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Modal parameters identification with environmental tests and advanced numerical analyses for masonry bell towers: a meaningful case study

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Abstract

In the first part, a dynamic monitoring for non-destructive evaluation of heritage structures is discussed with reference to a case study, namely the Pomposa Abbey belfry, located in the Ferrara Province (Italy). The main dynamic parameters constitute an important reference to define an advanced numerical model, discussed in the second part, based on Non-Smooth Contact Dynamics (NSCD) method. Schematised as a system of rigid blocks undergoing frictional sliding and plastic impacts, the tower has exhibited complex dynamics, because of both geometrical nonlinearity and the non-smooth nature of the contact laws. First, harmonic oscillations have been applied to the basement of the tower and a systematic parametric study has been conducted, aimed at correlating the system vulnerability to the values of amplitude and frequency of the assigned excitation corroborated by the dynamic identification results. In addition, numerical analyses have been done to highlight the effects of the friction coefficient and of the blocks geometries on the dynamics, in particular on the collapse modes. Finally, a study of the tower stability against seismic excitations has been addressed and 3D simulations have been performed with a real earthquake.

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1. Introduction

Recent seismic events have highlighted that ancient masonry towers are particularly susceptible to damage and the safety assessment of these structures against earthquakes appears to be of relevant importance for historical, social and artistic reasons. Ancient masonry towers very often exhibit unique peculiar morphologic and typological features, which might affect their structural behaviour under horizontal loads and they were usually conceived mainly to withstand vertical loads. Lately, national (Ministero per i Beni e le Attività Culturali, 2011) and international standards (EN 1998-1, 2004) have imposed the evaluation of the structural performance in the presence of horizontal loads, which simulate earthquake excitations, encouraging the use of sophisticated analyses.

When real historical masonry constructions are considered and the problem of assessing their dynamic vulnerability is addressed, only simplified numerical models can be used to handle complex geometries, intricate typologies of the masonry textures and uncertainties on the material properties. For this purpose ambient vibration survey (AVS) has become the main experimental method available to evaluate the dynamic behaviour of full-scale structures (Gentile and Saisi, 2007), especially for Cultural Heritage (CH), with the possibility to measure the modal parameters of monitored structures in real ambient conditions without artificial excitations (Clementi et al., 2018).

In this context, the information obtained from the AVS can be used to calibrate and/or to test the numerical model to draw an exhaustive picture of the seismic vulnerability of the investigated structure. With such complex structures with a strongly nonlinear behaviour, we can distinguish two different numerical modelling. The first one considers masonry buildings as continuous structures and it discretizes them using finite elements (Quagliarini et al., 2017). Finite Element (FE) schemes have been applied to many ancient constructions, such as churches (Clementi et al., 2017c; Milani and Valente, 2015a, 2015b; Monni et al., 2017), monasteries (Clementi et al., 2017a, 2016), bridges (Fanning and Boothby, 2001), towers (Acito et al., 2016; Valente and Milani, 2016) and historical city centres (Formisano, 2016). The second approach, alternative to the first, considers masonry as an assembly of blocks subject to unilateral frictional contacts and distinct element formulations are used to model them. The Non-Smooth Contact Dynamics (NSCD) method has been developed by (Jean, 1999; Jean et al., 2001) and it applies the Signorini's and Coulomb's laws to take into account the impenetrability and friction between blocks although more complex models have been developed (Dubois et al., 2018). It provides a time-stepping scheme to solve dynamical problems with many blocks and their unilateral frictional contacts. The NSCD method has been numerically implemented in the LMGC90[®] code, by using an implicit algorithm.

In this research, an experimental and numerical methodology is proposed, to perform the dynamic identification of a historical building lying in low-medium seismic hazard zones by using a wired sensor network. The measurements are performed with high sensitivity piezoelectric sensors and with a data acquisition system able to record Ambient Vibrations (AV) with very low amplitude range (10^{-6} m/sec²). Furthermore, the dynamics of the masonry belfry is investigated numerically using the NSCD method. First, harmonic oscillations have been applied to the basement of the tower and a systematic parametric study has been conducted, aimed at correlating the system vulnerability to the values of amplitude and frequency of the assigned excitation corroborated with the results of the AVS. Also, numerical analyses have been done to highlight the influences of the friction coefficient of the blocks geometry on the dynamics and the effects on the collapse modes. Finally, the study of the tower stability against seismic excitations has been addressed. Attention has been paid to the occurrence of out-of-plane torsional overturning mechanisms as observed in the last Italian earthquakes.

This proposed study aims to achieve four goals: (i) to show the capabilities of the NSCD method in predicting the failure mechanisms of complex masonry structures and to verify that it is a valuable numerical tool, which adds to the usual FE models; (ii) to assess the seismic vulnerability of the Pomposa Belfry; (iii) to verify if the seismic sequence of central Italy would have caused damage to the structure; (iv) if the AVS could be an excellent tool to validate the DEM.

2. The belfry of Pomposa Abbey

The bell tower of Pomposa Abbey (Fig. 1) is located in the municipality of Codigoro in Ferrara Province in Italy. It is very high (48 meters) compared to the rest of the Abbey and it was built in 1063 in Romanesque-Lombarde style under the supervision of architect Deusededit. Proceeding from the base towards the top of the bell tower,

windows increase in number, and they become broader than previous ones to lighten the weight of the tower and to better propagate the sound of the bells. From the bottom to the top there are *monofore*, *bifore*, *trifore* and *quadrifore* windows. The geometry is very regular with a stone square base of 7 m x 7 m and the bell tower rises on nine floors separated by frames. The red and yellow bricks retain very rare ceramic basin inserts from various Mediterranean countries.

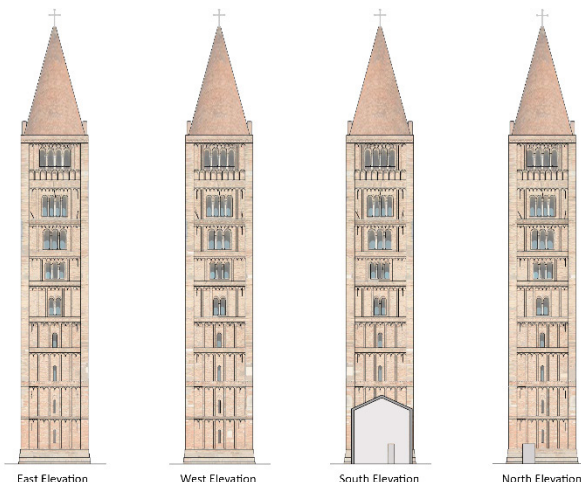


Fig. 1. Main façades of Pomposa belfry in Codigoro (Ferrara, Italy).

3. Ambient Vibration Survey (AVS)

The method for vibration-based identification of modal parameters used in the presented work operates in the time domain, and it is based on a state-space description of the dynamic problem (Van Overschee and De Moor, 1996a) using the Covariance Stochastic Subspace Identification (SSI-Cov) algorithm.

Using the procedure described in (Peeters and De Roeck, 1999), the modal parameters (frequencies, modal shapes and damping ratios) can be extracted by the Eigen-decomposition of the system matrix $[A]$. For the sake of brevity, the complete formulation is not included in the discussion.

3.1. The equipment

The AVS response of the towers was measured at different elevations (Fig. 2) and with different acquisitions. The accelerometers were fixed directly in contact with the structural elements (Fig. 2) and parallel to the main directions of the belfry, in order to get both translational and torsional modes (Catinari F. et al., 2017; Clementi et al., 2017b; Pierdicca et al., 2017, 2016, 2015).

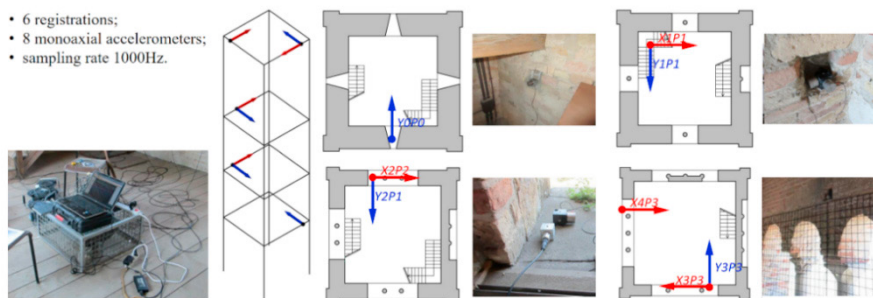


Fig. 2. The layout of the accelerometers at each floor of the analysed belfry of Pomposa Abbey.

Each level was instrumented at least in two corners. At each corner, two high sensitive accelerometers, measuring in two orthogonal directions, were placed. Other couples of accelerometers were put in different positions at various levels to obtain more information about the dynamic behaviour of the whole tower.

A wired sensor network was used, composed of two types of piezoelectric sensors (Integrated Electronic Piezoelectric - IEPE):

- KS48C-MMF with voltage sensitivity of 1V/g and measurement range of $\pm 6g$;
- KB12VD-MMF with voltage sensitivity of 10V/g and measurement range of $\pm 0.6g$.

The digital recorder (DaTa500) is composed by a 24-bit Digital Signal Processor (DSP), an analogue anti-aliasing filter and a high-frequency acquisition range (0.2Hz to 200kHz). RG58 coaxial cables link accelerometers and recorder. M28 and M32 signal conditioners with a frequency range of 0.1 to 100kHz and selectable gain are also used for each record. Some images of the used equipment for AVS are reported in Fig. 2.

3.2. Dynamic measurement and estimation of structural dynamic properties

The collected measurements were initially sampled at 1000 Hz. A factor of 40 decimated them before processing to have the final data of 12.5 Sample per Second (SPS) (Singh et al., 2014). The record duration varied between forty minutes to one hour: it should be long enough to eliminate the influence of possible non-stochastic excitations during the test (McConnell, 2001). The described procedure was used for each AVS.

A modal parameter extractor developed in Labview[®] programming language carried out data processing. It can perform analyses in the time domain according to the SSI-Cov procedure mentioned before.

The stabilisation diagram obtained from the analysis of the collected data through Cov-SSI is reported in Fig. 3.

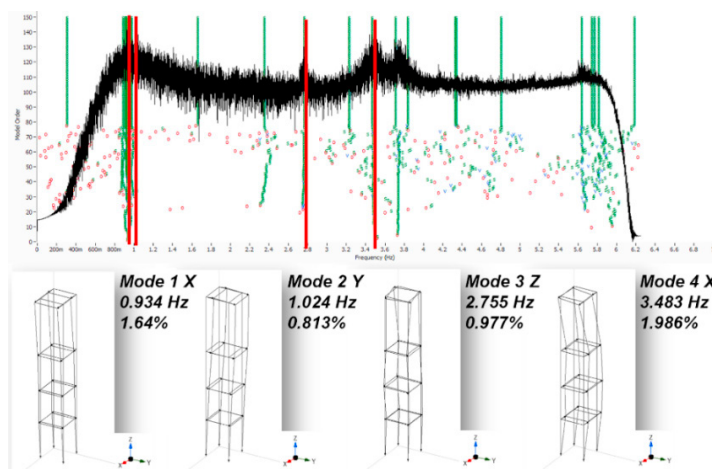


Fig. 3. Stabilisation diagrams (Cov-SSI) for Pomposa bell tower in Codigoro (Ferrara).

It shows the alignments of stable poles, for increasing order models, and it allows the determination of the n eigenvectors of dynamic matrix $[A]$ which are representative of structural modes, and how many are instead purely numeric (due to their redundancy of calculation or noise). Red points indicate negative results of the stability test, while green points represent the positive ones: considering that natural modes show intrinsic characteristics of the towers, they are invariant to the process and the order of the NM (Van Overschee and De Moor, 1996b). Then, it is possible to isolate the natural modes from the numerical ones by increasing the order of the model and checking the stability of the results. The stability of a pole is defined as follow:

- the estimated frequency is considered stable if it does not change more than 2%;
- the damping for different orders should not deviate more than 15%;
- the modal shape obtained by a certain order is compared to the same one obtained by a minor order by Modal Assurance Criterion (MAC) that must be at least equal to 90%.

The identified frequencies appear slightly spaced and local modes show only for values upper to the 3rd (Fig. 3).

4. The numerical model

In this section, the principal peculiarities of Non-Smooth Contact Dynamics (NSCD) model and the main modelling assumptions are highlighted. The problem parameters and the seismic excitation applied to the base of the bell tower are also briefly reported.

4.1. Non-Smooth Contact Dynamics method

The dynamics of a system of rigid bodies is governed by the equation of motion and by the frictional contact conditions. To limit the length of the paper, for an exhaustive description of the method see (Jean, 1999; Moreau, 1988) and (Lee and Fenves, 1998). It is only mentioned that Signorini's law of impenetrability and the dry-friction Coulomb's law are used and the equation of motion in integral form is solved numerically by using a time-stepping approach. Regarding the contacts between bodies, the model does not account for elasto-plastic impacts governed by restitution laws on velocities (Newton law) or impulses (Poisson law) (Pfeiffer and Glocker, 1996), or energetic impact laws (Nordmark et al., 2009).

The Signorini's law implies perfectly plastic impact, i.e., the Newton law with restitution coefficient equal to zero and therefore the impossibility to describe, for instance, bouncing phenomena, and, furthermore, it overestimates the energy dissipated during impacts. However, in the case of systems of bricks or stones, the restitution coefficient has low values and thus bouncing phenomena are secondary and negligible. More sophisticated impact laws would lead to more accurate and complex models such as that proposed in (De Lorenzis et al., 2007), but they would not be feasible for large systems.

Furthermore, the deformability of blocks is neglected. This is a reasonable approximation since the expected operating compressive stresses at the base of the masonry walls of the tower are reasonably low. Since deformability drastically increases the computational complexity, practically it cannot be applied to large 3D structures like the tower of this study. On the other hand, simplified two-dimensional schemes rule out a crucial aspect of the dynamics of box-shaped structures such as houses, churches and towers that is, interactions between adjacent walls laying on different planes, which mutually exchange considerable inertia forces.

Since we are interested in the dynamical interactions between the different parts of the belfry, we consider 3D schemes, but we neglect blocks deformability. It follows that the numerical results obtained depict an overall picture of the tower dynamics and describe the failure mechanisms of the whole tower, due to blocks rocking and sliding, but, apparently, they do not give a description of the stresses and strain distributions within each block.

Since experimental data were not available, the friction coefficients were selected from standard values in literature, with values of μ ranging from 0.3 to 1.2, according to different combinations of units and mortars (Vasconcelos and Lourenço, 2009). As a first attempt, we assume two different values for these coefficients for the interface block/block, i.e. $\mu = 0.3$ and 0.5 , and $\mu = 0.9$ for the interface block/foundation to observe, mainly, the dynamics of the tower without the structure-foundation interaction. Furthermore, it is important to highlight that in real old masonry buildings, the degradation of the mortar over time contributes to deteriorate the friction coefficient and thus confirms the hypotheses of the first attempt.

Finally, we observe that damping is not considered here, and only friction and perfectly plastic impacts dissipate energy.

4.2. Belfry dynamics

Because of the above simplifying hypotheses, the NSCD method is straightforward, as also shown by the fact that it requires only two constitutive parameters: the friction coefficient μ and the mass density ρ . For this reason, firstly, harmonic oscillations of different amplitudes and frequencies are assigned to the basement, according to structural dynamic properties obtained by AVS data processing, analysing the influence of the friction coefficient on the global response of the tower.

We assume a basement oscillation with acceleration (in the three-main direction X, Y, Z) equal to $a(t) = A \cdot \sin(2\pi f \cdot t)$. Since the problem unknowns are the velocities of the blocks, in the numerical code, the velocity $v(t) = (A/2\pi f) \cdot \sin(2\pi f \cdot t)$ is assigned to the basement. The integration time in the simulations is 40 s. Last

instants of the simulation at varying of frequency f and amplitude A are reported in Figures 4(a) and 4(b) for different μ .

The primary consideration is that in those Figures are reported the final instants of the simulations except for the $f=1\text{Hz}$, where $t\sim 20\text{sec}$. Then, Pomposa tower is very vulnerable at low frequencies, i.e. 1 Hz (see Figs 4(a) and 4(b)), which it is about the frequency of the first two modes identified in Sect. 3 and reported in Fig. 3. The belfry considered here does not consider any deformability, however, due to its high mobility since blocks are just laid one over the other (a large number of degrees of freedom), it produces the same dynamical effect of deformability, that is a specific vulnerability to low-frequency excitations. In general, for a fixed amplitude A , the collapse instant increases with increasing frequency. As a result, the system is very resistant to high frequencies. Otherwise, increasing A , the collapse instant decreases for $f=1\text{Hz}$, and amplitude does not affect the tower dynamics with high frequencies. We notice that for $f\rightarrow 0$ (here not reported for brevity issue) the problem is reduced to a static problem (tower at rest in its static configuration) and the collapse instant tends to infinity.

Finally, we underline that, as expected, increasing μ , the system resistance increases. Indeed, fixed amplitude A and frequency f , the collapse instants increase for growing μ . This aspect agrees with what has been shown by recent earthquakes in Italy: towers have high periods of oscillations and many damages were observed for low-frequency contents of earthquakes. In the next section, it is shown that μ influences the mode of collapse, too.

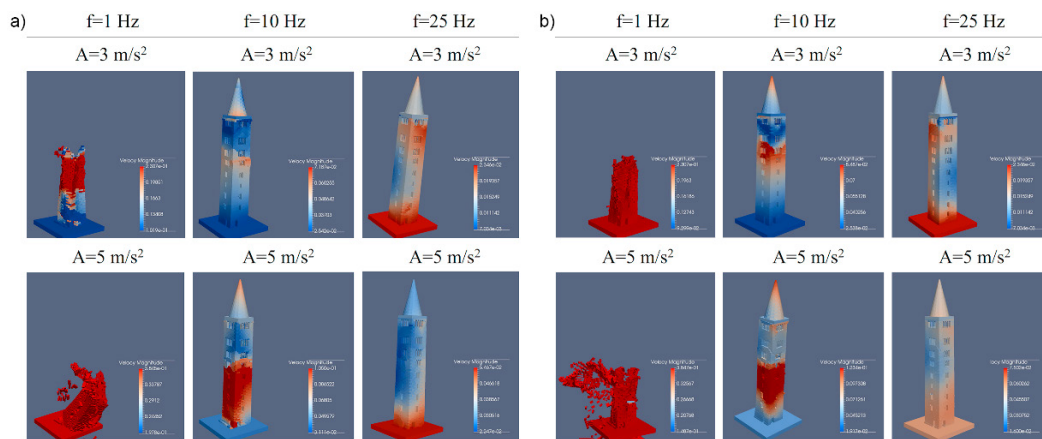


Fig. 4. Simulation results with harmonic excitation for: (a) $\mu = 0.3$; (b) $\mu = 0.5$.

4.3. Collapse mechanisms with real earthquakes

The parametric analysis of the previous sections allowed us to draw a complete picture of the tower response to harmonic oscillations. In this section, the excitation of a real earthquake is applied to the basement and the tower seismic vulnerability is investigated, considering the accelerations of Forca Canapine (Norcia, Italy) of the 30th October 2016 earthquake. During that earthquake with epicentre in Norcia a Peak Ground Acceleration (PGA) of 910.367 cm/s^2 and a Peak Ground Velocity (PGV) of 77.318 cm/s have been registered in FCC recording station (see the website: <http://strongmotioncenter.org>). In numerical simulations, velocities are applied to DE model at the base where the tower is laid. The three velocities components in the three main coordinate directions are determined by direct integration of accelerations in a time interval of 40 s, during which the maximum amplitudes are attained, without using any correlation method. In the simulations, the time step $dt = 0.005$ s has been used.

For $\mu=0.3$ (Fig. 5(a)) the blocks of the tower mainly experience relative sliding, which contributes to misalign them. For $\mu=0.5$ (Fig. 5(b)), the major value of μ limits sliding mechanisms and it facilitates the relative rotations.

In the first case, the initial damages are observable starting from $t=10$ sec, where the columns of the windows collapse. Subsequently, for about $t=20$ sec., there is a horizontal slip at the roof and a similar crack in the middle of the tower always produced by sliding into the block interfaces. After $t=30$ sec, the tower remains without any further damage as the seismic time history is practically nil. Likewise, for $\mu=0.5$, the same cracks are observable on the roof, but smaller cracks respect the previous case appear in the middle part of the tower, in correspondence of the corners of the tower. Further study with different seismic inputs will be the subject of future work.

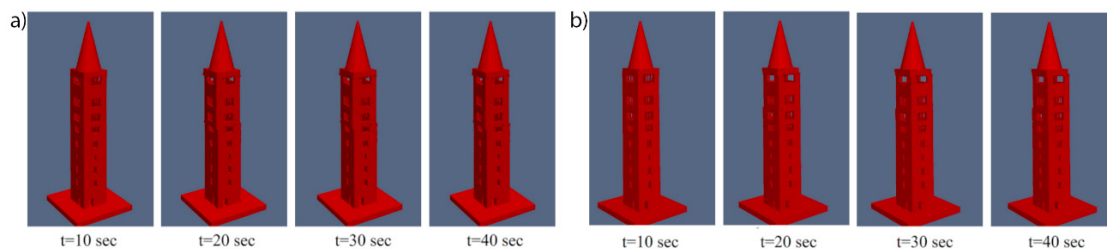


Fig. 5. Simulation results for the earthquake in Forca Canapine (Norcia, Italy) at 30th Oct. 2016 for different time steps: (a) $\mu=0.3$; (b) $\mu=0.5$.

5. Conclusions

Management, maintenance and preservation of heritage structures are usually very complex tasks because of old construction techniques, unique structural schemes, construction defects and limited destructive investigations for structural characterisation. Therefore, the availability of experimental estimations of modal properties becomes relevant for structural and seismic assessment processes.

Considering the increasing interest towards the opportunities provided by the dynamic monitoring as a tool for non-invasive techniques, this study would like to be a contribution to the development of rational and sustainable procedures for non-destructive investigation according to the current codes for heritage structures. In particular, the main results such as frequencies are used as input data for subsequent numerical analyses of the Belfry of Pomposa.

The NSCD method implemented in the LMGC90[®] code has been then applied to the study of the tower and, by combining modelling simplicity and great predictive capabilities, it turned out to be a powerful tool for exploring the dynamics of ancient masonry structures. It is based on only two constitutive parameters, the mass density ρ and the friction coefficient μ . This is a significant modelling advantage since the identification of constitutive parameters of ancient structures is always extremely uncertain.

A parametric study has been done by assigning a harmonic horizontal acceleration of the tower basement, varying its amplitude and frequency according to those identified. We have found that low-frequency excitation between the first two linear modes, i.e. 1 Hz, is much more threatening than the high frequencies. Then, the belfry seismic vulnerability is analysed and a clear picture of the expected damaged under high seismic excitation is obtained.

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