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ScienceDirect

Procedia Engineering 199 (2017) 1050–1055

Procedia Engineering

www.elsevier.com/locate/procedia

X International Conference on Structural Dynamics, EURODYN 2017

Quasi-isospectral Sturm-Liouville operators and applications to system identification

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Abstract

Quasi-isospectral Sturm-Liouville operators play an important role in inverse spectral theory and are typically used for determining exact solutions to suitable classes of eigenvalue problems with variable coefficients. In this work we investigate on alternative applications of quasi-isospectral operators as key tool for structural identification purposes. We review some recent results concerned with the construction of rods with a given finite number of natural frequencies and we present some generalization to beams under bending vibration and to the identification of damages from natural frequency data.

Keywords: Vibrating beams; eigenvalues; Darboux's Lemma; structural identification; inverse problems.

1. Introduction

Second order Sturm-Liouville operators play an important role in Structural Dynamics. As an example, the small longitudinal vibration of straight elastic beams and the small transverse vibration of taut strings are governed by Sturm-Liouville operators. Two Sturm-Liouville operators are said to be quasi-isospectral when, under specific set of end conditions, they share all the eigenvalues with the exception of a single eigenvalue, which usually is free to move in a specified open interval. An important theoretical feature is the possibility to explicitly construct families of quasi-isospectral Sturm-Liouville operators. The main mathematical tool is a classical lemma by Darboux [1]. It can be shown that this explicit construction plays an important role in the solution of inverse problems in vibration for Sturm-Liouville operators, as it was pointed out, among others, by Pöschel and Trubowitz [2].

In this paper we explore possible applications of quasi-isospectral operators for structural identification purposes. We review some results on the explicit construction of axially vibrating rods having prescribed values of the first N natural frequencies, and we present new applications to structural damage identification from resonant frequency data. Other recent investigations concern with the determination of beams with prescribed values of the buckling loads [3] and the reconstruction of blockages in acoustic ducts by eigenfrequency data [4]. The potential of the method for

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1877-7058 © 2017 The Authors. Published by Elsevier Ltd.
Peer-review under responsibility of the organizing committee of EURODYN 2017.
10.1016/j.proeng.2017.09.288

structural and damage identification purposes has been tested on an extended series of simulations, and its stability to errors has been checked both for noisy and experimental data. The interested reader is refereed to the references for more details on the applications.

2. The Darboux Lemma

The main mathematical tool of our analysis is the following result, known as *Darboux's Lemma* [1]. Let us denote $(\cdot)' \equiv \frac{d(\cdot)}{dx}$. Let μ be a real number, and suppose $g = g(x)$ is a non-trivial solution of the Sturm-Liouville equation $-g'' + \widehat{q}g = \mu g$ with potential $\widehat{q} = \widehat{q}(x)$. If f is a non-trivial solution of $-f'' + \widehat{q}f = \lambda f$ and $\lambda \neq \mu$, then $y = g^{-1}[g, f] \equiv g^{-1}(gf' - g'f)$ is a non-trivial solution of the equation $-y'' + \check{q}y = \lambda y$ with $\check{q} = \widehat{q} - 2(\ln(g(x)))''$. Moreover, the general solution of the equation $-y'' + \check{q}y = \mu y$ is $y = g^{-1}\left(a + b\int_0^x g^2(s)ds\right)$, where a and b are arbitrary constants. In particular, $y = g^{-1}$ is a solution of $-y'' + \check{q}y = \mu y$.

This general result allows to associate to every equation of the form $-g'' + \widehat{q}g = \mu g$, that one knows how to integrate for all values of μ, another equation of the same form that one also knows how to integrate for all the values of the parameter μ. In particular, the addition law of the logarithm makes iteration of the procedure simple. We refer to [2] and [5] for a formal justification in a more abstract context based on a *commutation formula* for Sturm-Liouville operators. It should be noted that if g vanishes, then the equation $-y'' + \check{q}y = \lambda y$ is understood to hold between the roots of g. One can show that these singular situations disappear by applying the Darboux Lemma twice, see also the general method presented in [6].

3. Construction of rods with a given finite set of eigenvalues

In this section we shall show how to explicitly construct a rod which has prescribed values of the first N natural frequencies.

3.1. Main ideas of the method

In order to present the main ideas of the construction procedure, we shall refer to rods under supported end conditions. The analysis of this case is simpler than that for other sets of end conditions and allows for a clear presentation of the key aspects of the method. The free (undamped, infinitesimal) longitudinal vibrations $u(x)$ of frequency ω, of a thin straight rod of unit length are governed by the Sturm-Liouville equation

$$\begin{cases} (\widehat{A}(x)u'(x))' + \lambda\widehat{A}(x)u(x) = 0, \qquad x \in (0,1), & (1) \\ u(0) = 0 = u(1), & (2) \end{cases}$$

where $\lambda = \sigma\omega^2E^{-1}$. Here E is Young's modulus, σ is the volume mass density, both assumed constant; $\widehat{A} = \widehat{A}(x)$ is the cross-sectional function. We shall assume throughout that $\widehat{A}(x)$ is a strictly positive, twice continuously differentiable function of x in $[0, 1]$. It is well-known that there exists an infinite sequence $\{\lambda_m\}_{m=1}^{\infty}$ of eigenvalues, with $0 < \lambda_1 < \lambda_2 < ..., \lim_{m\to\infty} \lambda_m = \infty$, for which the eigenvalue problem (1)–(2) has a non-trivial solution $u(x)$.

Let $n \geq 1$ be given. The key step of the method is the explicit construction of a new rod quasi-isospectral to the given rod, that is a rod with cross-sectional profile $A(x)$ having the same Dirichlet eigenvalues as the given rod $\widehat{A}(x)$, with the exception of the nth eigenvalue. In fact, by keeping fixed all the eigenvalues λ_m with $m \neq n$ and moving the nth eigenvalue λ_n to the desired value, say $\widetilde{\lambda}_n$, and using repeatedly the procedure, after N steps we will produce a rod with the first N given eigenvalues $\{\widetilde{\lambda}_m\}_{m=1}^{N}$, and the construction is finished. We will see in the next section that the reconstruction procedure works only if the rod to be determined is *not far* from the initial rod. Moreover, even if the initial rod is fixed, it is evident that the construction is not unique, since the flow from the initial rod depends also on the particular order chosen to move every individual eigenvalue to the target value.

The main steps of our construction of rods $A(x)$ quasi-isospectral to a given rod $\widehat{A}(x)$, under Dirichlet end conditions, are the following. First, equation (1) is reduced to canonical form with Schrödinger potential $\widehat{q}(x)$ by a standard Sturm-Liouville transformation. Second, the Darboux Lemma is used to construct explicit families of Schrödinger potentials $q(x)$ quasi-isospectral to the initial potential $\widehat{q}(x)$. Third, the Darboux Lemma is applied once more in iterate form to determine rods $A(x)$ corresponding to the quasi-isospectral potentials $q(x)$ and, ultimately, to find rods $A(x)$ quasi-isospectral to the initial rod $\widehat{A}(x)$.

3.2. Quasi-isospectral rods under Dirichlet end conditions

Let us denote

$$\widehat{A}(x) = \widehat{a}^2(x), \quad y(x) = \widehat{a}(x)u(x), \tag{3}$$

where $\widehat{a} = \widehat{a}(x)$ can be chosen of one-sign in $[0, 1]$, say positive. Then, the eigenvalue problem (1)–(2) can be transformed to the Sturm-Liouville canonical form

$$\begin{cases} y''(x) + \lambda y(x) = \widehat{q}(x)y(x), & x \in (0, 1), \quad (4) \\ y(0) = 0 = y(1), & \quad (5) \end{cases}$$

where the potential $\widehat{q}(x) = \widehat{a}''(x)\widehat{a}(x)^{-1}$ is a continuous function in $[0, 1]$. Let us denote by $\{z_m\}_{m=1}^{\infty}$ the eigenfunctions of (4)–(5), normalized so that $z'_m(0) = 1$, $m = 1, 2, ...$ Let n, $n \geq 1$, be a given integer and let $t \in \mathbb{R}$ be such that

$$\lambda_{n-1}(\widehat{q}) < \lambda_n(\widehat{q}) + t < \lambda_{n+1}(\widehat{q}), \tag{6}$$

with $\lambda_0(\widehat{q}) = 0$. Following [2], denote by $y_\alpha = y_\alpha(x, \widehat{q}, \lambda_n + t)$, $\alpha = 1, 2$, the solution to the initial value problem

$$\begin{cases} y''_\alpha + (\lambda_n + t)y_\alpha = \widehat{q}y_\alpha, & x \in (0, 1), \quad (7) \\ y_\alpha(0) = \delta_{1\alpha}, \quad y'_\alpha(0) = \delta_{2\alpha} = 0, & \quad (8) \end{cases}$$

where $\delta_{\alpha\beta} = 1$ if $\alpha = \beta$ and $\delta_{\alpha\beta} = 0$ otherwise, $\alpha, \beta = 1, 2$. Moreover, we introduce the function

$$w_{n,t}(x, \widehat{q}, \lambda_n + t) = y_1(x, \widehat{q}, \lambda_n + t) + \frac{y_1(1, \lambda_n) - y_1(1, \lambda_n + t)}{y_2(1, \lambda_n + t)} y_2(x, \widehat{q}, \lambda_n + t) \tag{9}$$

solution to

$$\begin{cases} w''_{n,t} + (\lambda_n + t)w_{n,t} = \widehat{q}w_{n,t}, & x \in (0, 1), \quad (10) \\ w_{n,t}(0) = 1, \quad w_{n,t}(1) = y_1(1, \widehat{q}, \lambda_n), & \quad (11) \end{cases}$$

for $t \neq 0$. Finally, let $\omega_{n,t}(x, \widehat{q}, \lambda_n + t) = [w_{n,t}, z_n]$. The function $\omega_{n,t}$ is a continuous and strictly positive function in $[0, 1]$ for every $\widehat{q} \in C([0, 1])$ and for every t satisfying (6), $n \geq 1$. Moreover, $\omega_{n,t}$ is a C^2-function of the variable x in $[0, 1]$, see [2] (p. 109). Under the above notation, and by adapting the arguments shown in [7], it can be shown that for a given $n \geq 1$ the supported rod with profile $A(x) = a^2(x)$, where

$$a = \widehat{a} - t\frac{w_{n,t}}{\lambda_n \omega_{n,t}}[z_n, \widehat{a}], \quad \lambda_{n-1} < \lambda_n + t < \lambda_{n+1}, \tag{12}$$

has exactly the same eigenvalues $\{\lambda_m\}$ of the supported rod $\widehat{A}(x) = \widehat{a}^2(x)$, with the exception of the nth, which is fixed to the value $\lambda_n + t$, e.g., $\lambda_m(\widehat{A}) = \lambda_m(A)$ for every $m \geq 1$ with $m \neq n$, and $\lambda_n(A) = \lambda_n(\widehat{A}) + t$. The supported rod $A = A(x)$ is the wished rod quasi-isospectral to the initial supported rod $\widehat{A} = \widehat{A}(x)$.

For the sake of completeness, we sketch the main steps of the construction of the profiles a's shown in (12). In a first step, the analysis is based on a double application of the Darboux Lemma to obtain potentials $q(x)$ quasi-isospectral to the initial potential $\widehat{q}(x)$ in the eigenvalue problem (3)–(4), see also [2]. Next, in the second step, the Darboux Lemma is applied once more in an iterated form to determine the expression (12) of a solution to $q = a''a^{-1}$, that is, to reconstruct the quasi-isospectral rod operator with profile $A(x) = a^2(x)$ [7]. It should be noticed that the function $a = a(x)$ in (12) corresponds to a "physical" rod, since it can be proved that the function a is C^2-regular and strictly positive in $[0, 1]$ for every value of t satisfying the two inequalities in (12).

3.3. The reconstruction procedure

In this section we show how to use the previous results to construct a supported rod which has prescribed values of the first N natural frequencies.

Consider a supported rod with given cross sectional profile $A_0(x) = a_0^2(x)$ and eigenvalues $\{\lambda_m(a_0)\}_{m=1}^{\infty}$, $0 < \lambda_1(a_0) < \lambda_2(a_0) < \cdots$. We now ask whether it is possible to construct from this rod a new rod having prescribed values of the first N eigenvalues $\{\widetilde{\lambda}_m\}_{m=1}^{N}$ under the same set of end conditions, with $0 < \widetilde{\lambda}_1 < \widetilde{\lambda}_2 < \cdots < \widetilde{\lambda}_N$. By the above analysis we know how to construct from the rod $A_0(x)$ a new rod, say $A_1(x) = a_1^2(x)$, so that the Dirichlet eigenvalues $\{\lambda_m(a_0)\}$, $m \geq 2$, are kept fixed while $\lambda_1(a_1)$ is moving to the desired value $\widetilde{\lambda}_1$. More precisely, by (12), the function $a_1 = a_1(x)$ given by $a_1 = a_0 - t\frac{w_{1,t}}{\lambda_1(a_0)\omega_{1,t}}[z_1(a_0), a_0]$ corresponds to a one-parameter family of rods such that $\lambda_m(a_1) = \lambda_m(a_0) + \delta_{m1}t$, $m \geq 1$, for t such that $0 < \lambda_1(a_0) + t < \lambda_2(a_0)$. If $\widetilde{\lambda}_1 < \lambda_2(a_0)$, then we can determine the parameter t, say $t = t_1$, such that $\lambda_1(a_1) = \widetilde{\lambda}_1$, i.e., $t_1 = \widetilde{\lambda}_1 - \lambda_1(a_0)$. The rod $A_1 = a_1^2$ has eigenvalues $\{\widetilde{\lambda}_1, \lambda_2(a_0), \lambda_3(a_0), \cdots\}$, with $0 < \widetilde{\lambda}_1 < \lambda_2(a_0) < \lambda_3(a_0) < \cdots$, and can be used as starting point for the next step of the procedure.

By repeating the same arguments, it is possible to modify a_1 so as to keep $\lambda_m(a_1)$ fixed for $m \neq 2$ and to move $\lambda_2(a_0)$ to the desired value $\widetilde{\lambda}_2$, i.e., $a_2 = a_1 - t_2\frac{w_{2,t_2}}{\lambda_2(a_0)\omega_{2,t_2}}[z_2(a_1), a_1]$, with $t_2 = \widetilde{\lambda}_2 - \lambda_2(a_0)$. The eigenvalues of the new rod $a_2(x)$ are $\{\widetilde{\lambda}_1, \widetilde{\lambda}_2, \lambda_3(a_0), \lambda_4(a_0), \cdots\}$. Using repeatedly this procedure, after N steps we produce a rod, with cross-sectional profile area $A_N(x) = a_N^2(x)$, such that $\lambda_m(a_N) = \widetilde{\lambda}_m$, for $1 \leq m \leq N$, and the construction is finished. We note that the choice of the initial rod $a_0(x)$ is restricted by the conditions $\widetilde{\lambda}_1 < \lambda_2(a_0)$, $\widetilde{\lambda}_2 < \lambda_3(a_0)$, $\cdots$, $\widetilde{\lambda}_{N-1} < \lambda_N(a_0)$, $\widetilde{\lambda}_N < \lambda_{N+1}(a_0)$,, which allow to determine the numbers $t_1, t_2, \cdots, t_N$.

The above construction, which is based on a finite number of eigenfrequencies, is obviously not unique, since the flow from the initial rod a_0 to a rod with prescribed first N Dirichlet eigenvalues depends on the particular order chosen to move every individual eigenvalue to the target value. Similarly, the compatibility conditions on the initial rod a_0 may change depending on the sequence of eigenvalue shifts.

3.4. Extensions to different end conditions

We recall that the end conditions of the rod are assumed to be Elastically Restrained when $A(0)u'(0) - ku(0) = 0 = A(1)u'(1) + Ku(1)$, where $k, K \geq 0$.

The cases of Free and Mixed end conditions are obtained as limit cases when $k = 0 = K$ (e.g., $u'(0) = 0 = u'(1)$) or $k = \infty$, $0 \leq K < \infty$ (e.g., $u(0) = 0$, $A(1)u'(1) + Ku(1) = 0$), respectively. It is possible to prove that the analysis developed in the previous sections can be adapted to deal with rods under Free and Mixed conditions. The main difference between the Dirichlet and Mixed case is that, in the former case, the parameter t appearing in the definition of the cross-sectional profile needs to satisfy the condition (6) only. Conversely, it can be shown that in the Mixed case (for $K > 0$) there is a further restriction on the parameter t, namely t must belong to a *sufficiently* small neighborhood of zero. This last condition is necessary in order to construct one-sign profiles $a(x)$ and, simultaneously, to recover physical end condition at $x = 1$ (e.g., positive stiffness of the elastic spring at the right end). We refer to [7] for more details.

The case of Elastically Restrained end-conditions has some own specific peculiarity. It turns out that it is possible to construct only families of quasi-isospectral rods which preserve the higher spectrum (e.g., omitting the first eigenvalue), and change the first eigenvalue in a prescribed manner. This case is considered in [7] and the analysis is based on the determination of quasi-isospectral potentials under Robin-Robin end conditions presented in [8].

4. Generalizations and applications to Structural Identification

This section is devoted to present some generalization and applications of the results found in Section 3. For brevity, we recall the main results and we refer to the references for technical details and numerical implementation of the method.

4.1. Construction of beams with a finite number of given natural frequencies

It has been remarked that the key mathematical tool used in constructing axially vibrating rods with given natural frequencies is the Darboux Lemma. At the best of our knowledge, it is not known whether an analogue of the Darboux Lemma applies also to the fourth order operator governing the bending vibrations of an Euler-Bernoulli beam. Here we show that a partial result can be obtained working on a special class of pinned-pinned beams.

The free (undamped, infinitesimal) bending vibration with frequency ω of a thin straight elastic beam of unit length is governed by the fourth order eigenvalue problem

$$\begin{cases} (\widehat{I}(x)v''(x))'' - \lambda^2\widehat{\gamma}(x)v(x) = 0, \quad x \in (0,1), & (13)\\ v(0) = v''(0) = 0, \qquad v(1) = v''(1) = 0, & (14)\end{cases}$$

where $\lambda^2 = \frac{\omega^2}{E}$ and $v = v(x)$ is the transverse displacement of the beam axis. The function $\gamma = \gamma(x)$ is the mass density per unit length, and $I = I(x)$ is the second moment of the cross-sectional area. $I = I(x)$ and $\gamma = \gamma(x)$ are assumed to be uniformly positive and smooth in $[0, 1]$. We shall consider the class of beams $\mathcal{B}$ for which the product between the moment of inertia and the mass density is equal to a given constant $C > 0$:

$$\mathcal{B} = \{(I(x), \gamma(x)) \mid I(x)\gamma(x) \equiv C \text{ in } [0,1]\}. \tag{15}$$

In [9] it was proved that, starting from a given pinned-pinned beam $(\widehat{I},\widehat{\gamma}) \in \mathcal{B}$, it is possible to explicitly construct a new pinned-pinned beam $(I, \gamma) \in \mathcal{B}$ having prescribed values of the first N natural frequencies.

As it was explained in Section 3, the key step is the explicit determination of a beam $(I, \gamma) \in \mathcal{B}$ quasi-isospectral to the given beam $(\widehat{I},\widehat{\gamma}) \in \mathcal{B}$. Without going into the details of the proof, we simply sketch the main steps of the procedure. First, we noticed the spectral equivalence between the family of Euler-Bernoulli beams $\mathcal{B}$ and a suitable family of strings, namely, if $(\lambda^2, v(x))$ is an eigenpair of (13)–(14), then $(\lambda, v(x))$ is an eigenpair of a taut supported string having unit length, traction equal to $\sqrt{C}$ and mass density equal to $\gamma(x)$; and vice versa. Next, the equivalent string eigenvalue problem is reduced to Sturm–Liouville canonical form, and the Darboux Lemma is iterated twice to construct explicit families of potentials quasi-isospectral to the initial potential. Then, the Darboux Lemma is applied once more to determine string mass densities corresponding to the quasi-isospectral potentials. Finally, the spectral equivalence between beams and strings is used to find pinned-pinned beams $(I, \gamma) \in \mathcal{B}$ quasi-isospectral to the initial pinned-pinned beam $(\widehat{I},\widehat{\gamma})$.

4.2. Damage identification in rods

The use of quasi-isospectral operators has been recently proposed for damage identification in rods [10]. Let us consider a thin straight rod, having both the ends supported and unit length, in its undamaged state. The free, undamped, infinitesimal longitudinal vibrations are governed by the $(\widehat{p},\widehat{\rho})$-eigenvalue problem

$$\begin{cases} (\widehat{p}u')' + \widehat{\lambda}\widehat{\rho}u = 0, \quad \text{in } (0,1), & (16)\\ u(0) = 0 = u(1), & (17)\end{cases}$$

where $u = u(x)$ is the amplitude and $\sqrt{\widehat{\lambda}}$ is the radian frequency of the vibration. The axial stiffness $\widehat{p} = \widehat{p}(x)$ and the mass density $\widehat{\rho} = \widehat{\rho}(x)$ are uniformly positive, smooth functions in $[0, 1]$. Moreover, we consider rods which are symmetrical with respect to the mid-point $x = 2^{-1}$, namely $\widehat{p}(x) = \widehat{p}(1 - x)$, $\widehat{\rho}(x) = \widehat{\rho}(1 - x)$ in $[0, 1]$. Let us denote by $\widehat{u}_m(x)$ the eigenfunction associated to the mth eigenvalue $\widehat{\lambda}_m$.

Let us introduce the damaged configuration of the rod. Structural damage usually reflects into a variation (reduction) of the axial stiffness of the rod, leaving the mass density unchanged. Therefore, the $(p,\widehat{\rho})$-eigenvalue problem for the damaged rod is assumed of the form

$$\begin{cases} (pu')' + \lambda\widehat{\rho}u = 0, \quad \text{in } (0,1), & (18)\\ u(0) = 0 = u(1), & (19)\end{cases}$$

where the axial stiffness $p = p(x)$ satisfies the same regularity and positivity conditions of $\widehat{p}(x)$. Moreover, we restrict the analysis to symmetrical damaged rods, that is $p(x) = p(1 - x)$ in $[0, 1]$. The eigenvalues λ_m of the problem (18)–(19) are such that $\widehat{\lambda}_{m-1} \le \lambda_m \le \widehat{\lambda}_m$, $m \ge 1$, with $\widehat{\lambda}_0 = 0$.

The diagnostic problem can be formulated as an inverse problem in vibration with finite eigenfrequency data: given the undamaged configuration of the rod $(\widehat{p}(x), \widehat{\rho}(x))$, we wish to determine the axial stiffness coefficient $p = p(x)$ from the knowledge of the first N eigenvalues $\{\lambda_m\}_{m=1}^N$ of the damaged rod. General results on the inverse Sturm–Liouville eigenvalue problem [11] show that the full Dirichlet spectrum $\{\lambda_m\}_{m=1}^\infty$ is needed in order to have uniqueness. Therefore, since N is a finite number, our method will provide an estimate of the unknown coefficient p.

The proposed identification method is of constructive type and is based on three main steps. In a first step, the $(\widehat{p}, \widehat{\rho})$-eigenvalue problem (16)–(17) is transformed into an impedance-type eigenvalue problem (1)–(2) with coefficient $\widehat{A}$ by means of a Liouville transformation. Next, the theory of quasi-isospectral Sturm–Liouville operators in impedance form illustrated in Section 3 can be applied to the eigenvalue problem for $\widehat{A}$ to construct a new impedance coefficient A which has the prescribed values of the first N eigenvalues of the damaged rod. Finally, the Liouville transformation used in the first step can be reversed to come back to a $(p, \widehat{\rho})$-eigenvalue problem of the type (18)–(19) and, therefore, to determine the axial stiffness p of the damaged rod.

The reconstruction procedure was numerically implemented and tested for the identification of single and multiple localized damages, both for rods under supported or free end conditions. The sensitivity of the technique to the number of frequencies used and to the shape, intensity and position of the damages, as well as to the presence of noise in the data, was evaluated and discussed on the basis of an extended series of numerical simulations. The identification method has been also tested on experimental data. A complete description of the identification procedure and the results of numerical/experimental applications are reported in [10].

5. Conclusions

The theory of quasi-isospectral Sturm-Liouville operators is a classical tool for determining exact solutions to suitable classes of eigenvalue problems with variable coefficients. In this paper we have presented some applications to structural identification issues and, particularly, to the determination of axially vibrating rods with specified natural frequencies and to damage identification in rods from natural frequency data.

References

[1] G. Darboux, Sur la répresentation sphérique des surfaces (On the spherical representation of the surfaces), Cr. Acad. Sci. I-Math. 94 (1882) 1343–1345.
[2] J. Pöschel, E. Trubowitz, Inverse Spectral Theory, Academic Press, London, 1987.
[3] A. Morassi, M. Pressacco, A. Vrech, Explicit determination of pinned-pinned beams with a finite number of given buckling loads, Int. J. Struct. Stab. Dy. (to appear) (2017).
[4] A. Bilotta, A. Morassi, E. Turco, Reconstructing blockages in a symmetric duct via quasi-isospectral horn operators, J. Sound Vib. 366 (2016) 149–172.
[5] P.A. Deift, Application of a commutation formula, Duke Math. J. 45 (1978) 267–310.
[6] G. Gesztesy, G. Teschl, On the double commutation method, P. Am. Math. Soc. 124 (1996) 1831–1840.
[7] A. Morassi, Constructing rods with given natural frequencies, Mech. Syst. Signal Pr. 40 (2013) 288–300.
[8] E.L. Isaacson, H.P. Mc Kean, E. Trubowitz, The inverse Sturm-Liouville problem II, Commun. Pur. Appl. Math. XXXVII (1984) 1–11.
[9] A. Morassi, Exact construction of beams with a finite number of given natural frequencies, J. Vib. Control 21 (2015) 591–600.
[10] A. Bilotta, A. Morassi, E. Turco, The use of quasi-isospectral operators for damage detection in rods, Meccanica (submitted) (2017).
[11] H. Hochstadt, The Inverse Sturm-Liouville Problem, Commun. Pur. Appl. Math. XXVI (1973) 715-729.