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Lateral-torsional buckling of compressed and highly variable cross section beams

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Abstract: In the critical state of a beam under central compression a flexural-torsional equilibrium shape becomes possible in addition to the fundamental straight equilibrium shape and the Euler bending. Particularly, torsional configuration takes place in all cases where the line of shear centres does not correspond with the line of centres of mass. This condition is obtained here about a z-axis highly variable section beam; with the assumptions that shear centres are aligned and line of centres is bound to not deform.
For the purpose, let us evaluate an open thin wall C-cross section with flanges width and web height linearly variables along z-axis in order to have shear centres axis approximately aligned with gravity centres axis.
Thus, differential equations that govern the problem are obtained.
Because of the section variability, the numerical integration of differential equations that gives the true critical load is complex and lengthy. For this reason, it is given an energetic formulation of the problem by the theorem of minimum total potential energy (Ritz-Rayleigh method).
It is expected an experimental validation that proposes the model studied.

Keywords: Buckling analysis; theorem of minimum total potential energy, Ritz-Rayleigh method; variable cross section beam; coupled flexural-torsional buckling; shear centre position in variable section beams

1 Introduction

The stability analysis of highly variable section beam subjected to normal compressive stress is a suggestive and crucial topic of engineering applications. In particular, it is very important in structural, mechanical and aeronautical engineering.

The buckling analysis of non uniform columns under its own weight, known as Greenhill's problem, is a subject of considerable scientific studies.

Another typical example of buckling failure by combination of torsion and bending in continuously varying cross section mechanical component is given by drive shaft.

If the ends of the compressed beam were restrained to some degree, but the deflection of z-axis is free and if shear centre does not correspond with centre of mass there is a possibility of torsional deflected form of equilibrium.

In this case it is difficult to find the exact closed-form solutions for the buckling problems.

If cross section varies according to a power of the distance along the beam, it is generally difficult to find the exact analytical solution because the basic differential equation for bending of beam

$$E \cdot I \cdot \eta'^{V} + F \cdot \eta'' = 0 \qquad (1)$$

isn't anymore an homogeneous fourth-order linear constant coefficient ordinary differential equation.

In the most simple case, where the line of shear centres is straight and parallel to the line of centres of mass, the equation becomes an homogeneous fourth-order linear non-constant coefficient ordinary differential equation

$$\eta'^{V} + c \cdot f(z) \cdot \eta'' = 0 \qquad (2)$$

Solving non-constant coefficient differential equations is quite difficult so that it is advisable to use one of the approximate methods: VIM (Variational Iteration Method), EPT (Energy method Total Potential Energy principle based) etc.

Exact buckling solution for thin walled beams with constant open cross section were obtained by many researchers including Franciosi [1, 2], Timoshenko and Gere [3], Vlasov [4], Trahair [5, 6], Li [7].

Coşkun and Atay [8] have analyzed elastic stability for continuously restrained Euler columns, they have obtained an exact solution by means of Variational Iteration Method.

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In recent years the attention of many authors was concentrated on the stability problems of beams with variable cross section. For example, Zhang and Tong [9, 10] investigated the linear stability of doubly symmetric I tapered beams; many other authors have dealt with torsional flexural buckling of members with symmetric cross section: Kováč [11, 12], Benyamina [13], Qiao [14]. Asgarian, Soltani, Mohri [15, 16] investigated the lateral buckling stability of tapered thin-walled beams with arbitrary cross sections and boundary condition by means energetic method; Eisenberger [17] investigated the flexural torsional buckling of variable and open cross section members. Janevski [18] analyzes the transverse vibration of a Timoshenko beam with one-step change in cross-section when subjected to an axial force. Prokić [19] studied the influence of the bimoment induced by external axial loads on the elastic torsional buckling of thin-walled beams with open cross-section.

In this paper, the solution of the problem is obtained by an approximate method. It examines the case of beam hinged at the ends under the hypothesis of tapered thin-walled C-cross section, constant thickness, flanges width and web height both linearly variables along z-axis.

In this way the shear centres axis is approximately aligned with line of gravity centres and the hypothesis underlying the theory adopted are satisfied.

It's interesting verify the possibility to apply the theory of De Saint Venant regarding non-uniform torsion when that such alignment is not rigorously satisfied.

It is crucial to determine the solution of this case study to avoid disastrous torsional effects. In fact in real applications a perfect alignment of the centres axis is not feasible, except through the choice of sections with elaborate geometries and little interest in engineering applications.

2 Problem definition

2.1 Rigorous approach

Examine the case of simply supported beam of thin-walled open cross section with variable thickness along the arc length s, that is the middle line.

The length of the thin-walled beam is larger compared to the cross section dimensions.

The ξ, η, ς is the principal centroidal co-ordinate system of the cross section, it has invariant direction along the beam.

A direct rectangular co-ordinate system is chosen, with origin of axis located in shear centre C. Axis z is parallel to initial longitudinal axis and x, y are parallel to main bending axes. For both reference systems it is assumed that remains straight during deflection.

According to hypotheses of De Saint Venant's theory, it is assumed that the cross-section rotates as a rigid body and there isn't distortion of cross-section shape in x, y plane. This last hypothesis can be ensured through any suitable transverse stiffeners.

Cross section has any shape and any variability along z-axis as long as the shear and rotation centres coincide; the shear centres axis therefore remains straight during torsion.

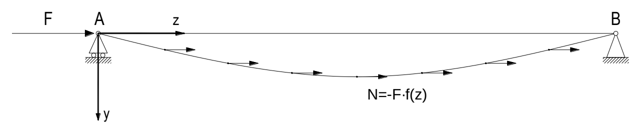

Figure 1: Geometric scheme of the beam.

If longitudinal compressive forces are continuously distributed along a beam and parallel to gravity axis, as shown in Fig. 1, the normal force to the generic abscissa z is

$$N = -F \cdot f(z) \tag{3}$$

If shear and gravity centres not coincide and if they are aligned on two parallel straight lines, equations of static equilibrium can be derived.

The most general equation for non uniform torsion is

$$C_2 \cdot \vartheta'^{V} + C'_2 \cdot \vartheta''' - C_1 \cdot \vartheta'' - C'_1 \cdot \vartheta' = m_t \tag{4}$$

where C_1 is the warping rigidity

$$C_1 = G \cdot \frac{\int \delta^3 ds}{3} \tag{5}$$

C_w is the warping constant

$$C_2 = E \cdot C_w = E \cdot \int_s \omega_s \cdot \delta ds \tag{6}$$

m_t is the torque per unit length applied on the lateral surface of the beam. In this particular case it's valued at zero.

In formulas (5) and (6) G is shear modulus, E is Young's modulus, ω_s is the sectorial area and δ is the section thickness.

The equation (4) is deducted, according to Vlasov torsion theory, under the hypotheses that the shear centre axis is straight and the shear centre is also centre of twist and last the cross-section rotates as a rigid body.

Given the normal share force N(z) the equation (4) becomes

$$C_2 \cdot \vartheta'^{V} + C'_2 \cdot \vartheta''' - C_1 \cdot \vartheta'' - C'_1 \cdot \vartheta' = -\frac{dM_t}{dz} \quad (7)$$

The torque M_t is sum of two contributes

$$M_t = M_{tn} + M_{ts} \quad (8)$$

If T_x and T_y are the shear force to the generic abscissa z evaluated in the undeflected configuration and if bending moment is assumed positive when the top of the beam is compressed and if proceeding from z to z+dz, by considering the equilibrium of an element dz of the beam can be derived

$$\begin{aligned} dM_x &= -N \cdot dv + T_y \cdot dz = F \cdot f \cdot dv + T_y \cdot dz \\ dM_y &= N \cdot du - T_x \cdot dz = -F \cdot f \cdot du - T_x \cdot dz \end{aligned} \quad (9)$$

or

$$\begin{aligned} \frac{dM_x}{dz} &= F \cdot f \cdot \frac{dv}{dz} + T_y \\ \frac{dM_y}{dz} &= -F \cdot f \cdot \frac{du}{dz} - T_x \end{aligned} \quad (10)$$

and then

$$\begin{aligned} \left(E \cdot I_\eta \cdot u''\right)' &= -F \cdot f \cdot u' - T_x \\ \left(E \cdot I_\xi \cdot v''\right)' &= -F \cdot f \cdot v' - T_y \end{aligned} \quad (11)$$

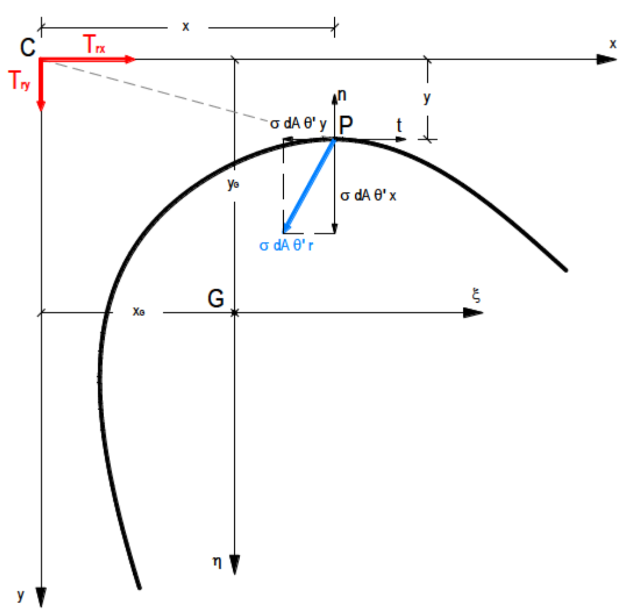

Figure 2: Cross section of the beam.

If the shear centre axis is forced to remains straight during torsion (for example by constraints applied along the beam), the normal force N(z) produces reactions r(z) parallel to the x, y plane. Assuming that the gravity axis is a straight line, the reactions produce in all section of beam the shears

$$\begin{aligned} T_{rx} &= -\sigma \cdot \vartheta' \cdot \int_A y dA = -\sigma \cdot \vartheta' \cdot S_x \\ T_{ry} &= \sigma \cdot \vartheta' \cdot \int_A x dA = \sigma \cdot \vartheta' \cdot S_y \end{aligned} \quad (12)$$

as shown in Fig. 2.

Because shear centre axis remains straight during torsion, it must be applied to it share forces equal and opposite

$$\begin{aligned} T_x &= \sigma \cdot \vartheta' \cdot A \cdot y_G \\ T_y &= -\sigma \cdot \vartheta' \cdot A \cdot x_G \end{aligned} \quad (13)$$

Ultimately, we get that the displacements u and v are governed by equations

$$\begin{aligned} \left(E \cdot I_\eta \cdot u''\right)' - N \cdot u' + N \cdot y_G \cdot \vartheta' &= 0 \\ \left(E \cdot I_\xi \cdot v''\right)' - N \cdot v' - N \cdot x_G \cdot \vartheta' &= 0 \end{aligned} \quad (14)$$

The deflection of components u(z) and v(z) entails that the normal axis to cross section have a deflection to z axis.

This strain state produces torque. Taking the moment about the shear-centre axis of the force F component in u and v direction it's given by the expression

$$M_{ts} = N \cdot u' \cdot y_G - N \cdot v' \cdot x_G \quad (15)$$

Let us assume that we can neglect variability of the cross section in an element of length dz between two cross sections taken normal to the original, undeflected, axis of the beam. During buckling, the cross section will undergo translation and rotation. Translation is defined by the deflections u and v in the x and y directions, respectively, of the shear centre C. The rate of change of the angle of twist of cross section along the axis of the beam is denoted by $\vartheta' = \frac{d\vartheta}{dz}$.

Thus, during rotation of the cross section, point P moves to P', as shown in Fig. 3. The plane P'AP is normal to the straight line PC that is in turn normal to the undeflected line AP.

Taken an element of cross section with area $\delta \cdot ds$, it acts on it the elementary force $\sigma \cdot \delta \cdot ds$. Deflection occurred, the forces $\sigma \cdot \delta \cdot ds$ are again parallel to undeflected z axis but they are applied on A and P' points. According to moment equilibrium equation it produces a torque $\sigma \cdot \delta \cdot ds \cdot \vartheta' \cdot r \cdot dz$, corresponding to bending moment due to the couple formed by two shearing forces $\sigma \cdot \delta \cdot ds \cdot \vartheta' \cdot r$ applied in P' and in A, along P'P.

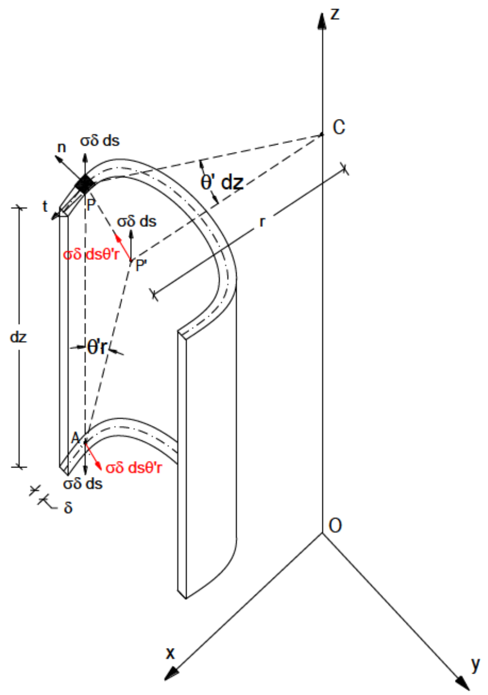

Figure 3: Rotation of the cross section.

If $\sigma > 0$ and $\vartheta' > 0$, the force $\sigma\cdot\delta\cdot ds\cdot\vartheta'\cdot r$, acting on the positive cross section, produces, in according to the right-hand rule of signs, about point C, the negative torque

$$M_{tn} = -\frac{N}{A}\cdot\vartheta'\int_A \delta\cdot ds\cdot r^2 = -\frac{N}{A}\cdot\vartheta'\cdot I_C \qquad (16)$$

where A is the cross section area and I_c is the polar moment of inertia of the cross section about the shear centre C.

Ultimately the basic equilibrium equation, when $m_t = 0$, is

$$\begin{aligned} &C_2\cdot\vartheta'^{V} + C'_2\cdot\vartheta''' - \left(C_1 + \frac{N\cdot I_C}{A}\right)\cdot\vartheta'' \\ &-\left(C_1 + \frac{N\cdot I_C}{A}\right)'\cdot\vartheta' + N\cdot u''\cdot y_G - N\cdot v''\cdot x_G = 0 \end{aligned} \qquad (17)$$

Under the further hypothesis that the beam is subjected to only constant axial compressive force $N = -F$, the basic equations for the analysis become

$$\begin{aligned} &\left(E\cdot I_\eta\cdot u''\right)' + F\cdot u' - F\cdot y_G\cdot\vartheta' = 0 \\ &\left(E\cdot I_\xi\cdot v''\right)' + F\cdot v' + F\cdot x_G\cdot\vartheta' = 0 \\ &C_2\cdot\vartheta'^{V} + C'_2\cdot\vartheta''' - \left(C_1 - \frac{F\cdot I_C}{A}\right)\cdot\vartheta'' \\ &-\left(C_1 - \frac{F\cdot I_C}{A}\right)'\cdot\vartheta' - F\cdot u''\cdot y_G + F\cdot v''\cdot x_G = 0 \end{aligned} \qquad (18)$$

They are three simultaneous differential equations for buckling by bending and torsion and can be used to determine the critical loads.

Solving this system of equations is quite difficult. A further difficulty is the identification of the location of the shear centre along the beam. For a constant cross section, its position is univocally determined by the geometrical characteristics. When the cross section varies appreciably along the beam, this is no longer true and the position of the shear centre also depends on the applied stress.

Because of the symmetry of the section with respect to the x axis, the shear centre lies on this axis. Its ordinate is then uniquely determined. Then, it remains to determine the abscissa.

The stress distribution is given through Navier's formula

$$\sigma_z = \frac{N}{A} + \frac{M_n}{I_n}\cdot d_n \qquad (19)$$

in the present case

$$\sigma_z = \frac{N}{A} + \frac{M_x}{I_x}\cdot y \qquad (20)$$

differentiating with respect to z is obtained

$$\frac{\partial\sigma_z}{\partial z} = \frac{1}{A}\cdot\frac{dN}{dz} - \frac{N}{A^2}\cdot\frac{dA}{dz} + \frac{T_y}{I_x}\cdot y + M_x\cdot y\cdot\frac{d}{dz}\left(\frac{1}{I_x}\right) \qquad (21)$$

where y is the distance of the centre of gravity from the centroid of the areola dA.

Assuming we load the beam with constant axial compressive force fixed in centroid G

$$\frac{\partial\sigma_z}{\partial z} = -\frac{N}{A^2}\cdot\frac{dA}{dz} + \frac{T_y}{I_x}\cdot y + M_x\cdot y\cdot\frac{d}{dz}\left(\frac{1}{I_x}\right) \qquad (22)$$

The Gauss's law, in the domain A^-, defined by the area of portion of the member's cross-sectional that precedes the section where thickness δ is measured, it allows to write

$$\tau\cdot\delta = \int_{A^-} div\,\tau_z dA = -\int_{A^-}\frac{\partial\sigma_z}{\partial z}dA \qquad (23)$$

then

$$\tau = -\frac{T_y\cdot S_x}{I_x\cdot\delta} + \frac{N}{2\cdot A^2\cdot\delta}\cdot\frac{d}{dz}(A^-)^2 - \frac{M_x}{\delta}\cdot\frac{d}{dz}\left(\frac{S_x}{I_x}\right) \qquad (24)$$

Therefore, we can make the maximum stress on each flange

$$\begin{aligned} \tau_{l,\max} = &-\frac{T_y\cdot B'\cdot H'}{2\cdot I_x} + \frac{N\cdot\delta}{2\cdot A^2}\cdot\frac{d}{dz}(B')^2 \\ &-\frac{M_x}{\delta}\cdot\frac{d}{dz}\left(\frac{6\cdot B'\cdot H'}{6\cdot B'\cdot H'^2 + H'^3}\right) \end{aligned} \qquad (25)$$

According to the hypothesis of Jourawsky's theory, the shear stress on each flange of cross section is

$$T_f = \left(-\frac{T_y}{I_x}\cdot\frac{B'\cdot H'}{2}+\frac{N\cdot\delta}{2\cdot A^2}\cdot\frac{dB'}{dz} - \frac{M_x}{\delta}\cdot\frac{d}{dz}\left(\frac{6\cdot B'\cdot H'}{6\cdot B'\cdot H^2+H^3}\right)\right)\cdot\frac{B'\cdot\delta}{2} \tag{26}$$

then

$$T_f = T_f'\left(B',H',\delta,T_y\right)+T_f''\left(B',H',\delta,M_x,N\right) \tag{27}$$

where:

$$T_f' = -\frac{T_y}{I_x}\cdot\frac{B^2\cdot H'\cdot\delta}{4} \tag{28}$$

$$T_f'' = \frac{N\cdot\delta}{2\cdot A^2}\cdot\frac{dB'}{dz}\cdot\frac{B'\cdot\delta}{2} - \frac{M_x}{\delta}\cdot\frac{d}{dz}\left(\frac{6\cdot B'\cdot H'}{6\cdot B'\cdot H^2+H^3}\right)\cdot\frac{B'\cdot\delta}{2} \tag{29}$$

In Fig. 4 is shown the shear stress distribution, with the following meaning of the terms:

$$\tau_f' = -\frac{T_y\cdot S_x}{I_x\cdot\delta} \tag{30}$$

$$\tau_f'' = \frac{N}{2\cdot A^2\cdot\delta}\cdot\frac{d}{dz}(A^-)^2 - \frac{M_x}{\delta}\cdot\frac{d}{dz}\left(\frac{S_x}{I_x}\right) \tag{31}$$

T_f is the shear stress on flange
T_w is the shear stress on web

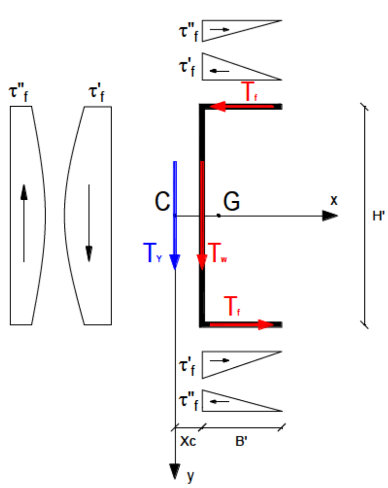

Figure 4: Shear stress distribution.

The equilibrium of the element gives

$$T_f\cdot H' = T_y\cdot x_c \tag{32}$$

and

$$x_c = -\frac{B'^2\cdot H'^2\cdot\delta}{4\cdot I_x}+\frac{H'}{T_y}\cdot T_f''(B',H',\delta,N,M_x) \tag{33}$$

with respect to the formula used for constant section beam

$$x_c = -\frac{B^2\cdot H^2\cdot\delta}{4\cdot I_x} \tag{34}$$

there is an additional term to be taken into account, induced by the presence of axial compressive force and torque.

2.2 Approximate approach

In a such complicated problem, energy method (Ritz-Rayleigh method) gives a very satisfactory approximation to the true critical load, provided the shape of the assumed curve is reasonably close to the exact curve.

For any generalized elastic body the theorem of minimum total potential energy asserts that: of all the displacements satisfying compatibility and the prescribed boundary condition, those that satisfy the equilibrium equation make the potential energy a minimum.

Assuming a shape for the deflection curve, it is essential to approximate the displacement function in a way that is consistent with the boundary conditions and satisfy certain minimum continuity requirements.

If the displacements are expressed in terms of a set of coefficients (Lagrange multipliers), then the coefficients become the unknown variables, and the correct values of the coefficients are those which minimize the total potential energy.

Minimizing the total potential energy with respect to the coefficients is equivalent to setting the variation in the total potential energy with respect to the coefficients equal to zero.

The energy method always gives values of the critical load which are larger than the true value unless the assumed deflection curve happens to be the correct one.

This follows from the fact that the true shape is the only one which represents a deflection configuration for which each element of the beam is in equilibrium. To have the beam in equilibrium with an incorrect shape of buckling, requires that additional constraints be introduced in order to maintain that shape. The addition of constraints increases the rigidity of the beam and hence the critical load becomes larger than its true value.

Thus if several assumed deflection curves are used, the lowest critical load found from those assumed curves will be the most accurate.

In the particular case of simply supported beam, if during deformation the ends of the beam can rotate freely with respect to the principal axes of inertia parallel to the x and y axes, while rotation with respect to z axis is prevented by some constraints, the conditions that deflection curve has to satisfy are

$$u = u'' = 0 \qquad (35)$$
$$v = v'' = 0$$
$$\vartheta = \vartheta'' = 0$$

We can take the components of deflection curve u, v, ϑ in the form of a trigonometric series

$$u_C = \sum u_n \cdot sen\frac{n \cdot \pi \cdot z}{l} \qquad (36)$$
$$v_C = \sum v_n \cdot sen\frac{n \cdot \pi \cdot z}{l}$$
$$\vartheta_C = \sum \vartheta_n \cdot sen\frac{n \cdot \pi \cdot z}{l}$$

in which each term together with its second derivative vanishes at the ends of the beam as required by the conditions of constraint.

The components of deflection curve in the centroidal axis directions during buckling are

$$u = u_c - y \cdot \vartheta \qquad (37)$$
$$v = v_c + x \cdot \vartheta$$
$$w = 0$$

Applying Clapeyron's theorem, the potential energy of deformation of a beam can be calculated by

$$L_2 = \int_V \sigma_z \cdot \delta\varepsilon_z^{(2)} dV \qquad (38)$$

being

$$\delta\varepsilon_z^{(2)} = \frac{1}{2} \cdot \left[\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right] \qquad (39)$$

The strain energy of bending is

$$W = \frac{1}{2} \cdot \left[\int_0^l E \cdot I_\eta \cdot u''^2 dz + \int_0^l E \cdot I_\xi \cdot v''^2 dz + \int_0^l M_t \cdot \vartheta' dz \right] \qquad (40)$$

where

$$M_t = C_1 \cdot \vartheta' - C_2 \cdot \vartheta''' \qquad (41)$$

The total potential energy

$$\delta_2 E = L_2 + W \qquad (42)$$

gives a second order homogeneous equation in u_n, v_n, ϑ_n.

The minimum condition is expressed by

$$\frac{\partial \delta_2 E}{\partial q_i} = 0 \qquad (43)$$

this equation makes three simultaneous differential equations in m Lagrange multipliers q_i (u_n, v_n, ϑ_n).

If that homogeneous system has a singular matrix, it has farther not trivial solution. This condition gives a m^{th} degree equation in F; the lower solution is the critical load.

By taking two or more terms of the series increases the number of Lagrange multipliers and the solution will be closer than the true value.

3 Example of experimental investigation

Examine the case of a steel beam of open thin wall C-cross section with flanges width and web height variables along z-axis.

The laws governing shape variability of the beam were chosen in order to have the same distance between shear centre and centroid along the beam. If we choose linear variability laws for flanges width and web height, the condition of alignment of the centres axis is not rigorous feasible.

On the other hand, the choice of a section with elaborate geometries infringes the fundamental principle of Simple Design.

For this reason we prefer to approximate the variability laws for flanges width and web height to appropriate linear function in order to reach the best approximate alignment of the centres axis. Thus, the solution is less close to real value but it gives a dramatic simplification of the problem.

Let us consider a simply supported beam. In particular during deformation, the ends of the beam can rotate freely with respect to the principal axes of inertia parallel to the x and y axes, while rotation with respect to z axis is prevented by some constraints. The beam is subjected to a constant axial compressive force.

The geometry of the beam is shown in Fig. 5.

The variability range for the critical load was obtained by the approximate approach shown in 2.2 by taking one, two and three terms of the series. The Ritz Rayleigh algorithm was developed with Mathematica® software [20].

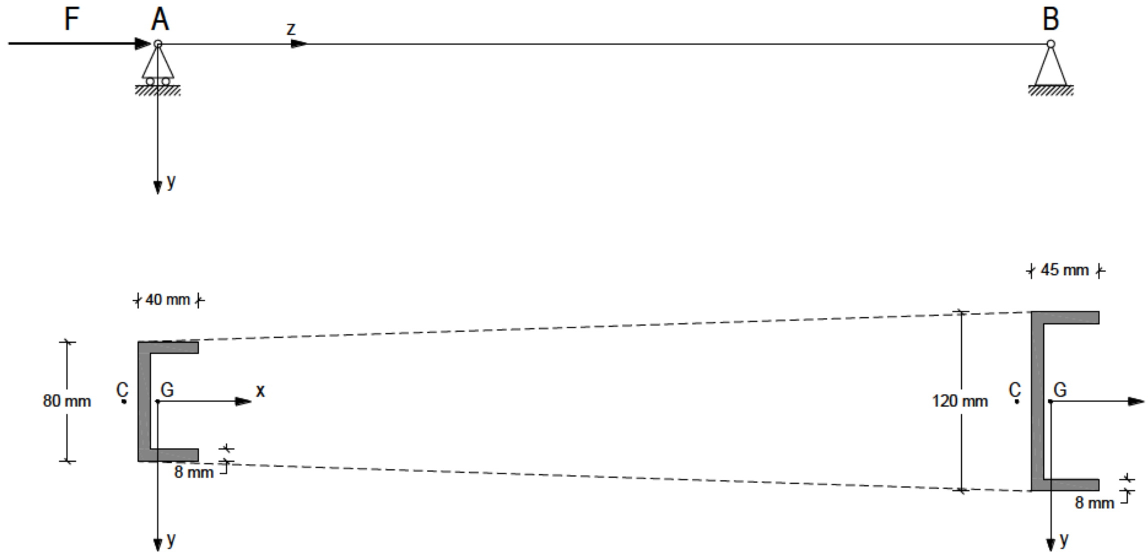

Figure 5: Beam geometry.

4 Conclusions and future work

Using energy method we have approximate components of deflection curve by taking one, two and finally three terms of a trigonometric series and then we have approximate the deflection curve by a one, two and three sine waveforms. In this manner we have obtained a critical load close to the true value.

If we take more terms of series and introduce more Lagrange multipliers, we can investigate that the solution accuracy doesn't improve significantly.

We have obtain that lower limit of the critical load is F_{cr} = 110,18 kN.

The gains that have been made are very satisfactory. The energy method gives a very accurate result that is lower than the value obtained by Euler buckling stress equation.

Indeed, applied Euler buckling stress equation to a beam of equal length, with same constraints but constant cross section and chosen as cross sections the ends cross sections of the original beam, we can obtain a first approximate large range of variability for critical load. Remembering that

$$F_x = \frac{\pi^2 \cdot E \cdot I_\xi}{l^2} \qquad (44)$$
$$F_y = \frac{\pi^2 \cdot E \cdot I_\eta}{l^2}$$
$$F_\vartheta = \frac{A}{I_C} \cdot \left(C_1 + \frac{\pi^2}{l^2} \cdot C_2 \right)$$

we obtain

$$231,58\,\text{kN} \le F_{cr} \le 371,41\,\text{kN} \qquad (45)$$

It is worth pointing out that the method used in this paper for the research of the critical load requires the alignment of the centroid and shear centre axes. This alignment is not rigorously realizable in the real cases. It will be interesting to see how the theoretical value is close to the true value.

In particular, it will be interesting to understand how the variability of the position of the shear centre with the applied stress influences the behaviour of the beam.

For interest in subject the theoretical results will be verified and validated with beam test, which are being developed by the authors.

Conflict of Interests: The authors declare that there is no conflict of interests regarding the publication of this paper.

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