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Emission spectrum and geometric phase in deformed Jaynes-Cummings model

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ABSTRACT

The emission spectrum of a qubit (two-level atom) system that interacts with a field in the framework of parity deformations is investigated in this paper. The model consists of a qubit coupled to a single-mode field within the parity deformed Jaynes-Cummings model (PDJCM) based on the λ-analog of the quantum harmonic oscillator algebra. We numerically evaluate the atomic emission spectrum (AES), by considering the influence of the deformed parameter and half-band-width of the spectrometer. Moreover, the dependence of the spectrum peaks on the detuning parameter is discussed. Finally, we study the variation of the geometric phase of the whole system state modelled by the PDJCM in terms of the main physical parameters.

Introduction

The JCM which has been utilized largely in quantum optics (QO) prescribes the interaction between a 2-level atom with a single radiation field. The application and solvability of this model has extensively been discussed in detail [1,2]. This model identifies a set of several phenomena in the quantum mechanics theory, such as, entanglement of atom-field states [3], Rabi oscillations [4,5], revival and collapse phenomena of the population inversion [5]. Especially, it carries out a significant role in latest quantum information processing (QIP) [6–10,18,11–17]. Moreover, JCM is one of the number of potential schemes for generating the nonclassical states [19–22]. According to the experiments of Rydberg atoms with high quality cavity, the dynamics of the JCM has predicted [23]. In view of the fact that the JCM is considered as an optimal model in QO, its several expansions, as for example, transition of 2 photons or multi photons, 2 or 3-cavity modes for 3-level atom, intensity dependent coupling, and the Tavis-Cummings model were examined [24,25]. By using the method of the algebraic operator, the above mentioned models have studied [26]. At the same time, the exact solvability of the JCM can be realized through the supergroup theoretical method and the theory of super-algebras [27]. Lately, the ordinary annihilation and creation operators of the field in the JCM may be replaced by the deformed operators to describe, so-called the deformed JCM [28]. Subsequently, the effect of Kerr nonlinearity in the JCM has been considered in the context of the f-oscillator formalism [29]. On the other hand, it is shown that the generalized JCM can achieve through a set of shape-invariant bound state problem [30].

ES is considered as one of the fundamental processes in QO, and its effective control is essential for many potential applications in miniature lasers, light emitting diodes, and solar cells to single-photon sources for QIP. It is well known that there are two basic methods to control the AES. One is to place atoms into suitable surroundings, as various optical wave guides and cavities [31,32]. The other way is to drive atoms with a coherent external field [33–37]. More particularly, ES does not only depend on the structures of the energy levels for the atom but also on the nature of embedded environment, precisely on both the density of radiation-field states and on the electric field per photon. In the high-Q cavity, the coupling strength between the atom and the field can be modified, which leads to change the electric field per photon. The AES light is simultaneously viewed as an intrinsic property of materials with processes were uncontrollable. The consideration of a cavity to control the coupling of the dipole-field system and the photonic density, leads to design the spontaneous emission rate of radiating dipoles [38]. Purcell's idea had been intensely followed with diverse cavities in theoretical and experimental studies [39,40], and the enhancement [41] and the inhibition [42] of ES and the quantum throbs of atomic populations [41] had been examined. The ES

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properties of large sets of atomic systems have studied based on the quantum statistics [43]. In recent years, with the development of solid-state QO applications, more and more attention has been paid to photonic crystals [44,45], which is a new kind of artificial optical material with photonic band gaps.

A widespread knowledge of the essential description of the quantum mechanics has been carried out after Berry's inspiration of a geometric characteristic involved with the dynamics of a system under unitary evolutions [46–53]. Quantum evolutions of states with non-unitary, noncyclic and nonadiabatic mechanisms have been widely introduced [51,53]. Berry's asserted that in the complex phase argument, the quantum state retains the memory of its evolution, on which it relies on the geometry of the path through which the quantum system describes. The factor of the geometric phase (GP) was originated from the core of quantum mechanics, which can be characterized by the Stokes theory as an integral part of the path and may be modified to an integral part of the surface and then it carries like a geometric region. The characteristics of the GP are considered as a key to perform the execution of fault-tolerant quantum (FTQ) computation. This provides the advantages of the inherent durability provided by the topological characteristics of some typical quantum systems in order to build FTQ logic gates. Lately, significant investigations have been considered to realize the GP in various tasks QIP, such as in NMR experiments [54], ion traps [55], atoms in cavity QED [56], Josephson quantum dots [57], junction devices [58], etc.

In this manuscript, the main aim is to explore and examine the ES of a 2-level atom coupled to a λ-deformed field in the context of the JCM. We will show whether the characteristics and structures of the ES could be affected by the quantum features of the deformed field and other parameters of the model. More precisely, we will explore the dependance of the peaks corresponding to the AES on the deformed parameter, detuning parameter and half-band-width. Finally, we will study the GP of the whole system state modelled by the parity deformed JCM in terms of the main parameters of the model.

The paper is structured as follows. In Section "Model and theoretical framework" we give the important steps of the JCM model in the context of parity deformations and its dynamics of a 2-level atom system with a the single mode field. In Section "Physical quantities", we prescribes the concept of the AST and GP. In Section "Results and discussion" we show and discuss the results. Finally, discussions and conclusions are given in Section "Conclusion".

Model and theoretical framework

To describe the parity extension of the standard JCM, we introduce the parity deformed Heisenberg algebra (PDHA) which is considered as applications of para-statistics and para-fields [59–63]. This quantum algebra is described by the generators $\{I, A, A^+, R\}$ with the commutation relations [64]

$$[A, A^+] = 1 + 2\lambda R, \quad \{R, A^+\} = \{R, A\} = 0. \tag{1}$$

Here R is the parity operator and the λ is a real deformed parameter. The operator R obeys the properties

$$R^+ = R^{-1} = R. \quad R^2 = I, \tag{2}$$

and acts on a Fock state as

$$R|n\rangle = (-1)^n|n\rangle. \tag{3}$$

The N-number operator, such that $N|n\rangle = n|n\rangle$, is different from A^+A and it satisfies

$$A^+A = N + \lambda(1 - R), \quad [N, A] = -A, \quad [N, A^+] = A^+, \tag{4}$$

with

$$A|2n\rangle = \sqrt{2n}|2n-1\rangle, \quad A^+|2n\rangle = \sqrt{2n+2\lambda+1}|2n+1\rangle$$
$$A|2n+1\rangle = \sqrt{2n+2\lambda+1}|2n+1\rangle, \quad A^+|2n+1\rangle = \sqrt{2n+2}|2n+2\rangle.$$

The generators A and A^+ are given in terms of the usual annihilation and creation operators a and a^+ as follows [65]:

$$A = a - \frac{\lambda}{\sqrt{2}x}R, \quad A^+ = a^+ + \frac{\lambda}{\sqrt{2}x}R \tag{5}$$

with the deformed coordinate representation for the canonical pair $\hat{p}$ and position $\hat{x}$

$$\hat{p} = \frac{A - A^+}{i\sqrt{2}}, \quad \hat{x} = \frac{A + A^+}{\sqrt{2}}. \tag{6}$$

The PDJCM, parity extension of the standard JCM is introduced as [64]

$$\begin{aligned} H_P &= \tfrac{\omega_F}{2}\{A, A^+\} + \tfrac{\omega_A}{2}\sigma_z + g(A^+\sigma_- + A\sigma_+) \\ &= H_S + \tfrac{\omega_F}{2}\tfrac{\lambda(\lambda-1)}{x^2} - i\tfrac{\sqrt{2}g\lambda}{x}\hat{R}\sigma_y, \end{aligned} \tag{7}$$

where H_S is the standard Jaynes-Cummings model defined as

$$H_S = \frac{\omega_F}{2}\left\{a, a^+\right\} + \frac{\omega_A}{2}\sigma_z + g(a^+\sigma_- + a\sigma_+). \tag{8}$$

In the $\lambda \to 0$ limit, the parity model reduces to the standard JCM. The operators σ_- (σ_+) and σ_z denoting the lowering (rising) and inversion operators for the qubit states, $|\pm\rangle$, that satisfy the commutations $[\sigma_+, \sigma_-]$ and $[\sigma_z, \sigma_\pm] = 2\sigma_\pm$. a (a^+) indicates the annihilation (creation) operator for the mode field with the commutation relation $[a, a^+] = 1$. The term g presents the coupling strength between the 2-level atom and the field mode with the external mode frequency ω_F and the atomic transition frequency ω_A. λ characterizes the strength of the field and the coupling between the atom and the external classical field is determined by the constant $\varepsilon_{ext} r \equiv \frac{\sqrt{2}\lambda g}{x}\hat{R}$, where ε_{ext} indicates the amplitude of the external field and r refers to the qubit dipole matrix element. At the exact-resonance situation ($\omega_F = \omega_A$) with the absence of coupling ($g = 0$), it is worth mentioning that the hamiltonian operator H_P is super-symmetric. This exactly solvable model considers the inverse quadratic potential, $\frac{1}{2}\frac{\lambda(\lambda-1)}{x^2}$, that is firstly introduced by Post [66] in the context of the one-dimensional particles in the presence of the interaction of the pair forces.

The eigenvalues and eigenvectors of the Hamiltonian operators H_p are given by

$$E_\pm^{n,\lambda} = \left(2n + \lambda + 1\right)\omega_F \pm \frac{\Omega^{n,\lambda}}{2} \tag{9}$$

$$|E_\pm^n\rangle = c_\pm|2n, +\rangle \pm c_\mp|2n+1, -\rangle, \tag{10}$$

where $\Omega^{n,\lambda} = \sqrt{\Delta^2 + 4g^2(1 + 2n + 2\lambda)}$ is the generalized Rabi frequency and $\Delta = \omega_A - \omega_F$ is the detuning parameter.

The atom-field state, $|\psi(t)\rangle$, at subsequent times can be determined by using the Schrödinger equation

$$i\hbar\frac{\partial|\psi(t)\rangle}{\partial t} = H_P\left|\psi(t)\right\rangle. \tag{11}$$

The state $|\psi(t)\rangle$ can be extended as a superposition of the Fock states $|2n, +\rangle$ and $|2n+1, -\rangle$:

$$\left|\psi(t)\right\rangle = \sum_{n=0}^{\infty}[C_{2n,+}(t)|2n, +\rangle + C_{2n+1,-}(t)|2n+1, -\rangle], \tag{12}$$

where the coefficients $C_{2n,+}(t)$ and $C_{2n+1,-}(t)$ can be easily determined by substituting the state $|\psi(t)\rangle$ into Eq. 11 and replaced the initial conditions. Here, we consider the qubit system is initially defined in an upper state, $|+\rangle$ and the λ-deformed field is described by the coherent states [64]

$$\left|\xi\right\rangle_{\lambda} = \left[\frac{\left(\frac{|\xi|}{\sqrt{2}}\right)^{2\lambda-1}}{\mathscr{I}_{\lambda-\frac{1}{2}}(|\xi|^2)}\right]^{\frac{1}{2}} \sum_{n=0}^{\infty} \frac{\xi^{2n}}{\left[2^{2n}n!\Gamma\left(\frac{1}{2}+n+\lambda\right)\right]^{1/2}} \left|2n\right\rangle, \tag{13}$$

where $\mathscr{I}_{\lambda}(x)$ indicates the modified Bessel function first kind.

Physical quantities

The ES is introduced as a 2-time integration of the correlation function over the time t. For obtaining the correlation function, $f(t_1, t_2) = \langle\psi(0)|\sigma_+(t_1)\sigma_-(t_2)|\psi(0)\rangle$, of the 2-level atom, we reformulate the operator of the atomic raising as $\sigma_+(0) = |+\rangle_a\langle -| \otimes I_F$ at time $t = 0$, where $I_F = \sum_n |n\rangle\langle n|$. Subscripts F and a denoting the single-mode field and the 2-level atom system, respectively. The operator of the atomic raising $\sigma_+(t)$ at subsequent times can be determined by using the Heisenberg equation

$$i\frac{\partial\sigma_+(t)}{\partial t} = [\sigma_+(t), H_P]. \tag{14}$$

Since $\sigma_-(t)$ is the complex conjugate of $\sigma_+(t)$, one can simply find the atomic operators $\sigma_+(t_1)$ and $\sigma_-(t_2)$ at different times. Consequently, one may obtain the atomic correlation function with respect to the choice the initial state. Here, we numerically evaluate the ES of the 2-level atom coupled to the purity field and interpret the influence of the main parameters of the model on the ES. The time-dependent spectrum of a qubit system coupled to single-mode field cavity is defined by [67]

$$S(\omega, t, \Lambda) = 2\Lambda\int_0^{\infty} dt_1 \int_0^{\infty} e^{-(\Lambda - i\omega)(t-t_1)} e^{-(\Lambda + i\omega)(t-t_2)} \times \langle\psi(0)|\sigma_+(t_1)\sigma_-(t_2)|\psi(0)\rangle dt_2, \tag{15}$$

where ω defines the frequency of the probe field, Λ indicates the half-band-width of the spectrometer, t is the measured time, and $|\psi(0)\rangle$ denotes of the atom-field state at $t = 0$.

For noncyclic evolutions, the quantum systems are described by final wave functions that are different and not be gotten from the initial wave functions through a multiplication with a complex factor. Assuming that the initial state $|\psi(0)\rangle$varies with time to the state $|\psi(t)\rangle$, then we define the noncyclic phase that verifies

$$\langle\psi(0)|\psi(t)\rangle = \Gamma\exp(i\gamma), \tag{16}$$

where Γ is a real scalar. This noncyclic phase includes the cyclic GP because the latter can be considered a special case for $\Gamma = 1$. In general, determining the phase between the two varying states during the evolution is not self-evident. Pancharatnam stands the phase obtained during an the evolution of a state from $|\psi(0)\rangle$to $|\psi(t)\rangle$as $\Phi_G(t) = \arg[\Gamma\exp(i\gamma)]$.

Results and discussion

To show the effect of various physical parameters related to the whole system state on the AES, we plot $S(\omega)$ with different values of the deformed parameter λ, half-band-width Λ, and detuning Δ. Figs. 1 and 2 are for various values of λ and Λ in the resonance $\Delta = 0$ and off-resonance $\Delta = 0.75$, respectively. The 2-level atom system will have a considerable probability for emitting photon in the presence of a frequency smaller than ω_F. Generally, we can see two peaks for the atomic AES that are symmetric with the same height in the resonance case (Fig. 1). The symmetric structure of the ES is demolished by the detuning parameter Δ, providing a shift in the spacing of two adjacent peaks in the off-resonance regime, one is high and the other one is low in the off-resonance case (Fig. 2). On the other hand, when the parameter λ decreases, the two peaks increase and the spectrums move away from each other. This feature can be explained as the increasing parameter λ leads to reduce the frequency shift of the cavity field and the atom-field frequency is not detuned from the resonance. Therefore, the distance separated the neighboring peaks decreases. In addition, the

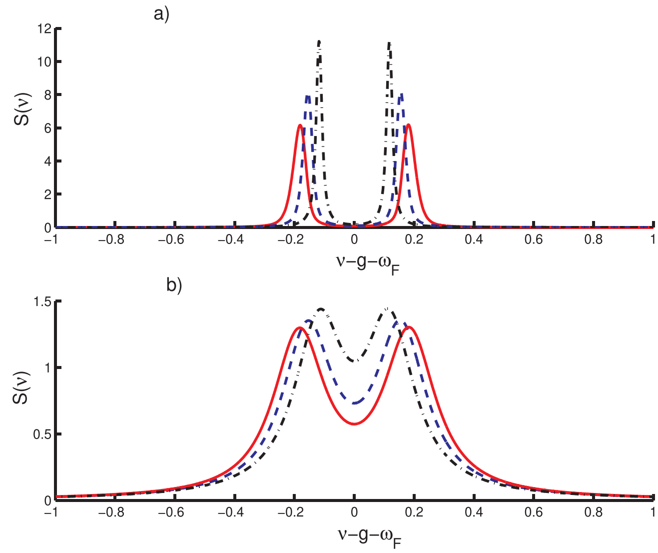

Fig. 1. Variation of the AES as a function of $\nu - g - \omega_F$ for various values of the parameters λ and Λ in the resonance case $\Delta = 0$ with $|\xi| = \sqrt{30}$. Fig. 1(a) is for $\Lambda = 0.01$ and Fig. 1(b) is for $\Lambda = 0.1$. The red line is for $\lambda = 0$, blue line is for $\lambda = 10$ and black line is for $\lambda = 30$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

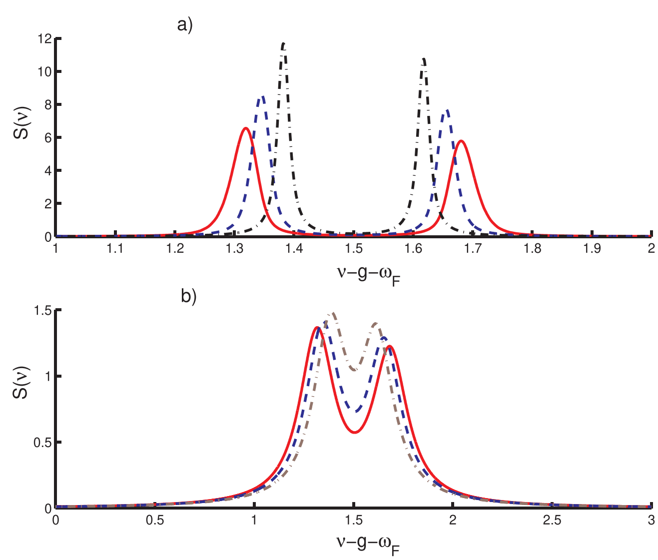

Fig. 2. Variation of the AES as a function of $\nu - g - \omega_F$ for various values of the parameters λ and Λ in the off-resonance case $\Delta = 0.75$ with $|\xi| = \sqrt{30}$. Fig. 2(a) is for $\Lambda = 0.01$ and Fig. 2(b) is for $\Lambda = 0.1$. The red line is for $\lambda = 0$, blue line is for $\lambda = 10$ and black line is for $\lambda = 30$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

value the two adjacent peaks becomes large as Λ augments with the increase in the height of the peaks in the ES. When the parameter Λ attains really a very large value, the spectrum will be appear with two sharp peaks.

The results of the GP when the field is initially starts from a standard state ($\lambda = 0$) and deformed state ($\lambda \neq 0$), are displayed versus the dimensionless time gt in Fig. 3(a) and (b) for both cases of resonance and off-resonance regimes, respectively. We can see from the plots, the GP exhibits a periodic behavior and reveals of the well-known phenomenon of collapse and revival of the GP. The increase of the parameters λ leads to appear a time shift in the evolution of the GP and reduce the duration of the collapse. On the other hand, the presence of the

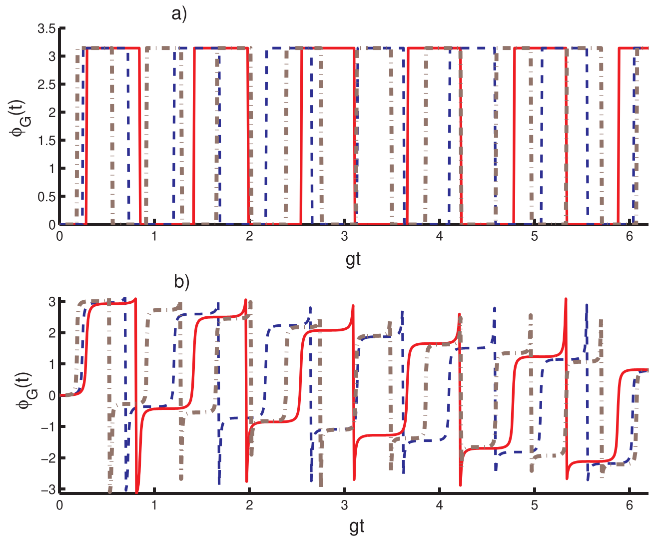

Fig. 3. Variation of the GP as a function of gt for various values of the parameter λ with $|\xi| = \sqrt{30}$. Fig. 3(a) is for $\Delta = 0$ and Fig. 3(b) is for $\Delta = 0.75$. The red line is for $\lambda = 0$, blue line is for $\lambda = 10$ and black line is for $\lambda = 30$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

detuning, the structures of the periodic oscillations of the GP disappeared with positive and negative values of the GP. For large times, these structures become much more complex. From these results. The deformed and detuning parameters can help to realize and stabilize the system during the evolution.

Conclusion

In summary, we have numerically examined the ES of a qubit (two-level atom system) coupled to deformed bosonic field in the framework of the PDJCM. We have shown how an considerable choice of the main parameters allowing to control the AES. We have seen that the ES of the qubit system reveals interesting quantum features depending on the main physical parameters. As a result, the spectrum structure presents symmetric peaks with a height and separation distance that are depending on the value of the deformed parameter λ in the resonance case. This symmetric of the atomic spectrum is destroyed in the off-resonance case. Finally, we have examined the dependence of the GP on the main parameters of the PDJCM. We have found that the GP is very sensitive to the deformed and detuning parameters. It is shown that the GP exhibits a periodic oscillations, that depends on the parameter λ, with reveals and collapse phenomena during the time evolution in the resonance case. On the other hand, the structures of the periodic oscillations of the GP disappeared with positive and negative values of the GP in the off-resonance case. From these results. the deformed and detuning parameters can help to realize and stabilize the system during the evolution.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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