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Research Article

Defining Reserve Times for Metro Systems: An Analytical Approach

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The aim of this paper is to provide an analytical approach for determining operational parameters for metro systems so as to support the planning and implementation of energy-saving strategies. Indeed, one of the main targets of train operating companies is to identify and implement suitable strategies for reducing energy consumption. For this purpose, researchers and practitioners have developed energy-efficient driving profiles with the aim of optimising train motion. However, as such profiles generally entail an increase in travel times, the operating parameters in the planned timetable need to be appropriately recalibrated. Against this background, this paper develops a suitable methodology for estimating reserve times which represent the main rate of extra time needed to put ecodriving strategies in place. Our proposal is to exploit layover times (i.e., times spent by a train at the terminus waiting for the next trip) for energy-saving purposes, keeping buffer times intact in order to preserve the flexibility and robustness of the timetable in case of delays. In order to show its feasibility, the approach was applied in the case of a real metro context, whose service frequency was duly taken into account. In particular, after stochastic analysis of the parameters involved for calibrating suitable buffer times, different operating schemes were simulated by analysing the relationship between layover times, number of convoys, and feasible headway values. Finally, some operation configurations are analysed in order to quantify the amount of energy that can be saved.

1. Introduction

Rail transport systems are undoubtedly the best solution to satisfy high travel demand in densely populated areas: they offer high transport capacity, are unaffected by traffic congestion, and are able to reduce, capturing users from road transport, greenhouse gas (GHG) and air pollutant emissions. The importance of rail transport systems has increased in recent decades since environmental issues assumed a crucial role in transport policy at all territorial levels, whether worldwide, continental, country-wide, or at the more local scale. For instance, most of the European Union's [1] transport policy targets environmental issues and especially transport emissions. Indeed, to achieve the EU's objective to reduce transport emissions by 60% by the year 2050, two main strategies are involved: promotion of rail transport for both passengers and goods and improvement in vehicle energy efficiency.

Since 72.8% of GHG emissions in the transportation sector are produced by road traffic [2], it is important to increase the attractiveness of rail transport. This can be partly achieved by promoting intermodality between trains and buses [3–5]. Other strategies for promoting the use of rail systems are to increase service quality [6–8], reliability [9–11], and competitiveness with personal cars, by optimising service schedules and operations [12–15].

Energy efficiency in rail systems can be achieved by operating on vehicles, engines, and/or driving styles. In this paper we refer to driving styles that affect and interact with schedules: an energy-efficient driving style (or eco-driving) may need to modify running times and schedules.

As will be clarified in the next section, eco-drive strategies are mainly based on available reserve times on each railway section (between two stations) or, in some cases, at the terminus. Indeed, eco-driving increases train running times and, if the scheduled timetables are to be maintained, only the available reserve time can be used.

When a timetable is designed, reserve times are usually defined according to the expected regularity of the service, without considering their (possible) utility in energy-efficient driving strategies. The aim of this paper is to provide an analytical approach for defining operational parameters for metro systems, including reserve times, considering explicitly their utility for implementing energy-saving strategies.

The paper is organised as follows: in Section 2, railway energy-efficient driving is examined; in Section 3 the analytical approach of our proposal is described; in Section 4 the methodology is applied in the case of a real metro line; finally, conclusions and research prospects are summarised in Section 5.

2. Railway Energy-Efficient Driving

Reducing the energy consumption of railway systems is currently considered an important objective in the transportation sector. Several approaches can be adopted to achieve this result: the design of low-consumption engines, the provision of efficient energy recovery systems, and the optimisation of both schedules, so as to reduce peaks in energy demand and train diving styles.

Energy-efficient driving, or eco-driving, has been widely studied in the literature. Most approaches are based on the application of optimal control theory in the case of railway contexts [16–18].

The effectiveness of energy-efficient driving strategies was underlined in [19, 20]. Interesting results were obtained in [21], where driving profiles were optimised by considering as constraints the operational requirements of railways. In [22], an optimisation framework based on a genetic algorithm was proposed for minimising energy consumption.

Other papers focusing on this issue are [23–26]; real-time optimisation was studied in [27, 28]. In [29], the potential effects of energy-efficient driving profiles on consumption were studied with a parametric approach in an Italian region; the results showed that up to 20% of total traction energy can be saved by optimising speed profiles and up to 35% if trains are equipped with braking energy recovery systems. Simulation-based methods can be found in [30–33].

The impact and integration of such procedures within the more complex problem of optimal train scheduling were studied in [34, 35]. Finally, in [36] energy-saving driving methods were applied to freight trains.

The starting point of any analysis is to define the mechanical kinetic energy required to move a rail convoy during a time interval which may be calculated as follows:

$$E(\Delta t) = \int_0^{\Delta t} dE(\tau) = \int_0^{\Delta t} P(\tau) \cdot d\tau = \int_0^{\Delta t} T(v(\tau)) \cdot v(\tau) \cdot d\tau, \tag{1}$$

where $E(\Delta t)$ is the kinetic energy required during time interval Δt; Δt is the generic time interval; $dE(\tau)$ is the increase in kinetic energy at time τ; τ is the generic time instant; $P(\tau)$ is the instantaneous power at time τ; $d\tau$ is the generic infinitesimal time interval; $v(\tau)$ is the instantaneous speed at time τ; and $T(\cdot)$ is the tractive effort (i.e., tractive force) at rail wheels which depends on instantaneous speed $v(\cdot)$.

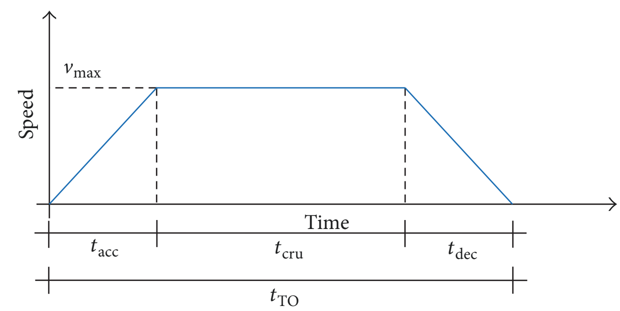

Figure 1: Speed profile in the case of a *Time Optimal* (TO) strategy.

Most approaches proposed in the literature (see, e.g., [37–39]) are based on the definition of a reference scenario, indicated as the *Time Optimal* (TO) scenario, which consists in considering the movement of the rail convoy in the case of maximum performance: the train first accelerates with maximum acceleration to reach the maximum speed *(acceleration phase)*, then travels at constant maximum speed *(cruising phase)*, and finally brakes at maximum deceleration *(deceleration phase)*. This scenario provides the lowest travel time but the maximum energy consumption.

Figure 1 provides a generic speed profile in the case of a TO strategy with the simplifying assumption of instantaneous variation of acceleration (i.e., the jerk value equal to $+\infty$), while in real cases, the jerk value is a finite quantity.

The total travel time between two successive stops (i.e., stations or red signals) in the case of strategy TO may be formulated as follows:

$$t_{TO} = t_{acc} + t_{cru} + t_{dec}, \tag{2}$$

where t_{TO} is the travel time in the case of a TO strategy; t_{acc} is the time duration of the acceleration phase; t_{cru} is the time duration of the cruising phase; and t_{dec} is the time duration of the deceleration phase.

Since parameter fluctuations are generally stochastic (e.g., a driver may adopt different values of acceleration, speed, and deceleration), when designing timetables UIC [40] suggests increasing the *total travel time* (i.e., the minimum running time) by a *running time reserve* of about 3–8% in order to provide a reserve in the case of reduced performance, thereby avoiding the onset of delays. Most *Energy-Saving* (ES) strategies proposed in the literature are based on adopting different speed profiles which consume less energy but require more time. The additional time may be obtained by consuming a part of the reserve time. In particular, these strategies may be classified according to two approaches:

(i) The first, where between the cruising and deceleration phases an additional phase is inserted, referred to as the *coasting phase*, where the engine is stopped and the train moves by inertia

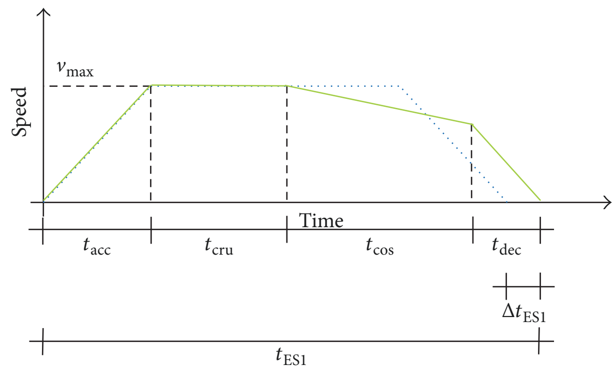

FIGURE 2: Speed profile in the case of *Energy-Saving* (ES) strategy 1.

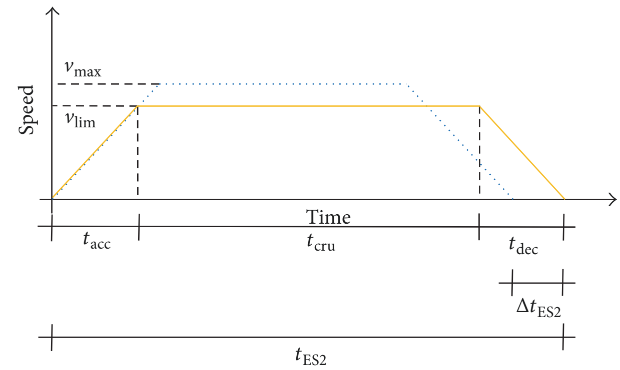

FIGURE 3: Speed profile in the case of *Energy-Saving* (ES) strategy 2.

(ii) The second where a lower maximum speed is adopted.

Figure 2 provides a generic speed profile in the case of the first ES strategy with the same assumption of jerk values adopted in the case of strategy TO. In this case, the total travel time between two successive stops (i.e., stations or red signals) may be formulated as

$$t_{ES1} = t_{acc} + t_{cru} + t_{cos} + t_{dec} \tag{3}$$

or equivalently

$$t_{ES1} = t_{TO} + \Delta t_{ES1}, \tag{4}$$

where t_{ES1} is the travel time in the case of the first ES strategy; t_{cos} is the time duration of the coasting phase; and Δt_{ES1} is the increase in travel time in the case of the first ES strategy with respect to strategy TO.

The increase in travel time due to implementing the first ES strategy may be calculated by imposing the constancy of the section length (i.e., the travel distance is independent of the implemented strategy); that is,

$$\Delta s = \int_0^{t_{TO}} v_{TO}(\tau) \cdot d\tau = \int_0^{t_{ES1}} v_{ES1}(\tau) \cdot d\tau, \tag{5}$$

where Δs is the track length between the two successive stops analysed, $v_{TO}(\cdot)$ is the speed profile in the case of the TO strategy as shown in Figure 1, and $v_{ES1}(\cdot)$ is the speed profile in the case of the first ES strategy as shown in Figure 2. Hence, by replacing (4) in (5), we obtain

$$\int_{t_{TO}}^{t_{TO}+\Delta t_{ES1}} v_{ES1}(\tau) \cdot d\tau = \int_0^{t_{TO}} \left(v_{TO}(\tau) - v_{ES1}(\tau)\right) \cdot d\tau, \tag{6}$$

where the increase in travel time, that is, term Δt_{ES1}, represents the unknown variable to be determined.

Likewise, Figure 3 provides a generic speed profile in the case of the second ES strategy with the same assumption of jerk values adopted in the case of the TO strategy. In this scenario, the total travel time between two successive stops may be formulated as

$$t_{ES2} = t_{acc} + t_{cru} + t_{dec} \tag{7}$$

or equivalently

$$t_{ES2} = t_{TO} + \Delta t_{ES2}, \tag{8}$$

where t_{ES2} is the travel time in the case of the second ES strategy and Δt_{ES2} is the increase in travel time in the case of the second ES strategy with respect to the TO strategy.

Also in the case of the second ES strategy, the increase in travel time may be calculated by imposing the constancy of the section length; that is,

$$\Delta s = \int_0^{t_{TO}} v_{TO}(\tau) \cdot d\tau = \int_0^{t_{ES2}} v_{ES2}(\tau) \cdot d\tau, \tag{9}$$

where $v_{ES2}(\cdot)$ is the speed profile in the case of the second ES strategy, as shown in Figure 3.

In this case, by replacing (8) in (9), we obtain

$$\int_{t_{TO}}^{t_{TO}+\Delta t_{ES2}} v_{ES2}(\tau) \cdot d\tau = \int_0^{t_{TO}} \left(v_{TO}(\tau) - v_{ES2}(\tau)\right) \cdot d\tau, \tag{10}$$

where the increase in travel time, that is, term Δt_{ES2}, represents the unknown variable to be determined.

Adoption of the first or second approach depends on the technological level of the rail system. Indeed, the first approach requires the need to communicate to the train the precise point at which the coasting phase begins. By contrast, the second approach requires simply communicating a different speed limit and hence is more straightforward to implement. However, most of the contributions in the literature are based on considering the implementation of ES strategies between two successive stops in order to preserve arrivals and departures at each station.

Since a metro system may be considered a frequency service [41, 42] where it is necessary to observe the average headway between two successive convoys rather than a timetable generally unknown to users, we propose to apply the ES strategies by considering as mainstay the arrival and departure times only at the terminus and not at the intermediate stations. Hence, we suggest implementing any strategy by considering the entire outward and return trip. Obviously, implementation of any optimisation approach for ensuring lower energy consumption requires the knowledge and estimation of reserve times according to the formulation proposed in the paper.

3. The Analytical Approach

The aim of this paper is to determine the value of reserve times in a metro system in order to quantify expendable time resources for reducing energy consumption. The use of an analytical approach arises from the need to have a system of equations which allows a priori exclusion of all unfeasible operative service configurations, as shown below. However, since both ES strategies described in the previous section are based on the extension of travel time, the following analytical formulation can be applied in both cases. Moreover, in the case of more complex layouts, such as in the case of rail systems, the proposed approach may be easily integrated with suitable train timetabling optimisation techniques.

Generally, we may define the minimum cycle time, indicated as $\mathrm{CT}_{\mathrm{min}}$, as the minimum time required by a convoy to complete the outward trip and the following return trip and achieve the initial condition. It can be calculated as follows:

$$\mathrm{CT}_{\mathrm{min}} = \sum_{\mathrm{lot}} \mathrm{tt}_{\mathrm{lot}} + \sum_{\mathrm{sot}} \mathrm{dt}_{\mathrm{sot}} + \mathrm{it}_{\mathrm{ot}} + \sum_{\mathrm{lrt}} \mathrm{tt}_{\mathrm{lrt}} + \sum_{\mathrm{srt}} \mathrm{dt}_{\mathrm{srt}} + \mathrm{it}_{\mathrm{rt}}, \tag{11}$$

where $\mathrm{tt}_{\mathrm{lot}}$ and $\mathrm{tt}_{\mathrm{lrt}}$ are the travel times associated, respectively, with links lot and lrt; lot and lrt are the generic links (i.e., track sections) associated, respectively, with the outward trip (ot) and return trip (rt); $\mathrm{dt}_{\mathrm{sot}}$ and $\mathrm{dt}_{\mathrm{srt}}$ are the dwell times associated, respectively, with platforms sot and srt; sot and srt are the generic platforms of station s for, respectively, the outward trip (ot) and return trip (rt); and $\mathrm{it}_{\mathrm{ot}}$ and $\mathrm{it}_{\mathrm{rt}}$ are the inversion times (i.e., preparation times for the subsequent trip) associated, respectively, with the outward trip (ot) and return trip (rt).

However, in the ordinary operations to avoid delay propagation, the adopted cycle time, indicated as $\mathrm{CT}_{\mathrm{ord}}$, may be obtained by increasing the minimum value as follows:

$$\mathrm{CT}_{\mathrm{ord}} = \mathrm{CT}_{\mathrm{min}} + \mathrm{bt}_{\mathrm{ot}} + \mathrm{bt}_{\mathrm{rt}}, \tag{12}$$

where $\mathrm{bt}_{\mathrm{ot}}$ and $\mathrm{bt}_{\mathrm{rt}}$ are the buffer times used for recovery of delays, respectively, in the case of the outward trip (ot) and return trip (rt) due to variation in travel, dwell, and inversion times.

Finally, in order to satisfy timetable requirements, the planned cycle time, indicated as $\mathrm{CT}_{\mathrm{plan}}$, may be calculated as follows:

$$\mathrm{CT}_{\mathrm{plan}} = \mathrm{CT}_{\mathrm{ord}} + \mathrm{lt}_{\mathrm{ot}} + \mathrm{lt}_{\mathrm{rt}}, \tag{13}$$

where $\mathrm{lt}_{\mathrm{ot}}$ and $\mathrm{lt}_{\mathrm{rt}}$ are the layover times (i.e., times spent by a train at the terminus waiting for the subsequent trip) associated, respectively, with the outward trip (ot) and return trip (rt).

In this context, in order to maintain a constant headway between two successive convoys, it is necessary to impose the following constraints:

$$0 \le \mathrm{bt}_{\mathrm{ot}} + \mathrm{lt}_{\mathrm{ot}} \le H \tag{14}$$

$$0 \le \mathrm{bt}_{\mathrm{rt}} + \mathrm{lt}_{\mathrm{rt}} \le H, \tag{15}$$

where H is the headway between two successive rail convoys.

The number of convoys, indicated as NC, required to perform the service may be calculated as

$$\mathrm{NC} = \frac{\mathrm{CT}_{\mathrm{plan}}}{H}. \tag{16}$$

By substituting (13) into (16), we obtain

$$\mathrm{lt}_{\mathrm{ot}} + \mathrm{lt}_{\mathrm{rt}} = \mathrm{NC} \cdot H - \mathrm{CT}_{\mathrm{ord}}. \tag{17}$$

Likewise, since the layover times must be non-negative, combining (14) and (15) with the non-negativity condition produces

$$0 \le \mathrm{lt}_{\mathrm{ot}} + \mathrm{lt}_{\mathrm{rt}} \le 2 \cdot H - \left(\mathrm{bt}_{\mathrm{ot}} + \mathrm{bt}_{\mathrm{rt}}\right). \tag{18}$$

Finally, by substituting (17) into (18), we obtain

$$\frac{\mathrm{CT}_{\mathrm{ord}}}{H} \le \mathrm{NC} \le 2 + \frac{\mathrm{CT}_{\mathrm{ord}}}{H} - \frac{\mathrm{bt}_{\mathrm{ot}} + \mathrm{bt}_{\mathrm{rt}}}{H}. \tag{19}$$

Therefore, since NC must be an integer value, we obtain the following constraints:

$$\begin{aligned} \mathrm{NC}_{\mathrm{min}} &= \mathrm{int}\left(\frac{\mathrm{CT}_{\mathrm{ord}}}{H}\right) + 1 \\ \mathrm{NC}_{\mathrm{max}} &= \mathrm{int}\left(2 + \frac{\left(\mathrm{CT}_{\mathrm{ord}} - \left(\mathrm{bt}_{\mathrm{ot}} + \mathrm{bt}_{\mathrm{rt}}\right)\right)}{H}\right), \end{aligned} \tag{20}$$

where $\mathrm{NC}_{\mathrm{min}}$ and $\mathrm{NC}_{\mathrm{max}}$ are, respectively, the minimum and the maximum number of rail convoys required to perform the service.

A number of convoys lower than $\mathrm{NC}_{\mathrm{min}}$ are unable to ensure a headway of H due to lack of rolling stock. A number of convoys higher than $\mathrm{NC}_{\mathrm{max}}$ provide an interference in terms of movements among convoys which prevents them from maintaining their cycle time.

According to the definition of reserve time provided by UIC [40], in our approach it is the sum of two quantities, the buffer time and the layover time; that is,

$$\begin{aligned} \mathrm{rt}_{\mathrm{ot}} &= \mathrm{bt}_{\mathrm{ot}} + \mathrm{lt}_{\mathrm{ot}} \\ \mathrm{rt}_{\mathrm{rt}} &= \mathrm{bt}_{\mathrm{rt}} + \mathrm{lt}_{\mathrm{rt}}, \end{aligned} \tag{21}$$

where $\mathrm{rt}_{\mathrm{ot}}$ and $\mathrm{rt}_{\mathrm{rt}}$ are the reserve times for, respectively, the outward and return trips.

Since the layover times (i.e., terms $\mathrm{lt}_{\mathrm{ot}}$ and $\mathrm{lt}_{\mathrm{rt}}$) represent quantities that a train loses to maintain the headway, our proposal consists in adopting these times as storage tanks for incrementing travel times without affecting service frequencies (expressed in terms of H) so as to implement suitable ES strategies. Hence, we may define the total usable reserve time, that is, the term turt, which represents the sum of the two layover times; that is,

$$\mathrm{turt} = \mathrm{lt}_{\mathrm{ot}} + \mathrm{lt}_{\mathrm{rt}} \tag{22}$$

since in our approach we assume that buffer times (i.e., $\mathrm{bt}_{\mathrm{ot}}$ and $\mathrm{bt}_{\mathrm{rt}}$) used for recovery of delays are kept intact and are not used to implement ES strategies.

In this context, (17) may be expressed as

$$\text{turt} = \text{NC} \cdot H - \text{CT}_{\text{ord}}. \quad (23)$$

Theoretically, the turt value could be split arbitrarily between the outward and the return trip. Hence, we may define a new parameter, indicated as α, which expresses the partition rate as follows:

$$\text{lt}_{\text{ot}} = \alpha \cdot \text{turt} \quad (24)$$

$$\text{lt}_{\text{rt}} = (1 - \alpha) \cdot \text{turt} \quad (25)$$

with $\alpha \in [0; 1]$.

By substituting (24) into (14), we obtain

$$0 \leq \alpha \leq \frac{(H - \text{bt}_{\text{ot}})}{\text{turt}}. \quad (26)$$

Likewise, by substituting (25) into (15), we obtain

$$1 - \frac{(H - \text{bt}_{\text{rt}})}{\text{turt}} \leq \alpha \leq 1. \quad (27)$$

Since parameter α has to satisfy jointly (26) and (27), and it is not possible to state a priori whether $(H - \text{bt}_{\text{ot}})/\text{turt}$ is greater or lower than $1 - (H - \text{bt}_{\text{rt}})/\text{turt}$, we need to identify two feasible conditions which are mutually exclusive (i.e., disjoint):

(i) Condition 1: $1 - (H - \text{bt}_{\text{rt}})/\text{turt} \leq (H - \text{bt}_{\text{ot}})/\text{turt}$

(ii) Condition 2: $(H - \text{bt}_{\text{ot}})/\text{turt} < 1 - (H - \text{bt}_{\text{rt}})/\text{turt}$.

In the first case, it is possible to identify the feasibility set of parameter α as follows:

$$\max\left\{0; 1 - \frac{H - \text{bt}_{\text{rt}}}{\text{turt}}\right\} \leq \alpha \leq \min\left\{\frac{H - \text{bt}_{\text{ot}}}{\text{turt}}; 1\right\}, \quad (28)$$

while, in the second case, the feasibility set of parameter α is an empty set since (26) and (27) identify disjointed sets. However, by reductio ad absurdum, we may state that the second condition never occurs. Indeed, condition 2 may be formulated as

$$\text{turt} > 2 \cdot H - (\text{bt}_{\text{ot}} + \text{bt}_{\text{rt}}). \quad (29)$$

By substituting (23) into (29), we obtain

$$\text{NC} \cdot H - \text{CT}_{\text{ord}} > 2 \cdot H - (\text{bt}_{\text{ot}} + \text{bt}_{\text{rt}}). \quad (30)$$

Moreover, by substituting (12) into (30), we obtain

$$\text{NC} > 2 + \left(\frac{\text{CT}_{\min}}{H}\right). \quad (31)$$

Since, by substituting (12) and (13) into (16), we obtain

$$\text{NC} = \frac{(\text{CT}_{\min} + (\text{bt}_{\text{ot}} + \text{bt}_{\text{rt}}) + (\text{lt}_{\text{ot}} + \text{lt}_{\text{rt}}))}{H}, \quad (32)$$

it is possible to substitute (32) into (31), thus obtaining

$$\frac{(\text{bt}_{\text{ot}} + \text{bt}_{\text{rt}}) + (\text{lt}_{\text{ot}} + \text{lt}_{\text{rt}})}{H} > 2. \quad (33)$$

Combining (14) and (15) results in the following:

$$0 \leq \frac{(\text{bt}_{\text{ot}} + \text{bt}_{\text{rt}}) + (\text{lt}_{\text{ot}} + \text{lt}_{\text{rt}})}{H} \leq 2 \quad (34)$$

which expresses a contradiction between condition 2 (which provides (33)) and the constant headway constraint (which provides (34)). Therefore, this contradiction implies that condition 2 never occurs.

In conclusion, although parameter α belongs to set $[0; 1]$, its value cannot be fixed arbitrarily but has to satisfy condition (28). Hence, we define a minimum, $\alpha_{\min}$, and a maximum, $\alpha_{\max}$, value of parameter α, that is,

$$\begin{aligned} \alpha_{\min} &= \max\left\{0; 1 - \frac{H - \text{bt}_{\text{rt}}}{\text{turt}}\right\} \\ \alpha_{\max} &= \min\left\{\frac{H - \text{bt}_{\text{ot}}}{\text{turt}}; 1\right\}. \end{aligned} \quad (35)$$

Importantly, having fixed the infrastructure and the signalling system, the minimum headway between two successive convoys depends on inversion times and the main features of the signalling system, that is,

$$H_{\min} = \max\left\{\text{ts}_{\text{ot}}^{\text{inv}}; \text{ts}_{\text{rt}}^{\text{inv}}; \text{ts}_{\text{ot}}^{\text{ls}}; \text{ts}_{\text{rt}}^{\text{ls}}; \text{ts}_{\text{min-ss}}\right\}, \quad (36)$$

where $H_{\min}$ is the minimum value of H; $\text{ts}_{\text{ot}}^{\text{inv}}$ is the time spacing due to the inversion of the rail convoy at the final terminus of the outward trip; $\text{ts}_{\text{rt}}^{\text{inv}}$ is the time spacing due to the inversion of the rail convoy at the final terminus of the return trip; $\text{ts}_{\text{ot}}^{\text{ls}}$ is the time spacing of the rail convoy at the last section of the outward trip (to access the first station of the return trip); $\text{ts}_{\text{rt}}^{\text{ls}}$ is the time spacing of the rail convoy at the last section of the return trip (to access the first station of the subsequent trip which is an outward trip); and $\text{ts}_{\text{min-ss}}$ is the minimum time spacing allowed by the signalling system along the line which has to take into account dwell times at stations and circulation rules (such as the criterion of station releasing).

Term $\text{ts}_{\text{ot}}^{\text{inv}}$ (or equivalently $\text{ts}_{\text{rt}}^{\text{inv}}$) and term $\text{ts}_{\text{ot}}^{\text{ls}}$ (or equivalently $\text{ts}_{\text{rt}}^{\text{ls}}$) are strictly related to the framework of the infrastructure, signalling system, and the service organisation. Indeed, these values depend on the place where trains stop for the buffer and layover times. For instance, if these times are spent on the inversion track, terms $\text{ts}_{\text{ot}}^{\text{inv}}$ and $\text{ts}_{\text{rt}}^{\text{inv}}$ have to include travel times along inversion tracks, stop times for waiting for technical inversion operations (such as the transfer of the driver from the previous head to the new head which was the previous tail), the buffer time for recovery delays, the layover time for achieving the planned headway, and, finally, the times related to the signalling system functioning (such as the release time for unlocking the block system). Alternatively, if buffer and layover times are spent at the first station of the subsequent trip, these times have to be included in the definition of terms $\text{ts}_{\text{ot}}^{\text{ls}}$ and $\text{ts}_{\text{rt}}^{\text{ls}}$, together with the dwell time planned for that station.

Therefore, in both cases, the allocation of layover times between the outward and return trips (i.e., the value of parameter α) affects the definition of the minimum headway (i.e., $H_{\min}$) by means of (36).

The feasibility of a planned headway H may be verified by means of the following condition:

$$H \geq H_{\min}. \quad (37)$$

Since the value of H and the adopted number of convoys NC provide univocally the term turt according to (23) and the allocation of turt between the outward and return trips (i.e., the value of parameter α) affects the definition of the minimum headway (i.e., $H_{\min}$) by means of (36), formally (36) may be expressed as follows:

$$H_{\min} = H_{\min}\left(\text{turt}\left(H, \text{NC}\right), \alpha\right) \quad (38)$$

subject to

$$\text{NC} \in \left[\text{NC}_{\min}\left(H\right); \text{NC}_{\max}\left(H\right)\right] \quad (39)$$

$$\alpha \in \left[\alpha_{\min}\left(H\right); \alpha_{\max}\left(H\right)\right], \quad (40)$$

where (39) concisely expresses constraints (20) and (40) concisely expresses (35).

In this context, in order to verify the feasibility of a generic headway H to be planned, for any value of NC which satisfies condition (39), we may identify an optimal value of α, indicated as α_{opt}, which minimises $H_{\min}$, that is,

$$\alpha_{\text{opt}} = \arg\min_{\alpha} H_{\min}\left(\alpha\right) \quad (41)$$

with

$$\alpha \in \left[\alpha_{\min}; \alpha_{\max}\right] \quad (42)$$

and we can verify whether condition (37) is satisfied.

However, if trains spend buffer and layover times on the inversion track, variation in parameter α

(i) does not affect the value of terms $\text{ts}_{\text{ot}}^{\text{ls}}$, $\text{ts}_{\text{rt}}^{\text{ls}}$, or $\text{ts}_{\text{min-ss}}$;

(ii) affects term $\text{ts}_{\text{ot}}^{\text{inv}}$, according to a linear strictly increasing function;

(iii) affects term $\text{ts}_{\text{rt}}^{\text{inv}}$, according to a linear strictly decreasing function.

Alternatively, if trains spend buffer and layover times at the first station of the subsequent trip, variation in parameter α

(i) does not affect the value of terms $\text{ts}_{\text{ot}}^{\text{inv}}$, $\text{ts}_{\text{rt}}^{\text{inv}}$, or $\text{ts}_{\text{min-ss}}$;

(ii) affects term $\text{ts}_{\text{ot}}^{\text{ls}}$, according to a linear strictly increasing function;

(iii) affects term $\text{ts}_{\text{rt}}^{\text{ls}}$, according to a linear strictly decreasing function.

In both cases, function $H_{\min}(\alpha)$ is convex, that is,

$$\begin{aligned} &H_{\min}\left(\lambda \cdot \alpha_1 + (1-\lambda) \cdot \alpha_2\right) \\ &\quad \leq \lambda \cdot H_{\min}\left(\alpha_1\right) + (1-\lambda) \cdot H_{\min}\left(\alpha_2\right) \\ &\qquad \forall \alpha_1, \alpha_2 \in \left[\alpha_{\min}; \alpha_{\max}\right] \;\; \forall \lambda \in [0; 1]. \end{aligned} \quad (43)$$

Moreover, if

$$\begin{aligned} &\max\left\{\text{ts}_{\text{ot}}^{\text{inv}}; \text{ts}_{\text{rt}}^{\text{inv}}\right\} > \max\left\{\text{ts}_{\text{ot}}^{\text{ls}}; \text{ts}_{\text{rt}}^{\text{ls}}; \text{ts}_{\text{min-ss}}\right\} \\ &\qquad \forall \alpha \in \left[\alpha_{\min}; \alpha_{\max}\right] \end{aligned} \quad (44)$$

in the case of buffer and layover times spent at the inversion track, or

$$\begin{aligned} &\max\left\{\text{ts}_{\text{ot}}^{\text{ls}}; \text{ts}_{\text{rt}}^{\text{ls}}\right\} > \max\left\{\text{ts}_{\text{ot}}^{\text{inv}}; \text{ts}_{\text{rt}}^{\text{inv}}; \text{ts}_{\text{min-ss}}\right\} \\ &\qquad \forall \alpha \in \left[\alpha_{\min}; \alpha_{\max}\right] \end{aligned} \quad (45)$$

in the case of buffer and layover times spent at the first station of the subsequent trip, function $H_{\min}(\alpha)$ is strictly convex, that is,

$$\begin{aligned} &H_{\min}\left(\lambda \cdot \alpha_1 + (1-\lambda) \cdot \alpha_2\right) \\ &\quad < \lambda \cdot H_{\min}\left(\alpha_1\right) + (1-\lambda) \cdot H_{\min}\left(\alpha_2\right) \\ &\qquad \forall \alpha_1, \alpha_2 \in \left[\alpha_{\min}; \alpha_{\max}\right] \;\; \forall \lambda \in [0; 1]. \end{aligned} \quad (46)$$

The convexity (or the strict convexity) of function $H_{\min}(\alpha)$ allows us to solve the constrained optimisation problem (41) subject to (42) by adopting traditional solution algorithms for convex objective function problems.

However, the proposed approach, being considered a decision support system, has to be applied in planning phases, that is, by considering ordinary conditions. Obviously, an extension to the disruption conditions may be easily obtained by affecting operating values with the corresponding disrupted values.

4. Application of the Proposed Methodology

In order to validate the proposed formulation, we applied it to the case of a real metro line: Line 1 of the Naples metro system in southern Italy. This line, which is about 18 km long, connects the suburbs with the city centre, and its outward and return routes are strongly asymmetric in terms of elevations (Figure 4), which also entails asymmetry in terms of energy consumption (as shown in Table 1).

The first step of the application was to calibrate the commercial software OpenTrack® [43] in order to provide a mathematical model which allows simulation of all phases of the metro service (i.e., travel, dwell, and inversion times) and also exploration of different operational configurations without the need to apply them physically (i.e., on the real line). In particular, all infrastructure, rolling stock, and signalling system data were appropriately adopted for tuning the model (details on the calibration techniques may be found in [33, 44, 45]).

Travel, dwell, and inversion times were obtained by implementing a deterministic simulation of the metro service (see Table 1).

Likewise, by means of 200 stochastic simulations (for details on the use of stochastic simulations see, e.g., [46]), we were able to determine the statistical distribution of all service parameters so as to provide buffer times and related

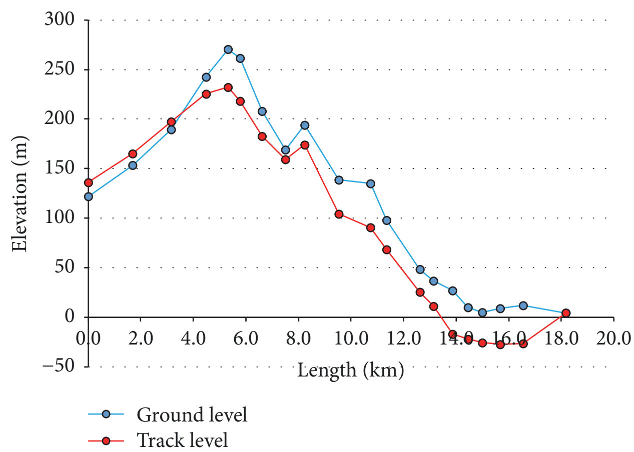

Figure 4: Elevation profile of Line 1 (from Piscinola to Garibaldi).

Table 1: Operational parameters of Line 1.

Parameter	Value	
	Piscinola-Garibaldi direction	Garibaldi-Piscinola direction
Travel distance	18.791 km	18.616 km
Total travel time	1,463 s (24.4 min)	1,485 s (24.8 min)
Total dwell time	400 s (6.7 min)	400 s (6.7 min)
Inversion time	307 s (5.1 min)	268 s (4.5 min)
Buffer time [90th percentile]	116 s (1.9 min)	103 s (1.7 min)
Buffer time [95th percentile]	131 s (2.2 min)	116 s (1.9 min)
Buffer time [99th percentile]	159 s (2.7 min)	141 s (2.4 min)
Adopted cycle time CT_{ord} [90th percentile]	4,542 s (75.7 min)	
Adopted cycle time CT_{ord} [95th percentile]	4,570 s (76.2 min)	
Adopted cycle time CT_{ord} [99th percentile]	4,623 s (77.1 min)	
Minimum time spacing along the line $ts_{min\text{-}ss}$	110 s (1.83 min)	
Energy consumption	279.01 kWh	386.21 kWh

cycle times as functions of an assumed confidence level. Indeed, differences in performance between the stochastic and deterministic travel times may be calculated as follows:

$$\delta_{ot}^{i} = \left(\sum_{lot} tt_{lot}^{i,STOC} + \sum_{sot} dt_{sot}^{i,STOC} + it_{ot}^{i,STOC} \right) - \left(\sum_{lot} tt_{lot}^{DET} + \sum_{sot} dt_{sot}^{DET} + it_{ot}^{DET} \right)$$

$$\delta_{rt}^{i} = \left(\sum_{lrt} tt_{lrt}^{i,STOC} + \sum_{srt} dt_{srt}^{i,STOC} + it_{rt}^{i,STOC} \right) - \left(\sum_{lot} tt_{lrt}^{DET} + \sum_{srt} dt_{srt}^{DET} + it_{rt}^{DET} \right), \tag{47}$$

where δ_{ot}^{i} and δ_{rt}^{i} represent, respectively, the difference in the case of outward and return trips at the ith stochastic simulation; $X^{i,STOC}$ represents the value of variable X (where X represents the travel time, dwell time, or inversion time) in the case of the ith stochastic simulation; and X^{DET} represents the value of variable X in the case of a deterministic simulation.

Since stochastic simulations are based on reductions in train performance, we may assume that

$$X^{i,STOC} \geq X^{DET} \quad \forall i \; \forall X \tag{48}$$

which implies that

$$\delta_{ot}^{i} \geq 0 \quad \forall i$$
$$\delta_{rt}^{i} \geq 0 \quad \forall i. \tag{49}$$

Hence, if we assume that δ_{ot}^{i} and δ_{rt}^{i} are distributed according to a normal (i.e., Gaussian) distribution, we can calibrate function parameters (i.e., mean and variance) so as to reproduce the observed data by solving the following minimisation problems:

$$\left[\widehat{\mu}_{ot}, \widehat{\sigma}_{ot}^{2}\right] = \arg\min_{\mu_{ot}, \sigma_{ot}^{2}} Z_{ot}\left(\delta_{ot}^{i}, \mu_{ot}, \sigma_{ot}^{2}\right)$$
$$\left[\widehat{\mu}_{rt}, \widehat{\sigma}_{rt}^{2}\right] = \arg\min_{\mu_{rt}, \sigma_{rt}^{2}} Z_{rt}\left(\delta_{rt}^{i}, \mu_{rt}, \sigma_{rt}^{2}\right) \tag{50}$$

with

$$\sigma_{ot}^{2} \geq 0,$$
$$\sigma_{rt}^{2} \geq 0, \tag{51}$$

where μ_{ot} and μ_{rt} are the means of the normal distributions in the case of the outward trip (ot) and return trip (rt); $\widehat{\mu}_{ot}$ and $\widehat{\mu}_{rt}$ are optimal values of μ_{ot} and μ_{rt}; σ_{ot}^{2} and σ_{rt}^{2} are the variances of the normal distributions in the case of the outward trip (ot) and return trip (rt); $\widehat{\sigma}_{ot}^{2}$ and $\widehat{\sigma}_{rt}^{2}$ are the optimal values of σ_{ot}^{2} and σ_{rt}^{2}; Z_{ot} is an objective function which expresses the distance between the cumulative distribution of observed values δ_{ot}^{i} and the cumulative distribution of the normal function of parameters μ_{ot} and σ_{ot}^{2}; Z_{rt} is an objective function which expresses the distance between the cumulative distribution of observed values δ_{rt}^{i} and the cumulative distribution of the normal function of parameters μ_{rt} and σ_{rt}^{2}.

The results of the calibration phases (i.e., solution of minimisation problems (50)) are shown in Table 2, and comparisons between the cumulative distribution of observed values

Table 2: Normal distribution parameters.

μ_{ot}	σ^2_{ot}	μ_{rt}	σ^2_{rt}
64.114	40.803	56.922	36.247

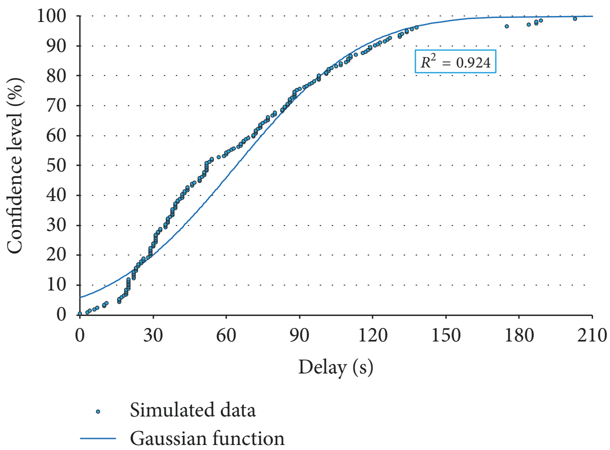

Figure 5: Comparison between cumulative distributions in the case of the outward trip.

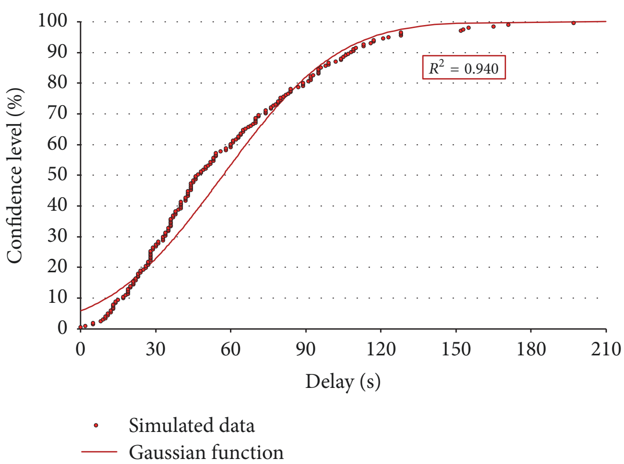

Figure 6: Comparison between cumulative distributions in the case of the return trip.

and that of corresponding normal functions are proposed in Figures 5 and 6.

These values were adopted to determine buffer times in the case of confidence levels equal to 90th, 95th, and 99th percentiles (values are shown in Table 1). Obviously, the adopted cycle times, that is, CT_{ord}, were calculated by assuming three different confidence levels associated with buffer time calculation, as shown in Table 1. In particular, the adoption of buffer times calculated by means of the proposed statistical approach allows any feasible fluctuation in operating service parameters to be handled.

The aim of the second test consisted in analysing and testing different operative schemes, and verifying their feasibility analytically. Indeed, since the planned headway H affects the value of the minimum headway of the line (see (38)), it is necessary to verify whether condition (37) is satisfied.

First, having fixed an operative scheme based on trains spending buffer and layover times at the first station of the subsequent trip, the lower bound of H_{min} (i.e., the minimum headway) may be identified as the maximum among constant terms, that is, ts^{inv}_{ot}, ts^{inv}_{rt}, and $ts_{min\text{-}ss}$. This preliminary analysis provided a lower bound value of 5.1 min due to the inversion time of the outward trip (i.e., at Garibaldi station), as shown in Table 1.

Hence, for any planned headway H greater than 5.1 minutes, it is possible to fix a corresponding number of convoys NC satisfying constraints (20) and then calculate turt in a closed form formulation by means of (23). Since the adoption of different split rates for layover times affects the value of H_{min} (as shown by (38)), we solved the optimisation problem (41) subject to (42) for identifying the minimum value of H_{min} to be compared with the planned headway H so as to verify the feasibility test (37).

The results of the proposed procedure are summarised in Tables 3–5, where the italic values indicate the unfeasible operating schemes (i.e., combinations of planned headway H and adopted number of convoys NC) which do not satisfy condition (37). However, tests were performed by considering different confidence levels in the buffer time estimation.

The results show that the increase in total buffer time (i.e., the sum of bt_{ot} and bt_{rt}) was 0.47 min from the 90th to 95th confidence level, 0.88 min from the 95th to 99th confidence level, and 1.34 min from the 90th to 99th confidence level.

Moreover, numerical results show that, for any operating scheme, if the sum of layover times was higher than buffer time increases, the increase in buffer time was offset by the reduction in layover time such that their sum (i.e., total reserve time according to UIC [40] or equivalently the sum of (21)) and the minimum headway was kept constant. Obviously, if the turt value was lower than the increase in buffer times, their sum could not be kept constant and the configuration would be unfeasible (see, e.g., headway values of 5.5 and 7.0 in the case of the 99th confidence level).

Since, in real cases, service frequencies may be different during a day (e.g., the headway during the peak hours may be lower than that during off-peak hours), the feasibility combination of planned headway H and adopted number of convoys NC has to be verified for any frequency configuration.

Finally, in order to show the utility of the proposed approach, we calculated reductions in energy consumption in the case of some operative schemes.

First of all, we assumed that the ES strategy was implemented by imposing a reduction in maximum speed during the outward and return trips. Indeed, this approach, which is described as ES strategy 2 in Section 2 (see Figure 3), may be easily implemented by affecting signalling system features (e.g., from the operative centre) without the awareness, knowledge, and/or preparation of drivers or changes in the ground-train communication system (i.e., information related to the beginning of the coasting phase or the new start-breaking point).

Table 3: Feasible configuration calculation in the case of the 90th percentile.

H [min]	NC_{min}	NC_{max}	NC	turt [min]	α_{min}	α_{max}	α_{opt}	H_{min} [min]	Test
5.5	**14**	**15**	**14**	**1.30**	**0.0%**	**100.0%**	**41.0%**	**5.12**	**OK**
5.5	*14*	*15*	*15*	*6.80*	*44.4%*	*52.5%*	*48.3%*	*6.87*	*NO*
6.0	**13**	**14**	**13**	**2.30**	**0.0%**	**100.0%**	**44.9%**	**5.12**	**OK**
6.0	*13*	*14*	*14*	*8.30*	*48.4%*	*49.0%*	*48.6%*	*7.62*	*NO*
7.0	**11**	**12**	**11**	**1.30**	**0.0%**	**100.0%**	**41.0%**	**5.12**	**OK**
7.0	*11*	*12*	*12*	*8.30*	*36.3%*	*61.0%*	*48.6%*	*7.62*	*NO*
8.0	**10**	**11**	**10**	**4.30**	**0.0%**	**100.0%**	**47.3%**	**5.62**	**OK**
8.0	*10*	*11*	*11*	*12.30*	*48.9%*	*49.3%*	*49.1%*	*9.62*	*NO*
9.0	**9**	**10**	**9**	**5.30**	**0.0%**	**100.0%**	**47.8%**	**6.12**	**OK**
9.0	*9*	*10*	*10*	*14.30*	*49.1%*	*49.4%*	*49.2%*	*10.62*	*NO*
10.0	**8**	**9**	**8**	**4.30**	**0.0%**	**100.0%**	**47.3%**	**5.62**	**OK**
10.0	*8*	*9*	*9*	*14.30*	*42.1%*	*56.4%*	*49.2%*	*10.62*	*NO*
12.0	**7**	**8**	**7**	**8.30**	**0.0%**	**100.0%**	**48.6%**	**7.62**	**OK**
12.0	*7*	*8*	*8*	*20.30*	*49.3%*	*49.6%*	*49.4%*	*13.62*	*NO*
14.0	**6**	**7**	**6**	**8.30**	**0.0%**	**100.0%**	**48.6%**	**7.62**	**OK**
14.0	*6*	*7*	*7*	*22.30*	*44.9%*	*54.1%*	*49.5%*	*14.62*	*NO*
15.0	**6**	**6**	**6**	**14.30**	**7.1%**	**91.4%**	**49.2%**	**10.62**	**OK**

Table 4: Feasible configuration calculation in the case of the 95th percentile.

H [min]	NC_{min}	NC_{max}	NC	turt [min]	α_{min}	α_{max}	α_{opt}	H_{min} [min]	Test
5.5	**14**	**15**	**14**	**0.83**	**0.0%**	**100.0%**	**34.0%**	**5.12**	**OK**
5.5	*14*	*15*	*15*	*6.33*	*43.7%*	*52.4%*	*47.9%*	*6.87*	*NO*
6.0	**13**	**14**	**13**	**1.83**	**0.0%**	**100.0%**	**42.7%**	**5.12**	**OK**
6.0	*13*	*14*	*14*	*7.83*	*48.1%*	*48.7%*	*48.3%*	*7.62*	*NO*
7.0	**11**	**12**	**11**	**0.83**	**0.0%**	**100.0%**	**34.0%**	**5.12**	**OK**
7.0	*11*	*12*	*12*	*7.83*	*35.3%*	*61.5%*	*48.3%*	*7.62*	*NO*
8.0	**10**	**11**	**10**	**3.83**	**0.0%**	**100.0%**	**46.5%**	**5.62**	**OK**
8.0	*10*	*11*	*11*	*11.83*	*48.7%*	*49.2%*	*48.9%*	*9.62*	*NO*
9.0	**9**	**10**	**9**	**4.83**	**0.0%**	**100.0%**	**47.2%**	**6.12**	**OK**
9.0	*9*	*10*	*10*	*13.83*	*48.9%*	*49.3%*	*49.0%*	*10.62*	*NO*
10.0	**8**	**9**	**8**	**3.83**	**0.0%**	**100.0%**	**46.5%**	**5.62**	**OK**
10.0	*8*	*9*	*9*	*13.83*	*41.7%*	*56.5%*	*49.0%*	*10.62*	*NO*
12.0	**7**	**8**	**7**	**7.83**	**0.0%**	**100.0%**	**48.3%**	**7.62**	**OK**
12.0	*7*	*8*	*8*	*19.83*	*49.2%*	*49.5%*	*49.3%*	*13.62*	*NO*
14.0	**6**	**7**	**6**	**7.83**	**0.0%**	**100.0%**	**48.3%**	**7.62**	**OK**
14.0	*6*	*7*	*7*	*21.83*	*44.7%*	*54.1%*	*49.4%*	*14.62*	*NO*
15.0	**6**	**6**	**6**	**13.83**	**5.5%**	**92.7%**	**49.0%**	**10.62**	**OK**

Our proposal was based on two different speed limits between the outward and the return trips, indicated, respectively, as v_{ot}^{LIM} and v_{rt}^{LIM}. Hence, for any feasible operative scheme (i.e., bold values in Tables 3–5) and for any assumed confidence levels (i.e., 90th, 95th, and 99th), we calculated the maximum reductions in energy consumption in the case of three values of parameter α (i.e., split rates):

(i) α_{min}, which represents the minimum layover time for the outward trip or, equivalently, the maximum layover time for the return trip (see (35))

(ii) α_{opt}, which represents the layover times providing the minimum headway (see (41))

(iii) α_{max}, which represents the maximum layover time for the outward trip or, equivalently, the minimum layover time for the return trip (see (35)).

Obviously, in the case of $\alpha_{min} = 0$, no ES strategy can be defined for the outward trip (i.e., it is not possible to define term v_{ot}^{LIM}). Likewise, in the case of $\alpha_{max} = 100\%$, it is not possible to perform an ES strategy for the return trip (i.e., it is not possible to define term v_{rt}^{LIM}).

Table 5: Feasible configuration calculation in the case of the 99th percentile.

H [min]	NC_{min}	NC_{max}	NC	turt [min]	α_{min}	α_{max}	α_{opt}	H_{min} [min]	Test
5.5	*15*	*15*	*15*	*5.45*	*42.2%*	*52.3%*	*47.1%*	*6.87*	*NO*
5.5	*15*	*15*	*16*	*10.95*	*71.2%*	*26.0%*	*48.6%*	*9.62*	*NO*
6.0	**13**	**14**	**13**	**0.95**	**0.0%**	**100.0%**	**33.3%**	**5.12**	**OK**
6.0	*13*	*14*	*14*	*6.95*	*47.5%*	*48.2%*	*47.7%*	*7.62*	*NO*
7.0	*12*	*12*	*12*	*6.95*	*33.1%*	*62.6%*	*47.7%*	*7.62*	*NO*
7.0	*12*	*12*	*13*	*13.95*	*66.7%*	*31.2%*	*48.9%*	*11.12*	*NO*
8.0	**10**	**11**	**10**	**2.95**	**0.0%**	**100.0%**	**44.6%**	**5.62**	**OK**
8.0	*10*	*11*	*11*	*10.95*	*48.4%*	*48.9%*	*48.6%*	*9.62*	*NO*
9.0	**9**	**10**	**9**	**3.95**	**0.0%**	**100.0%**	**46.0%**	**6.12**	**OK**
9.0	*9*	*10*	*10*	*12.95*	*48.6%*	*49.0%*	*48.8%*	*10.62*	*NO*
10.0	**8**	**9**	**8**	**2.95**	**0.0%**	**100.0%**	**44.6%**	**5.62**	**OK**
10.0	*8*	*9*	*9*	*12.95*	*40.9%*	*56.8%*	*48.8%*	*10.62*	*NO*
12.0	**7**	**8**	**7**	**6.95**	**0.0%**	**100.0%**	**47.7%**	**7.62**	**OK**
12.0	*7*	*8*	*8*	*18.95*	*49.1%*	*49.3%*	*49.2%*	*13.62*	*NO*
14.0	**6**	**7**	**6**	**6.95**	**0.0%**	**100.0%**	**47.7%**	**7.62**	**OK**
14.0	*6*	*7*	*7*	*20.95*	*44.4%*	*54.2%*	*49.2%*	*14.62*	*NO*
15.0	**6**	**6**	**6**	**12.95**	**2.3%**	**95.4%**	**48.8%**	**10.62**	**OK**

Since, for any α value, it is possible to determine layover times for the outward and the return trips, the speed limits were fixed by solving the following optimisation problem in the case of the outward trip:

$$v_{\mathrm{ot}}^{\mathrm{LIM}} = \arg\min_{v_{\mathrm{ot}}^{*}} \Delta t_{\mathrm{ot}} \left(v_{\mathrm{ot}}^{*}\right) \tag{52}$$

subject to

$$\begin{gathered} \int_{0}^{t_{\mathrm{ot}}^{\mathrm{TO}}} v_{\mathrm{ot}}^{\mathrm{TO}} (\tau) \cdot d\tau = \int_{0}^{t_{\mathrm{ot}}^{\mathrm{TO}}+\Delta t_{\mathrm{ot}}} v_{\mathrm{ot}}^{\mathrm{ES}} (\tau) \cdot d\tau \\ \Delta t_{\mathrm{ot}} \leq \alpha \cdot \mathrm{turt} \\ v_{\mathrm{ot}}^{\mathrm{ES}} \leq v_{\mathrm{ot}}^{\mathrm{LIM}} \end{gathered} \tag{53}$$

or equivalently in the case of the return trip

$$v_{\mathrm{rt}}^{\mathrm{LIM}} = \arg\min_{v_{\mathrm{rt}}^{*}} \Delta t_{\mathrm{ot}} \left(v_{\mathrm{rt}}^{*}\right) \tag{54}$$

subject to

$$\begin{gathered} \int_{0}^{t_{\mathrm{rt}}^{\mathrm{TO}}} v_{\mathrm{rt}}^{\mathrm{TO}} (\tau) \cdot d\tau = \int_{0}^{t_{\mathrm{rt}}^{\mathrm{TO}}+\Delta t_{\mathrm{rt}}} v_{\mathrm{rt}}^{\mathrm{ES}} (\tau) \cdot d\tau \\ \Delta t_{\mathrm{rt}} \leq (1-\alpha) \cdot \mathrm{turt} \\ v_{\mathrm{rt}}^{\mathrm{ES}} \leq v_{\mathrm{rt}}^{\mathrm{LIM}}, \end{gathered} \tag{55}$$

where v_{ot}^{*} and v_{rt}^{*} are the generic speed limit in the case of the outward trip (ot) and return trip (rt); Δt_{ot} and Δt_{rt} are the increase in total travel time (i.e., the sum of travel times and dwell times) due to the implementation of the ES strategy in the case of outward trip (ot) and return trip (rt); $t_{\mathrm{ot}}^{\mathrm{TO}}$ and $t_{\mathrm{rt}}^{\mathrm{TO}}$ are the total travel time (i.e., the sum of travel times and dwell times) in the *Time Optimal* (TO) condition in the case of outward trip (ot) and return trip (rt); $v_{\mathrm{ot}}^{\mathrm{TO}}$ and $v_{\mathrm{rt}}^{\mathrm{TO}}$ are the travel speeds in the *Time Optimal* (TO) condition in the case of outward trip (ot) and return trip (rt); and $v_{\mathrm{ot}}^{\mathrm{ES}}$ and $v_{\mathrm{rt}}^{\mathrm{ES}}$ are the travel speeds in the case of ES strategy implementation in the case of outward trip (ot) and return trip (rt).

Once the new speed limits have been fixed, it is possible to calculate the new energy consumption by (1) and hence the energy consumption reductions as follows:

$$\begin{gathered} \Delta E_{\mathrm{ot}} = E_{\mathrm{ot}}^{\mathrm{ES}} - E_{\mathrm{ot}}^{\mathrm{TO}}, \\ \Delta E_{\mathrm{rt}} = E_{\mathrm{rt}}^{\mathrm{ES}} - E_{\mathrm{rt}}^{\mathrm{TO}}. \end{gathered} \tag{56}$$

Obviously, in the case of an undetermined speed limit (i.e., $\alpha_{\mathrm{min}} = 0$ in the case of an outward trip and $\alpha_{\mathrm{max}} = 100\%$ in the case of a return trip), reductions in energy consumption are null.

The reduction in energy consumption for each cycle may be calculated as follows:

$$\Delta E_{\mathrm{TOT}} = \Delta E_{\mathrm{ot}} + \Delta E_{\mathrm{rt}}. \tag{57}$$

Finally, in order to show the order of magnitude of the amount of energy saved, we calculated the daily reduction in energy consumption by adopting the following assumptions:

(i) There are 17 hours (i.e., from 6.00 to 23.00) of metro service in a day.

(ii) The service frequency is constant throughout the day.

(iii) There is lack of interference among operating trains and trains moving from or to the depot.

(iv) The transition phase in the morning (TP_m) to reach the planned headway H, since initially all convoys are at the depot, and the transition phase in the

Table 6: Energy variations in the case of the 90th percentile.

H [min]	NC	α	$v_{\mathrm{ot}}^{\mathrm{LIM}}$	$v_{\mathrm{rt}}^{\mathrm{LIM}}$	ΔE_{ot} [kWh]	ΔE_{rt} [kWh]	ΔE_{TOT} [kWh]	$\Delta E_{\mathrm{Daily}}$ [kWh]	Reduction in energy consumption
5.5	14	0.0%	–	57	–	32.7	32.7	5,487.9	4.91%
5.5	14	41.0%	65	61	20.5	20.3	40.8	6,852.5	6.13%
5.5	14	100.0%	58	–	39.8	–	39.8	6,688.6	5.98%
6.0	13	0.0%	–	51	–	44.2	44.2	6,888.6	6.64%
6.0	13	44.9%	61	57	34.0	32.7	66.7	10,402.0	10.02%
6.0	13	100.0%	53	–	56.1	–	56.1	8,751.5	8.43%
7.0	11	0.0%	–	57	–	32.7	32.7	4,311.9	4.91%
7.0	11	41.0%	65	61	20.5	20.3	40.8	5,384.1	6.13%
7.0	11	100.0%	58	–	39.8	–	39.8	5,255.4	5.98%
8.0	10	0.0%	–	44	–	44.1	44.1	5,290.4	6.63%
8.0	10	47.3%	54	52	50.2	38.9	89.1	10,687.9	13.39%
8.0	10	100.0%	45	–	67.5	–	67.5	8,104.5	10.15%
9.0	9	0.0%	–	42	–	57.8	57.8	6,246.2	8.69%
9.0	9	47.8%	51	49	59.3	49.1	108.4	11,709.0	16.30%
9.0	9	100.0%	65	–	20.5	–	20.5	2,215.4	3.08%
10.0	8	0.0%	–	44	–	44.1	44.1	4,232.3	6.63%
10.0	8	47.3%	54	52	50.2	38.9	89.1	8,550.3	13.39%
10.0	8	100.0%	45	–	67.5	–	67.5	6,483.6	10.15%
12.0	7	0.0%	–	36		68.0	68.0	5,233.1	10.22%
12.0	7	48.6%	45	44	67.5	44.1	111.6	8,595.1	16.78%
12.0	7	100.0%	37	–	95.2	–	95.2	7,328.9	14.31%
14.0	6	0.0%	–	36	–	68.0	68.0	4,485.5	10.22%
14.0	6	48.6%	45	44	67.5	44.1	111.6	7,367.2	16.78%
14.0	6	100.0%	37	–	95.2	–	95.2	6,281.9	14.31%
15.0	6	7.1%	61	30	34.0	90.1	124.1	7,447.5	18.66%
15.0	6	49.2%	39	38	80.3	70.7	150.9	9,056.9	22.69%
15.0	6	91.4%	31	57	108.0	32.7	140.7	8,439.3	21.14%

evening (TP_e) to lead all convoys to the depot may be calculated as follows:

$$\mathrm{TP}_m = \mathrm{TP}_e = (\mathrm{NC} - 1) \cdot H. \tag{58}$$

It is worth noting that since in the considered line the depot is located next to Piscinola station (the suburb terminus) and is connected to the line by means of two different tracks (one for each direction), the introduction, variation, and exit of convoys do not provide any interference to the service.

Obviously, the daily energy saving could be easily calculated also by removing some of the above assumptions (such as the variation in service frequency during the day).

Tables 6–8 provide the energy variation for any feasible operating scheme and for the three confidence levels adopted in the computation of buffer times. The results showed that the greater the confidence level, the lower the energy saving since there is a reduction in total usable reserve time.

Moreover, since an increase in headway (frequency reduction) provides, on the one hand, an increase in energy saving for each convoy, but, on the other, a reduction in the number of convoys required for performing the service, it is not possible to provide a monotonic trend of the energy-saving function.

5. Conclusions and Research Prospects

In this paper, we proposed an analytical methodology for determining all operating parameters (including reserve times) in the case of metro systems. In particular, calculating the amount of reserve time is a fundamental step for implementing suitable *Energy-Saving* (ES) strategies. Indeed, as shown in the literature, reduction in energy consumption may be achieved by reducing performance of convoys (e.g., by reducing the maximum travel speed). Obviously, in order to preserve service quality, it is necessary to compensate the reduction in performance by consuming dead times (i.e., times spent by a train in a stop condition) such as buffer and layover times.

Generally, buffer times are adopted for recovery of delays, layover times for waiting for the subsequent trip. Our proposal consists in consuming layover times for energy-saving

Table 7: Energy variations in the case of the 95th percentile.

H [min]	NC	α	v_{ot}^{LIM}	v_{rt}^{LIM}	ΔE_{ot} [kWh]	ΔE_{rt} [kWh]	ΔE_{TOT} [kWh]	ΔE_{Daily} [kWh]	Reduction in energy consumption
5.5	14	0.0%	–	61	–	20.3	20.3	3,406.3	3.05%
5.5	14	34.0%	69	63	13.5	16.4	30.0	5,036.9	4.51%
5.5	14	100.0%	62	–	32.9	–	32.9	5,523.1	4.94%
6.0	13	0.0%	–	54	–	34.4	34.4	5,374.1	5.18%
6.0	13	42.7%	62	59	32.9	24.6	57.4	8,958.8	8.63%
6.0	13	100.0%	55	–	49.8	–	49.8	7,771.6	7.49%
7.0	11	0.0%	–	61	–	20.3	20.3	2,676.4	3.05%
7.0	11	34.0%	69	63	13.5	16.4	30.0	3,957.5	4.51%
7.0	11	100.0%	62	–	32.9	–	32.9	4,339.6	4.94%
8.0	10	0.0%	–	45	–	48.1	48.1	5,768.5	7.23%
8.0	10	46.5%	56	53	46.4	42.0	88.4	10,602.6	13.28%
8.0	10	100.0%	46	–	68.4		68.4	8,204.8	10.28%
9.0	9	0.0%	–	43	–	56.4	56.4	6,086.2	8.47%
9.0	9	47.2%	53	50	56.1	40.7	96.8	10,457.5	14.56%
9.0	9	100.0%	43	–	79.1	–	79.1	8,546.4	11.90%
10.0	8	0.0%	–	45	–	48.1	48.1	4,614.8	7.23%
10.0	8	46.5%	56	53	46.4	42.0	88.4	8,482.0	13.28%
10.0	8	100.0%	46	–.	68.4	–	68.4	6,563.8	10.28%
12.0	7	0.0%	–	37	–	67.6	67.6	5,202.5	10.16%
12.0	7	48.3%	47	45	69.2	48.1	117.2	9,026.7	17.62%
12.0	7	100.0%	38	–	96.4	–	96.4	7,421.4	14.49%
14.0	6	0.0%	–	37	–	67.6	67.6	4,459.3	10.16%
14.0	6	48.3%	47	45	69.2	48.1	117.2	7,737.2	17.62%
14.0	6	100.0%	38		96.4	–	96.4	6,361.2	14.49%
15.0	6	5.5%	62	30	32.9	90.1	123.0	7,379.2	18.49%
15.0	6	49.0%	39	38	80.3	70.7	150.9	9,056.9	22.69%
15.0	6	92.7%	31	59	108.0	24.6	132.5	7,952.5	19.92%

purposes, keeping buffer times intact in order to preserve the flexibility and robustness of the timetable in case of delays.

In order to verify the utility and feasibility of the proposed analytical approach, we applied it in the case of an Italian metro line in order to

(i) calculate buffer times as a function of the adopted confidence levels;

(ii) verify the feasibility of the operating scheme having fixed the planned headway H and the adopted number of convoys NC;

(iii) calculate the amount of energy consumption reduction for three different allocations of the total usable reserve time between the outward and return trips.

The first numerical applications showed that the higher the planned headway H, the lower the number of convoys required. Moreover, the higher the planned headway H, the higher the total usable reserve time (turt). Obviously, since the allocation of turt between the outward and return trips directly affects the minimum headway between two successive trains, the feasibility of any operating scheme may be verified by means of the proposed approach.

Moreover, having fixed a feasible operating scheme, an increase in confidence level provides an increase in buffer time. Hence, if the sum of layover times (i.e., turt) is higher than buffer time increases, the increase in buffer time is offset by the reduction in layover time such that their sum and the minimum headway are kept constant. Obviously, if the turt value were lower than the increase in buffer times, their sum could not be kept constant and the configuration would be unfeasible.

Finally, numerical tests for quantifying the reduction in energy consumption showed that an increase in headway provides an increase in the term turt and hence an increase in energy saving for the single convoy. Higher values of energy reduction were identified in the case of split rates (i.e., parameter α) providing the minimum headway where reductions are up to 22.69%. Hence, for future research, we propose to improve these performances further by optimising the allocation of reserve times between the outward and return trips in order to minimise energy consumption. Moreover,

Table 8: Energy variations in the case of the 99th percentile.

H [min]	NC	α	v_{ot}^{LIM}	v_{rt}^{LIM}	ΔE_{ot} [kWh]	ΔE_{rt} [kWh]	ΔE_{TOT} [kWh]	ΔE_{Daily} [kWh]	Reduction in energy consumption
6.0	13	0.0%	–	59	–	24.6	24.6	3,830.2	3.69%
6.0	13	33.3%	68	62	16.5	20.7	37.2	5,802.0	5.59%
6.0	13	100.0%	61	–	34.0	–	34.0	5,306.1	5.11%
8.0	10	0.0%	–	49	–	49.1	49.1	5,894.9	7.38%
8.0	10	44.6%	58	55	39.8	36.3	76.1	9,133.8	11.44%
8.0	10	100.0%	50	–	60.7	–	60.7	7,285.0	9.13%
9.0	9	0.0%	–	45	–	48.1	48.1	5,191.6	7.23%
9.0	9	46.0%	55	52	49.8	38.9	88.7	9,581.7	13.34%
9.0	9	100.0%	45	–	67.5	–	67.5	7,294.0	10.15%
10.0	8	0.0%	–	49	–	49.1	49.1	4,715.9	7.38%
10.0	8	47.7%	58	55	39.8	36.3	76.1	7,307.0	11.44%
10.0	8	100.0%	50	–	60.7	–	60.7	5,828.0	9.13%
12.0	7	0.0%	–	38	–	70.7	70.7	5,443.3	10.63%
12.0	7	47.7%	48	46	65.5	49.9	115.4	8,887.8	17.35%
12.0	7	100.0%	39	–	80.3	–	80.3	6,179.7	12.06%
14.0	6	0.0%	–	38	–	70.7	70.7	4,665.7	10.63%
14.0	6	47.7%	48	46	65.5	49.9	115.4	7,618.1	17.35%
14.0	6	100.0%	39	–	80.3	–	80.3	5,296.9	12.06%
15.0	6	2.3%	69	31	13.5	78.1	91.6	5,497.0	13.77%
15.0	6	48.8%	40	39	81.2	54.6	135.8	8,146.4	20.41%
15.0	6	95.4%	32	63	108.5	16.4	125.0	7,499.8	18.79%

stochastic simulations may be adopted to verify benefits in more realistic cases. Finally, we propose a reduction in the confidence level adopted in the buffer time calculation in order to increase the amount of available reserve time.

Obviously, in the case of rail systems (or metro systems with more complex layouts), it is necessary to integrate the proposed approach with timetable optimisation techniques.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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