

Research Article

A Note on Torsion of Nonlocal Composite Nanobeams

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The ERINGEN elastic constitutive relation is used in this paper in order to assess small-scale effects in nanobeams. Structural behavior is studied for functionally graded materials in the cross-sectional plane and torsional loading conditions. The governing boundary value problem has been formulated in a mixed framework. Torsional rotations and equilibrated moments are evaluated by solving a first-order differential equation of elastic equilibrium with boundary conditions of kinematic-type. Benchmark examples are briefly discussed, enlightening thus effectiveness of the proposed methodology.

1. Introduction

Assessments of stress and displacement fields in continuous media are a subject of special interest in the theory of structures. Numerous case studies have been examined in the current literature with reference to beams [1–6], half-spaces [7, 8], thin plates [9, 10], compressible cubes [11], and concrete [12, 13]. Several methodologies of analysis have been developed in the research field of geometric continuum mechanics [14, 15], limit analysis [16–19], homogenization [20], elastodynamics [21–25], thermal problems [26–28], random composites [29–32], and nonlocal and gradient formulations [33–38]. A comprehensive analysis of classical and generalized models of elastic structures, with special emphasis on rods, can be found in the interesting book by IËŞAN [39]. In particular, IËŞAN [40–42] formulated a method for the solution of SAINT-VENANT problems in micropolar beams with arbitrary cross-section. Detailed solution of the torsion problem for an isotropic micropolar beam with circular cross-section is given in [43, 44]. Experimental investigations are required for the evaluation of the behavior of composite structures [45].

In the context of the present research, particular attention is devoted to the investigation of scale effects in nanostructures; see, for example, [46–55] and the reviews [56, 57]. Recent contributions on functionally graded materials have been developed for nanobeams under flexure [58, 59] and torsion [60].

Unlike previous treatments on torsion of gradient elastic bars (see, e.g., [61]) in which higher-order boundary conditions have to be enforced, this paper is concerned with the analysis of composite nanobeams with nonlocal constitutive behavior conceived by ERINGEN in [62]. Basic equations governing the ERINGEN model are preliminarily recalled in Section 2. The corresponding elastic equilibrium problem of torsion of an ERINGEN circular nanobeam is then formulated in Section 3. It is worth noting that only classical boundary conditions are involved in the present study. Small-scale effects are detected in Section 4 for two static schemes of applicative interest. Some concluding remarks are delineated in Section 5.

2. Eringen Nonlocal Elastic Model

Before formulating the elastostatic problem of a nonlocal nanobeam subjected to torsion, we shortly recall in the sequel some notions of nonlocal elasticity. To this end, let us consider a body \mathcal{B} made of a material, possibly composite, characterized by the following integral relation between the stress \mathbf{t}_{ij} at a point \mathbf{x} and the elastic strain field \mathbf{e}_{ij} in \mathcal{B} [62]:

$$\mathbf{t}_{ij}(\mathbf{x}) = \int_{\mathcal{B}} K(|\mathbf{x}' - \mathbf{x}|, \tau) \mathbb{E}_{ijhk}(\mathbf{x}') \mathbf{e}_{hk}(\mathbf{x}') dV. \quad (1)$$

The fourth-order tensor $\mathbb{E}_{ijhk}(\mathbf{x}')$, symmetric and positive definite, describes the material elastic stiffness at the point $\mathbf{x}' \in \mathcal{B}$.

The attenuation function K depends on the Euclidean distance $|\mathbf{x}' - \mathbf{x}|$ and on a nonlocal dimensionless parameter defined by

$$\tau = \frac{e_0 a}{l}, \quad (2)$$

where e_0 is a material constant, a is an internal characteristic length, and l stands for external characteristic length.

Assuming a suitable expression of the nonlocal modulus K in terms of a variant of the BESSEL function, we get the inverse differential relationship of (1) between the nonlocal stress and the elastic deformation

$$(1 - (e_0 a)^2 \nabla^2) \mathbf{t}_{ij} = \mathbb{E}_{ijhk} \mathbf{e}_{hk} \quad (3)$$

with ∇^2 the Laplacian. Note that (3) can be conveniently used in order to describe the law between the nonlocal shear stress field $\boldsymbol{\tau}_i$ on the cross-section of a nanobeam and the elastic shear strain $\boldsymbol{\gamma}_i$ as follows:

$$\boldsymbol{\tau}_i - (e_0 a)^2 \frac{d^2 \boldsymbol{\tau}_i}{dx^2} = \mu \boldsymbol{\gamma}_i, \quad (4)$$

where x is the axial direction and μ is the shear modulus.

3. Torsion of Nonlocal Circular Nanobeams

Let Ω be the cross-section of a circular nanobeam, of length L , subjected to the following loading conditions depicted in Figure 1:

$$\begin{aligned} m_t, \\ \text{distributed couples per unit length in the interval } [0, L], \\ \mathcal{M}_t, \\ \text{concentrated couples at the end cross-sections } \{0, L\}. \end{aligned} \quad (5)$$

The triplet (x, y, z) describes a set of Cartesian axes originating at the left cross-section centre \mathbf{O} .

Equilibrium equations are expressed by

$$\frac{dM_t}{dx} = -m_t, \quad \text{in } [0, L], \quad (6a)$$

$$M_t = \mathcal{M}_t, \quad \text{at } \{0, L\}, \quad (6b)$$

where M_t is the twisting moment.

Components of the displacement field, up to a rigid body motion, of a circular nanobeam under torsion write as

$$\begin{aligned} s_x(x, yz) &= 0, \\ s_y(x, yz) &= -\theta(x)z, \\ s_z(x, yz) &= \theta(x)y, \end{aligned} \quad (7)$$

where $\theta(x)$ is the torsional rotation of the cross-section at the abscissa x . Shear strains, compatible with the displacement field equation (7), are given by

$$\begin{aligned} \gamma_{yx}(x, y, z) &= -\frac{d\theta}{dx}xz, \\ \gamma_{zx}(x, y, z) &= \frac{d\theta}{dx}xy, \end{aligned} \quad (8)$$

where

$$\chi_t(x) = \frac{d\theta}{dx}(x) \quad (9)$$

is the twisting curvature. The shear modulus μ is assumed to be functionally graded only in the cross-sectional plane (y, z) .

The elastic twisting stiffness is provided by

$$k_t = \int_{\Omega} \mu(y^2 + z^2) dA, \quad (10)$$

where the symbol \cdot is the inner product between vectors.

The differential equation of nonlocal elastic equilibrium of a nanobeam under torsion is formulated as follows. Let us preliminarily multiply (4) by $\mathbf{Rr} := \{-z, y\}$ and integrate on Ω

$$\int_{\Omega} \mathbf{Rr} \cdot \boldsymbol{\tau} dA - (e_0 a)^2 \int_{\Omega} \mathbf{Rr} \cdot \frac{d\boldsymbol{\tau}}{dx^2} dA = \int_{\Omega} \mu \mathbf{Rr} \cdot \boldsymbol{\gamma} dA, \quad (11)$$

with the vector $\boldsymbol{\gamma} := \{\gamma_{yx}, \gamma_{zx}\}$ given by (8).

Enforcing (6a) and imposing the static equivalence condition

$$M_t = \int_{\Omega} (\tau_{zx}y - \tau_{yx}z) dA, \quad (12)$$

we get the relation

$$M_t(x) + (e_0 a)^2 \frac{dM_t}{dx}(x) = k_t \frac{d\theta}{dx}(x). \quad (13)$$

This equation can be interpreted as decomposition formula of the twisting curvature χ_t into elastic $(\chi_t)_{\text{EL}}$ and inelastic $(\chi_t)_{\text{IN}}$ parts

$$\chi_t = (\chi_t)_{\text{EL}} + (\chi_t)_{\text{IN}}, \quad (14)$$

with

$$(\chi_t)_{\text{EL}} = \frac{M_t}{k_t}, \quad (15a)$$

$$(\chi_t)_{\text{IN}} = \frac{(e_0 a)^2}{k_t} \frac{dM_t}{dx}. \quad (15b)$$

Accordingly, the scale effect exhibited by the torsional rotation function of a nonlocal nanobeam can be evaluated by solving a corresponding linearly elastic beam subjected to the twisting curvature distortion $(\chi_t)_{\text{IN}}$ (15b).

4. Benchmark Examples

Let us consider a nanocantilever and a fully clamped nanobeam of length L subjected to the following quadratic distribution of couples per unit length:

$$m_t = \frac{m}{L^2} x^2. \quad (16)$$

The cross-sectional torsional rotation is evaluated by following the methodology illustrated in the previous section.

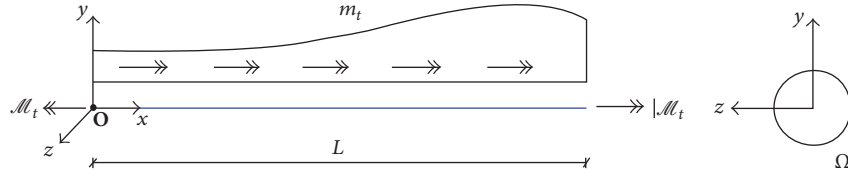
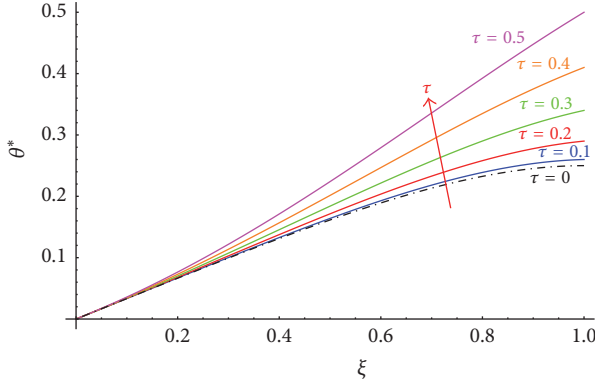
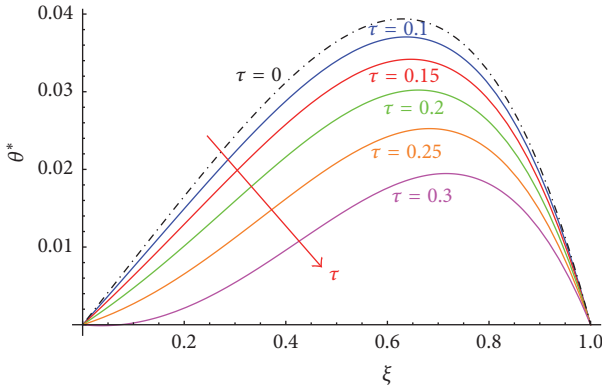


FIGURE 1: Sketch of a nanobeam under torsion.


 FIGURE 2: Torsional rotation θ^* for the nanocantilever versus ξ for increasing values of τ .

 FIGURE 3: Dimensionless torsional rotation θ^* for the fully clamped nanobeam versus ξ for increasing values of τ .

The nonlocality effect is assessed by prescribing the twisting curvature distortion equation (15b)

$$(\chi_t)_{,IN} = \frac{(e_0 a)^2}{k_t} \frac{2m}{L^2} x \quad (17)$$

on corresponding (cantilever and fully clamped) local nanobeams. Let us set $\xi = x/L$ and $\theta^*(\xi) = k_t/(mL^2)\theta(\xi)$. Torsional rotations θ^* versus ξ of both the nanobeams are displayed in Figures 2 and 3 for selected values of the nonlocal parameter $\tau := e_0 a/L$.

5. Concluding Remarks

The basic outcomes contributed in the present paper are listed as follows:

- (1) Size-effects in nanobeams under torsion have been evaluated by resorting to the nonlocal theory of elasticity.
- (2) Exact torsional rotations solutions of cross-sections of functionally graded nanobeams have been established for nanocantilevers and fully clamped nanobeams under a quadratic distribution of couples per unit length.
- (3) It has been observed that the stiffness of a nanobeam under torsional loadings is affected by the scale parameter and depends on the boundary kinematic constraints. Indeed, as shown in Figures 2 and 3, contrary to the nanocantilever structural behavior, the fully clamped nanobeam becomes stiffer for increasing values of the nonlocal parameter.

Competing Interests

There is no conflict of interests related to this paper.

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