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Distributions with given marginals: the beginnings

An interview with Giorgio Dall'Aglio

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Giorgio Dall'Aglio graduated in Mathematics from the University of Pisa. In 1956, he started his studies in Probability and Statistics under the supervision of Giuseppe Pompilj in Rome. Over the years he also studied in France and in the United States. In 1968, he was appointed Professor of Probability at La Sapienza (University of Rome), succeeding his mentor Pompilj, and additionally became Director of the Istituto di Calcolo delle Probabilità and the Scuola di Perfezionamento in Ricerca Operativa. He maintained these responsibilities for 15 years. Later, he also served as dean of the Department of Statistics.

In the fourth interview of the series, Dependence Modeling presents a conversation with Giorgio Dall'Aglio, an Italian mathematician and probabilist who is internationally acknowledged as one of the main contributors to the theory of *Distributions with Given Marginals*. In addition to describing his career path and his achievements in mathematics and probability, Giorgio Dall'Aglio portraits the several milestone mathematicians he met during his long career. In the following, our questions to Giorgio Dall'Aglio are typeset in bold-face.

1 Early career

How did you start your academic career? Which topic did you enjoy most during your university studies?

I completed my studies at the University of Pisa, starting in 1950, and at the same time I was a student of the Scuola Normale Superiore. It was a very good arrangement: we had room and board, which was very helpful since my parents were not rich (even more valuable considering the difficult times soon after the war), and could follow extra courses. It was an excellent education, aimed at an academic career and the fact that it included classes in humanities helped to open our minds and to give us a classical, as well as a scientific preparation. For this, I am indebted to an excellent teacher, Elodia Santollino, whom I had in the last year of high school. She noticed my potential in mathematics and prompted me to apply for the competition at the Scuola Normale. In fact, she acted as a mentor for me and provided extra classes, which turned out to be very valuable to enter the school. At the university, I was especially interested in analysis

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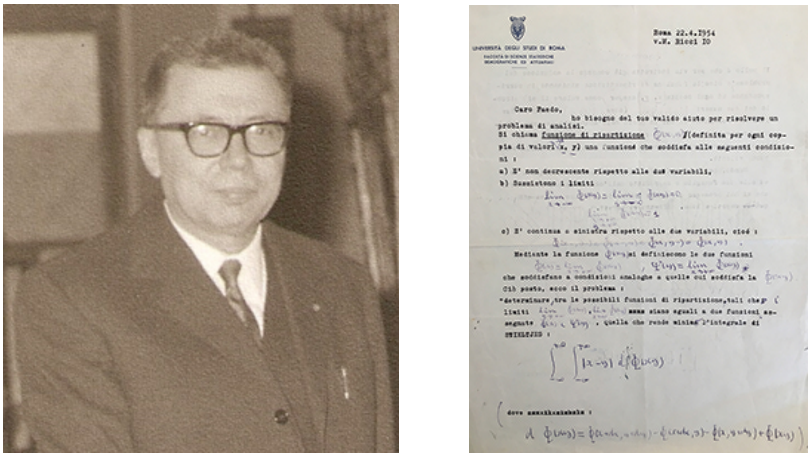


Figure 1: The letter by which Giuseppe Pompilj (on the left) asked Alesandro Faedo about the solution of problem (1).

and the foundations of mathematics (due to a very good old professor, Francesco Cecioni), while I did not particularly care about geometry or physics. The teaching of calculus was unfortunately not very complete, but supplementary courses were available at the Scuola Normale and I took advantage of discussions with older students. From them, I learnt about an interesting function, which helped me make a good impression on the professor of analysis Alessandro Faedo. Later, I asked him to be my supervisor for my graduation dissertation, and he proposed the search for extreme values of the integral

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} |x-y|^{\alpha}\, dF(x,y), \tag{1}$$

where $F(x,y)$ is a nondecreasing function having fixed, nondecreasing limit functions

$$F_1(x) = \lim_{y\to+\infty} F(x,y) \quad \text{and} \quad F_2(y) = \lim_{x\to+\infty} F(x,y),$$

and $\alpha \geq 1$. One recognizes at a glance a classical problem of *Distributions with Given Marginals* (DWGM). Professor Faedo provided general advice in finding the solution, and I obtained good results, which were subsequently published in a paper in the *Annali della Scuola Normale Superiore*; see [4]. The paper was very successful due, I must say, more to the formulation of the problem (which was not my idea) than to its solution. The problem had been posed to Faedo by his friend in Rome Giuseppe Pompilj. I still have Pompilj's letter, which Faedo showed me, adding that he already knew the solution for the discrete case; see Figure 1. This was my first encounter both with the theory of DWGM and with Giuseppe Pompilj. My big opportunity followed several months later when Pompilj asked Faedo to recommend a young researcher in analysis. I was chosen, and at the beginning of 1956 I moved to the Faculty of Statistics (the full name was "Facoltà di Scienze Statistiche Demografiche e Attuariali") in Rome. Thus, I had the double fortune of being offered an excellent path for my scientific career as well as finding a remarkable mentor in Giuseppe Pompilj.

When I arrived in Rome in 1956, I was hired (at first temporarily) as an assistant professor, and later asked to give some lectures in probability and operations research. In 1967, I won a competition for a chair in probability. It was the first examination on this subject ever held in Italy, and I was favoured because the committee was composed of Professors de Finetti and Pompilj, who knew me and appreciated my work, and three mathematical analysts, who, although not able to judge my skills in probability, knew me well and were satisfied with my mathematical skills. To be fair to myself, I should add that a friend of mine, whose works were specifically in mathematical statistics, was not admitted, even if he was supported by de Finetti and Pompilj as well. My first service was at the University of Catania, Sicily. After only one year, Professor Pompilj sadly passed away, and I was called up to succeed him in Rome. In those days, it was not common to give an important chair such as the one at the university in Rome to a fairly new and inexperienced professor, but

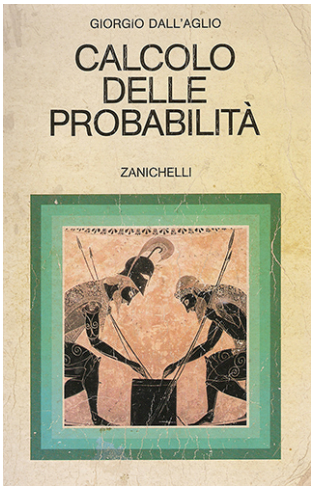


Figure 2: The cover of the book *Calcolo delle Probabilità* (*Probability Theory*, see [15]) by Giorgio Dall'Aglio, in its first edition as published by Zanichelli, Bologna, in 1987. On the original cover one can read the names of the players, Achilles and Ajax, and the words they pronounce, "four"and "three".

fortunately for me, the faculty of the university chose to maintain the unity of the excellent group of students that Pompilj had formed.

This early departure from Catania was not my intention, nor was the faculty of the University pleased, as they had expected me to stay for some years in order to launch the probability studies in the school; but the exceptional circumstances made the situation somewhat acceptable. As Pompilj's successor, I received, in addition to the chair, the direction of both the Istituto di Calcolo delle Probabilità and of the Scuola di Perfezionamento in Ricerca Operativa, two entities founded by Pompilj. I felt honoured and very pleased to be back in Rome, but these assignments were a challenge for an inexperienced professor. Other tasks were piled on top of the previous ones, for instance in the faculty board I was the only mathematician (except for an actuary) and I was also responsible for the courses of analysis and geometry. These commitments distracted me from my research, which afterwards never returned to its previous level.

I did gather great satisfaction from my book, *Calcolo delle Probabilità* ([15], see Figure 2), which was published in 1987. It grew out of a manuscript within the department over a period of five years and was very well received by both students and colleagues. Initially, it sold approximately 700–800 copies per year to practically all Italian students in probability. This was not surprising, considering that at that time there were still very few professors qualified enough to create a course of their own from scratch. The sales of course decreased over time, but there were also the photocopy rights. In 2012, ten years after I had retired from teaching, the book sold about one hundred copies, but the photocopies were generating approximately 400 EUR per year of revenues, more or less 6.000 pages copied.

What was the role of your family in the development of your early academic career?

My parents had very little schooling: my father finished middle school and my mother had only completed three years of primary school. In her small town Cavallino (in southern Italy, near Lecce) there were no schools available for further education and at that time in history in southern Italy, a girl was not allowed to attend school far from home (but some of her brothers did go to university). Despite their lack of formal schooling, they were intelligent and they gave my brothers and me a sound moral and cultural education. My father chose for me the *Liceo Classico*, which is the Italian high school that mainly focuses on Humanities (in those days, the choice was normally made by the parents). I very much liked classical culture; in fact, for many years I enjoyed reading classics in Latin. Even then, however, my favourite subject was mathematics, and although it may seem strange, the classical high school was the best choice for this as well. Derivatives and integrals (which would in any case be taught anew at university), were replaced by a more sound study of the fundamentals. In fact, when I arrived at the university, the professors advised the students coming from

the scientific high school to forget what they had learned and to start all over! It was also at my high school that I encountered Professor Santollino who helped me enter the Scuola Normale in Pisa. My parents made great sacrifices to send all of their children to university, we helped where we could by taking advantage of grants and tax exemptions (two of us were admitted to the Scuola Normale). And I believe that we made them proud: two of us became university professors, another one manager at ENI and one became a chief physician. Unfortunately, my youngest brother died at eighteen, shortly after obtaining his high school diploma with very good marks. We have always thought that he was the most brilliant of us and would surely have achieved great things in life. Our family career path is a good example of social mobility.

And I love my present family. My wife Laura was a high-school teacher of humanities. She is a wonderful mother for our three children and has various interests which she can cultivate better after retirement: classical music, art, human rights with Amnesty International. Our three children, two boys and a girl, are *bravi ragazzi*, affectionate with us. Only the eldest inherited his father's love for mathematics, and teaches with high dedication mathematics in high school. The others have different jobs: the second is a student of the Italian Renaissance and the third one, the girl, is an architect. Unfortunately, the present crisis is taking its toll on them as well.

My wife and I are practicing Catholics and are very interested in ecumenism. My first exposure to this movement was in 1960, before the II Vatican Council, in France (in Italy at that time this type of experience was not easily encountered). I remember a time in the Cité Universitaire, where the offerings at the mass were dedicated to the protestant chapel, which was in construction. Today, we continue practicing our faith, participating in a bible study group together with some Waldensians.

Which scientist would you consider most influential to your career?

From my previous answers, the important role played by Giuseppe Pompilj is clear, and I would like to spend some words on his accomplishments, which in my opinion, are not as well-known as they should be. Probability was still practically absent in Italian universities, despite the presence of such high-level students as de Finetti and Cantelli. The diffusion of the study of probability, mathematical statistics, and operations research can be credited to the work of Professor Pompilj.

Pompilj began his research in geometry, as a student of Federico Enriquez, but also of Francesco Severi and Guido Castelnuovo. Called up for military service during World War II, he was taken prisoner in a British camp in India, were he was detained under favourable conditions, which even allowed him to continue his studies. During his detention he found, in a small library of the camp, some writings in statistics, among which *Mathematical Statistics* by Aitken [1], which attracted his interest. Upon his return to Italy after the war, he succeeded in winning a competition for a chair in geometry. An encounter with Corrado Gini definitively changed the course of his scientific and academic life. Gini was an eminent statistician who, due to the autarchic attitude of fascism and the war, knew little about and did not appreciate the huge developments abroad. However, he recommended the hiring of Pompilj at the Faculty of Statistics in 1948. It is interesting to note that though Pompilj was officially assigned the chair of geometry, he in fact had to teach probability (which previously was taught only to students of actuarial science) and mathematical statistics. His main accomplishment no doubt is that he was able to foster the academic developments of these new, emerging fields of mathematics. Numerous foreign teachers and scientists were invited to Rome and he also managed to send his Italian students abroad; I was the first one, having received the opportunity to spend two years in France and one year in the USA. He also created within the school an Istituto di Calcolo delle Probabilità and, along with the Istituto Centrale di Statistica, an annual course in mathematical statistics for researchers in science and medicine.

Together with his students, Pompilj also did a considerable amount of more applied work. In particular, he very much supported statistical research related to medicine and science in general. One of his most important achievements was his collaboration with the Istituto Centrale di Statistica in the launch of sample surveys. He fought just as hard for the promotion of operations research, creating the Scuola di Perfezionamento in Ricerca Operativa. Pompilj was amongst the founders of the Italian Association in Operations Research. Pompilj's relevance is proven by the fact that a representative of the Istituto di Calcolo delle Probabilità was nominated as a permanent member of the scientific committee. I too, after him, received the honor of this position; but I was not interested, and quit soon after my appointment. In 1968, at 54, he died of an aneurysm.

In only twenty years, he had revolutionized statistics in Italian universities and created a solid group of students. More than ten of them attained a university chair; an unbelievable success considering that just one had succeeded before his death and that most of the appointed chairs were in statistics, where he never really had achieved academic authority.

I remember a chat I had once with Jimmie Savage, who remarked upon the scarce scientific works by Pompilj. I explained to him that most of the research conducted by his students (me in the first place) were in fact, started by him. Pompilj was also awarded a Gold Medal of the President of the Republic for his contribution to science and culture.

Let me conclude by mentioning an anecdote which demonstrates Pompilj's generous personality. Two years after my arrival, he asked me to write the third chapter of a booklet ([37]) which would discuss the plans of experiments. Although my contribution was only a small portion of the book, it was published with two authors; moreover, he said that he had received 300.000 ITL, and one third was for me. This type of generosity was not common in Italian universities.

2 Contributions to mathematics and probability

Do you feel you are more of a mathematician or a statistician? Or do you see no difference among these two sciences?

My formation in mathematics has always taken first place in my career. To be honest, I must say that I arrived in Rome with some reservations concerning statistics. Under the influence of Pompilj, I however did get involved with traditional Italian statistics, and more importantly with mathematical statistics. I should add that the Faculty of Statistics also had the important and interesting role of exposing me to the various sciences connected with real life: economy, demography, sociology, even some law and medicine; these underlined the function of statistics, which therefore was no longer just an abstract study to me. However, I always maintained my connection with the milieu of mathematics. For several years, starting in 1980, I was on the scientific committee of the National Group for Functional Analysis and Applications of the National Research Council (CNR). I was responsible for probability, which was already well established and already had some chairs at Italian universities. I managed to stimulate interest in mathematical statistics as well. For a time, I also directed a national research group for probability of the Mathematical Committee of the CNR.

I was also a member of the Council for Economic, Sociological and Statistical Sciences of the CNR and there is a funny anecdote regarding this period in my career. At noon, my colleagues and I usually had lunch in a trattoria near our office in Rome. We would ask for one bill, which we split, but the payment was annoying since finding the exact amounts of change took some time. Educated in subjective probability by Pompilj and de Finetti, I proposed that we would assign the payment each day to one of us chosen randomly. The proposal caused some argument among our group, but finally was accepted. Of course then it was always me that each time tossed a coin to choose the payer. So to come back to your question of mathematics versus statistics, in my career, I have been very fortunate to have been able to contribute to both areas. I am especially proud to have been able to support the development of probability theory and mathematical statistics in Italy.

In Italy, probability and statistics were not considered as scientific disciplines until recently. How was your personal path to them?

This judgment is too severe. In the first half of the twentieth century in Italy, statistics was well developed, in connection with demography and sociology, especially through Gini and his school. Its presence in universities was also good. The School of Statistics and Actuarial Science started in Rome in 1926, and again in Rome in 1936 the Faculty of Statistics was founded, through the hard work and commitment of Gini and Castelnuovo. What was missing was a connection to the important work which was being conducted abroad.

It was different for probability, which unfortunately did not have its own established coursework: it was included within the courses of Actuarial Science. Even de Finetti, although a full professor of Mathematics for Economics since 1946, attained the chair of Probability only in 1961. Research in probability was very active though, with Castelnuovo, Cantelli, de Finetti, and Ottaviani amply contributing to the area.

Both Castelnuovo and Cantelli were concerned with the organization and promotion of these fields. From a research point of view, Castelnuovo was interested in the mathematical foundations of probability and problems related to convergence. In [3], Cantelli introduced probability through functions on the measurable subsets of $[0, 1]$, in a way anticipating Kolmogorov's theory. Cantelli's contributions are by now well recognized, as for instance through the the famous Glivenko-Cantelli theorem and the Borel-Cantelli law. The liveliness of research in probability was further evidenced by the *Giornale dell'Istituto Italiano degli Attuari*, created by Cantelli and Castelnuovo, from its birth in 1930 until its suspension at the start of the war in 1941; Cantelli was president of the Istituto and director of the Giornale. It contained many important papers by the best probabilists: de Finetti, Doob, Fréchet, Glivenko, Khintchine, Kolmogorov, Lévy, Von Mises, besides numerous papers by Cantelli. For instance Glivenko and Kolmogorov established the convergence of distance between distributions (in the same issue and quoting each-other; see [29, 34]). Cantelli also introduced the convergence in probability $X_n \xrightarrow{p} c$ of a sequence of random variables X_n to a constant c; see [2]. Fréchet extended the definition of Cantelli in [24] as $|X_n - X| \xrightarrow{p} 0$, considering as limit an arbitrary random variable X.

At the time, I was surprised to note that the *Giornale* was written entirely in Italian; evidently then the authors believed that their work would find interested readers also in the Italian language. Later I was told that at that time Italian was actually the official language of the International Congress of Actuaries.

We suppose that you also did practical applications.

I recall one remarkable experience providing technical expertise with de Finetti on a case for the Court of Rome in 1972. There was a legal action about a public auction for road works involving the public company ANAS and some important people. A prosecutor noticed a strange accumulation of the bids and wished to explore what probability could say about it. He called me thinking that I, as director of a probability institute, was the best choice; but I knew better, and I hastened to ask Bruno de Finetti to join the case. I was happy to give him that satisfaction, since he was further convinced of the necessity of probabilistic expertise in the administration of justice. I had often heard him talking of that with Pompilj, who had also written a paper on the subject in the more widely read journal *Archimede*; see [36].

Sadly, Pompilj had died some years before; I am sure it would have been a great satisfaction for him to participate. We had numerous interesting discussions, mainly involving Bayes' theorem, about the case and very much thought of ways how to explain our findings to the judge involved. Unfortunately it never came so far as the trial was suspended. Jimmie Savage told us about a similar case in California, 1968; but some errors in the calculations discredited it. Following that event, Bayes theorem was often employed in the Anglo-Saxon world, however not without controversies often reported by the press. I explored also the historical use of probability in the administration of justice, which began in France shortly after the birth of probability; see [18].

Among the experts involved in the trial there was Giuseppe Ottaviani. He had proved in [35], shortly after Raikov (he actually learned about Raikov's work [38] when correcting the proofs of his article) that if the sum of two independent random variables has a Gaussian distribution, the same is true for the individual summands. He proved the same result for the Poisson distribution giving an elementary proof. Ottaviani was at the time president of the Italian Istitute of Actuaries and director of its *Giornale*.

I also had the opportunity to do some minor work for a company that wanted to start a lottery. They wanted to understand whether the income of a single day would be sufficient to pay the prizes of that same day, or whether they should constitute a reserve beforehand.

Also the Army and Navy posed problems which eventually were closely linked to Operations Research. During my military service, I had the occasion to enter an operations research group of the Army, which had been instituted because of the influence of foreign armed forces. I would later join a group established by the Navy together with the Istituto Centrale di Statistica, directed by Pompilj, which conducted several studies. Soon after I started teaching operations research, and I was for several years the lecturer of game theory and statistical decisions, a course for which I also wrote a book ([13]).

The paper *Fréchet classes and compatibility of distribution functions* (see [14]) continues to gather citations after 44 years. Will it be cited in 44 years, in 2060?

I presented the paper in 1971 at an international symposium organized by the Istituto di Alta Matematica, an institution for doctoral teaching, in which I too have given some courses. I recalled the origins and the

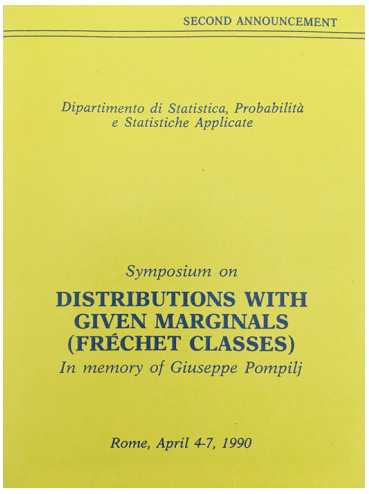


Figure 3: Giorgio Dall'Aglio (left) in 1990 during the symposium held in Rome on "Distributions with Given Marginals (Fréchet classes)". This picture has been taken by Abe Sklar. The poster announcing the symposium is shown on the right.

developments of DWGM, most of them by people with whom I was in contact. At the time, copula theory was not developed beyond the basic paper [42] by Sklar. This becomes clear when reading Berthold Schweizer's historical talk in Rome in 1990; see [41]. Sklar took up the topic again in 1973, see [43], but now benefitting from previous research on the modeling of dependence and association.

My contribution was, I think, a neat and complete exposition; it is not surprising that somebody still quotes it. The proceedings of the symposium held in Rome, which appeared 20 years later (see [20]), are much more exhaustive and detailed, but rather less well known. My paper [14] may be useful from a historical perspective of the field. Whether or not it will still be remembered in 44 years, time will tell.

Which other research work of yours would you like to mention?

My first papers, after the ones originating from my dissertation, were about regression and mean difference. I later examined the asymptotic behaviour of the estimators involved, generalizing a result by Hoeffding.

I further studied the asymptotic distribution when normality is not assumed, finding conditions for the χ^2 distribution in [10, 12]. I also worked on some problems of renewal processes, starting from the paper [9] written at Chapel Hill under the direction of Walter Smith. Some conditions for asymptotic normality were established and actuarial considerations elaborated in [8].

Several papers dealt with distributions; some specific distribution (as in the paper [7] written in Paris under the direction of Daniel Dugué); and other aspects, particularly their decomposability. By that, I mean that a distribution can be written as the convolution of two other distributions. In that case, I exploited some connection with DWGM as well as with copulas in [16]. Regarding convergence, I showed the connection between different forms of convergence of random variables (in law and in probability) and DWGM in [11]. I have also written some isolated papers on random walks, theory of games, and experimental design.

As I have said, I was also interested in teaching. In the sixties probability began to be taught in high school (before it was taught only in some professional schools). There were not many teachers sufficiently prepared, and the low level of the textbooks was evident, a fact which de Finetti had fiercely denounced. I collaborated with some journals and attended meetings with teachers.

You were the chairman of the organizing committee for a symposium held in Rome, 1990, on "Distributions with Given Marginals (Fréchet classes)"; see Figure 3. The proceedings [20] of this meeting provide a great overview of the historical developments and the most recent results until 1990. What are, in your opinion, the most important further developments since 1990?

This symposium was, for me, a major achievement. I got the idea during a conference, while chatting with Stamatis Cambanis who later became one of the invited speakers. I made some inquiries with people I

Figure 4: The participants of the conference *Distributions with Fixed Marginals, Doubly Stochastic Measures, and Markov Operators* held in Seattle (see the proceedings in [39]) in 1993. Giorgio Dall'Aglio is the first standing on the left of the front row.

knew to sound out the idea and received favorable answers: Berthold Schweizer in his lecture [41] recalled that he had responded with an enthusiastic yes. Most importantly, however, I wrote to Sam Kotz, whom I had met during my time at Chapel Hill, and who I thought could be the best guide for organizing the symposium because of his vast knowledge of research across the world, thanks also to his great work with Norman Johnson for the Encyclopedia of Statistical Sciences. He was glad to accept, and the scientific committee was then completed with Josef Štěpán from Prague. It had a large and geographically distributed attendance (eleven countries). Strangely, there was no one attending from France, although in the past French scientists had greatly contributed to the subject.

The main merit of the conference was the confluence of two streams, the one of DWGM and copulas. They are substantially the same object of study: the difference is that a copula is a distribution whose support is contained in $[0, 1]^n$; a functional transformation allows the passage between a copula and a general distribution, rendering the results more general. The main difference in the treatment is that copulas typically use the tools of probabilistic metric spaces. My introduction, *Frechét classes: the beginnings* (see [17]) gave an account of the history and results regarding the first item; the lecture by Schweizer, *Thirty years of copulas* (see [41]), was a deep and thorough analysis on the second item. It is not surprising then that there was successive work and a subsequent six symposia: Seattle (1993, see Figure 4), Prague (1996), Barcelona (2000), Québec (2004), Tartu (2007), and Sao Paolo (2010). I have attended only the first three and I have not followed the subsequent developments; I will only point out that the applications moved in various directions, among which I remember most clearly mathematical finance.

Copulas are a useful tool for studying DWGM. Many people believe that work on copulas all started with Sklar's theorem. It seems, however, that the notion of a copula is present in a work of Wassilly Hoeffding ([30]) and there are connections with early work of you, Féron, and Fréchet, to name a few. Can you clarify and discuss who else deserves credit in this regard?

As a matter of fact, Wassilly Hoeffding introduced and studied DWGM and also copulas, with the range $[-1/2, 1/2]$ instead of $[0, 1]$. Unfortunately, his paper [30] appeared in a virtually unknown German journal in 1940 and because of the war it went completely unnoticed. Therefore, it was left aside. In any case, the first

step was in the thirties, as part of the study by Gini and his school on the dissimilarity index

$$\frac{1}{n}\sum_{r=1}^{n}|x_r - y_r|.$$

This is a distance between the two sequences $\{x_r\}$ and $\{y_r\}$; with a suitable transformation one obtains a two-way table, which is a distribution function in the discrete setting. The minimal dissimilarity, i.e. the maximal association, was found, as well as the table giving it corresponding to the maximal distribution function. This table is called the "tabella di cograduazione", built by Tommaso Salvemini (1939) in [40]; it is known in operations research as the *North-West rule*. This is why Pompilj told Faedo that the solution of (1) was already known for discrete marginals. So the core of DWGM was already laid in these pieces of research, but that is where it stopped.

Modern research was started by Maurice Fréchet in 1951 with his famous paper *Sur les tableaux de corrélation dont les marges sont données* [26]. His interest came from talks with Paul Lévy about extreme values of a distance between distributions, about which Lévy inserted a section in Fréchet's book [25]. Fréchet introduced and studied DWGM in a rather simple context, finding the extreme distributions with their extreme association. He showed their relationship to linear programming as well.

But his major merit was to create interest in the problem among other students, especially Robert Féron, Abe Sklar, and Pompilj. In 1959, Sklar published his famous paper [42] which introduced copulas, entitled *Fonctions de répartition à n dimensions et leurs marges* which underlines the strict relationship between the two sectors. He took the subject up again some years later, making way for the huge work to be done on it.

Pompilj put his students to work – I was the first one – involving other researchers from the faculty as well, such as Alfredo Rizzi, Giuseppe Leti, Giampiero Landenna, Salvatore Bertino, and Amato Herzel.

My chief contributions were the generalization of Fréchet's results, together with some other topics, in particular related to compatibility. For instance the existence of 3-dimensional distributions with given bivariate marginal projections; see [5].

I came to an interesting conclusion in this field: I found that in the set of 3-dimensional distribution functions with fixed univariate marginals, a minimal distribution function exists under some restrictive assumptions. Obviously, this minimal distribution has minimal projections on each subspace; see [6]. It is a result that is contrary to intuition, since when we have two random variables both minimally associated to a third one, we expect a maximal association between them. This result was further generalized to n-variate distributions in [14] and I have also shown the relationship with convergence in probability; see [11].

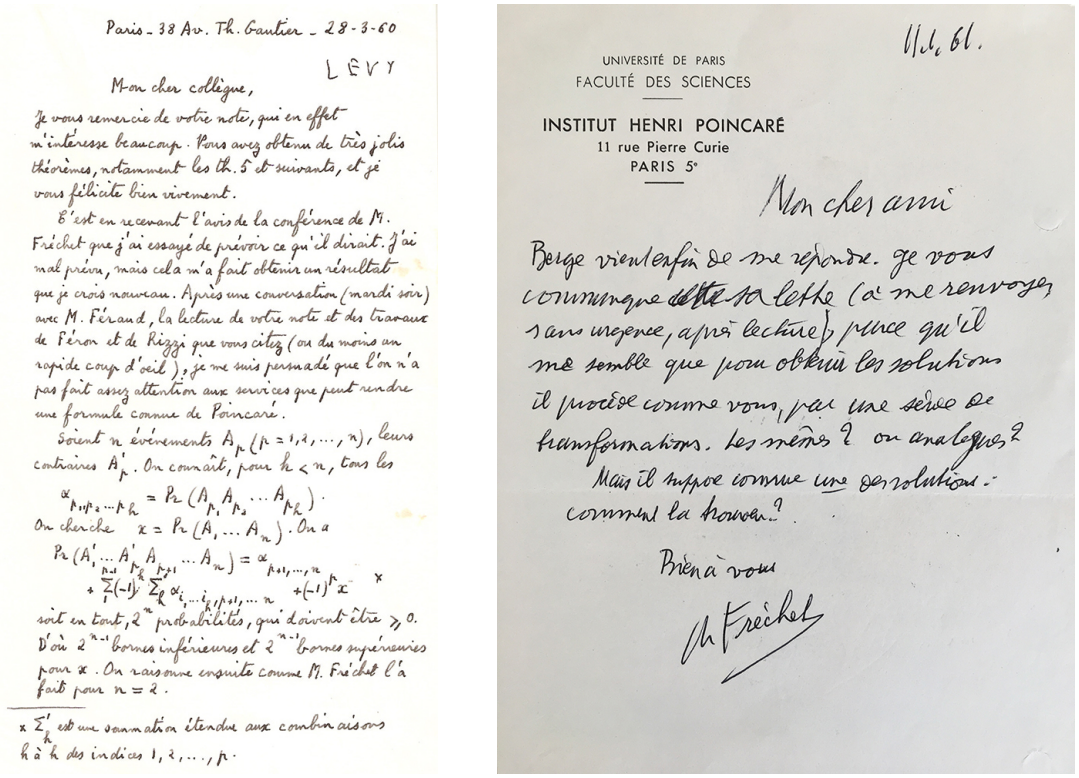
Without going further, I will only quote the early contributions [32, 33] by H.G. Kellerer and by V. Strassen, [44], who also dealt with stochastic processes. The convergence between the two streams occurred, as already said, with the symposium held in Rome.

3 Milestone personalities in Probability and Statistics

During your time at Chapel Hill and Paris you must have made contact with famous researchers such as Lévy, Fréchet, Hoeffding etc. What does that mean to you personally and what memories do you associate with them? Who is missing in this list?

I spent two academic years (1960-1962) at the Institut de Statistique de l'Université de Paris at la Sorbonne (at that time, if I remember correctly, the only university in Paris). Then one year (1962-1963) at the Department of Statistics at the University of North Carolina, Chapel Hill. It was part of the program to study abroad that Pompilj had organized. Other students took part in the program after me. During my stay, I attended some courses and gave some lectures, but mainly I continued my research.

In Paris, I was a student of Daniel Dugué, a remarkable person with regards to both his work and his human qualities. I was able to publish [7] under his direction. Maurice Girault was also part of the faculty with whom I worked. I met also Maurice Frechét, but I saw him more often in Rome. And I met Paul Lévy. He



Paris - 38 Av. Th. Gautier - 28-3-60

LÉVY

Mon cher collègue,

Je vous remercie de votre note, qui en effet m'intéresse beaucoup. Vous avez obtenu de très jolis théorèmes, notamment les th. 5 et suivants, et je vous félicite bien vivement.

C'est en recevant l'avis de la conférence de M. Fréchet que j'ai essayé de prévoir ce qu'il dirait. J'ai mal prévu, mais cela m'a fait obtenir un résultat que je crois nouveau. Après une conversation (mardi soir) avec M. Férand, la lecture de votre note et des travaux de Féron et de Rizzi que vous citez (ou du moins un rapide coup d'oeil), je me suis persuadé que l'on n'a pas fait assez attention aux services que peut rendre une formule comme de Poincaré.

Soient n événements A_p ($p = 1, 2, \dots, n$), leurs contraires A'_p. On connaît, pour $k < n$, tous les

$$\alpha_{p_1 p_2 \dots p_k} = P_2(A_{p_1} A_{p_2} \dots A_{p_k}).$$

On cherche $x = P_2(A_1 \dots A_n)$. On a

$$P_2(A'_1 \dots A'_{p_1} A_{p_1+1} \dots A_{p_2} A'_{p_2+1} \dots A_n) = \alpha_{p_1+1, \dots, n} + \sum_{1}^{p_2-p_1} (-1)^k \sum'_k \alpha_{i_1 \dots i_k, p_1+1, \dots, n} + (-1)^{p_2-p_1} x \quad *$$

soit en tout, 2^n probabilités, qui doivent être ≥ 0. D'où 2^{n-1} bornes inférieures et 2^{n-1} bornes supérieures pour x. On raisonne ensuite comme M. Fréchet l'a fait pour $n = 2$.

* $\sum'_k$ est une sommation étendue aux combinaisons k à k des indices $1, 2, \dots, p$.

11/4/61.

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Mon cher ami

Berge vient enfin de me répondre. Je vous communique sa lettre (à me renvoyer sans urgence, après lecture), parce qu'il me semble que pour obtenir les solutions il procède comme vous, par une série de transformations. Les mêmes ? ou analogues ?

Mais il suppose connue une des solutions. - comment la trouver ?

Bien à vous

M. Fréchet

Figure 5: A letter from Paul Lévy (left) and a letter from Maurice Fréchet (right) as received by Giorgio Dall'Aglio in 1960 and, respectively, 1961.

was still interested in DWGM – he pointed out to me an inequality of Poincaré (see Figure 5), and sent me a student with whom I was to work on this subject.

At the Department of Statistics in Chapel Hill I did some work on normal convergence with Walter Smith, but I met many remarkable statisticians there; besides Wassilly Hoeffding there were Harold Hotelling, Norman Johnson, and Samuel Kotz, whom I have already quoted for the symposium. I am certain that my research benefited from this stimulating environment.

I also met Joe Gani (who later would invite me to Sheffield). He surprised me by addressing me in Italian, and explained that he came from a Hebrew family of the Aegean islands, which had spoken and maintained the Italian language since the Venetian domination only to abandon it when Italy became allied with Germany.

However, I had many stimulating encounters in Rome as well. For instance Andrey Kolmogorov. I attended one of his lectures, which was followed by a short interval in order for him to conduct some tourism. He wished to visit Umbria; so two young people from the department and I took him on a one-day tour. Since my parents were living in Perugia, I thought it would be nice to have a simple dinner with them and I called my mother asking her to prepare her hand-made "tagliolini". My mother was not embarrassed to have the president of the Academy of Sciences of the USSR as a guest. We also accompanied him to the railway station, as he did not travel by air.

Fréchet came several times to Rome and, of course, we discussed DWGM; he said once that at this stage of his career he studied only easier subjects (he was past 80). I enjoyed his interesting conversation, not only on the topic of mathematics, such as one time in 1958 when we animatedly discussed the power take-over by Charles de Gaulle, whom most Italians considered authoritarian. He was interested in Esperanto; I once heard him and Savage talking in this artificial language.

Figure 6: Herman Chernoff receives a Laurea honoris causa from La Sapienza (University of Rome) in 1996. Giorgio Dall'Aglio is the first standing on the left.

Herman Chernoff came twice to Rome; he and his wife Judy became close friends with me and my wife. I was afterwards a guest of them in California. He received a Laurea honoris causa from my university in 1996; see Figure 6.

Henry Teicher was also a friend; I remember that he was often reading the second volume of Feller's book on probability, which had just appeared. Later we visited Naples with him and his wife Ann.

Kai Lai Chung was a well-preserved old man. He had resisted an assault by three young boy thieves. I liked his book, which also helped me for mine.

Jimmie Savage was never invited by Pompilj; he came several times to Rome to visit de Finetti, whom he admired. As a matter of fact, de Finetti's view of probability had oriented much of his work. He was a genial person, with whom I had interesting talks, especially on subjective probability. He was my sponsor in my application to the International Statistical Institute; other sponsors were Pompilj, de Finetti, Chernoff, and Fréchet; it is thus no wonder that I was elected.

I remember also, for operations research, Maurice Kendall, Philip Morse, and Steven Vajda.

As an important Italian mathematician, you have met important personalities like Corrado Gini and Bruno de Finetti and many others. Can you describe them and your collaboration with them?

Gini and de Finetti were extraordinary people and merit a larger discussion. Corrado Gini dominated statistics, demography and sociology in Italy for a long period of time. He was the president of ISTAT, the Italian Statistical Institute, founded by him, and from which he resigned after conflicts with Mussolini. He was for many years the dean of the Faculty of Statistics as well as a member of the Accademia dei Lincei, and of many other institutions both in Italy and abroad. He was a hard worker, and he demanded the same from his students (a Latin inscription on a bust which would describe him perfectly, reads: "to nobody he imposed higher burdens than to himself"). His lucidity of mind is apparent in his paper *I pericoli della statistica* [27], in which he underlines some not so evident errors; he also moves critics from a subjective probability viewpoint to the Anglo-Saxon statistical analysis, about which he had a correspondence with Fisher as reported in [28]. His most important statistical contributions were in descriptive statistics, with frequent references to probability and induction; Gini's index is well known and discussed in connection with the Lorenz curve; it is currently quoted in Italian newspaper articles on wealth inequality. Gini's mean difference is also very well known.

What is more interesting for me, however, is his work about the relationships between variables. He was moved by his dissatisfaction with the current indexes, especially the correlation coefficient, about which he criticized the variability of its range with given marginals. So he introduced the dissimilarity index and the

mean difference; and from them, as I have already said, the study of DWGM was born. Of course, I remember with appreciation and gratefulness his action in favour of Pompilj and statistics.

His renown was not only confined to Italy. He delivered lectures in France, at the London School of Economics, and at Harvard, where he received a Laurea honoris causa. I met with him frequently for a number of years; I remember him always arriving by bicycle. Once he asked me to complete an analysis of mean and variance for some data, telling me that he wanted to write a critical report. He also asked me to cooperate for a new edition of a book he was writing, but the project was never realized. I assisted a few times in his examinations.

Bruno de Finetti, on the other hand, was mostly interested in real life. He had a strong civil engagement towards social justice and against bad public administration. He was once even arrested, for his collaboration with a newspaper in a pacifist campaign. His research often originated from applications. He did some practical work as well, directing the installation of information systems at Assicurazioni Generali (AG), where he worked until he moved to University of Trieste in 1946. At AG, he was also the head of the office for rationalization, a duty very much in line with his personality. He wrote numerous important papers in analysis, statistics, economy, finance, and operations research. His first paper [21] (published at the age of twenty) was about Mendelian inheritance; it made such an impression on Gini that he promised to hire him at ISTAT when he graduated. It was also noticed in the USA by Alfred J. Lotka, at that time a true celebrity in the field. He also completed important work in probability. His introduction of subjective probability [22] is well known; for him it was the mathematical development of a way of thinking that should always be informative to people. Maybe less known are other important contributions such as the introduction of exchangeable events and random variables, as well as random processes with independent increments, two tools which permitted remarkable advancements in the field. And the use of characteristic functions, with important results in convergence, where he focused especially on the convergence to the Gaussian.

We should also remember his work in teaching, strongly criticizing many existing books. I had the honor of collaborating with him only on the didactic side of his work, but I considered him a mentor, as well as a friend.

4 Concluding words

Apart from your academic life, what are your interests?

Although I have never seriously practiced a sport, I have kept moving, which I think has helped me to reach the age of 84 in a reasonably satisfying physical condition. I recall that upon my return from foreign sojourns (1963) my weight was 113 kilos. In a few years, with little effort, through mountain walking, some tennis, and Mediterranean diet, I had lost 25 kilos. As a boy, during the summer holidays, I spent a lot of time swimming in the beautiful sea near Lecce (they say that it is still beautiful). When I was about 18 years old, I loved biking. I was then in Perugia with my family. Two of my brothers and I, (we were close in age), riding old bicycles upon which we had mounted the three-speed gears ourselves, took trips around Umbria and Tuscany, touching also the Adriatic sea (150 km).

I also have wonderful memories of two big trips, made rather poorly, with a tent for two, an Army surplus, and a few pans, camping in the meadows (organized campsites were rare at that time, especially in the south), with some elementary cooking on a fire of dry twigs, visiting sites and chatting with people. In 1952 we went south, arriving in Lecce where our mother's relatives lived. Back in Rome, my brothers and I were received by the Pope together with a group of a catholic association; see Figure 7.

In 1953 we went north, arriving at the Stelvio pass (near the border with Austria), made legendary for us by the exploits of cyclists Bartali and Coppi: 1500 km in a month.

For many years, I would go mountain hiking in the summer with my wife and children, and I did some climbing along with a guide. I also played some tennis, badly because I learned it at a later age, but still satisfying, and I continued that sport until just a few years ago.

Figure 7: Giorgio Dall'Aglio (first from left on the bottom row) with Pope Pio XII in 1952 in Rome.

Last, I must mention singing. When I was young, I really liked mountain and regional songs; Italian regions have different dialects (even some languages) and I knew songs for all of them. After retirement, I began to sing in a choir of baroque music, and each year we hold three concerts, mostly of Händel, the musician preferred by our director.

Any further remark from your side?

Before concluding, I would like to recall some passages from my farewell lecture (see [19]), held in June 2004, customary in the university as a speech directed to a larger (and not scientifically sophisticated) audience.

"In November 1954, soon after my master's degree, I received my first assignment, teaching exercises in analysis. So this academic year which I am concluding is the fiftieth year of my teaching. It is a long interval, which invites me to reflect, obviously not only upon my university life but on life in general. My most immediate thought is to thank God. I dare say, borrowing the words from the Gospel (Luke 1:49):

"For the Mighty One has done great things for me. Holy is His name."

This especially applies to the people in my life, starting with my parents, who showed me routes of culture, earnestness, generosity; my brothers, with whom I have always had excellent relationships of friendship and help; my children, with whom our mutual affection has always overcome the difficulties of the generational gap. However, the greatest gift I received is the love and the help of my wife Laura. There are of course also many friends and colleagues that during my university career have shown me respect and affection and I am thankful for them as well."

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