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Crack Propagation Analysis of Near-Surface Defects with Radial Basis Functions Mesh Morphing

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Abstract

Fracture mechanics analysis is nowadays adopted in several industrial fields to assess the capability of components to withstand fatigue loads. Finite Element Method (FEM) is a well-established tool for the evaluation of flaw Stress Intensity Factors (SIF) and for the survey of its propagation. Nevertheless the study of the growth of near-surface circular and elliptical cracks is still an arduous task to be faced with FEM. In fact, the interaction of the flaw with free surfaces leads the crack front to assume complex shapes, whose simulation cannot be easily accomplished. A possible answer to deal with such a problem is to use the mesh morphing technique, a nodal relocation methodology, that allows to cover different problems. In fact, with mesh morphing, it is possible to fit the baseline flaw front with the desired shape (generic shape) and to automatically simulate its evolution at a certain number of cycles. In the proposed work this approach is demonstrated exploiting ANSYS Mechanical as FEM tool and RBF Morph ACT Extension as mesh-morpher. The results of the proposed workflow are compared with those available in literature.

Keywords:
Near Surface Crack; Radial Basis Functions; Automatic Crack Propagation; Mesh Morphing

1. Introduction

The knowledge of fracture behaviour is of crucial relevance in several industrial fields in order to prevent sudden breaks. The fracture mechanics theory is based on the assumption that all real materials contain cracks of some size. For this reason the study of the fracture mechanics is of great importance for life prediction of components subjected

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to fatigue loads. Propagation of superficial cracks into notched elements was the research purpose of many authors. Carpinteri et al. (2003), Murakami and Keer (1986), Lin and Smith (1998) and Biancolini and Brutti (2002) faced such a problem making use of Finite Element (FE) models, useful to take into account the influence of the notch on the Stress Intensity Factors (SIFs). Another important research path concerns the propagation of internal defects. Chen and Roberts (1987) investigated the fatigue crack growth of internal flaws, that can be treated as edge cracks. This assumption is acceptable if the distance between the crack tip nearest the free surface and the free surface itself is lower than ten times the size of the plastic zone at the tip. However this hypothesis leads to excessive conservative results. Gilchrist and Smith (1991) made a review of a wide range of crack shapes and conditions. The object of the examination pertained to the case of subsurface defects and breakout growth, making use of a finite element technique, with automatic remeshing. Isida and Noguchi (1984) dealt with an elliptical crack embedded in a plate of finite thickness at an arbitrary position. This problem was treated for a wide range of crack parameters with numerical calculations. An alternative to Finite Element Method (FEM) is provided by Dai et al. (1998). Making use of the eigenstrain procedure, it is possible to find the SIF at any point of the crack front. A semi-elliptical and a circular sub surface cracks were inspected with this method, showing a life prediction in good agreement with those observed in the practice.

One of the most important issue with the use of FEM to simulate the propagation of a flaw is the requirement of remeshing, at each step of the growth. A single advancement of the front entails an updating of the mesh, which can result annoying and rather time-consuming if carried on by hand. A possible alternative to remeshing is represented by mesh morphing. This technique is a very powerful tool widely adopted in several engineering fields. Biancolini and Groth (2014) applied this methodology to simulate in a dynamic way the realistic ice shape growth on aircraft wings, allowing an optimized workflow from both time and quality points of view. A further example of mesh morphing application is the structural optimization of an automotive wheel rim. Costa et al. (2015) showed that with a shrinking of each spoke of the wheel rim it is possible to reduce the weight of 7%, respecting, on the other hand, all the imposed requirements. Mesh morphing can be used also for shape optimization exploiting a gradient based logic. Groth et al. (2018) demonstrated that an high computational and optimization efficiency can be achieved coupling the adjoint solver with the mesh morphing.

The mesh morphing procedure can be also adopted for the study of crack growth. As presented in Biancolini et al. (2018) the propagation of crack, in a notched bar, was simulated with a two degrees of freedom model and in an automatic way through an analysis-and-update procedure. Mesh morphing accomplished the nodes re-positioning at each step of the analysis, without the necessity of remeshing and therefore allowing a substantial reduction of spent time.

In this paper a further development of the two degrees of freedom model is presented. The progress herein introduced deals with the use of a Multi Degree of Freedom (MDOF) model for the evolution of planar cracks, using Radial Basis Functions (RBFs) morphing techniques for the mesh update. The procedure can be divided in four main steps: development of a representative three-dimensional FE model, calculation of the effective SIFs along the crack front, determination of the crack front advances employing the Paris-Erdogan law (Paris and Erdogan (1963)) and adjustment of the previous FE model according to the new crack front by means of mesh morphing. The new crack front definition with a MDOF model consists in dividing the crack front into a set of nodes and evaluating the new position of each node due to the fatigue contribution. These steps can be iterated to simulate the complete crack shape evolution during operation.

The MDOF model is applied to a sub-surface crack just after the breakout. Once reached the free surface, an initially circular flaw tends to assume a series of omega shapes before taking a semi-elliptical form. Main aim of the present work is to demonstrate that starting from a generic omega shaped crack, it is possible to obtain the semi-elliptical configuration with the proposed MDOF method, inside the ® Workbench ™ environment. A peculiarity of the described work is the employement of mesh-morphing to generate the initial omega crack front. As a matter of fact when the sub-surface flaw assumes a generic shape it cannot be reproduced with the Fracture Tool (FT), embedded in ANSYS Mechanical.

2. Mathematical Background

2.1. Radial Basis Functions

In this section a brief excursus about Radial Basis Functions is given in order to facilitate the understanding of the mesh morphing technique, adopted in the reminder of this paper. However a wide description of mathematical background on RBFs, along with their applications, are provided in dedicated textbooks Buhmann (2004), Fasshauer (2007), Biancolini (2018).

RBFs are a very powerful tool capable to interpolate a scalar quantity, known at a set of given points (source), everywhere in the space (target). An interpolation function s is a series of weighted radial basis φ; some of the most common expressions are reported in Table 1. A radial basis is a defined scalar function of the Euclidean distance between the source and target points. If N is the total number of contributing source points,γ_i are the weights of the radial basis, it is possible to write

$$s(x) = \sum_{i=0}^{N} \gamma_i \varphi(||\mathbf{x} - \mathbf{x_i}||) \tag{1}$$

To increase the accuracy of the interpolation, it is possible to add a polynomial term $h(x)$ to Equation 1. An example of polynomial in three-dimensional space is:

$$h(x) = \beta_1 + \beta_2 x + \beta_3 y + \beta_4 z \tag{2}$$

Given the scalar nature of the interpolation procedure, a vector field can be handled component-wise as described in the next set of equations (Equations 3).

Table 1. Most common RBF functions.

RBF	$\varphi\,(r)$
Spline type (Rn)	r^n , n odd
Thin plate spline (TPSn)	r^n log(r), n even
Multiquadratic (MQ)	$\sqrt{1+\epsilon^2 r^2}$
Inverse multiquadratic (IMQ)	$\frac{1}{\sqrt{1+\epsilon^2 r^2}}$
Inverse quadratic (IQ)	$\frac{1}{1+\epsilon^2 r^2}$
Gaussian (GS)	$e^{-\epsilon^2 r^2}$

$$\begin{cases} s_x(x) = \sum_{i=0}^{N} \gamma_i^x \varphi(||\mathbf{x} - \mathbf{x_i}||) + \beta_1^\mathbf{x} + \beta_2^\mathbf{x}\mathbf{x} + \beta_3^\mathbf{x}\mathbf{y} + \beta_4^\mathbf{x}\mathbf{z} \\ s_y(x) = \sum_{i=0}^{N} \gamma_i^y \varphi(||\mathbf{x} - \mathbf{x_i}||) + \beta_1^\mathbf{y} + \beta_2^\mathbf{y}\mathbf{x} + \beta_3^\mathbf{y}\mathbf{y} + \beta_4^\mathbf{y}\mathbf{z} \\ s_z(x) = \sum_{i=0}^{N} \gamma_i^z \varphi(||\mathbf{x} - \mathbf{x_i}||) + \beta_1^\mathbf{z} + \beta_2^\mathbf{z}\mathbf{x} + \beta_3^\mathbf{z}\mathbf{y} + \beta_4^\mathbf{z}\mathbf{z} \end{cases} \tag{3}$$

The mesh morphing technique imposes a displacement field to a selected group of nodes. It is possible to enclose a limited number of nodes for the morphing action, namely those of the desired zones of the mesh, imposing a null movement to the nodes that wrap the affected area. It is well said that the outcome of the morphing operation and the preservation of mesh quality depend on the skill of the user that performs the task.

2.2. Crack Propagation

The propagation of a crack with a Multiple Degrees of Freedom (MDOF) model is a complex task and different mathematical aspects need to be managed. The first topic concerns the values of displacement, related to the flaw growth, that have to be imposed to the nodes of crack front, by means of mesh morphing. In particular the local increment Δa_i of the $i-th$ node, is calculated using a Euler integration algorithm (Paris and Erdogan (1963)) based on the Paris-Erdogan law (Equation 4).

$$\frac{da}{dN} = C\left(\Delta K\right)^m \tag{4}$$

In which C and m are material properties. Making use of the effective Stress Intensity Factors (SIFs), extracted from the $j-th$ Finite Element Analysis (FEA), and defining a starting value for Δa_{max}, it is possible to evaluate the growth increment of each node of the front normalized with respect to $\Delta K_{max}^{(j)}$:

$$\Delta a_i^{(j)} = \left(\frac{\Delta K_i^{(j)}}{\Delta K_{max}^{(j)}}\right)^m \Delta a_{max}^{(j)} \tag{5}$$

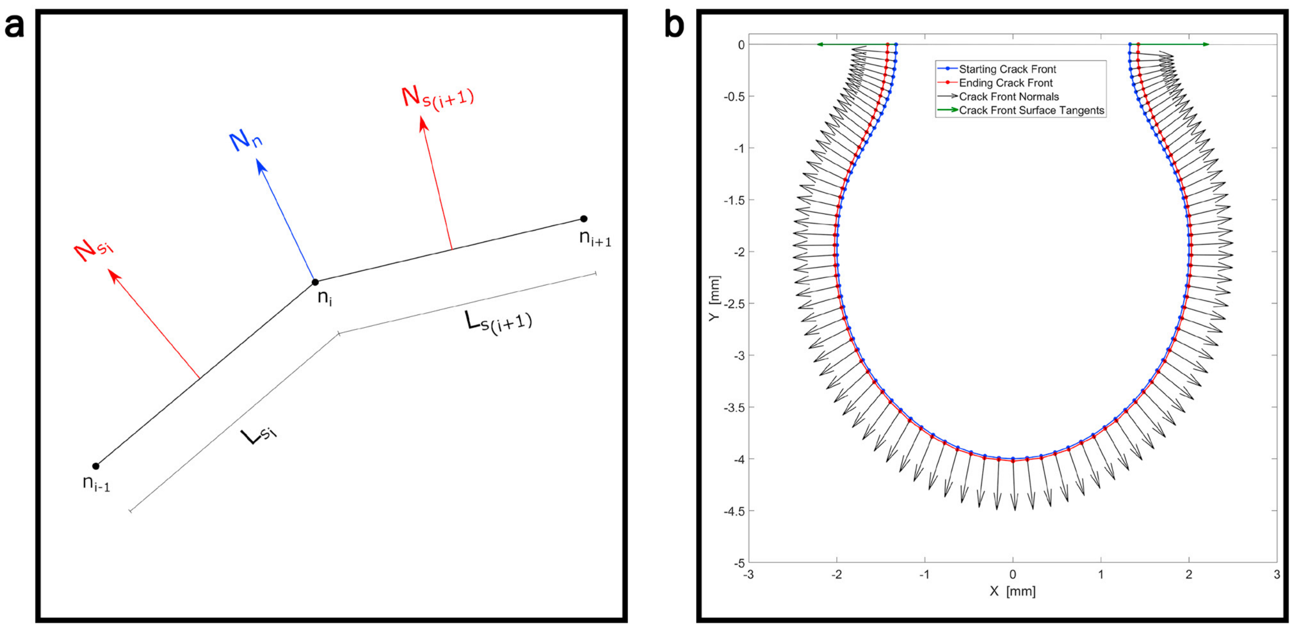

Fig. 1. (a) Crack front normal and growth direction; (b) Modulus and direction of the growth of crack front.

In addition, the maximum crack growth increment $\Delta a^{(j)}_{max}$, with the corresponding maximum SIFs $\Delta K^{(j)}_{max}$ are introduced in the following formula for the evaluation of the loading cycles:

$$\Delta N^j = \frac{\Delta a^{(j)}_{max}}{C(\Delta K^{(j)}_{max})^m} \tag{6}$$

So far, a question left open is the direction of the nodal displacements of Equation. 5. Each node on the crack front has two adjacent segments appertaining to the respective elements. The propagation direction is calculated as the weighted average of the normal vectors of each segment connecting to the node, being the weights the segments lengths. Referring to Fig. 1a, the nodal normal vector is evaluated with the following formula:

$$\overrightarrow{Nn} = \frac{\overrightarrow{Ns_i} \cdot Ls_i + \overrightarrow{Ns}_{(i+1)} \cdot Ls_{(i+1)}}{Ls_i + Ls_{(i+1)}} \tag{7}$$

where $\overrightarrow{Nn}$ is the growth direction on i^{th} node, $\overrightarrow{Ns_i}$ and $\overrightarrow{Ns}_{i+1}$ are the normal components of s_i and s_{i+1} segment respectively.

In Fig. 1b is depicted the growth of a generic crack, from the blue points (starting shape) to the red ones (final shape). The two nodes belonging to the free surface have growing directions (green arrow of Fig. 1b) tangential to the surface itself. In such a way, for small displacements the external nodes of the crack still remain on the surface.

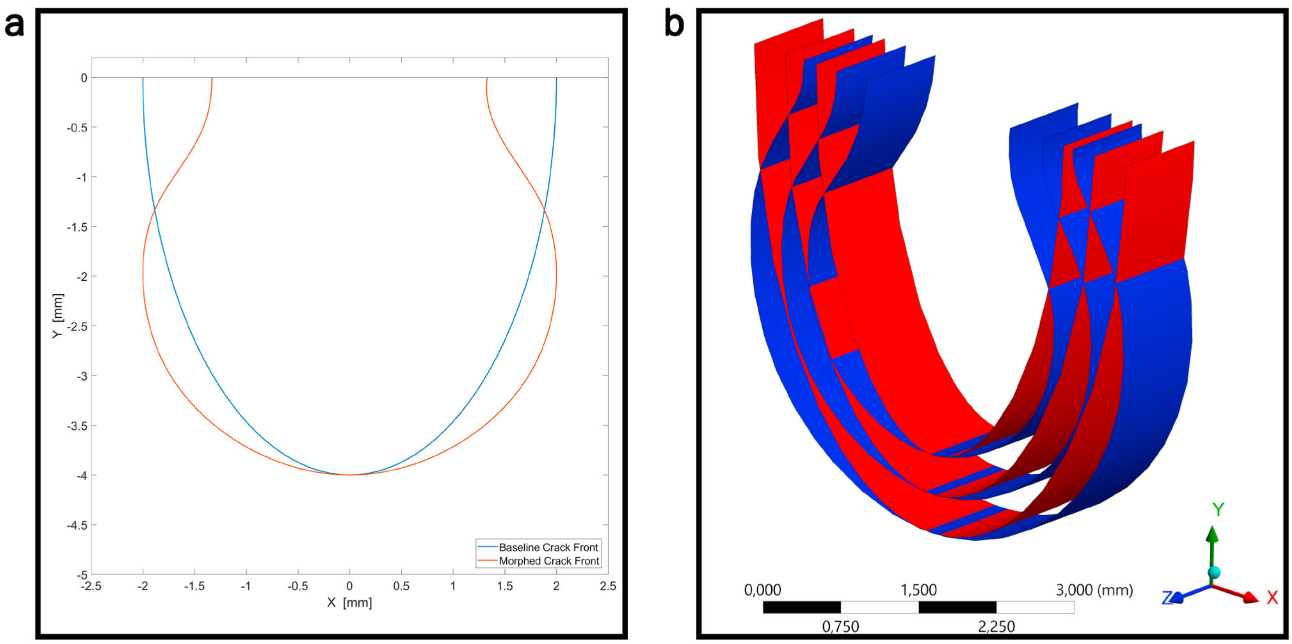

Fig. 2. (a) Semi-elliptical baseline (blue points) and morphed (red points) crack fronts; (b) Auxiliary surfaces adopted for morphing action: sources (blue surfaces) and targets (red surfaces).

3. Preliminary mesh morphing

The complexity of the growth of near-surfaces defects relies on the difficulty to model the interaction between the flaw and the free surface. In addition, as presented by Dai et al. (1998) the initial circular crack shape tends to assume a semi-elliptical aspect, passing through different omega shapes, because of the interaction with the free surface. The use of Fracture Tool embedded in ANSYS ® Mechanical, allows to model the initial or the final crack forms (semi-elliptical or circular shapes). Coupling this tool with RBFMorph™ ACT extension, it is possible to obtain the intermediate omega shapes Fig. 2a.

As in Dai et al. (1998), we assume an initial circular flaw with radius 1 mm, whose center is located 2 mm under the free surface. After the breakout at a certain number of cycles, the crack tends to assume a semi-elliptical shape with a minor radius of 2 mm and a major radius of 4 mm. This configuration, chosen as starting point for mesh morphing, was generated with Fracture Tool (FT) in ANSYS ® Mechanical. Quarter-point wedge elements are used around the crack front in order to model the stress field singularity.

Subsequently the mesh is morphed in order to obtain the desired omega shape. As reported in the next section, this flaw shape is used for the automatic growth of the defect, which ends with the crack assuming again a semi-elliptical shape. In order to avoid an unacceptable degradation of mesh quality because of the mesh deformation, it is convenient to provide information on the way the morphing operation was conducted. As previously said, wedge elements constitute the appropriate mesh topology to catch the singularity of the stress field. They are arranged in circles, with their tips converging on the line of the crack front. The intersection of the tube of elements along the flaw with the crack plane provides three curves: the central is the trace of the crack front, a tube radius shifts the inner and the outer offset lines from the centre. It is important to underline that an excessive deformation of the tube, in which the crack front is included, leads to an incorrect evaluation of the SIF values. To prevent the relative deformation of the cylinder, a set of auxiliary surfaces is employed. The three source surfaces, depicted in red in Fig. 2b, have the dimensions of the starting fracture (semi-elliptical shape). Each surface is located along one of the aforementioned curves. The target surfaces, the blue ones in Fig. 2b, have the desired omega shapes. These three surfaces are built preserving the relative distances, in order to achieve a proper tube deformation. The RBFs field is the one that moves each baseline surface onto its new position. The baseline flaw and the one subsequent to the mesh morphing, are depicted in Fig. 3

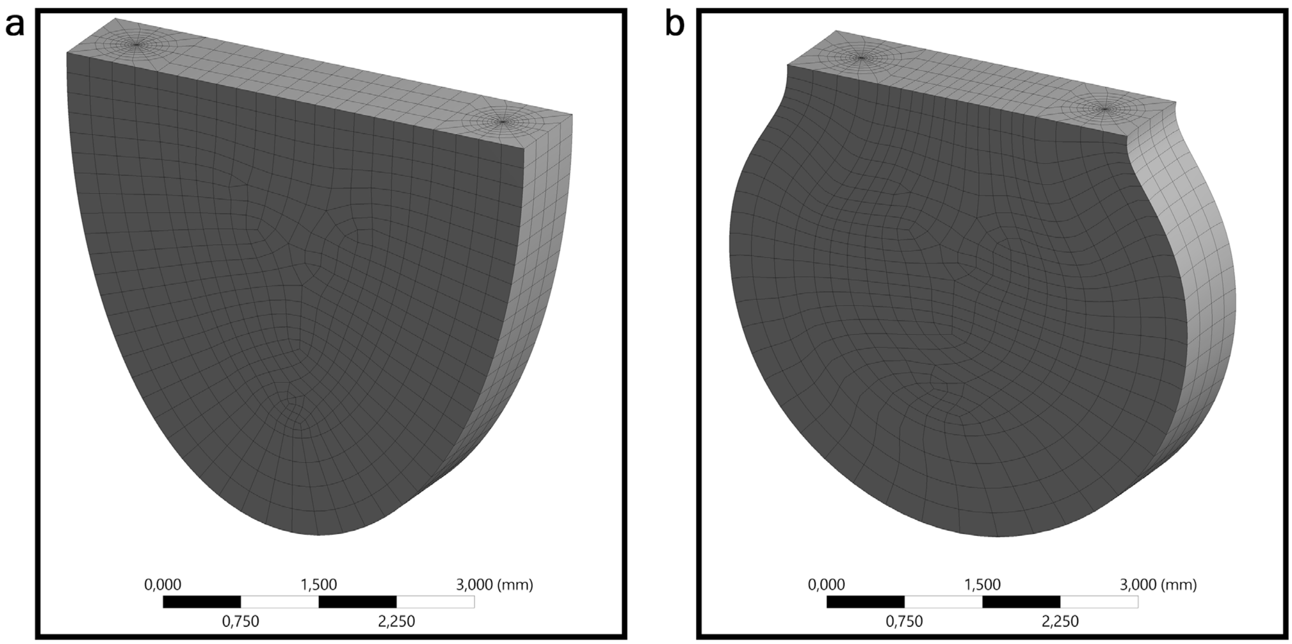

Fig. 3. (a) Baseline semi-elliptical crack shape; (b) Morphed omega crack shape.

4. Crack growth simulation

Once that the crack was morphed with the desired omega shape, it is possible to move forward with the simulation of its growth. Despite the symmetry of the model, the whole problem is represented in order to guarantee the presence of buffer elements all around the affected zone, for mesh deformation during the crack growth. The procedure can be divided in the next main steps: calculation of the effective stress intensity factors along the crack front by means of FEA, determination of crack front advances making use of Equation 5, mesh morphing to adjust the previous crack shape with the retrieved displacements. The proposed workflow is completely built in ANSYS ® Workbench ™ and allows an automatic growth of the defect. The main parameters (e.g. applied traction force and material properties) are extracted from Dai et al. (1998). Under a remote tensile force, the SIFs correspond to K_I in Anderson (2017): mode I of loading. Furthermore the dimensionless quantities described in Biancolini et al. (2018) are introduced in order to generalize the investigation. The dimensionless curvilinear abscissa and the dimensionless SIF, normalised with respect to h and reference stress respectively, are defined as follow:

$$\varsigma^* = \frac{\varsigma}{h} \tag{8}$$

$$K_I^* = \frac{K_I}{\sigma \cdot \sqrt{\pi a}} \tag{9}$$

Where σ is the remote applied stress range (i.e. $\Delta\sigma = 500MPa$) and a is the crack depth that is a function of the degree of advancement of the flaw. In Fig. 4a the progress of the crack front through 20 subsequent analyses is given. It is possible to notice that from the initial omega shape, the crack front finally tends to assume a semi-elliptical aspect,

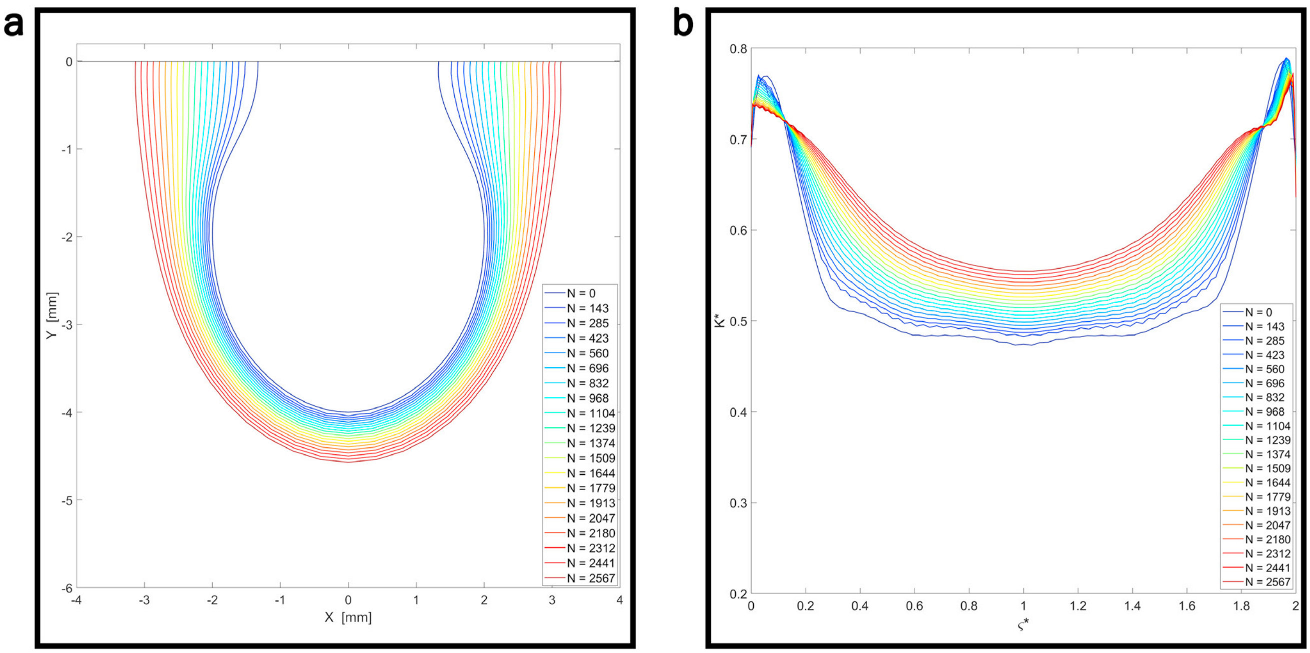

Fig. 4. (a) Crack front growth; (b) SIFs curves along crack front at different stages of the growth.

after 2567 cycles. Instead in Fig. 4b the normalized SIF curves, corresponding to the different crack fronts of Fig. 4a are depicted.

5. Conclusions

In the present paper an assessment of a growth simulation technique of near surface defects is presented. Due to the interaction of the flaw with the free surface, the crack front tends to assume complex shapes. An initial circular crack, completely located under the surface, after the breakout passes through a series of omega-shaped configurations converging to a semi-elliptical form. In this work, a generic omega shape was chosen for the crack front as starting point of the growth. To obtain such aspect, the mesh morphing strategy was adopted, the one of the tool RBFMorph ™, relying on Radial Basis Functions, and ANSYS ® Workbench ™ is the framework of investigation. The initial semi-elliptical crack was generated by means of the FT embedded in ANSYS Mechanical, subsequently used for the morphing actions. Once that the crack was morphed to assume the desired omega shape, it was possible to automatically predict the growth of the flaw, using a MDOF model. The displacements to be assigned to each node of the front can be calculated with the Paris-Erdogan law and applied to each point of the crack front according to the current value of nodal SIF. The final shape of the crack front after 2567 cycles (corresponding to 20 analyses) was semi-elliptical, as expected. It can be concluded that the proposed approach for the simulation of the generic crack growth leads to results consistent with literature. In addition, it can be stated that in the field of fracture mechanics, mesh morphing can give a substantial contribution, in particular with regards to the reduction of modelling time and the possibility to automate calculations.

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