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Procedia Engineering 199 (2017) 3332–3337

Procedia Engineering

www.elsevier.com/locate/procedia

X International Conference on Structural Dynamics, EURODYN 2017

Dynamic Characterization of the Eiffel Tower

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Abstract

The Eiffel tower is the most visited monument in the world. Millions of visitors have taken millions of pictures of it over the last century but apparently a dynamic picture (that is a dynamic characterization) does not exist or is not publicly available. In this paper we show the amount of information that can be extracted from a few recordings of ambient tremor collected on the tower and on the surrounding subsoil with a single pocket seismometer in a few minutes, during a leisure visit. We also propose a numerical model for the tower, capable to fit the observed data. This is interesting because the mass and stiffness distribution of the tower is unique and does not follow any modern construction rule. The dynamic model of the tower would also be important if Paris were a high seismic hazard town, which is not. According to our model, the tower could withstand peak ground accelerations >100% larger than the values prescribed by current seismic hazard estimates. The dynamic model of the tower is also important to better design the future interventions and to monitor the ageing of the structure.

Peer-review under responsibility of the organizing committee of EURODYN 2017.

Keywords: Eiffel tower; dynamic characterization

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1. Introduction

One of us (L.P.) is used to spend his vacations in Paris and one day, while watching his pictures of the town, another one of us (S.C.) proposed him to take a different picture of the town most famous structure. This said, on November 13th 2015, these two of us left for Paris with a pocket seismometer in the backpack and decided to climb the Eiffel Tower to measure its dynamic behavior, since surprisingly nobody has done it before, at least to our knowledge based on bibliographic research. In the literature, several studies about the tower shape [5, 11] and its foundations [1] can be found, as well as studies on the tower behavior to wind [3, 8, 9]. According to the French newspaper Le Moniteur [7], a 2 year project to create a numerical model of the tower was going to be realized by the Société d'Exploitation de la Tour Eiffel but we have not found any further information on this.

Peer-review under responsibility of the organizing committee of EURODYN 2017.
10.1016/j.proeng.2017.09.461

Before leaving, we bet with two structural engineers (M.B. and S.I.), expert in steel structures, about what to expect from the measurements and they liked the idea of joining us in the subsequent numerical modelling.
Countries characterized by high seismic hazard are used to measure the dynamic response of buildings in order to tune their numerical models and better plan retrofitting actions and/or to quantify the results of a retrofit. However, according to the Global Seismic Hazard Map [6] and its revisions (SHARE Consortium, [11]), seismic hazard in Paris is low (the bedrock peak ground acceleration with 10% exceedance probability in 50 years is equal to 0.02-0.04 g only) and this is possibly one of the reasons why a dynamic characterization of the Eiffel tower is not apparently available. Nevertheless, the dynamic characterization of a structure is relevant not only to the forecast or design its response to seismic actions but also, for example, to forecast or design its response to the wind actions. Gustave Eiffel studied the effect of wind on the tower but he could only approach this from a static point of view [4].
In addition to this, the dynamic characterization of the Eiffel tower is interesting because, beyond being the most visited paid monument in the world, the tower is very tall (more than 300 m) and light, which means that it is an unusual structure with a large stiffness and a small mass, whose behavior escapes any intuitive forecast.

In setting up a numerical model for the tower, another interesting fact arises: the non-structural masses of the tower changed over time. The original drawings of the Eiffel's engineering team are available [4]. In these drawings we could find the size and mass of every single structural and decorative element of the tower. However, it is easy to recognize that the non-structural masses are strongly different today, compared to 1889 and determining the distribution of the masses at present by fitting the numerical model with the experimental data is an interesting exercise.

2. Experimental survey

The Eiffel tower, one of the most iconic and popular buildings in the world, was designed by Gustave Eiffel and completed in 1889 for the World's Fair. From its construction to 1930, with its 324 m height, it was also the tallest building in the world. As anyone knows, it consists of a puddled iron lattice tower, divided into 3 levels. The foundations are 4 separate plinths for each leg, which means 16 separate foundations. These are set at a depth of 7 m, where, according to the trenches dug during the tower construction, sands and gravels are present. Before attempting any numerical modeling of the tower, we first characterized the dynamic behavior of the subsoil where it is founded and of the tower itself.

2.1. The subsoil

The excavations performed in 1887 for the construction of the tower foundations revealed silt and sand alternations in the first 20 m, with just a few meters thickness layer of chloritic limestone [4].
In order to get some quantitative information about the local subsoil properties, we performed a type of geophysical prospection that can be acquired with a single instrument (called H/V). This is based on the principle that the surface of the Earth is continuously excited by seismic waves produced by natural sources (e.g., atmospheric perturbations, wind, ocean waves etc.) and anthropic sources (traffic, human activities, etc.). These vibrations are enough to make the subsoil vibrate at its resonance frequencies, exactly as a building vibrates at its modal frequencies under any excitation. We applied this subsoil exploration technique around the Eiffel tower. Microtremor was recorded at 3 sites for 10 minutes at 512 samples per second with the 3-component tromometer Tromino® (MoHo s.r.l): a dominant resonance frequency at 2 Hz was clearly visible at all sites (Figure 1A).
A multichannel analysis of surface waves acquired in 2014 at a nearby site for a construction engineering work, was kindly provided by the company Miage (Laval, France, Figure 1B) and the joint fit of the H/V and dispersion curves allowed us to get a V_S (and therefore a stiffness) profile of the foundation subsoil of the Eiffel tower down to approximately 50 m, where the bedrock responsible for the 2 Hz resonance is located.

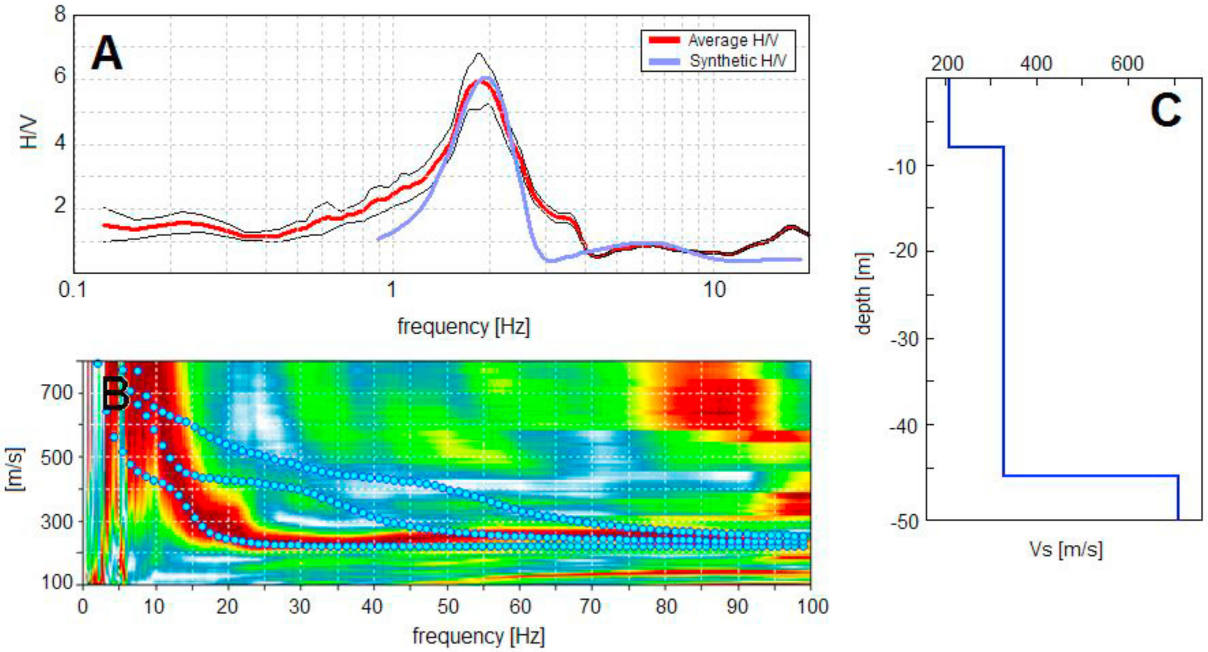

Figure 1. Joint fit of the H/V (panel A) and MASW (panel B) recordings to get a Vs profile of the subsoil (panel C).

2.2. The tower

The location, number and duration of the measurements inside the tower was conditioned by the accessibility (the centre of each platform, for example, is not accessible with the sole exception of level 2). The same portable seismometer used for the survey on the ground (10x7x14 cm, < 1 kg) was then placed inside a jewelry paper shopping bag (Figure 2) in order not to distract other visitors (whose transit close to the instrument would disturb the measurements) and we kept it stored inside the shopping bag during all the recordings.

We recorded ambient vibrations at the 3 platforms (55 m, 116, 270 m height), along the south pillar, as shown in Figure 2. At all levels we took two 10 minutes (1024 samples per second) recordings. On the second floor we managed to take an additional recording approximately in the centre of the platform, where the toilets are presently located. We used this recording to better recognize the torsion modes.

The observation of the spectral peak frequencies (Figure 3) allows us to identify the main vibration modes of the tower and the observation of the spectral inter-plane and intra-plane amplitudes allows us to make some inferences about the mode shapes. The first horizontal bending mode of the tower appears clearly at the frequency of 0.32 Hz. A second horizontal bending mode appears at 1 Hz and can be classified as bending because the linear velocity of motion of the central and peripheral locations is the same. A third mode appears at 0.8 Hz, which must be a torsion because the linear velocity of motion in the center tends to zero compared to the peripheral locations.

Then again we have two horizontal bending modes (1.4 and 1.7 Hz) and a second torsion mode (2.1 Hz). The experimental findings are summarized in Table 1. We computed that the fraction of the total mass moved in the horizontal component at mode 1 is 40%, followed by 5 modes with approximately the same importance (they move 5-7% of the total mass each).

3. Finite element model (FEM)

We constructed a geometrical model of the tower, faithfully reproducing the original drawing by Eiffel [4]. From these we derived both the structural and non-structural masses. Some of the latter were removed from the model because they do not exist anymore (e.g. some machineries installed between the second and the third floor that served for the elevators) while other non-structural masses were added because they did not exist in the original drawings (e.g., the radio antennas on the top and other devices). Details in the following discussion.

From the geometrical model, we built a 3-dimensional finite element model with the software MIDAS GEN rel. 8.4.0 (MIDASFEA), consisting in 3,472 nodes and 8,773 elements. The non-structural masses have been applied to 1,143 nodes. The beams have been divided only at the nodes connecting them with other beams, in order to avoid local vibration modes, not interesting in this study. The floors were modelled as stiff layers.

We used BEAM elements (6 degree of freedom end nodes grouped into 57 different sections), in place of the more common TRUSS elements, characterized by normal stress only, because some beams (e.g. the face diagonals) have a complex lattice structure with high flexion stiffness. The elements, which were built with puddled iron (i.e. low carbon content iron) were modelled as S355 according to the modern classification systems (European Norm 10025).

We connected the 4 foundations of each leg to the ground through 3-axial Winkler springs, characterized by vertical stiffness k = 3 kg/cm^3, applied to each of the 16 foundation elements (6 x 15 m each). However, we observed that the Winkler module does not affect the model result to a significant extent, probably because the vertical stress is only 0.8 kg/cm^2 (11,700 tons divided by 16 x 6 x 15 m^2), which can be supported by even poor quality subsoils at 7 m depth.

The model was tuned several times in order to match the experimental results. Our first attempt reproduced the original drawings of Eiffel (10,100 tons), with a dead load at the third level of 60 tons and the first bending mode that we got from this model was located at 0.45 Hz. The actual presence of the telecommunication installations on the top of the tower suggested a first increase of the mass on the top up to 250 tons, so that the first bending mode frequency shifted to 0.37 Hz, while upper modes remained substantially unaffected.

However, according to the article printed in Le Moniteur [7] the present mass of the tower is 11,700 tons. Since the structural elements have not changed since 1989, the extra mass has to be referred to dead loads at the different levels. We therefore distributed this dead load according to the surface of the levels, and also considering that the first floor is actually composed of 2 platforms. At the end, we increased the dead load at the first level of 800 tons, at the second level of 400 tons and at the top level of 360 tons. In this way we managed to reproduce the experimental modal frequencies, as described in Table 1 and illustrated in Figure 4.

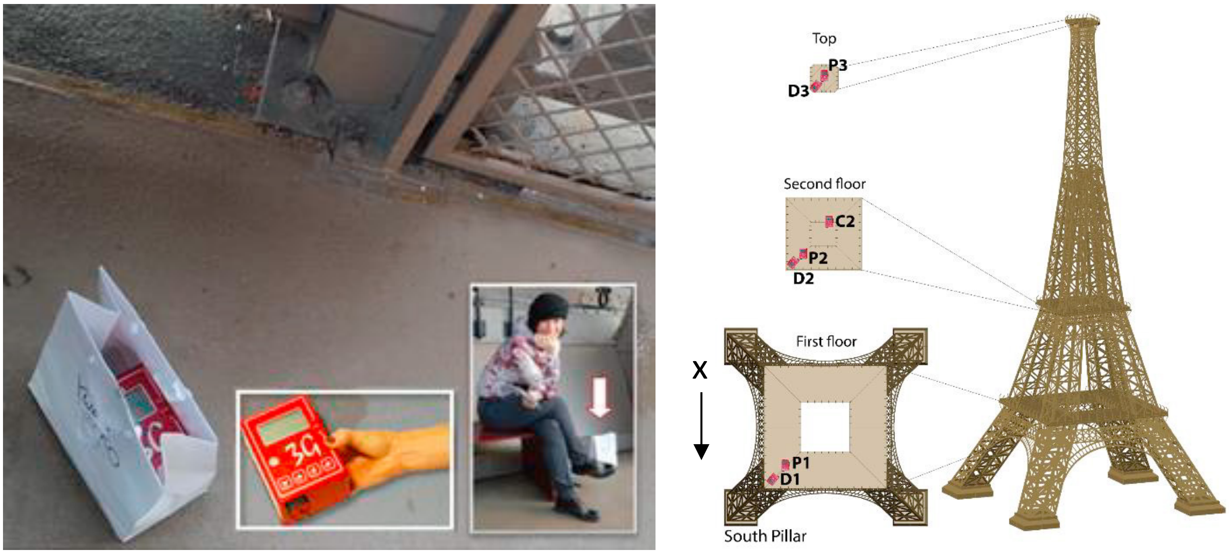

Figure 2. *Left:* the pocket seismometer (Tromino®) used in this study during a recording on the first level of the Eiffel tower. The frames illustrate the size of the device and the way we took the recordings. The instrument was placed inside a paper shopping bag to prevent visitors from getting close to it to ask questions. People transit too close to the instrument would affect its measurements. We sat/stood next to the bag for the whole duration of the recordings. *Right:* location of the measurement inside the tower and instrumental orientation. P stands for parallel (to the tower rim), D stands for diagonal, C for central. The number refers to the level.

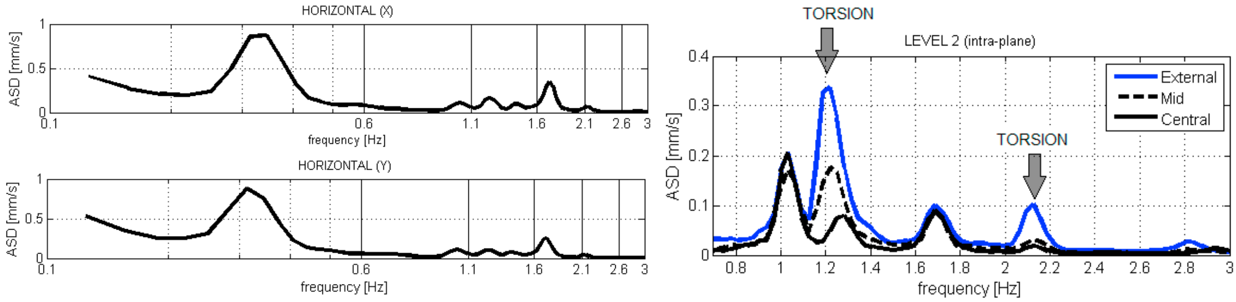

Figure 3. First bending/rocking mode of the tower (0.32 Hz) as measured at the top level. *Right:* horizontal spectra recorded at the second level of the tower in external, intermediate and central positions (D2, P2, C2 in Figure 1). Torsion modes can be easily recognized because the linear velocity at the centre of the level must be much lower compared to the peripheral positions. Bending modes, on the opposite, displace the same floor with the same amplitude. The mode sequence in this picture is therefore bending (1 Hz), torsion (1.3 Hz), bending (1.7 Hz) and torsion (2.1 Hz). ASD stands for Amplitude Spectral Density.

Table 1. Comparison of the experimental and the modelled modal frequencies and mechanisms of the Eiffel tower.

	EXPERIMENTAL		**MODELLED**	
No.	**Frequency [Hz]**	**Mode name/mechanism**	**Frequency [Hz]**	**Mode name/mechanism**
1	0.32	Horizontal bending (I)	0.33	Bending
2	1	Horizontal bending (II)	1.07	Bending
3	1.2	Torsion (I)	1.18	Torsion
4	1.4	Horizontal bending (III)	1.25	Bending
5	1.7	Horizontal bending (IV)	1.62	Torsion
6	2.1	Torsion (II)	1.8	Torsion
7			2.2	Torsion

4. Discussion and conclusions

The Eiffel tower is the most visited and popular monument in the world. It is also a unique structure. While there exist hundreds of architectural drawings of the tower, we could not find any dynamic characterization of its behavior. In this paper we show that such an important structure can be sufficiently characterized in a fully passive, fast and cheap way, under operative conditions. We achieved this goal by taking basically 4 measurements (1 per level at the external rim plus a central one) of 6-10' minutes, that is in less than one-hour work.

The modal frequencies of the tower appear as clear peaks in the spectra of the recorded motion. The observation of the peak amplitudes allowed us to reconstruct the mode shapes and the percentage of the total mass moved by each mode. While the mode frequencies are correctly identified even with a single instrument moved at different locations, the mode shapes are correctly reconstructed - in the passive analysis - only under the hypothesis of stationary vibration source. If this requirement does not hold, then a reference instrument is needed. We also note that working with a single instrument the phase information, that is needed to reconstruct the mode shapes beyond the first bending mode (characterized by displacements with the same sign at any level), is not available. What we got is therefore the absolute value of the mode shape, which is nonetheless informative.

We found that the first bending mode, which moves approximately the 40% of the tower mass in the horizontal plane, has a frequency of 0.32 Hz. The displacement on the top of the tower due to this motion is almost 2 orders of magnitude larger than on the second floor. Large rocking (which shows up as a large amplification of the vertical spectra at this frequency) is observed at all levels and implies the ground participation to the tower movement. The soil-structure interaction should therefore not be disregarded in the model.

The following 5 modes (bending, torsion, bending, bending, torsion) move 5-7% of the mass each in the horizontal plane.

We also explored the subsoil around the tower. According to the drillings of the late 1800, more than 15 m of alluvial sediments (sands and silts) are present under the tower. We actually measured a clear resonance frequency at 2 Hz that, modelled jointly with a surface wave dispersion curve available at a nearby site, stands for the local bedrock at approximately 50 m depth and for an average shear wave velocity of 250 m/s in the shallow alluvial layers.

Assessing the resonance frequencies of structures and soil is important in seismic countries to identify conditions of double resonance [2]. In this case the subsoil resonance frequency coincides with the frequency of the 6th mode, which is a torsion.

The modal analysis of the tower turns out to be interesting also because the tower is very tall and light, which means that it is an unusual structure with a large stiffness and a small mass, whose behavior escapes intuitive forecasts.

It is quite impressive what can nowadays be done in terms of structural characterization with a few minutes of 'stolen' recordings on a structure, with no need for bulky equipment, induced excitations, sophisticated software. Our 1-hour work on the tower is certainly not the most accurate survey than could be planned on such a structure but it shows to be enough to characterize the first 6 modes of the tower, that is what really matters in the standard engineering practice.

As many times in the past, the non-structural masses of the tower are going to change several more times in the future with the replacement of the elevators and of the antennas on the top, with different shops at the lower levels, etc. The knowledge of the dynamic response of the tower is important in the engineering practice to better design the future interventions and to monitor the ageing of the structure.

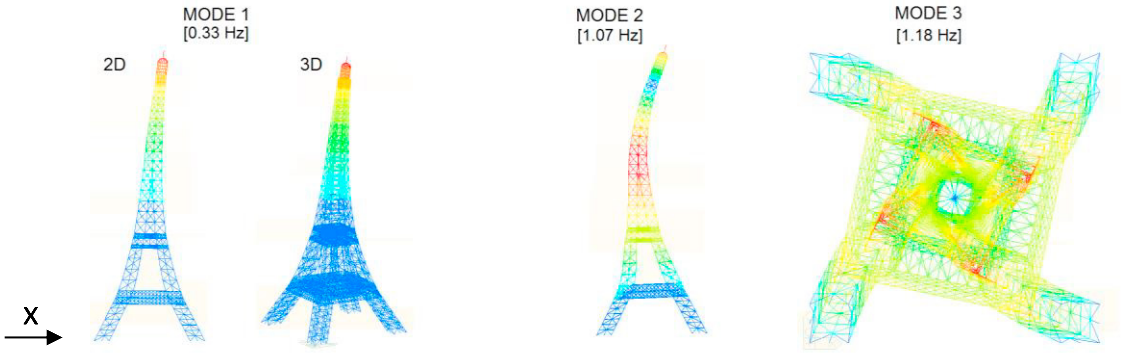

Figure 4. Mode shapes of the first 3 modes of the Eiffel tower from the FEM. Torsion modes (3) are shown in the horizontal plane. Bending modes (1, 2) are shown in the vertical plane.

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