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Zero-Dimensional Model for Dynamic Behavior of Engineered Rubber in Automotive Applications

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Abstract

This paper presents a zero-dimensional model for the simulation of the mechanical behavior of automotive engineered rubber components, such as flexible couplings. The objective is to develop a real-time-capable model, able to simulate the behavior of a driveline containing elastomer components: the engineered rubber model has to correlate stretch to stress, the mechanical behavior being represented by means of a hysteresis cycle. The study presents the implementation of Maxwell and Voigt models, showing their limits in the representation of the material behavior: elastomers present a nonlinear response in the relationship stress-strain. A combination of Maxwell and Voigt models, with stiffness and damping variable according to the stress and strain rate, to represent nonlinear material responses, is coupled to a relaxation model, in order to represent the Mullins effect (the rubber mechanical behavior also depends on load history).

Experimental tests have been carried out with different pre-load settings, stress amplitudes and stress frequencies. Tests results have been used to calibrate the parameters defining the simulation model, comparing the model outputs to experimental data: an optimization algorithm has been applied, with the aim of minimizing the results discrepancy with respect to experimental results. The optimization tool has been also used to reduce the number of parameters defining the model, in order to simplify the required computational power, avoiding at the same time over-parametrization.

In the second section of the paper, the model is used for the simulation of a different rubber component, whose behavior is identified using quasi-static load ramps, frequency and amplitude sweeps, steps and random cycles. An alternative model formulation, minimizing the degrees of freedom is then applied to the new dataset. The model parameters are separately optimized using different tests, in order to capture the specific mechanical behavior. Finally, the identified parameters are used to simulate the elastomer response in random tests, comparing the results to experimental data, to evaluate the simulation quality in terms of RMSE.

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1. Introduction

The main purpose of this work is the definition of a simulation model of elastomeric components for real-time applications. Real-time environment implies zero-dimensional modelling for simplicity and low computational cost. Modelling elastomeric components is required in many simulations tasks: from Hardware in The Loop simulations (where the driveline response should interact with the control actions of the device under test), to torsional models incorporated in engine/vehicle control systems [1, 2]. Control-oriented rubbers models are the first step in the process of a many model-based control systems development: the possibility to have a general model, capable to simulate different elements with different parameters, is mandatory in automotive driveline controls [3, 4]. Simple elastomeric models are also used for simulating shock absorber elements and for the study of vehicle dynamics behaviours [5, 6].

The analysis of experimental data on the frequency domain could also be used to highlight stiffness and damper dependence on the excitation dynamics: however, the main goal of this work is elastomer components real-time simulation, carried out on the time domain. The elastomeric component model has been developed for two different types of flexible couplings. Fig.1 shows an example of a generic flexible coupling made with elastomeric material:

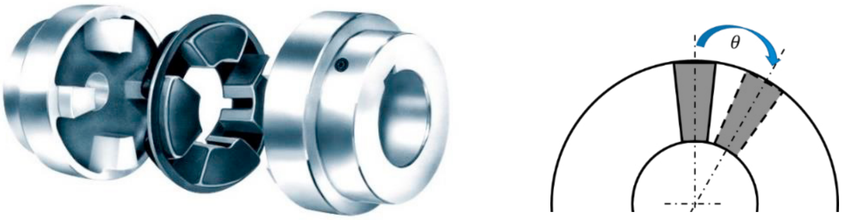


Fig. 1 a) example of a generic flexible coupling b) torsional elastomer scheme

Elements shown in Fig 1a)-b) react to an applied momentum with an angular stretch. As for other materials, elastomers behaviour is usually descripted by [7, 8] the relationship between stress (load applied in a specific area) and deformation. Main purpose of this paper is to describe this relationship with a zero-dimensional approach.

Due to the complex phenomena determining the behaviour of rubber materials, elastomers are usually simulated using 2D/3D models [9, 10], rarely with mono-dimensional modelling [11, 12].

The model definition is grounded on the literature, but also on experimental data analysis. Experimental data are available for both elements in the form of imposed deformation and corresponding material reaction in terms of load/torque response. Deformation velocity and acceleration are also usable for the model development. Tests have been carried out according to different combinations of load amplitude, preload, load frequency and load shape: the goal is to explore as extensively as possible the components behaviour.

In the proposed approach, flexible couplings are simulated by means of concentrated parameters models: the model's parameters are determined stimulating the simulated system with a set of displacement inputs and comparing the corresponding torque outputs with experimental data. The evaluation of the simulation quality is based on the RMSE resulting from the comparison. An auto-identification process is used to optimize the model response, minimizing RMSE: this improves the model accuracy leading rapidly to the choice of optimal parameters settings. Finally, the model is simplified, reducing the number of parameters, while maintaining a good simulation quality.

Nomenclature

σ	stress
ϵ	deformation
θ	angle
c	damping element constant
k	spring constant
$RMSE$	root mean square deviation

2. Experimental Setup

Tests have been carried out using hydraulic linear actuators connected to the elastomer under test by means of a rigid arm, while measuring the actuators displacement and force. These data can be easily converted into proper inputs and outputs for the model (in the case of a flexible coupling I/O are more likely angular displacement and torque). The actuators can be programmed in terms of displacement waveform as a function of time, and also average value (pre-load), amplitude and frequency can be set. A schematic of the test equipment is shown in Figure 2.

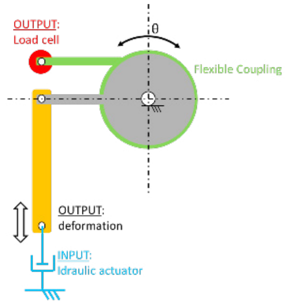


Fig. 2 schematic of the test equipment

In order to exhaustively describe the components behaviour, the first flexible coupling has been tested imposing sinusoidal displacements with different frequencies, amplitudes and offsets, while for the second elastomer also different profiles (steps, triangle waves, random profiles) have been introduced, for a better representation of the component actual use. This approach also allows assessing the model quality using different types of test with respect to those used for the model parameters optimization.

3. Elastomer behavior

The behaviour of elastomers is highly non-linear, with the output stress (force or torque) depending on time (memory effect), on the strain average value and amplitude (changes in the modulus of elasticity according to the level of deformation), and strain derivative (modulus of elasticity during loading phase is different with respect to the unloading phase). Figure 3 shows an example of the results pertaining to the tests carried out on the first elastomer: three different tests are presented with the same deformation frequency but different combinations of preload and stroke (each test is represented with a different colour in the two plots).

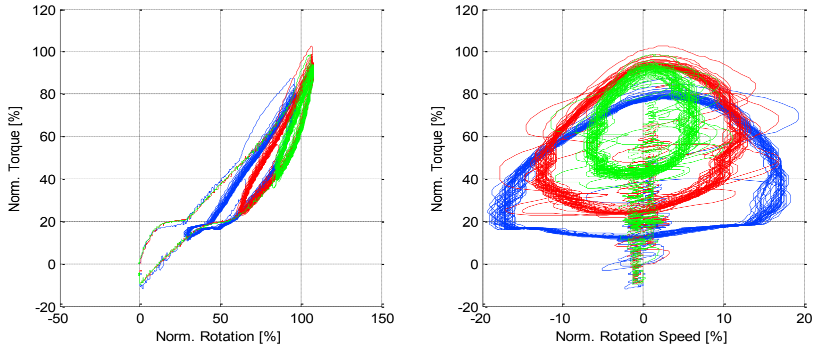


Fig. 3 a) torque response vs rotation b) torque response vs speed rotation

Available data show several typical elastomer behaviour characteristics:

- Presence of a load-deformation hysteresis loop;
- Quasi-linear response during loading phase;
- Non-linear response during unloading phase with decreasing of Young's modulus;
- Variable response with respect to time: firsts cycles are affected by higher stiffness than steady cycles;
- Translation of hysteresis loop depending on preload.

The literature [7, 9, 13] shows how all these behaviours are typical of elastomeric elements, which are affected by non-linear response: first phase of hysteresis loop is characterized by a decrease of Young's modulus (softening) followed by an increase (stiffening), also visible in Figure 3.

Another peculiarity is the response variation with time, also known as Mullins effect: during the first loading cycles, due to the material anisotropy, the elastomer shows a higher stiffness then in latter cycles. Mullins effect is more visible during loading phase and it is caused by the variation of the material structure which dynamically depends on the stress level: structure changes require time, thus the material reaction to strain may be different from cycle to cycle, especially for low strain cycle frequencies. All these facets need to be taken into account in the model development.

4. Basic simulation models

In order to develop a model that could be run in real-time, the present study starting point is one of the simplest model offered by the literature [7, 8]: the Maxwell model is based on a damping element in series with a spring.

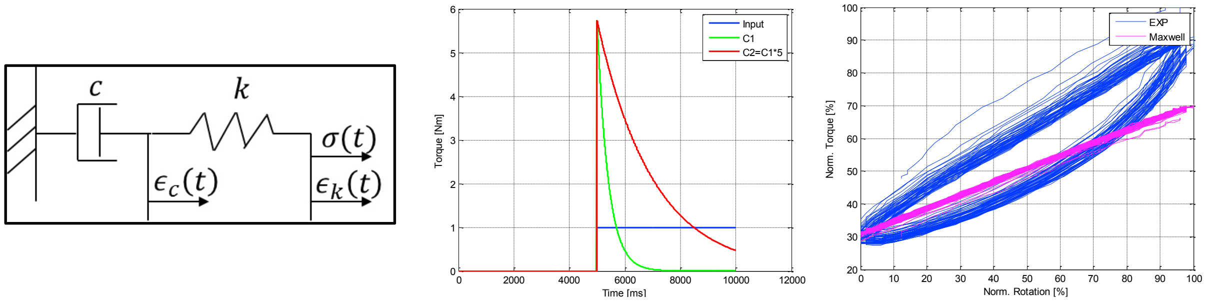


Fig. 4 a) Scheme of Maxwell model b) Torque response to an input impulse c) Simulation result vs experimental data

The model response is descripted by the equation

$$\sigma(t) = k(\epsilon_k - \epsilon_c) = c\dot{\epsilon}_c(t) \qquad \text{(Eq. 1)}$$

that means damper and spring are affected by the same stress.

Fig 4b shows why this model alone cannot be used represent the behaviour of the elastic coupling: the model has unlimited relaxation, because the torque response to a deformation step asymptotically decreases to zero.

The limit of this model is the impossibility to reproduce static stiffness: after a while the component reaction to the imposed strain damps out. The result does not change with the addiction of another Maxwell branch in series.

Another possible elementary approach is the Kelvin-Voigt model [14], based on a damper and a spring in parallel.

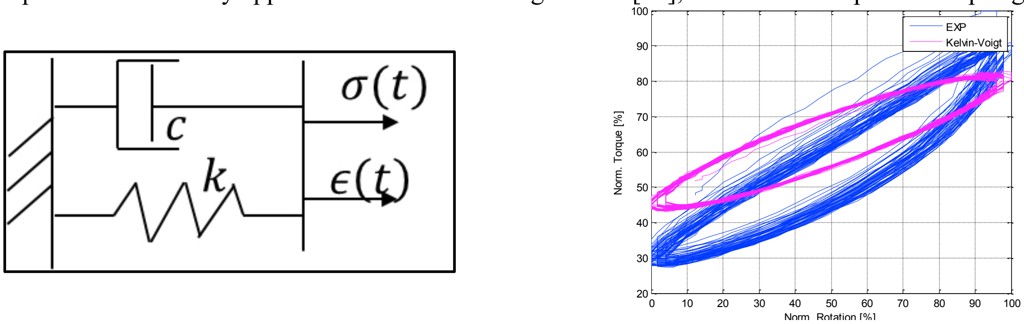


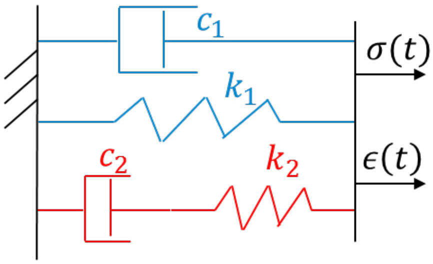
Fig. 5 a) Scheme of Kelvin-Voigt Model b) Simulation result vs experimental data

In this model, represented in Figure 5 a), both elements are subject to the same strain:

$$\sigma(t) = k\epsilon(t) + c\dot{\epsilon}(t) \qquad \text{(Eq. 2)}$$

The model can capture both static and dynamic stiffness separately, respectively by means of the spring constant and damping coefficient, but it is not able to describe the load/unload asymmetric behavior at the same time. With this implementation, it is easier than with the Maxwell model to reproduce the hysteresis loop, but it is not possible to reproduce the "relaxation effect". Results do not change with the addiction of another Kelvin-Voigt branch in parallel. To improve the model response, it is possible to add a mass element, but the result is not truly different: the main effect is to modify the shape of the hysteresis loop, without affecting the symmetrical shape of the loop.

A possible next step is to combine the Maxwell model with the Kelvin-Voigt one to capture the positive effects of both implementations [16] as shown in figure 6.



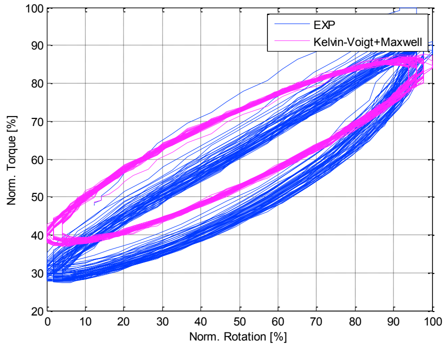


Fig. 6 a) Scheme of Simulation Model b) Simulation result vs experimental data

The Kelvin-Voigt model is able to generate static stiffness, while the Maxwell component reproduces the relaxation effect. Figure 6b compares simulation results to experimental data: the hysteresis loops are similar, even if Mullins effect and material non-linearity behaviours (asymmetry in the loops) are still not captured by the simulation.

Mullins effect [13, 17] consists in the time-based damped response of rubber, so the authors introduced in the model a first order transfer function for the simulation of this aspect. The stretch input is split into low frequency and high frequency contributions, the filtering action being managed by the transfer function. The simulated stress response is then given by the combination of two different factors: low frequency stress, related to low frequency stretch, and high frequency stress, that depends on the high frequency stretch component.

The model output able to simulate Mullins effect could then be computed as follows:

$$M_{est-K} = M_{LowFreq} + M_{HighFreq} = \left(\frac{1}{1+\tau s}\right) \theta * \mathrm{k}_1(\theta) + \theta \left(1 - \left(\frac{1}{1+\tau s}\right)\right) * k_2(\theta, \dot{\theta}) \qquad \text{(Eq. 3)}$$

A Kelvin-Voigt with a Maxwell component model and a first order transfer function can simulate the Mullins effect: Figure 7 shows that the first cycles have higher Torque for the same stress, if compared to steady state cycles.

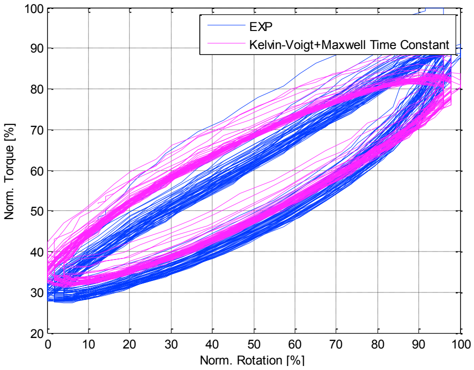


Fig. 7 Simulation result vs experimental data with Maxwell+Kelvin-Voigt

The model response can be improved with the insertion of variable parameters instead of constant values for the springs and dampers elements. Non-linear behavior means that stiffness and damping values depend on deformation and/or on its derivative.

4.1. Parameters Auto-Identification Process: Flexible Coupling

Due to the presence of lookup tables defining the dependence of stiffness and damping on strain and strain speed, a first tuning of the model defines 45 parameters (9 breakpoints for each lookup table), that have to be identified in order to fit experimental data. A high number of breakpoints allows fitting the dependency from deformation and velocity.

Experimental data have been used to implement the automatic identification of all the parameters with the support of Matlab-Simulink Toolbox Parameters Estimation. The auto-identification process is based on nonlinear least square optimization method and Trust-Region-Reflective algorithm. The difference between simulated and measured torque is the cost function to be minimized. The process optimization is focused on steady conditions, excluding the initial preload application and the beginning of the tests (Mullins effect is identified separately). Appropriate parameters starting points and sound constraints are fundamental to obtain a robust and reliable result.

The Toolbox has also been used to assess the sensitivity of the simulation results to the number of parameters. The simulation quality obtained after the identification, evaluated in terms of RMSE with respect to experimental data, is significantly improved compared to that obtained with first attempt values. The number of parameters has been decreased from 45 to 20, in the attempt to obtain the best compromise between model accuracy and complexity. Non-linearities, typical of elastomers, forced to maintain dependencies of stiffness and damping on deformation and deformation velocity.

Fig 8a shows RMSE of all available tests normalized with respect to the maximum RMSE of all three models, while Fig 8b shows RMSE percentage with respect to absolute maximum value of torque measured in all available tests: RMSE is significantly decreased after the optimization. The simulation accuracy obtained running the simplified model (using only 20 parameters) is still acceptable, being the RMSE always lower than 5% of the maximum stress. Taking into consideration the model objective (real-time simulation), the optimization effort and the quality of the results, the model based on 20 parameters seems a correct choice.

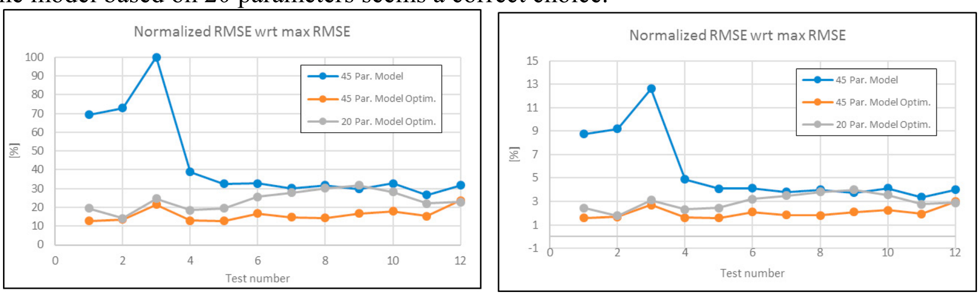


Fig. 8 a) RMSE normalized wrt maximum RMSE b) RMSE normalized wrt maximum torque's value

5. Simulation Model and Automatic Parameters Identification: Second Flexible Coupling

The previous model can be adapted to other rubber elements that have similar behavior in the stress-stretch relationship. The goal of the present section is to analyze the behavior of a different flexible coupling, using the same approach discussed in previous sections.

In this case, experimental data are available in the form of tests composed of several patterns: amplitude sweeps with different frequencies 5-15-25Hz; sine sweeps in 5-15-25Hz and different preload combinations; quasi-static load ramps; steps with different load and preload; sweeps of frequencies. Moreover, a random test is available, which is important for model validation.

Fig. 9 shows an example of available experimental data:

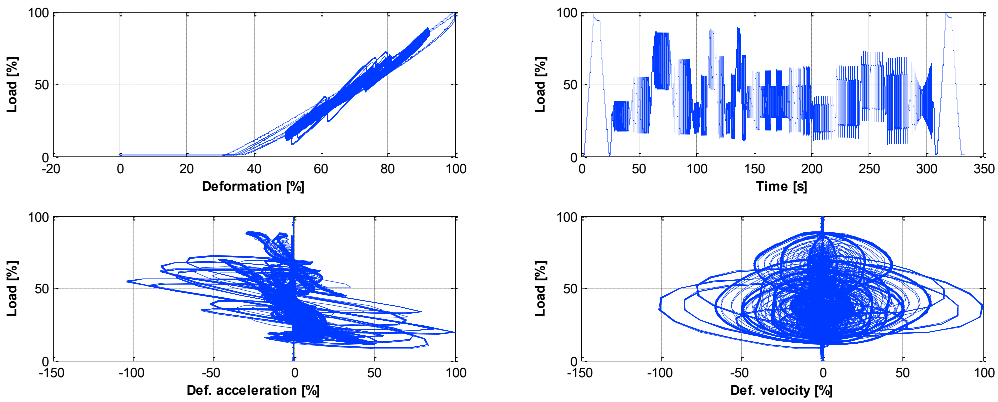


Fig. 9 Example of available test

Hysteresis load-deformation loops like those shown in previous figures can be easily observed in Figure 9: the model discussed previously could simulate this elastomer, provided that parameters are re-identified. Random test inputs will then be used to evaluate the simulation quality in general running conditions: Figure 10 shows that the model is perfectly able to reproduce the stress resulting from the application of a random strain. The outputs of the model with optimized parameters is superimposed with experimental data. Normalized indexes with respect to maximum torque value count $RMSE = 1.30\%$.

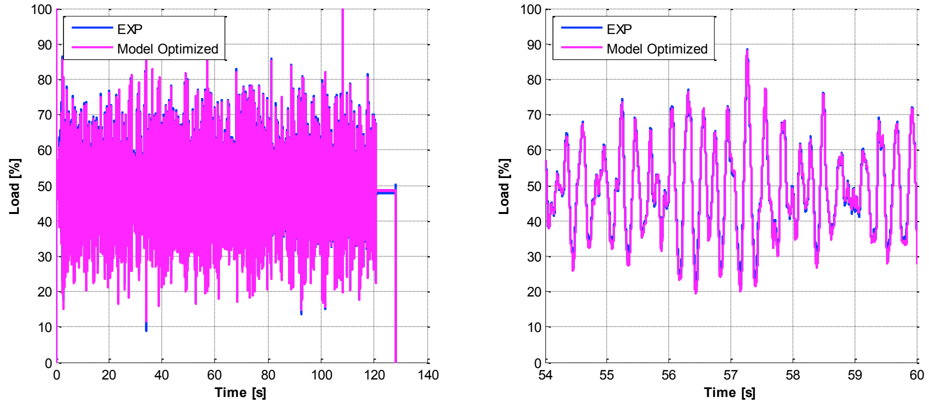


Fig. 10 a) Comparison between experimental data (random test) and simulation result b) High freq. zoom

Since results are reliable, the next step is to simplify the model definition, removing the Mullins effect, that for the considered elastomer is not significant, and reducing the parameters dependencies on deformation and deformation velocity.

The new model is defined according to Figure 11 and is made of two Maxwell branches and one spring in parallel. Only the second Maxwell branch has a stiffness value variable with deformation (linear dependency), while all the other parameters are kept constant. The three elements simulate different facets of the elastomer reaction to deformation: the static, quasi-static and the high-frequency behavior.

A different approach has then been used for parameters identification: every single branch is optimized on a specific test:

- K1 spring is estimated according to average values of stress and strain measured during the tests shown in Fig 11, aiming at the identification of a "static stiffness";
- K2-C2 Maxwell branch is optimized on slow ramps, to simulate the quasi-static behavior and relaxing effect;
- K3-C3 Maxwell branch is optimized on frequency and amplitude sweep, to catch high frequency effects. In this case, the spring stiffness linearly depends on deformation, thus it is defined by two parameters (offset and gain).

Dedicated parameters identification allows minimizing the DOF of the model without a relevant decrease of estimation accuracy: normalized estimators with respect to maximum torque value count $RMSE = 1.50\%$.

This result shows how it is possible to have a comparable torque estimation with only 6 parameters and with dedicated branches optimizations.

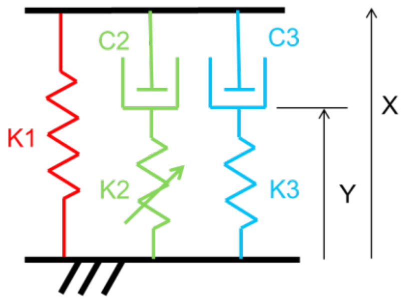


Fig. 11 Simulation model scheme for new elastomer model

6. Conclusions

The purpose of this study is to develop a zero-dimensional model for a generic elastomer component. Starting from simple models, a more complex concentrated parameters model has been developed. The main characteristics are the combination of Maxwell and Kelvin-Voigt models, with damping and stiffness depending and deformation and deformation speed, and the splitting of low-frequency and high-frequency behaviors, to highlight Mullins effect.

The introduction of automatic parameters identification allows decreasing the degrees of freedom of the model, while maintaining good accuracy. The parameters optimization and the comparison of different modeling solutions are carried out based on 12 different tests, evaluating RMSE. The best compromise solution (based on 20 parameters) is compatible with real-time simulations both in terms of accuracy and computational power.

The approach in easily extendable to the simulation of other rubber components, thus the model has been re-optimized against a new dataset pertaining to a different coupling, composed of quasi-static ramps, high frequency sweeps and random load variations). Simulation results are satisfying, leading to an average percentage RMSE of 1.3%. A further simplification is possible for the considered component: the simplified model has been coupled to a dedicated parameters identification strategy. Results show that the decreasing in the number of parameters (from 20 to 6) justifies the light increase in percentage RMSE (from 1.3% to 1.5%).

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