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On the application of a critical plane approach to the life assessment of welded joints

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Abstract

In the present work, the Fatemi-Socie approach is adopted in order to analyze the fatigue endurance of welded joints under multiaxial loads. This critical plane criterion has already been successfully applied to plain or notched components, however, it is not spread in the assessment of welded joints, yet. This work is focused on the practical implementation issues related to this particular application, which has not been discussed in the literature. The described procedure is adopted for the assessment of one hundred experimental tests and some preliminary results are shown. The specimen under investigation is a pipe-to-plate fillet joint made out of structural steel (S355JR). The tests were performed under both uniaxial and multiaxial, i.e. combined in-phase and out-of-phase bending and torsion, load conditions with a constant amplitude at the laboratories of the University of Pisa, Italy.

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Keywords: Critical plane approach, welded joints, multiaxial loads, steel.

1. Introduction

The fatigue life assessment of notched components under multiaxial loads is a problem of wide practical interest since most of the components in real applications show geometrical details that produce local stress concentrations.

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Also, many actual applications are characterized by the presence of multiple independent loads, which produce a multiaxial non-proportional state of stress/strain. Even when a single load is applied, we can generally observe a multiaxial state of stress/strain in the volume of material surrounding a notch, even though in this case it is usually proportional.

For the sake of simplicity experimental tests are often performed applying a constant amplitude uniaxial load and often times simple plain (un-notched) specimens are used in those investigations. Based on this fact, there is need to develop a procedure able to relate, in terms of fatigue damage, the actual multiaxial state of stress with the endurable (often uniaxial) stress state that is known from experiments, in order to estimate the fatigue life of real components.

Nomenclature

σ	normal stress
A	axial load amplitude value
τ	shear stress
B	torsion load amplitude value
φ	load phase angle
γ	shear strain
k	material constant in the FS parameter
σ_y	yield strength
σ_u	ultimate strength
K'	Ramberg-Osgood: cyclic strength coefficient
n'	Ramberg-Osgood: cyclic strain hardening exponent
f	Fatemi-Socie damage parameter value
N_f	number of cycles to failure
T_σ	scatter band amplitude

Among the earliest proposals for multiaxial fatigue, we can count the use of equivalent stresses or strains that are obtained through similarity invariant functions, such as the von Mises equivalent stress or the maximum principal stress. Many of those criteria were developed as yield theories and, successively, have been adopted as fatigue damage parameters. According to Eurocode [1] or IIW standards [2], the fatigue life assessment in multiaxial load conditions can be carried out using the Miner's linear damage rule combined with von Mises equivalent stress criterion. Experimental investigations [3, 4] when analyzed by this procedure give a total damage ranging from 0.2 to 0.5, where the lower values refer to non-proportional (out-of-phase) loads.

However, the use of stress/strain invariants to the fatigue life assessment has been questioned by many authors and a number of specifically defined damage parameters have been proposed [5, 6, 7, 8, 9]. A drawback related to the use of stress/strain invariants is the fact that under non-proportional loading they can even show a constant value (with respect to time) of the equivalent stress. As an example, a round bar tested with axial and torsion loads expressed by equations (1) and (2) shows a constant value of the von Mises equivalent stress if the load amplitudes are selected such as $A/B = \sqrt{3}$ and the phase angle is set to $\varphi = 90°$. Said load conditions would produce a fatigue damage in the component with a constant von Mises equivalent stress, implying infinite life regardless of the magnitude of the applied cyclic stresses. Generally speaking, condensing a second order tensor to a single scalar leads to a loss of information that may affect the accuracy of the fatigue damage assessment.

$$\sigma = A\ \sin(\omega t) \tag{1}$$
$$\tau = B\ \sin(\omega t + \varphi) \tag{2}$$

Critical plane approaches were developed in order to provide fatigue damage parameters not affected by the shortcomings related to stress/strain invariants. This class of methods was specifically designed in order to evaluate the fatigue endurance based on experimental cracking evidence. In particular, it was observed that the orientation of fatigue cracks is strictly related to the applied load conditions. For materials that show a ductile behavior cracks tend to nucleate on planes that undergo high tangential stress or strain. This is consistent in accordance with the permanent slip bands mechanism [10] which is dominated by the motion of dislocations. Those observations lead researchers to assume the actions on such planes as fatigue damage parameters. Such methods are, therefore, composed of two separate steps: the identification of the critical plane and the evaluation of the fatigue damage on that orientation.

One of the earliest works in this field was proposed by Brown and Miller [5] who proposed a damage parameter obtained from the combination of shear strain and normal strain. Later Fatemi and Socie [6] proposed a criterion where the damage parameter is evaluated by a combination of shear strain and the normal stress on the maximum shear plane. This formulation proved to provide a robust framework for fatigue life assessment under multiaxial loads for a wide range of ductile behaving materials. The Fatemi-Socie damage parameter (FS) can then be represented by the formulation of equation (3), where k is a material related constant, $\sigma_n^{\max}$ represents the maximum (over time) stress acting in a direction normal to the maximum shear plane, and σ_y is the yield strength of the material.

$$f = \gamma_{\max}(1 + k\, \sigma_n^{\max}/\sigma_y) \tag{3}$$

The critical plane based on the FS parameter can be chosen either as the orientation that maximizes the shear strain, or the orientation which maximizes the FS parameter variation over a load cycle. In the present work, the first definition is adopted. To the best of the authors' knowledge, this work, is one of the first applications of the Fatemi-Socie critical plane approach to welded joints, along with the recently published work in [11].

Welded joints show some peculiar characteristics that have to be accounted for in fatigue life evaluation. The main aspect is that severe notches can always be found along a weld seam. For a fillet weld, they are commonly located at the root and toe of the weld seam. Furthermore, the local geometry in the weld seam area is commonly affected by a wide variation in its dimensions, plus a number of defects and imperfections may be found in this area. A description of the weld seam actual geometry for the specimens under investigation and of its effects on fatigue endurance can be found in [12].

Another issue related to welding is the heating of the component during the welding process. It has an effect on the material properties of both the base and filler materials. The temperature history shows a large variation from point to point in the volume of the material that surrounds the weld seam and so do the mechanical properties. This makes the experimental determination of such properties a rather complex matter. Another effect related to the non-uniform heating is the generation of residual stresses that might be of relevant intensity and which are usually not taken into account, also due to the difficulties of their experimental evaluation. From the fatigue point of view the residual stresses can be assimilated to mean loads, however, it has to be kept in mind that the external load actions can cause a relaxation of said residual stresses.

Due to the above effects the results of fatigue experiments usually show an inherent scatter, which is greater if compared to the scatter of fatigue tests made on simpler un-welded specimens.

2. Specimen description and experimental set-up

The tested specimens were designed in order to reproduce a pipe-to-plate joint (Fig. 1) typically found in railway boogie components. The tube had an external diameter of 64 mm and a thickness of 10 mm, while the thickness of the plate was 25 mm. The tube is connected to the plate by fillet welding with a seam weld having a nominal dimension of 10 mm. The specimens were made of S355JR steel, for which $\sigma_y = 360$ MPa and $\sigma_u = 520$ MPa.

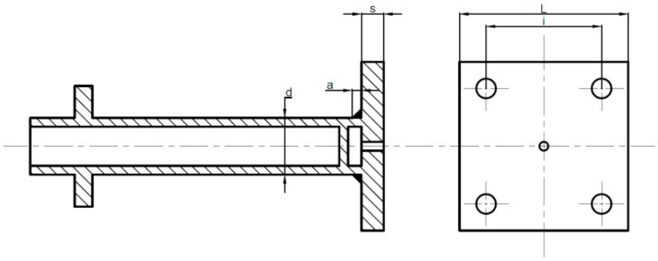


Fig. 1. Specimen geometry and dimensions.

Tests were performed on a custom designed test bench shown in Fig. 2 and described in [4]. The bench is composed of two independently controlled hydraulic actuators connected on each side of the specimen by means of a lever arm with a length of $2b$ = 600 mm. The lower plate of the specimen is fixed to the bench by four M20 bolts. The hydraulic actuators are controlled in order to produce sinusoidal forces with given amplitude, mean load and relative phase shift so that any desired bending and twisting moment can be obtained on the welded section.

The adopted failure criterion was the presence of a through-the-thickness crack. The occurrence of this kind of damage was easily detected by the sudden drop in air pressure imposed in the lower chamber of the specimen at the start of the test.

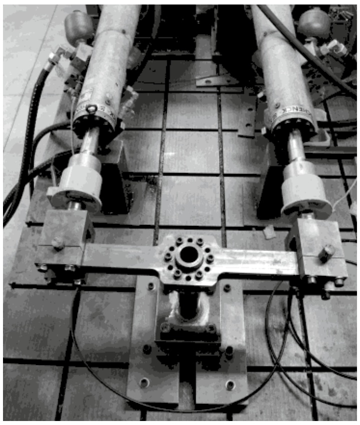


Fig. 2. Loading apparatus. The specimen is loaded by two hydraulic actuators attached at the extremities of a lever arm; this test rig was developed to have the possibility of applying complex combinations of bending and torsion.

3. Experimental tests

In the current investigation, a total of 91 experimental tests were analyzed, which can be grouped into twelve series based on the applied loading condition. The applied loading conditions can be summarized as follows:

- Bending
- Torsion
- In-phase bending and torsion with $\sigma/\tau = 3.2$
- In-phase bending and torsion with $\sigma/\tau = 0.88$
- Out-of-phase bending and torsion with $\sigma/\tau = 3.2$
- Out-of-phase bending and torsion with $\sigma/\tau = 0.88$

All test series were repeated with two different load ratios, i.e. $R = 0$ and $R = -1$. The combined loads are obtained through the simultaneous application of bending and torsion in the ratio indicated in terms of nominal stresses (i.e. σ/τ). The number of tests pertaining to each series is summarized in Table 1.

Table 1. Experimental tests summary.

			In-Phase		Out-of-Phase	
	Bending	Torsion	$\sigma/\tau = 3.2$	$\sigma/\tau = 0.88$	$\sigma/\tau = 3.2$	$\sigma/\tau = 0.88$
R = -1	10	13	8	5	5	5
R = 0	9	13	8	5	5	5

All the tests were conducted under constant amplitude loads in the "as-welded" conditions, meaning that no post-welding treatment was made on the specimens.

4. Fatigue life assessment

The fatigue evaluation process was composed of three independent steps. Firstly, the local elastic state of stress and strain was evaluated through a finite element model of the specimen. Then, the actual elasto-plastic behavior of the material was taken into account by means of the Neuber's rule. At last, the FS damage parameter was evaluated making use of an in-house developed *Python®* routine. In the following, each of these three steps will be discussed separately. The fatigue life assessment was performed analyzing the structural behavior of the weld root notch since most of the fatigue cracks were experimentally observed to nucleate from this detail.

4.1. Finite element simulations

The simulations utilized in this work were developed with the sub-modeling technique, they are composed of a coarse global and a detailed local model. This allowed a detailed description of the weld seam geometrical details while limiting the overall computational effort. This part of the work was conducted making use of the commercial software *Ansys®*.

An image of the global model is shown in Figure 3(a). The model was developed with first order elements and was designed in order to reproduce the structural behavior of the component in relation to the applied loads and constraint characteristics that reproduce the experimental apparatus; more details about the model calibration can be found in [12].

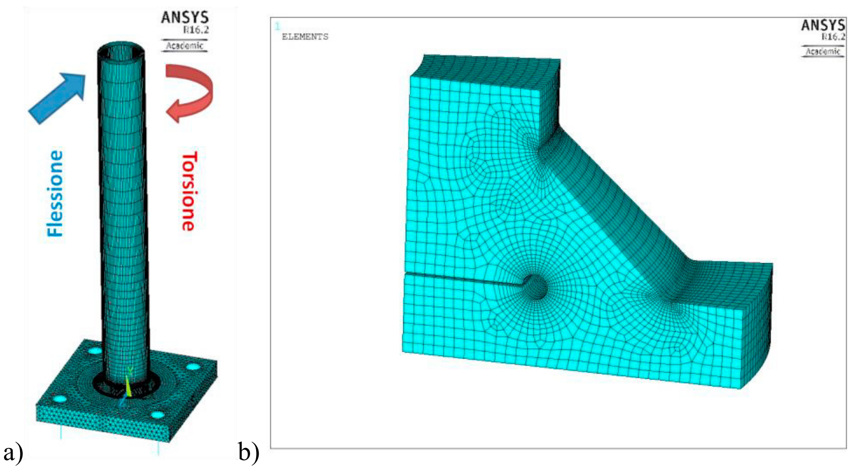


Fig. 3. Finite element model.

The sub-model, reported in Figure 1(b), represents a limited volume of the specimen. In particular, it reproduces a section of the weld seam where the highest values of stresses and strains are expected. Thanks to its small dimensions, an extremely detailed description of each geometrical detail was possible. In this work, it was decided to represent each of the weld notches with an imposed radius equal to $r = 1$ mm, even if the actual radius value was smaller. It is known that the actual geometry at these locations has a very significant effect on the stress/strain state surrounding the notch itself [1]. However, the actual radius value cannot be known at priori since it shows a large variation along the weld seam due to the characteristics of the joining process. The value of the radius adopted allows to partially take into account the gradient effect, similarly to the fictitious radius concept proposed by Radaj [13]. However, in the current application, there is no theoretical basis available for this choice and this aspect will be further investigated in the future. Furthermore, the assumption of a fixed radius value does not take into account the variations in weld seam geometry that can be observed in actual specimens.

An elastic material model was implemented in all of the described models, the Young's modulus was set to $E = 200$ GPa while the Poisson's modulus was $\nu = 0.3$.

Two separated simulations were performed, one for each load condition. For the assessment of the combined load tests the resulting stress/strain state was obtained through the superposition principle, applicable due to the linearity of the adopted material model.

a) b)

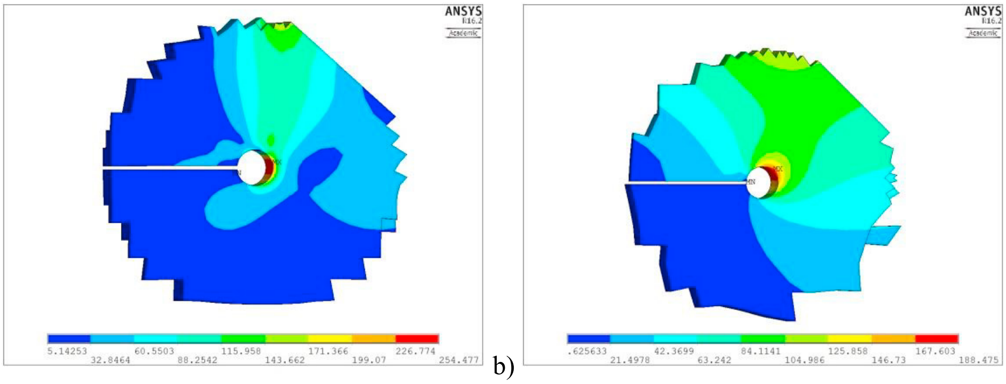


Fig. 4. Finite element results in the root notch area (equivalent von Mises stress).

The resulting von Mises equivalent stress in bending and torsion conditions is reported in Figures 4(a) and 4(b), respectively. It is interesting to notice how the maximum stress is achieved in different locations depending on the applied external load. In case of combined out-of-phase loads, the maximum stress point is, then, expected to shift its position on the notch circumference during the loading cycle.

4.2. Elasto-plastic stress/strain estimation

Plastic deformations, when present, have a significant effect on the fatigue damage process. Their evaluation is a crucial aspect of any assessment procedure that aims to a good estimation of the fatigue life. An accurate description of the stress/strain fields could be obtained through an elasto-plastic finite element simulation when a more complex non-linear material model is adopted. However, this approach is generally avoided due to its high cost in terms of computational effort. It has to be considered that a wide number of simulations would be needed (one for each load configuration) since the superposition principle does not apply in the case of non-linear systems.

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K'}\right)^{1/n'} \tag{4}$$

Many analytical procedures have been proposed in the literature in order to analytically derive the stress/strain field on a notch in order to avoid numerical computation. The more spread among them are based on either the Neuber's rule [14] or on the strain energy density rule [15]. At the current time, the Neuber's rule has been adopted in combination with a Ramberg-Osgood stress-strain relationship (i.e. equation 4) describing the cyclic material properties. The material related parameters that appear in equation (4) were determined starting from the yield and ultimate stresses of the base material (S355JR steel) according to the empirical relations given in [16]. The resulting constants were: $K' = 1231$ MPa and $n' = 0.187$. The Neuber's rule for converting elastically calculated stress and strain to inelastic stress and strain is given by:

$$\Delta\sigma \, \Delta\varepsilon = (\Delta\sigma)_{el} (\Delta\varepsilon)_{el} \tag{5}$$

As described in Figure (5), Neuber's rule is described by an hyperbole (equation 5), in the practical application it allows to obtain the elasto-plastic stress/strain (i.e. point B in Fig. 5) starting from the calculated elastic one (i.e. point A in Fig. 5) if the constitutive material law is known (blue line in Fig. 5).

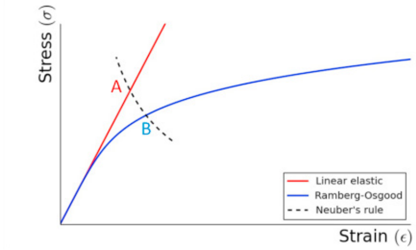


Fig. 5. Neuber's rule application.

The Neuber's rule is formulated in terms of the equivalent stress (i.e. von Mises), while the complete stress and strain tensors are needed for the FS damage parameter evaluation. Therefore, a further hypothesis was needed in order to obtain the elasto-plastic tensors starting their equivalent values. As a preliminary approach, it was assumed that the principal directions of both stress and strain do not change going from the purely elastic to the actual elasto-plastic condition. The error caused by this assumption is expected to be relatively small if the amount of plastic deformation is small. This approach proved to be in fairly good accordance with the results of non-linear finite element simulations when the maximum bending loads used in the experiments are applied.

The Neuber's rule, however, is formulated for static load conditions. The practical implementation, in this work, consists in the application of the Neuber's rule in order to obtain the elasto-plastic stress and strain starting from their elastic counterparts for each time instant during the load cycle. This approach is not based on any plasticity model and leads to a loss of accuracy since the residual stresses induced by plastic deformation and the constraint effects of the surrounding material are not taken into account. As an example, we can think of a loading history where the load goes back to zero after plastic deformation was produced. At that time instant, we can expect to find residual stresses and strains in the real material, which cannot be predicted by the linear elastic model. Also, the Neuber's rule when applied at this time instant predicts a null state of stress and strain, since the input equivalent stress and strain are both equal to zero. A more accurate analytical model is currently under development. Also, residual stresses deriving from the welding process are not taken into account in this work.

4.3. Fatemi-Socie damage parameter evaluation

The evaluation of the FS damage parameter is obtained by means of an in-house designed *Python*® routine. In the current work, the method is applied in its classic (simpler) formulation, although a modification has been recently proposed [17]; the evaluation consists of two main steps: firstly the critical plane has to be identified, then the damage parameter can be computed on the said plane. The critical plane is assumed as the plane that undergoes the maximum shear strain variation ($\Delta\gamma_{\max}$) over a load cycle (an alternative definition consists in placing the critical plane in the direction that endures the maximum FS damage parameter [11]).

The designed routine takes as input the plastic strain tensor at several time instants during the cycle and rotates it searching for the direction that corresponds to the maximum shear strain variation. The matrix rotation was obtained through the composition of three subsequent rotations following the Euler angles sequence *x-y-z* (extrinsic rotations). Each angle was varied with a step of 2.5° in the range between zero and 90°, obtaining 36 possible values for each angle and 46656 plane orientations. The coverage of any possible critical plane orientation results to be complete and sufficiently accurate, although it is not uniformly distributed. An example of the non-uniform spacing can be observed in Figure 6, where the tip of the y vector axis for the tested plane orientation is plotted as a red dot. This leads to a non-optimized computation procedure but does not affect the accuracy of the obtained results.

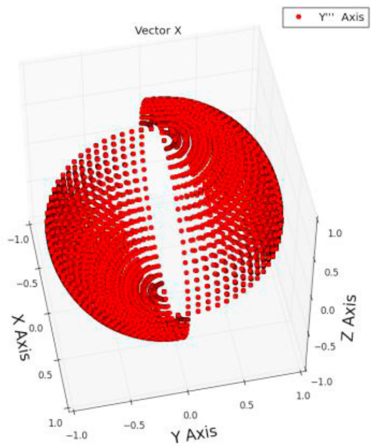


Fig. 6. Spatial distribution of the y axis tip according to the implemented rotation sequence.

Once the critical plane is obtained, its orientation can then be expressed by the three Euler angles. Applying the same rotation to the stress tensor it is possible to obtain the maximum normal stress value acting on the critical plane during the cycle. Then the FS damage parameter can be computed. In the current work, the material constant in the FS formulation was fixed to $k = 0.4$, this value was derived in [11] for a similar material.

Since the position of the maximum damaged point over the notch is not known at priori, the FS damage parameter was computed on a total of 32 points along the notch circumference, each point corresponding to a node of the finite element model.

5. Discussion and conclusions

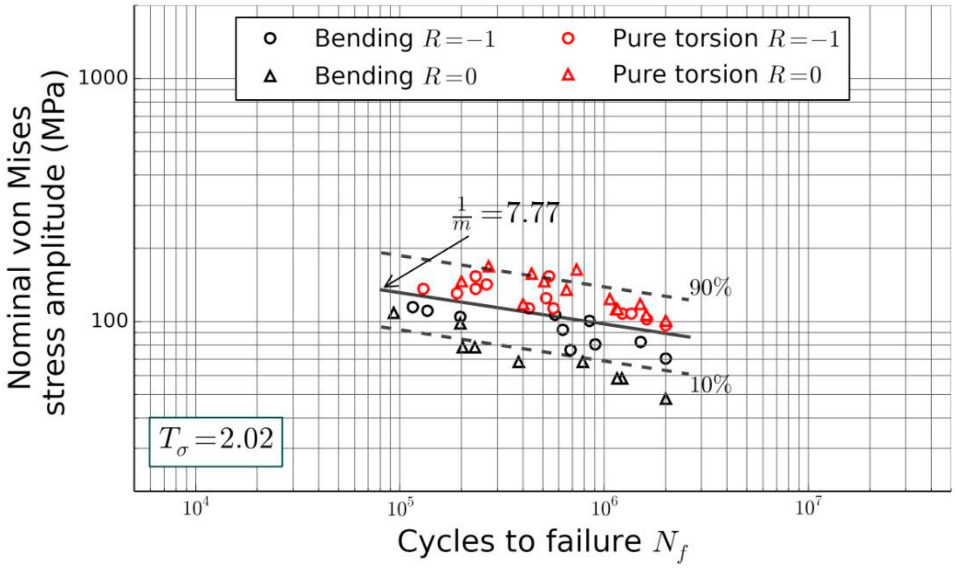
The Fatemi-Socie critical plane approach provides a robust framework for the fatigue life assessment in multiaxial load conditions. Its characteristics have been already described in literature (e.g. [18]). In this section, some considerations about the load parameters effects on the current dataset are presented. Several data sub-sets are analyzed, each one composed of two test series differing by one load parameter, each one allows to observe a particular load effect on fatigue life, including mean stress, phase angle, and the normal to shear stress ratio.

5.1. Mean stress in bending and torsion

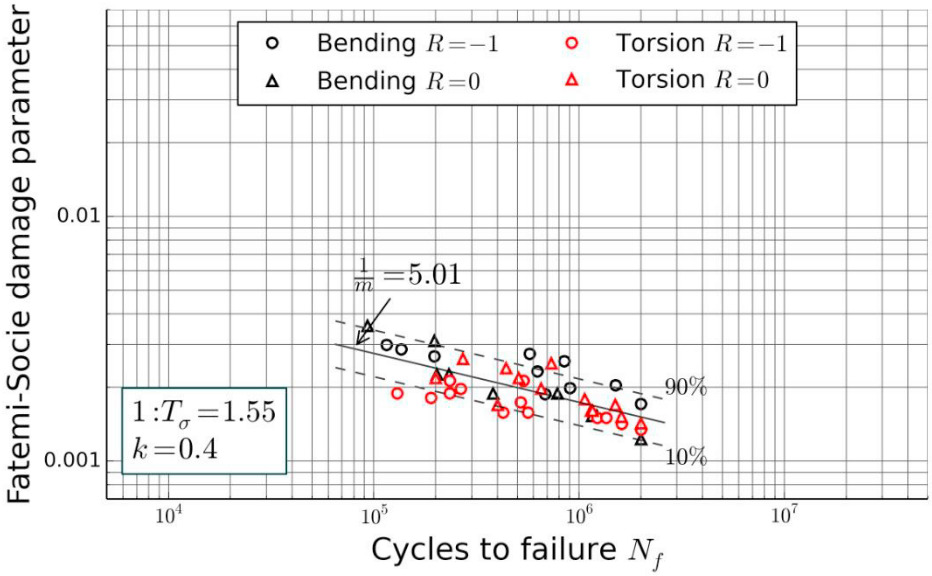
Experimental observations generally show a decrease in terms of fatigue life in bending tests for increasing mean loads (e.g. going from $R = -1$ to $R = 0$), while the fatigue endurance to torsion loads proves to be dependent only on load amplitude with little or no mean load effects, as long as the maximum shear stress does not exceed shear yield strength [4].

In case of welded joints, the mean stress is generally not taken into account due to the high residual stresses that are generally present in the weld area. However, some design codes [2] suggest empirical correction factors to be applied in case of stress relieved or thin joints or anywhere the amount of residual stresses is limited.

The FS formulation incorporates the mentioned mean load effect in its formulation. In fact, this is due to the normal stress term that appears in equation (3). The maximum (over time) stress acting normal to the critical plane is directly proportional to the applied mean load in bending since the critical plane is oriented at a 45° angle with respect to the principal stress direction. In pure torsion, instead, an equal biaxial stress state is produced. In this case, the critical plane is oriented along the shear stress direction and no normal stress acts on it.



a)



b)

Fig. 7. Mean load effect on fatigue test results evaluated through: a) the nominal stress method, b) the FS based approach

The experimental test results evaluated through the nominal stress method and the FS-based approach in bending and torsion are reported in Figures 7(a) and 7(b), respectively. Figure 7(a) shows a clear separation between the endurable nominal stress in bending tests with different load ratios, while the mean stress seems to produce little or no effects on the fatigue endurance when shear stresses from torsion loads are involved.

The same experimental evidences are reported in Figure 7(b), this time analyzed by means of the FS critical plane approach. It is evident that the data correlation improved considerably. Different stress ratios, with this approach, show a comparable fatigue endurance. Also, the two different load conditions (i.e. bending and torsion) prove to produce close results in terms of damage parameter, indicating that the chosen value of $k = 0.4$ reflect the actual material behavior.

5.2. Phase angle and nominal to shear stress ratio effects on combined load tests

A load parameter that commonly produces relevant effects on fatigue life is the phase angle $\emptyset$ between the two load signals. This parameter differentiates between proportional (in-phase or $\emptyset = 0°$) and non-proportional loads (out-of-phase or $\emptyset \neq 0°$). The latter two terms refer to the proportionality of the stress tensor over the loading cycle. In facts, a proportional load produces a stress state with fixed principal axes with respect to time, while a rotation of said axes is observed under non-proportional load conditions.

Many researchers have observed a decrease in fatigue endurance with increasing phase angles [4, 7] and pointed out the shortcomings of invariant based methods when approaching this kind of analysis since their structure does not allow to account for the effects of principal axes rotation. Critical plane approaches, instead, were expressly designed to deal with non-proportional load histories and proved themselves to give fairly better life estimations on plain and notched components [6, 17]. Their application in case of welded joints is expected to produce a similar improvement.

The last load parameter investigated here is the normal to shear stress ratio, which is expressed here in terms of nominal von Mises equivalent values. Two ratio values were imposed during the experimental investigation: $\sigma/\tau = 3.20$ and $\sigma/\tau = 0.88$, as described in [4]. The application of a critical plane approach to these experimental results is also expected to improve the assessment quality, introducing a more accurate analysis of the effects produced by the interaction of normal and shear stresses.

In the present work, the application of the Fatemi-Socie critical plane approach to the fatigue life assessment of welded joints was analyzed. What is described here is the actual status of a research activity that is still under development regarding a complex problem, where its implementation required the formulation of several assumptions and hypothesis. Those assumptions have been described in the previous sections. They can be briefly pointed out as:

- The material properties were estimated based on an empirical formulation and are referred to the base material.
- A radius equal to $r = 1$ mm was assumed in the finite element modeling of the notch root;
- The residual stresses generated by the welding process were not accounted for;
- The analytical plasticity model is still incomplete;
- The damage parameter has been evaluated only at the notch root and not at the two toe notches;
- The current investigation is based on a fatigue nucleation approach (the FS), while the experimental lives are also composed of a propagation fraction;
- More recent FS formulations could be adopted.

The presented assessment procedure is still under development in many of its components, in order to decrease the amount of simplifying assumptions related to its application and improve its reliability. As an example, experimental investigations have been planned to investigate the actual material stress/strain relationship and evaluate the residual stress field. At the same time, an analytical plasticity model is under development.

The issues concerning the weld seam geometry and the stress gradient effect, however, remain open points for discussion. The presented critical plane approach is based on a point formulation, or rather the damage parameter is evaluated looking at the stress/strain state at a specific physical point. While the actual root notch proves to be of irregular shape, with relevant variations in its dimensions between different specimens and along the seam itself. In this work, it has been decided to substitute the actual notch geometry with $r = 1$ mm. This happens to be a pretty useful solution since the experimental investigation of the actual notch geometry is avoided. Also, the enlarged radius offers the possibility to account for the gradient effect to a certain degree in analogy with the cited fictitious radius approach.

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