the model parameters. Fig. 4 shows the effect of the Kerr medium parameter, detuning, and dipole–dipole strength on the evolution of the entanglement of the SC-qubits-field state in the case of $\lambda(t) = \lambda$. The figure results some attractive dynamical features of the von Neuman entropy. By a suitable choosing for the optimal initial conditions remarkably affect the evolution shape of the entanglement of the two SC-qubits-field state. Generally, the von Neumann entropy exhibits random behavior with very rapidly oscillations and shows a tendency to a steady behavior at large values of the time. Interestingly, we find that the presence of the Kerr medium has a significant impact on the von Neumann entropy, contrary what happens in the presence of the effect of detuning and dipole–dipole interaction, where the behaviour of the entropy does not certainty affected during the dynamics. While the case of $\lambda(t) \neq \lambda$ leads to reduce the oscillations of the von-Neumann entropy (see Fig. 5). We note that the dynamical behavior of the von Neumann entropy in the absence of the physical parameters is deeply influenced the time-dependent coupling effect (see Fig. 5(a)), where the von Neumann entropy shows periodic oscillations, exhibiting sudden and sudden birth phenomena of the entanglement. Such as an entanglement occurs via the single-mode field due to the periodic emissions and absorptions of photons by SC-qubits [66]. Thus, there exist an optimal choice of the model parameters to extend the nonlocal correlation between the two SC-qubits and the quantized field.

For the sake of completeness, we investigate the dynamics of the photon distributions of the single mode field by plotting the different situations of the Q_P parameter. In Figs. 6 and 7, we display the behaviour of the Q_P parameter versus λt according to the initial conditions of the model parameters. Generally, as we can see from the figures, the initial values of the parameters χ, Δ_i, and J remarkably influence the photon distribution of the single mode field, exhibiting oscillation behaviour during the time-evolution with sub-Poissonian distribution at smallest time intervals, verifying the inequality $-1 \leqslant Q_p \leq 0$. While for other cases, the value of the Q parameter rises with the time, showing Poissonian and super-Poissonian distributions of the field photons with the inequality $Q_p \geqslant 0$, for large values of the time. When the coupling strength varies with the time, the oscillations of the Mandel's parameter will be reduced during the dynamics. We note also that the photon statistics of the field is basically not changed by the detuning effect between the SC-qubits and the field.

It is worthwhile not only exploring the statistical properties of the field, but also includes the information of quantum entanglement for

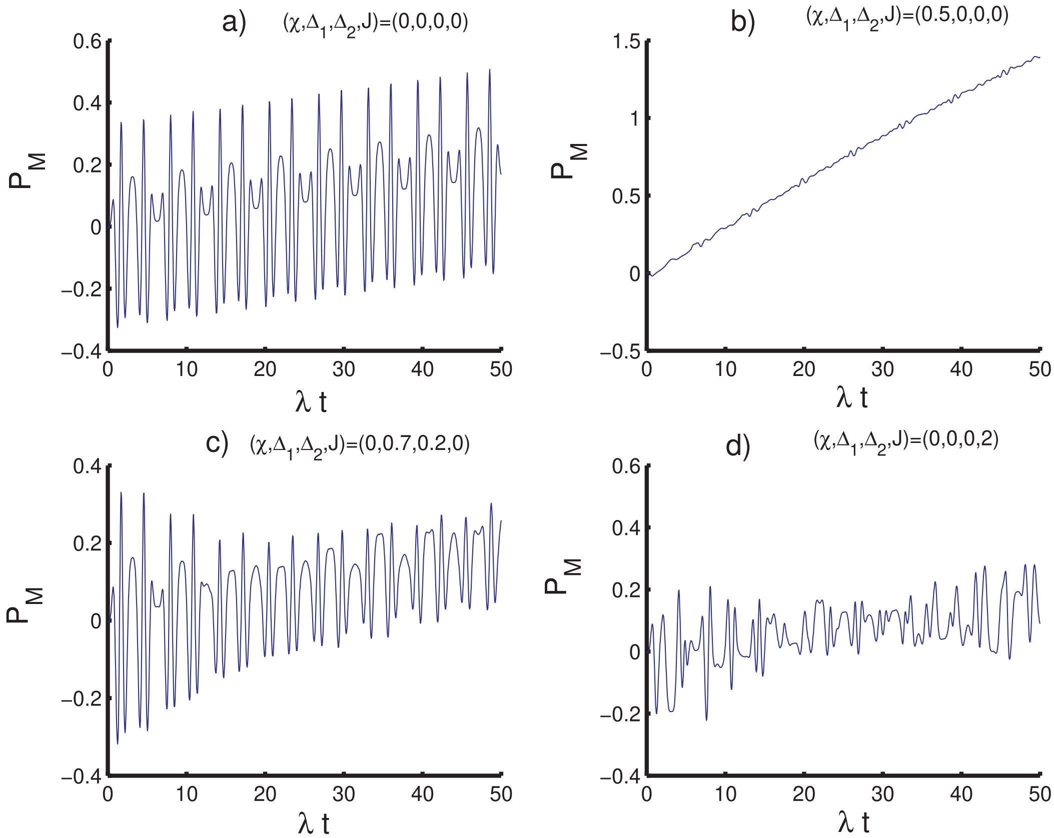

Fig. 7. The same as Fig.5 under the effect of SC-qubits motion for $\lambda(t) = \lambda \sin(t)$.

the subsystem states. The comparison between the time-evolution of the entanglement of subsystem states and Mandel's parameter with and without time-dependent coupling strength shows that the evolution of the photon distribution of the quantized field definitely affects the dynamical behaviour of the state entanglement among the subsystems. We note that the entanglement measures and Mandel's parameter exhibit the same behaviour according to the initial choice of the model parameters. The obtained results show that the quantumness in the single node field could be used as a tool for controlling and manipulating of the entanglement for the subsystem states by a suitable choosing of the model parameters.

4. Conclusion

In summary, we have examined in detail the entanglement properties of a system consists of two SC-qubits interacting with a single-mode field in the presence of a Kerr-like medium and dipole–dipole interaction. We have discussed the influence of these terms on SC-qubit–SC-qubit and SC-qubit-field entanglement without and with time-dependent coupling effect. For resonant or off-resonant cases, we have shown that the dipole interaction and Kerr-like term lead to a decrease in the amount of the entanglement. We obtained that the presence of the time dependence in the coupling term leads to reduce the oscillations of the entanglement measures for subsystem states and stabilize its behavior as the time becomes significantly large. We have examined the dynamics of the photon statistics of the single mode field through the calculation of the Mandel's parameter. On the other hand, we have demonstrated the time-dependent correlation between the entanglement of subsystem state and the photon statistics according to the initial conditions of the model parameters. The present paper is potentially to be efficacious to give some significance for the empirical achievements in the presence the time variation of the coupling strength with a like-Kerr medium.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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