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Identification of Dynamic Parameters for Robots with Elastic Joints

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This paper presents a novel method for identifying dynamic parameters of robot manipulators with elastic joints. A procedure based on the Lagrangian formulation of the dynamic model is proposed. Each term is inspected to search for a linear relationship with the dynamic parameters, thus enabling the linearization of robot dynamic model. Hence, the torque vector is expressed as the product of a regressor matrix, suitably defined by the vector of dynamic parameters. A parametric identification based on a least-squares technique is applied to determine dynamic parameters of robots with elastic joints. The correctness of the proposed procedure has been tested in simulation on two robotic structures with elastic joints of different complexity, that is, a 2-degree-of-freedom (dof) and a 6-dof manipulator, controlled with a PD control in the joint space.

1. Introduction

Robot dynamics describes the relationship between motion and forces by means of a number of parameters, including link and motor masses, inertias, and friction. Dynamic parameters can be grouped into the following three main categories, based on the contribution they provide to generate motor torques [1]:

(i) unidentifiable parameters: if a variation of the parameter does not modify the robot dynamic behavior;

(ii) linearly independent parameters: if a variation of the parameter modifies the robot dynamic behavior in a way that is not repeatable by varying another parameter (or a set of other parameters);

(iii) linearly dependent parameters: if a variation of the parameter modifies the robot dynamic behavior in a way that is repeatable by varying another parameter (or a set of other parameters).

The knowledge of robot dynamic parameters is helpful for simulation purposes [2], for control purposes [3, 4], and for mechatronic design purposes (i.e., to optimize robot design by studying the interaction between the robot and its environment [5]). Notwithstanding, a proper estimation is often unavailable. Hence, the need emerges of resorting to methods for estimating them; for clarity, a schematic classification of these methods is provided in Figure 1, with a brief description.

This paper is focused on one specific branch of the tree in Figure 1, that is, the *Parameters identification procedures* belonging to the category of offline estimation.

They consist of *"exciting"* the dynamics of a manipulator making it move along optimized trajectories through proper torque commands [1, 6]. These methods are generally used to identify parameters of open kinematic chains, generally neglecting coupled dynamics and transmission elasticity. They are grounded on three possible approaches. The first approach exploits the *Newton-Euler formulation* (N.-E.) of the dynamic model [1, 7, 8]. The second approach is based

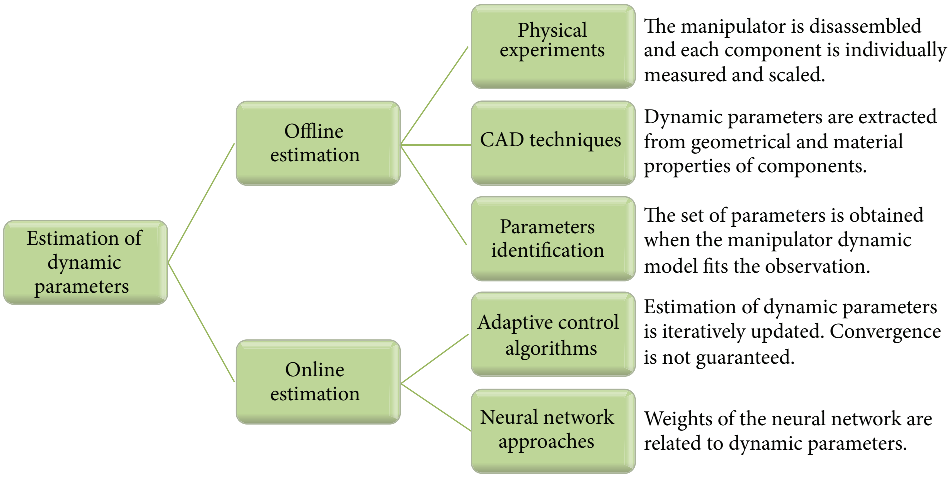

Figure 1: Methods for dynamic parameters identification (readapted from [26]).

on the *Lagrangian formulation* of the dynamic model [9, 10]. The third one exploits again *energetic* considerations [11, 12], as in the second approach based on the *Lagrangian formulation*. However, it evaluates the energy of each configuration without relating the external torques to the Lagrangian of the system. In [13] a comparative analysis of the computational burden of the Lagrangian formulation with respect to the energetic formulation is provided.

All the aforementioned approaches are formulated for robots with rigid joints and links and cannot be easily extended to the case of robots with mechanical elasticity because of the different dynamic model. On the other hand, when existing, elastic phenomena cannot be neglected because of the degradation of robot performance they may cause [3, 14, 15].

It is worth noticing that only a few works cope with the identification of the elasticity in single joints due to the transmission systems and, to our knowledge, no work does exist on the identification of the complete dynamics of robots with mechanical elasticity. For instance, in [16] a method to estimate one actuator elasticity is proposed, but motor inertia is supposed to be known. The work in [17] presents the parameter identification of one elastic joint based on a nonlinear model, and at least four series of experiments are needed to apply the proposed approach.

This paper intends to propose a novel procedure for identifying the dynamic parameters of robots with elastic joints. It is grounded on the Lagrange formulation of the dynamic model of the manipulator, thus accounting for robot elastic energy in addition to kinetic and gravitational energy contributions. Open kinematic chains and absence of coupled dynamics are supposed. Furthermore, joint elasticity is assumed to be concentrated in the transmission system between the motor and the joint. This entails that 14 dynamic parameters should be estimated for each joint. They are link mass, three coordinates of the center of mass, six components of the inertia tensor, motor inertia, static and viscous friction coefficients (assuming the Coulomb friction model for static friction), and transmission elasticity. Special attention is paid to the extraction of the regressor matrix, in order to linearize robot dynamic model and facilitate parametric identification. Also a method is proposed to identify the category to which a parameter belongs (i.e., unidentifiable, linearly dependent, or else independent).

The paper is structured as follows. Section 2 introduces the notation that is adopted in this paper. In Section 3, linearization of robot dynamic model with respect to dynamic parameters is presented. The regressor matrix relating external and actuation torques with dynamic parameters is extracted in Section 4. Parameter identification is carried out in Section 5; it also proposes a method to discriminate the category to which a parameter belongs. Finally, the proposed procedure is validated on two simulated robots (a planar and a 6-dof robot) in Section 6. Section 7 summarizes the main achievements of this work, presenting conclusions and future works.

2. Notation

In this paper, the following notation is used. All the parameters are defined in the robot base frame, that is, "0" frame, if not specified by other apexes. Regarding kinematic parameters we have the following:

(i) θ_i: angular position of motor i, divided by reduction gear k_{m_i} of transmission i;

(ii) q_i: angular position of link i, which is different from θ_i due to transmission elasticity;

(iii) $\mathbf{p}_i$, $\dot{\mathbf{p}}_i$, $\mathbf{R}_i$, $\boldsymbol{\omega}_i$: position, velocity, rotation matrix, and angular velocity of reference frame i on link i;

(iv) $\mathbf{z}_{m_i}$: rotation axis of rotor of motor i, expressed in frame $i-1$.

For defining robot dynamic parameters, it is useful to introduce the concept of "*body*." Body i is the system composed of link i and the stator of the motor rigidly connected to it. Thus, the following notation applies to dynamic parameters:

(i) m_i: mass of body i;

(ii) $\mathbf{c}_i$: product of mass and center of mass of body i, represented in frame i;

(iii) $\mathbf{I}_i$: inertia tensor of body i represented in frame i and relatively to its frame origin;

(iv) Υ_i: inertia of rotor of motor i;

(v) K_i: elasticity of transmission i;

(vi) $\mathbf{f}_{si}, \mathbf{f}_{vi}$: static and viscous friction coefficients of motor i. Coulomb friction model is assumed. Friction is supposed to be concentrated in motors, as proposed in the literature [18, 19]. In other words, it is assumed that all the components of link i prone to friction rotate with angular velocity θ_i.

3. Dynamic Model of Robot Manipulators with Elastic Joints

The following assumptions are provided:

(1) The robot has an open kinematic chain of rigid bodies, driven by electrical actuators through elastic joints undergoing small deformations in the domain of linear elasticity.

(2) Rotors of motors are rigid bodies with uniform density around their rotation axes.

(3) Stator of motor i is rigidly connected to joint $i-1$.

Robot kinetic energy T, gravitational potential energy U_g, and elastic potential energy U_e can be computed as follows:

$$\begin{aligned} T &= \sum_{i=1}^{n}\left(\frac{1}{2}m_i\dot{\mathbf{p}}_i^T\dot{\mathbf{p}}_i + \dot{\mathbf{p}}_i^T\boldsymbol{\omega}_i\times(\mathbf{R}_i\mathbf{c}_i) + \frac{1}{2}\boldsymbol{\omega}_i^T\mathbf{R}_i\mathbf{I}_i\mathbf{R}_i^T\boldsymbol{\omega}_i \right.\\ &\qquad \left. + k_{m_i}\dot{\theta}_i\Upsilon_i\mathbf{z}_{m_i}^T\mathbf{R}_{i-1}^T\boldsymbol{\omega}_{i-1} + \frac{1}{2}k_{m_i}^2\dot{\theta}_i^2\Upsilon_i\right), \\ U_g &= -\sum_{i=1}^{n}\left(m_i\mathbf{g}_0^T\mathbf{p}_i + \mathbf{g}_0^T\mathbf{R}_i\mathbf{c}_i\right), \\ U_e &= \sum_{i=1}^{n}K_i\left(q_i-\theta_i\right)^2, \end{aligned} \tag{1}$$

where n is the number of robot joints and $\mathbf{g}_0$ represents the gravity vector. For the analytical formulation of T, refer to [20] and Appendix A; for U_g, refer to [20]; for U_e, refer to [21].

By applying the Lagrangian formulation, dynamic model of robots with elastic joints can be expressed as [3, 21]

$$\begin{aligned} \boldsymbol{\tau} &= \mathbf{B}(\mathbf{q})\ddot{\overline{\mathbf{q}}} + \mathbf{C}\left(\mathbf{q},\dot{\overline{\mathbf{q}}}\right)\dot{\overline{\mathbf{q}}} + \mathbf{f}_v\dot{\overline{\mathbf{q}}} + \mathbf{f}_s\ \mathrm{sign}\left(\dot{\overline{\mathbf{q}}}\right) \\ &\quad + \mathbf{g}(\mathbf{q}) + \mathbf{K}_e\overline{\mathbf{q}}, \end{aligned} \tag{2}$$

where $\boldsymbol{\tau}^T = [(\mathbf{J}(\mathbf{q})^T\mathbf{h}_e)^T \ \ \mathbf{u}^T]^T \in \mathbb{R}^{(2n\times1)}$ is the external torque vector, $\mathbf{J}(\mathbf{q}) \in \mathbb{R}^{(6\times n)}$ is the robot Jacobian matrix, $\mathbf{h}_e \in \mathbb{R}^{(6\times1)}$ is the generalized vector of external forces, and $\mathbf{u} \in \mathbb{R}^{(n\times1)}$ is the actuation torque vector; $\overline{\mathbf{q}} = \left[\mathbf{q}^T \ \ \boldsymbol{\theta}^T\right]^T \in \mathbb{R}^{(2n\times1)}$ is the generalized joint vector, and

(1) $\mathbf{B}(\mathbf{q}) \in \mathbb{R}^{(2n\times2n)}$ is the inertia matrix, related to total robot kinetic energy T as follows:

$$T = \frac{1}{2}\dot{\overline{\mathbf{q}}}^T\mathbf{B}(\mathbf{q})\dot{\overline{\mathbf{q}}}; \tag{3}$$

(2) matrix $\mathbf{C}(\mathbf{q},\dot{\overline{\mathbf{q}}}) \in \mathbb{R}^{(2n\times2n)}$ is the centrifugal and Coriolis torque matrix, related to elements of $\mathbf{B}(\mathbf{q})$ as

$$\mathbf{C}\left(\mathbf{q},\dot{\overline{\mathbf{q}}}\right) = \sum_{k=1}^{n}\dot{q}_k\mathbf{H}^{(k)}(\mathbf{q}) - \mathrm{asym}\left(\begin{bmatrix}\dot{\overline{\mathbf{q}}}^T\mathbf{H}^{(1)}(\mathbf{q}) \\ \vdots \\ \dot{\overline{\mathbf{q}}}^T\mathbf{H}^{(n)}(\mathbf{q}) \\ \mathrm{zeros}(n,2n)\end{bmatrix}\right), \tag{4}$$

being operator asym($\mathbf{M}$) defined as asym($\mathbf{M}$) $= (\mathbf{M}-\mathbf{M}^T)/2$ and $\mathbf{H}^{(k)}(\mathbf{q}) = \partial\mathbf{B}(\mathbf{q})/\partial q_k$; zeros$(n,2n)$ represents the $(n\times 2n)$ null matrix. The relationship between matrix $\mathbf{C}(\mathbf{q},\dot{\overline{\mathbf{q}}})$ and matrices $\mathbf{H}^{(k)}(\mathbf{q})$, $k = 1\ldots n$, is shown in Appendix B;

(3) the generalized static and viscous friction matrices $\mathbf{f}_s$ and $\mathbf{f}_v$ are given by

$$\begin{aligned} \mathbf{f}_s &= \begin{bmatrix}\mathrm{zeros}(n,n) & \mathrm{zeros}(n,n) \\ \mathrm{zeros}(n,n) & \mathrm{diag}\left\{f_{s_1},\ldots,f_{s_n}\right\}\end{bmatrix}, \\ \mathbf{f}_v &= \begin{bmatrix}\mathrm{zeros}(n,n) & \mathrm{zeros}(n,n) \\ \mathrm{zeros}(n,n) & \mathrm{diag}\left\{f_{v_1},\ldots,f_{v_n}\right\}\end{bmatrix}; \end{aligned} \tag{5}$$

(4) $\mathbf{g}(\mathbf{q}) = \partial U_g/\partial\mathbf{q}$ is the gravity vector, where U_g is the gravitational potential energy;

(5) $\mathbf{K}_e\overline{\mathbf{q}} = \partial U_e/\partial\overline{\mathbf{q}}$ is the contribution of elasticity, being $\mathbf{K}_e$ expressed as [21]

$$\mathbf{K}_e = \begin{bmatrix}\mathbf{K} & -\mathbf{K} \\ -\mathbf{K} & \mathbf{K}\end{bmatrix}, \tag{6}$$

where U_e is the elastic potential energy and $\mathbf{K} = \mathrm{diag}\{K_1,\ldots,K_n\}$ is the joints elasticity matrix.

As shown in Appendix A, by exploiting the relationships between linear and angular velocity with joint velocity, inertia matrix $\mathbf{B}(\mathbf{q})$ can be written as follows:

$$\mathbf{B}(\mathbf{q}) = \begin{bmatrix}\mathbf{B}_m(\mathbf{q}) + \mathbf{B}_c(\mathbf{q}) + \mathbf{B}_I(\mathbf{q}) & \mathbf{B}_d(\mathbf{q}) \\ \mathbf{B}_d(\mathbf{q})^T & \mathbf{B}_\Upsilon\end{bmatrix}, \tag{7}$$

where

$$\mathbf{B}_m(\mathbf{q}) = \sum_{i=1}^{n} m_i \mathbf{J}_P^{(i)^T} \mathbf{J}_P^{(i)},$$

$$\mathbf{B}_c(\mathbf{q}) = \sum_{i=1}^{n} \left(\mathbf{J}_O^{(i)^T} (\mathbf{R}_i \mathbf{c}_i)_{[x]} \mathbf{J}_P^{(i)} - \mathbf{J}_P^{(i)^T} (\mathbf{R}_i \mathbf{c}_i)_{[x]} \mathbf{J}_O^{(i)} \right),$$

$$\mathbf{B}_I(\mathbf{q}) = \sum_{i=1}^{n} \mathbf{J}_O^{(i)^T} \mathbf{R}_i \mathbf{I}_i \mathbf{R}_i^T \mathbf{J}_O^{(i)},$$

$$\mathbf{B}_d(\mathbf{q}) = \left[\Upsilon_1 k_{m_1} \mathbf{J}_O^{(0)^T} \quad \mathbf{R}_0 \mathbf{z}_{m_1} \quad \cdots \quad \Upsilon_n k_{m_n} \mathbf{J}_O^{(n-1)^T} \quad \mathbf{R}_{n-1} \mathbf{z}_{m_n} \right],$$

$$\mathbf{B}_\Upsilon = \operatorname{diag}\left\{ \Upsilon_1 k_{m_1}^2, \ldots, \Upsilon_n k_{m_n}^2 \right\}. \tag{8}$$

Symbol $(\cdot)_{[x]}$ denotes the skew-symmetric matrix corresponding to the cross product. Also note that $(\cdot)_{[x]}^T = -(\cdot)_{[x]}$. $\mathbf{J}_P^{(i)}$ and $\mathbf{J}_O^{(i)}$ are the position and orientation Jacobians of body i.

It is worth observing that submatrix $\mathbf{B}_m(\mathbf{q})$ is linear with body masses m_i, $\mathbf{B}_c(\mathbf{q})$ is linear with products of body masses and body centers of mass $\mathbf{c}_i$, $\mathbf{B}_I(\mathbf{q})$ is linear with body inertia tensors $\mathbf{I}_i$, and submatrices $\mathbf{B}_d(\mathbf{q})$ and $\mathbf{B}_\Upsilon$ are linear with rotor inertias Υ_i.

Once matrix $\mathbf{B}(\mathbf{q})$ is defined, matrices $\mathbf{H}^{(k)}(\mathbf{q})$ can be easily evaluated as

$$\mathbf{H}^{(k)}(\mathbf{q}) = \begin{bmatrix} \mathbf{H}_m^{(k)}(\mathbf{q}) + \mathbf{H}_c^{(k)}(\mathbf{q}) + \mathbf{H}_I^{(k)}(\mathbf{q}) & \mathbf{H}_d^{(k)}(\mathbf{q}) \\ \mathbf{H}_d^{(k)}(\mathbf{q})^T & \operatorname{zeros}(n,n) \end{bmatrix}, \tag{9}$$

where

$$\mathbf{H}_m^{(k)}(\mathbf{q}) = \sum_{i=1}^{n} m_i \left(\mathbf{A}_{P,(k)}^{(i)^T} \mathbf{J}_P^{(i)} + \mathbf{J}_P^{(i)^T} \mathbf{A}_{P,(k)}^{(i)} \right),$$

$$\begin{aligned} \mathbf{H}_c^{(k)}(\mathbf{q}) &= \sum_{i=1}^{n} \Big(\mathbf{A}_{O,(k)}^{(i)^T} (\mathbf{R}_i \mathbf{c}_i)_{[x]} \mathbf{J}_P^{(i)} + \mathbf{J}_O^{(i)^T} \left(\left(\mathbf{J}_{O,k}^{(i)}\right)_{[x]} \mathbf{R}_i \mathbf{c}_i \right)_{[x]} \mathbf{J}_P^{(i)} \\ &\quad + \mathbf{J}_O^{(i)^T} (\mathbf{R}_i \mathbf{c}_i)_{[x]} \mathbf{A}_{P,(k)}^{(i)} - \mathbf{A}_{P,(k)}^{(i)^T} (\mathbf{R}_i \mathbf{c}_i)_{[x]} \mathbf{J}_O^{(i)} \\ &\quad - \mathbf{J}_P^{(i)^T} \left(\left(\mathbf{J}_{O,k}^{(i)}\right)_{[x]} \mathbf{R}_i \mathbf{c}_i \right)_{[x]} \mathbf{J}_O^{(i)} - \mathbf{J}_P^{(i)^T} (\mathbf{R}_i \mathbf{c}_i)_{[x]} \mathbf{A}_{O,(k)}^{(i)} \Big), \end{aligned}$$

$$\begin{aligned} \mathbf{H}_I^{(k)}(\mathbf{q}) &= \sum_{i=1}^{n} \Big(\mathbf{A}_{O,(k)}^{(i)^T} \mathbf{R}_i \mathbf{I}_i \mathbf{R}_i^T \mathbf{J}_O^{(i)} + \mathbf{J}_O^{(i)^T} \left(\mathbf{J}_{O,k}^{(i)}\right)_{[x]} \mathbf{R}_i \mathbf{I}_i \mathbf{R}_i^T \mathbf{J}_O^{(i)} \\ &\quad - \mathbf{J}_O^{(i)^T} \mathbf{R}_i \mathbf{I}_i \mathbf{R}_i^T \left(\mathbf{J}_{O,k}^{(i)}\right)_{[x]} \mathbf{J}_O^{(i)} + \mathbf{J}_O^{(i)^T} \mathbf{R}_i \mathbf{I}_i \mathbf{R}_i^T \mathbf{A}_{O,(k)}^{(i)} \Big) \end{aligned}$$

$$\mathbf{H}_d^{(k)}(\mathbf{q}) = \left[\Upsilon_1 \mathbf{r}_{a_0}^{(k)} \quad \cdots \quad \Upsilon_n \mathbf{r}_{a_n}^{(k)} \right], \tag{10}$$

being

$$\mathbf{r}_{a_i}^{(k)} = k_{m_i} \left(\mathbf{A}_{O,(k)}^{(i-1)^T} \mathbf{R}_{i-1} + \mathbf{J}_O^{(i-1)^T} \left(\mathbf{J}_{O,k}^{(i-1)} \right)_{[x]} \mathbf{R}_{i-1} \right) \mathbf{z}_{m_i}, \tag{11}$$

and $\mathbf{A}_{P,(k)}^{(i)} = \partial \mathbf{J}_P^{(i)} / \partial q_k$, $\mathbf{A}_{O,(k)}^{(i)} = \partial \mathbf{J}_O^{(i)} / \partial q_k$, and $(\mathbf{J}_{O,k}^{(i)})_{[x]} \mathbf{R}_i = \partial \mathbf{R}_i / \partial q_k$.

Finally, coefficient j of gravity vector $\mathbf{g}(\mathbf{q})$ can be easily shown to be equal to

$$\mathbf{g}(\mathbf{q})_j = \frac{\partial U_g}{\partial q_j} = \begin{cases} \mathbf{g}(\mathbf{q})_{m,j} + \mathbf{g}(\mathbf{q})_{c,j} & j \le n \\ 0 & j > n, \end{cases} \tag{12}$$

where $\mathbf{g}(\mathbf{q})_{m,j} = -\sum_{i=1}^{n} m_i \mathbf{g}_0^T \mathbf{J}_{P,j}^{(i)}$ depends only on masses, while $\mathbf{g}(\mathbf{q})_{c,j} = -\sum_{i=1}^{n} \mathbf{g}_0^T (\mathbf{J}_{O,j}^{(i)})_{[x]} \mathbf{R}_i \mathbf{c}_i$ depends only on products of masses and centers of gravity.

4. Linearization of Robot Dynamic Model

Formulation of robot dynamics as in Section 3 makes each term depend only on one dynamic parameter. This has the consequent advantage of simplifying the linearization of the dynamic model and the evaluation of regressor matrix $\mathbf{Y}(\bar{\mathbf{q}}, \dot{\bar{\mathbf{q}}}, \ddot{\bar{\mathbf{q}}})$, which relates the measured generalized external torques $\boldsymbol{\tau}$ to the robot dynamic parameters. Hence, robot dynamic model in (2) can be linearized as

$$\boldsymbol{\tau} = \left(\mathbf{Y}_B\left(\mathbf{q}, \ddot{\bar{\mathbf{q}}}\right) + \mathbf{Y}_C\left(\mathbf{q}, \dot{\bar{\mathbf{q}}}\right) + \mathbf{Y}_{\mathrm{oth}}\left(\bar{\mathbf{q}}, \dot{\boldsymbol{\theta}}\right) \right) \boldsymbol{\Pi} = \mathbf{Y}\left(\bar{\mathbf{q}}, \dot{\bar{\mathbf{q}}}, \ddot{\bar{\mathbf{q}}}\right) \boldsymbol{\Pi}, \tag{13}$$

where $\mathbf{Y}_B(\mathbf{q}, \ddot{\bar{\mathbf{q}}})$ is the regressor of the inertia matrix, $\mathbf{Y}_C(\mathbf{q}, \dot{\bar{\mathbf{q}}})$ is the regressor of the centrifugal and Coriolis torque matrix, $\mathbf{Y}_{\mathrm{oth}}(\bar{\mathbf{q}}, \dot{\boldsymbol{\theta}})$ is the regressor of all the remaining terms in (2), and

$$\boldsymbol{\Pi} = \left[\boldsymbol{\Pi}_m^T \quad \boldsymbol{\Pi}_c^T \quad \boldsymbol{\Pi}_I^T \quad \boldsymbol{\Pi}_\Upsilon^T \quad \boldsymbol{\Pi}_{f_s}^T \quad \boldsymbol{\Pi}_{f_v}^T \quad \boldsymbol{\Pi}_K^T \right]^T \tag{14}$$

is the parameter vector composed of the subvectors of mass parameters $\boldsymbol{\Pi}_m = [m_1 \ \cdots \ m_n]^T$, product of mass and center of mass parameters $\boldsymbol{\Pi}_c = [\mathbf{c}_1^T \ \cdots \ \mathbf{c}_n^T]^T$, body inertia parameters $\boldsymbol{\Pi}_I = [\mathbf{I}_1^{*T} \ \cdots \ \mathbf{I}_n^{*T}]^T$, rotor inertia parameters $\boldsymbol{\Pi}_\Upsilon = [\Upsilon_1, \ldots, \Upsilon_n]^T$, static friction coefficient parameters $\boldsymbol{\Pi}_{f_s} = [f_{s,1} \ \cdots \ f_{s,n}]^T$, viscous friction coefficient parameters $\boldsymbol{\Pi}_{f_v} = [f_{v,1} \ \cdots \ f_{v,n}]^T$, and elasticity parameters $\boldsymbol{\Pi}_K = [K_1 \ \cdots \ K_n]^T$. Body inertia $\mathbf{I}_i^*$ is expressed as $\mathbf{I}_i^* = [I_{xx,i} \ I_{yy,i} \ I_{zz,i} \ I_{xy,i} \ I_{xz,i} \ I_{yz,i}]^T$, where the first three parameters represent the inertia moments and the last three parameters represent the inertia products.

In the following the expression of the regressor matrix related to each term of the dynamic model is presented.

First, product $\mathbf{B}(\mathbf{q})\ddot{\overline{\mathbf{q}}}$ is considered. It can be rewritten as

$$\mathbf{B}(\mathbf{q})\ddot{\overline{\mathbf{q}}} = \begin{bmatrix} \mathbf{Y}_{B_m}(\mathbf{q},\ddot{\mathbf{q}})\mathbf{\Pi}_m + \mathbf{Y}_{B_c}(\mathbf{q},\ddot{\mathbf{q}})\mathbf{\Pi}_c + \mathbf{Y}_{B_I}(\mathbf{q},\ddot{\mathbf{q}})\mathbf{\Pi}_I + \mathbf{Y}_{B_{\Upsilon 1}}(\mathbf{q},\ddot{\boldsymbol{\theta}})\mathbf{\Pi}_\Upsilon \\ \mathbf{Y}_{B_{\Upsilon 2}}(\mathbf{q},\ddot{\overline{\mathbf{q}}})\mathbf{\Pi}_\Upsilon \end{bmatrix}. \quad (15)$$

The regressor matrices in (15) can be obtained from (8) as[1]

$$\mathbf{Y}_{B_m}(\mathbf{q},\ddot{\mathbf{q}}) = \begin{bmatrix}\mathbf{Y}_{B_{m,1}} & \cdots & \mathbf{Y}_{B_{m,n}}\end{bmatrix} \in \mathbb{R}^{n\times n}$$
$$\text{being}\quad \mathbf{Y}_{B_{m,i}} = \mathbf{J}_P^{(i)^T}\mathbf{J}_P^{(i)}\ddot{\mathbf{q}} \in \mathbb{R}^{n\times 1} \quad (16)$$

$$\mathbf{Y}_{B_c}(\mathbf{q},\ddot{\mathbf{q}}) = \begin{bmatrix}\mathbf{Y}_{B_{c,1}} & \cdots & \mathbf{Y}_{B_{c,n}}\end{bmatrix} \in \mathbb{R}^{n\times 3n}$$
$$\text{being}\quad \mathbf{Y}_{B_{c,i}} = \left(\mathbf{J}_P^{(i)^T}\left(\mathbf{J}_O^{(i)}\ddot{\mathbf{q}}\right)_{[x]} - \mathbf{J}_O^{(i)^T}\left(\mathbf{J}_P^{(i)}\ddot{\mathbf{q}}\right)_{[x]}\right)\mathbf{R}_i \in \mathbb{R}^{n\times 3} \quad (17)$$

$$\mathbf{Y}_{B_I}(\mathbf{q},\ddot{\mathbf{q}}) = \begin{bmatrix}\mathbf{Y}_{B_{I,1}} & \cdots & \mathbf{Y}_{B_{I,n}}\end{bmatrix} \in \mathbb{R}^{n\times 6n}$$
$$\text{being}\quad \mathbf{Y}_{B_{I,i}} = \mathbf{J}_O^{(i)^T}\mathbf{R}_i\left(\mathbf{R}_i^T\mathbf{J}_O^{(i)}\ddot{\mathbf{q}}\right)_{[\triangleright\triangleleft]} \in \mathbb{R}^{n\times 6} \quad (18)$$

$$\mathbf{Y}_{B_{\Upsilon 1}}(\mathbf{q},\ddot{\boldsymbol{\theta}}) = \begin{bmatrix}\mathbf{Y}_{B_{\Upsilon 1,1}} & \cdots & \mathbf{Y}_{B_{\Upsilon 1,n}}\end{bmatrix} \in \mathbb{R}^{n\times n}$$
$$\text{being}\quad \mathbf{Y}_{B_{\Upsilon 1,i}} = \ddot{\theta}_i k_{m_i}\mathbf{J}_O^{(i-1)^T}\mathbf{R}_{i-1}\mathbf{z}_{m_i} \in \mathbb{R}^{n\times 1} \quad (19)$$

$$\mathbf{Y}_{B_{\Upsilon 2}}(\mathbf{q},\ddot{\overline{\mathbf{q}}}) = \operatorname{diag}\left\{Y_{B_{\Upsilon 2,1}} \ \cdots \ Y_{B_{\Upsilon 2,n}}\right\} \in \mathbb{R}^{n\times n}$$
$$\text{being}\quad Y_{B_{\Upsilon 2,i}} = k_{m_i}\mathbf{z}_{m_i}^T\mathbf{R}_{i-1}^T\mathbf{J}_O^{(i-1)}\ddot{\mathbf{q}} + k_{m_i}^2\ddot{\theta}_i \in \mathbb{R}^{1\times 1}. \quad (20)$$

From (16)–(20) it follows that $\mathbf{B}_m\ddot{\mathbf{q}} = \mathbf{Y}_{B_m}\mathbf{\Pi}_m$, $\mathbf{B}_c\ddot{\mathbf{q}} = \mathbf{Y}_{B_c}\mathbf{\Pi}_c$, $\mathbf{B}_I\ddot{\mathbf{q}} = \mathbf{Y}_{B_I}\mathbf{\Pi}_I$, $\mathbf{B}_d\ddot{\boldsymbol{\theta}} = \mathbf{Y}_{B_{\Upsilon 1}}\mathbf{\Pi}_\Upsilon$, and $\mathbf{B}_d^T\ddot{\mathbf{q}} + \mathbf{B}_\Upsilon\ddot{\boldsymbol{\theta}} = \mathbf{Y}_{B_{\Upsilon 2}}\mathbf{\Pi}_\Upsilon$.

Consequently, product $\mathbf{B}(\mathbf{q})\ddot{\overline{\mathbf{q}}}$ can be expressed as $\mathbf{Y}_B(\mathbf{q},\ddot{\overline{\mathbf{q}}})\mathbf{\Pi}$, where

$$\mathbf{Y}_{\mathbf{B}}(\mathbf{q},\ddot{\overline{\mathbf{q}}}) = \begin{bmatrix} \mathbf{Y}_{\mathbf{B}_m}(\mathbf{q},\ddot{\mathbf{q}}) & \mathbf{Y}_{\mathbf{B}_c}(\mathbf{q},\ddot{\mathbf{q}}) & \mathbf{Y}_{\mathbf{B}_I}(\mathbf{q},\ddot{\mathbf{q}}) & \mathbf{Y}_{\mathbf{B}_{\Upsilon 1}}(\mathbf{q},\ddot{\boldsymbol{\theta}}) & \text{zeros}(n,3n) \\ \text{zeros}(n,n) & \text{zeros}(n,3n) & \text{zeros}(n,6n) & \mathbf{Y}_{\mathbf{B}_{\Upsilon 2}}(\mathbf{q},\ddot{\overline{\mathbf{q}}}) & \text{zeros}(n,3n) \end{bmatrix} \in \mathbb{R}^{(2n\times 14n)}. \quad (21)$$

Analogously, product $\mathbf{C}(\mathbf{q},\dot{\overline{\mathbf{q}}})\dot{\overline{\mathbf{q}}}$ can be written as (see Appendix B)

$$\mathbf{C}(\mathbf{q},\dot{\overline{\mathbf{q}}})\dot{\overline{\mathbf{q}}} = \mathbf{Y}_C(\mathbf{q},\dot{\overline{\mathbf{q}}})\mathbf{\Pi} = \left(\sum_{k=1}^{n}\left(\dot{q}_k\mathbf{Y}_{\overline{\mathbf{H}}^{(k)}}(\mathbf{q},\dot{\overline{\mathbf{q}}})\right) - \frac{1}{2}\begin{bmatrix}\dot{\overline{\mathbf{q}}}^T\mathbf{Y}_{\mathbf{H}^{(1)}}(\mathbf{q},\dot{\overline{\mathbf{q}}}) \\ \vdots \\ \dot{\overline{\mathbf{q}}}^T\mathbf{Y}_{\mathbf{H}^{(n)}}(\mathbf{q},\dot{\overline{\mathbf{q}}}) \\ \text{zeros}(n,2n)\end{bmatrix}\right)\mathbf{\Pi}, \quad (22)$$

where $\mathbf{Y}_{\mathbf{H}^{(k)}}(\mathbf{q},\dot{\overline{\mathbf{q}}})$ is defined so that $\mathbf{H}^{(k)}(\mathbf{q})\dot{\overline{\mathbf{q}}} = \mathbf{Y}_{\mathbf{H}^{(k)}}(\mathbf{q},\dot{\overline{\mathbf{q}}})\mathbf{\Pi}$. Similarly to matrix $\mathbf{Y}_{\mathbf{B}}(\mathbf{q},\ddot{\overline{\mathbf{q}}})$, matrix $\mathbf{Y}_{\mathbf{H}^{(k)}}(\mathbf{q},\dot{\overline{\mathbf{q}}})$ is given by

$$\mathbf{Y}_{\mathbf{H}^{(k)}}(\mathbf{q},\dot{\overline{\mathbf{q}}}) = \begin{bmatrix} \mathbf{Y}_{\mathbf{H}_m^{(k)}}(\mathbf{q},\dot{\mathbf{q}}) & \mathbf{Y}_{\mathbf{H}_c^{(k)}}(\mathbf{q},\dot{\mathbf{q}}) & \mathbf{Y}_{\mathbf{H}_I^{(k)}}(\mathbf{q},\dot{\mathbf{q}}) & \mathbf{Y}_{\mathbf{H}_{\Upsilon 1}^{(k)}}(\mathbf{q},\dot{\boldsymbol{\theta}}) & \text{zeros}(n,3n) \\ \text{zeros}(n,n) & \text{zeros}(n,3n) & \text{zeros}(n,6n) & \mathbf{Y}_{\mathbf{H}_{\Upsilon 2}^{(k)}}(\mathbf{q},\dot{\mathbf{q}}) & \text{zeros}(n,3n) \end{bmatrix} \in \mathbb{R}^{(2n\times 14n)}, \quad (23)$$

where

$$\mathbf{Y}_{\mathbf{H}_m^{(k)}}(\mathbf{q},\dot{\mathbf{q}}) = \begin{bmatrix}\mathbf{Y}_{\mathbf{H}_{m,1}^{(k)}} & \cdots & \mathbf{Y}_{\mathbf{H}_{m,n}^{(k)}}\end{bmatrix} \in \mathbb{R}^{n\times n}$$
$$\text{being}\quad \mathbf{Y}_{\mathbf{H}_{m,i}^{(k)}} = \mathbf{A}_{P,(k)}^{(i)^T}\mathbf{J}_P^{(j)}\dot{\mathbf{q}} + \mathbf{J}_P^{(j)^T}\mathbf{A}_{P,(k)}^{(i)}\dot{\mathbf{q}} \in \mathbb{R}^{n\times 1} \quad (24)$$

$$\mathbf{Y}_{\mathbf{H}_c^{(k)}}(\mathbf{q},\dot{\mathbf{q}}) = \begin{bmatrix}\mathbf{Y}_{\mathbf{H}_{c,1}^{(k)}} & \cdots & \mathbf{Y}_{\mathbf{H}_{c,n}^{(k)}}\end{bmatrix} \in \mathbb{R}^{n\times 3n}$$
$$\begin{aligned}\text{being}\quad \mathbf{Y}_{\mathbf{H}_{c,i}^{(k)}} = \Big(&\mathbf{A}_{P,(k)}^{(i)^T}\left(\mathbf{J}_O^{(i)}\dot{\mathbf{q}}\right)_{[x]} + \mathbf{J}_P^{(i)^T}\left(\mathbf{A}_{O,(k)}^{(i)}\dot{\mathbf{q}}\right)_{[x]} \\ &+ \mathbf{J}_P^{(i)^T}\left(\mathbf{J}_O^{(i)}\dot{\mathbf{q}}\right)_{[x]}\left(\mathbf{J}_{O,k}^{(i)}\right)_{[x]} - \mathbf{A}_{O,(k)}^{(i)^T}\left(\mathbf{J}_P^{(i)}\dot{\mathbf{q}}\right)_{[x]} \\ &- \mathbf{J}_O^{(i)^T}\left(\mathbf{A}_{O,(k)}^{(i)^T}\dot{\mathbf{q}}\right)_{[x]} \\ &- \mathbf{J}_O^{(i)^T}\left(\mathbf{J}_P^{(i)}\dot{\mathbf{q}}\right)_{[x]}\left(\mathbf{J}_{O,k}^{(i)}\right)_{[x]}\Big)\mathbf{R_i} \in \mathbb{R}^{n\times 3},\end{aligned} \quad (25)$$

$$\begin{aligned}\mathbf{Y}_{\mathbf{H}_I^{(k)}}(\mathbf{q},\dot{\mathbf{q}}) &= \left[\mathbf{Y}_{\mathbf{H}_{I,1}^{(k)}} \ \cdots \ \mathbf{Y}_{\mathbf{H}_{I,n}^{(k)}}\right] \in \mathbb{R}^{n\times 6n} \\ \text{being} \quad \mathbf{Y}_{\mathbf{H}_{I,i}^{(k)}} &= \mathbf{A}_{O,(k)}^{(i)^T}\mathbf{R}_i\left(\mathbf{R}_i^T\mathbf{J}_O^{(i)}\dot{\mathbf{q}}\right)_{[\triangleright\triangleleft]} \\ &\quad + \mathbf{J}_O^{(i)^T}\left(\mathbf{J}_{O,k}^{(i)}\right)_{[x]}\mathbf{R}_i\left(\mathbf{R}_i^T\mathbf{J}_O^{(i)}\dot{\mathbf{q}}\right)_{[\triangleright\triangleleft]} \\ &\quad - \mathbf{J}_O^{(i)^T}\mathbf{R}_i\left(\mathbf{R}_i^T\left(\mathbf{J}_{O,k}^{(i)}\right)_{[x]}\mathbf{J}_O^{(i)}\dot{\mathbf{q}}\right)_{[\triangleright\triangleleft]} \\ &\quad + \mathbf{J}_O^{(i)^T}\mathbf{R}_i\left(\mathbf{R}_i^T\mathbf{A}_{O,(k)}^{(i)}\dot{\mathbf{q}}\right)_{[\triangleright\triangleleft]} \in \mathbb{R}^{n\times 6},\end{aligned} \tag{26}$$

$$\begin{aligned}\mathbf{Y}_{\mathbf{H}_{Y1}^{(k)}}\left(\mathbf{q},\dot{\boldsymbol{\theta}}\right) &= \left[\mathbf{Y}_{\mathbf{H}_{Y1,1}^{(k)}} \ \cdots \ \mathbf{Y}_{\mathbf{H}_{Y1,n}^{(k)}}\right] \in \mathbb{R}^{n\times n} \\ \text{being} \quad \mathbf{Y}_{\mathbf{H}_{Y1,i}^{(k)}} &= \dot{\theta}_i\mathbf{r}_{a_i}^{(k)} \in \mathbb{R}^{n\times 1}\end{aligned} \tag{27}$$

$$\begin{aligned}\mathbf{Y}_{\mathbf{H}_{Y2}^{(k)}}(\mathbf{q},\dot{\mathbf{q}}) &= \operatorname{diag}\left\{\mathbf{Y}_{\mathbf{H}_{Y2,1}^{(k)}} \ \cdots \ \mathbf{Y}_{\mathbf{H}_{Y2,n}^{(k)}}\right\} \in \mathbb{R}^{n\times n} \\ \text{being} \quad \mathbf{Y}_{\mathbf{H}_{Y,i}^{(k)}} &= \mathbf{r}_{a_i}^{(k)^T}\dot{\mathbf{q}} \in \mathbb{R}^{1\times 1},\end{aligned} \tag{28}$$

where vectors $\mathbf{r}_{a_i}$ are defined as in (11).

As regards the remaining conservative terms, that is, $\mathbf{g}(\mathbf{q}) + \mathbf{K}_e\overline{\mathbf{q}}$, and the dissipative terms, that is, $\mathbf{f}_v\dot{\overline{\mathbf{q}}} + \mathbf{f}_s\,\mathrm{sign}(\dot{\overline{\mathbf{q}}})$, they can be written as

$$\mathbf{g}(\mathbf{q}) + \mathbf{K}_e\overline{\mathbf{q}} + \mathbf{f}_v\dot{\overline{\mathbf{q}}} + \mathbf{f}_s\,\mathrm{sign}\left(\dot{\overline{\mathbf{q}}}\right) = \mathbf{Y}_{\mathrm{oth}}\left(\overline{\mathbf{q}},\dot{\boldsymbol{\theta}}\right)\boldsymbol{\Pi}, \tag{29}$$

where

$$\mathbf{Y}_{\mathrm{oth}}\left(\overline{\mathbf{q}},\dot{\boldsymbol{\theta}}\right) = \begin{bmatrix}\mathbf{Y}_{g_m}(\mathbf{q}) & \mathbf{Y}_{g_c}(\mathbf{q}) & \mathrm{zeros}(n,7n) & \mathrm{zeros}(n,n) & \mathrm{zeros}(n,n) & \mathbf{Y}_K(\overline{\mathbf{q}}) \\ \mathrm{zeros}(n,n) & \mathrm{zeros}(n,3n) & \mathrm{zeros}(n,7n) & \mathbf{Y}_{f_s}\left(\dot{\boldsymbol{\theta}}\right) & \mathbf{Y}_{f_v}\left(\dot{\boldsymbol{\theta}}\right) & -\mathbf{Y}_K(\overline{\mathbf{q}})\end{bmatrix} \in \mathbb{R}^{(2n\times 14n)}, \tag{30}$$

being both $\mathbf{Y}_{g_m}(\mathbf{q})$ and $\mathbf{Y}_{g_c}(\mathbf{q})$ upper triangular matrices satisfying relations $\mathbf{g}(\mathbf{q})_{m,j} = \mathbf{Y}_{g_m}(\mathbf{q})\boldsymbol{\Pi}_m$ and $\mathbf{g}(\mathbf{q})_{c,j} = \mathbf{Y}_{g_c}(\mathbf{q})\boldsymbol{\Pi}_c$ with

$$\begin{aligned}\mathbf{Y}_{g_m}(\mathbf{q}) &= \begin{bmatrix}-\mathbf{g}_0^T\mathbf{J}_{P,1}^{(1)} & \cdots & -\mathbf{g}_0^T\mathbf{J}_{P,1}^{(n)} \\ & \ddots & \vdots \\ 0 & & -\mathbf{g}_0^T\mathbf{J}_{P,n}^{(n)}\end{bmatrix} \in \mathbb{R}^{n\times n}, \\ \mathbf{Y}_{g_c}(\mathbf{q}) &= \begin{bmatrix}-\mathbf{g}_0^T\left(\mathbf{J}_{O,1}^{(1)}\right)_{[x]}\mathbf{R}_1 & \cdots & -\mathbf{g}_0^T\left(\mathbf{J}_{O,1}^{(n)}\right)_{[x]}\mathbf{R}_n \\ & \ddots & \vdots \\ 0 & & -\mathbf{g}_0^T\left(\mathbf{J}_{O,n}^{(n)}\right)_{[x]}\mathbf{R}_n\end{bmatrix} \in \mathbb{R}^{n\times 3n}.\end{aligned} \tag{31}$$

On the other hand, $\mathbf{Y}_{f_s}$, $\mathbf{Y}_{f_v}$, and $\mathbf{Y}_K$ are diagonal matrices defined as

$$\mathbf{Y}_{f_s}\left(\dot{\boldsymbol{\theta}}\right) = \operatorname{diag}\left\{\mathrm{sign}\left(\dot{\theta}_1\right),\ldots,\mathrm{sign}\left(\dot{\theta}_n\right)\right\} \in \mathbb{R}^{n\times n}, \tag{32}$$

$$\mathbf{Y}_{f_v}\left(\dot{\boldsymbol{\theta}}\right) = \operatorname{diag}\left\{\dot{\theta}_1,\ldots,\dot{\theta}_n\right\} \in \mathbb{R}^{n\times n}, \tag{33}$$

$$\mathbf{Y}_K(\overline{\mathbf{q}}) = \operatorname{diag}\left\{q_1-\theta_1,\ldots,q_n-\theta_n\right\} \in \mathbb{R}^{n\times n}. \tag{34}$$

5. Parametric Identification Procedure

A parametric identification procedure for robots with elastic joints is proposed to identify vector of dynamic parameters $\boldsymbol{\Pi}$. As for the rigid case [7, 10], it exploits the property of linearity of robot dynamic model with respect to the vector of dynamic parameters in (13). The parametric identification procedure requires recording manipulator actuation torques $\mathbf{u}$, and motion variables $\overline{\mathbf{q}}$, $\dot{\overline{\mathbf{q}}}$, and $\ddot{\overline{\mathbf{q}}}$, while moving the robot in the free space along suitable trajectories. For each measurement k, with $k = 1,\ldots,w$, matrix $\mathbf{W}_k = \mathbf{Y}(\overline{\mathbf{q}}_k,\dot{\overline{\mathbf{q}}}_k,\ddot{\overline{\mathbf{q}}}_k) \in \mathbb{R}^{2n\times 14n}$ and vector $\mathbf{T}_k = \left[\mathbf{0}^T \ \ \mathbf{u}_k^T\right]^T \in \mathbb{R}^{2n\times 1}$ can be evaluated and, by resorting to (13), the following relation can be written:

$$\begin{bmatrix}\mathbf{T}_1 \\ \vdots \\ \mathbf{T}_w\end{bmatrix} = \begin{bmatrix}\mathbf{W}_1 \\ \vdots \\ \mathbf{W}_w\end{bmatrix}\boldsymbol{\Pi}, \tag{35}$$

or, alternatively,

$$\mathbf{T} = \mathbf{W}\boldsymbol{\Pi}, \tag{36}$$

where

$$\mathbf{W} = \begin{bmatrix}\mathbf{W}_1 \\ \vdots \\ \mathbf{W}_w\end{bmatrix} \in \mathbb{R}^{(2nw\times 14n)}, \qquad \mathbf{T} = \begin{bmatrix}\mathbf{T}_1 \\ \vdots \\ \mathbf{T}_w\end{bmatrix} \in \mathbb{R}^{(2nw\times 1)}. \tag{37}$$

If matrix $\mathbf{W}$ is of full rank and the number of rows is greater than (or at least equal to) the number of columns, parameter vector $\boldsymbol{\Pi}$ can be identified as

$$\boldsymbol{\Pi} = \mathbf{W}^{\dagger}\mathbf{T}, \tag{38}$$

where $\mathbf{W}^{\dagger} = (\mathbf{W}^T\mathbf{W})^{-1}\mathbf{W}^T$ is the left pseudoinverse matrix of $\mathbf{W}$. It is worth noticing that, regarding relation (36) as a system of $2nw$ equations in $14n$ variables, at least 7 measurements are required for the identification of the vector $\boldsymbol{\Pi}$; thus $w \geq 7$.

Otherwise, if rank of matrix $\mathbf{W}$ is not full, it is possible to identify the category to which a single parameter belongs [1] with the procedure explained as follows.

(1) If column i of matrix $\mathbf{W}$ is a null vector, then parameter π_i is unidentifiable and does not affect robot dynamics.

(2) If column i of matrix $\mathbf{W}$ is not null, but it can be obtained as a linear combination of other columns, then parameter π_i is identifiable only in linear combination with other parameters.

(3) If column i of matrix $\mathbf{W}$ is not null and cannot be obtained as a linear combination of other columns, then parameter π_i is independently identifiable.

In order to evaluate whether parameters are independently identifiable or not, Gauss-Jordan elimination procedure can be applied to evaluate a base matrix $\mathbf{L}$ for matrix $\mathbf{W}$, that is, a set of linearly independent columns of matrix $\mathbf{W}$. Then, matrix $\mathbf{K}$ can be introduced, composed of not-null columns of matrix $\mathbf{W}$ which are not columns of matrix $\mathbf{L}$. Thus, each column $\mathbf{K}_i$ of matrix $\mathbf{K}$ can be written as a linear combination of columns $\mathbf{L}_j$ of matrix $\mathbf{L}$ as follows:

$$\mathbf{K}_i = a_{i,1}\mathbf{L}_1 + \cdots + a_{i,m}\mathbf{L}_m = \mathbf{L}\mathbf{A}_i \quad \mathbf{A}_i = \left[a_{i,1} \ \cdots \ a_{i,m}\right]^T, \tag{39}$$

where m is the number of vectors composing the base of $\mathbf{W}$.

Eventually, by grouping vector $\mathbf{\Pi}$ into two subvectors $\mathbf{\Pi_L}$ and $\mathbf{\Pi_K}$, the following relations can be obtained:

$$\mathbf{W\Pi} = \left[\mathbf{L} \ \ \mathbf{K}\right]\begin{bmatrix}\mathbf{\Pi_L} \\ \mathbf{\Pi_K}\end{bmatrix}, \tag{40}$$

and, due to relation (39),

$$\mathbf{W\Pi} = \mathbf{L}\left[\mathbf{I}_{(m\times m)} \ \ \mathbf{A}\right]\begin{bmatrix}\mathbf{\Pi_L} \\ \mathbf{\Pi_K}\end{bmatrix} = \mathbf{L\Pi}^*, \tag{41}$$

where $\mathbf{I}_{(m\times m)}$ is the $(m \times m)$ identity matrix, $\mathbf{A} = \left[\mathbf{A}_1 \ \cdots \ \mathbf{A}_{14n-m}\right]$ is the matrix composed of columns $\mathbf{A}_i$, and $\mathbf{\Pi}^* = \mathbf{\Pi_L} + \mathbf{A\Pi_K}$ is the vector of parameters to be identified. Thus, parameter $\mathbf{\Pi}_{\mathbf{L}_j}$ is independently identifiable if row j of matrix $\mathbf{A}$ is a null vector; otherwise, parameter $\mathbf{\Pi}_{\mathbf{L}_j}$ is identifiable only in linear combination with some parameters of vector $\mathbf{\Pi_K}$. Parameters of vector $\mathbf{\Pi_K}$ are identifiable only in linear combination.

Further, vector $\mathbf{\Pi}^*$ can be computed through a least-squares approach as

$$\mathbf{\Pi}^* = \left(\mathbf{L}^T\mathbf{L}\right)^{-1}\mathbf{L}^T\mathbf{T}. \tag{42}$$

6. Validation and Results

Simulation tests have been performed to validate the proposed identification procedure of dynamic parameters on robots with elastic joints. To this purpose, two robotic structures of different complexity have been modeled. The former is a planar manipulator with 2 dofs and elastic joints (see Figure 2). The latter is a 6-dof manipulator modeled as a PUMA 560 with additional elastic joints.

The planar manipulator moves in the vertical plane; hence, gravitational acceleration $\mathbf{g}_0$ is directed along axis $-y_0$ with respect to the base reference system ($\mathbf{g}_0 = \left[0 \ \ -9.81 \ \ 0\right]^T$ m/s^2).

Kinematic and dynamic parameters of the planar robot are as follows: $a_1 = a_2 = 0.5$ m (a_i: link length), $l_1 = l_2 = 0.25$ m (l_i: center of mass distance; see Figure 2), $m_1 = 20$ kg, $m_2 = 10$ kg (m_i: link mass), $m_{m_1} = m_{m_2} = 1$ kg (m_{m_i}: motor mass), $k_{m_1} = k_{m_2} = 0.1$ (k_{m_i}: transmission reduction gear),

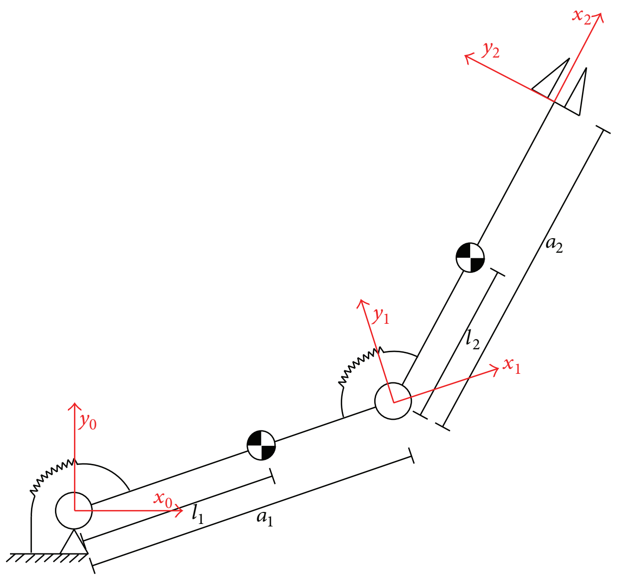

Figure 2: 2-dof planar robot with elastic joints. Gravity vector is directed along $-y_0$ axis.

$K_1 = 3000$ N · m/rad, $K_2 = 1800$ N · m/rad (K_i: transmission elasticity), $I_{l_1} = 0.4667$ kg · m^2, $I_{l_2} = 0.2333$ kg · m^2 (I_{l_i}: link inertia around rotation axis), $\Upsilon_1 = 21.18$ kg · m^2, and $\Upsilon_2 = 12.10$ kg · m^2 (Υ_i: motor inertia). It is supposed that friction torque F_i acts on joint i. It accounts for the Stribeck effect and is modeled as follows [22]:

$$\begin{aligned} F_i = {} & f_{1,i}\dot{\theta}_i + f_{2,i}\operatorname{sign}\left(\dot{\theta}_i\right) - f_{3,i}\operatorname{sign}\left(\dot{\theta}_i\right)e^{-|\dot{\theta}_i|/f_{4,i}} \\ & - f_{5,i}\operatorname{sign}\left(\dot{\theta}_i\right)e^{-1/f_{6,i}|\dot{\theta}_i|}, \end{aligned} \tag{43}$$

where

$$\begin{aligned} f_{1,1} &= 0.1434 & f_{1,2} &= 0.1391 \\ f_{2,1} &= 0.3302 & f_{2,2} &= 0.3576 \\ f_{3,1} &= 0.2499 & f_{3,2} &= 0.2451 \\ f_{4,1} &= 0.0537 & f_{4,2} &= 0.1073 \\ f_{5,1} &= 0.2991 & f_{5,2} &= 0.3389 \\ f_{6,1} &= 16.624 & f_{6,2} &= 10.097. \end{aligned} \tag{44}$$

In accordance with model in (5), only the static and viscous components of the friction torque are taken into account, thus neglecting the nonlinear effects as shown, for example, in Figure 3 for the Coulomb friction. In the identification procedure, static and viscous components will be identified (as shown in (32) and (33)), while the nonlinear effects will be regarded as external disturbances.

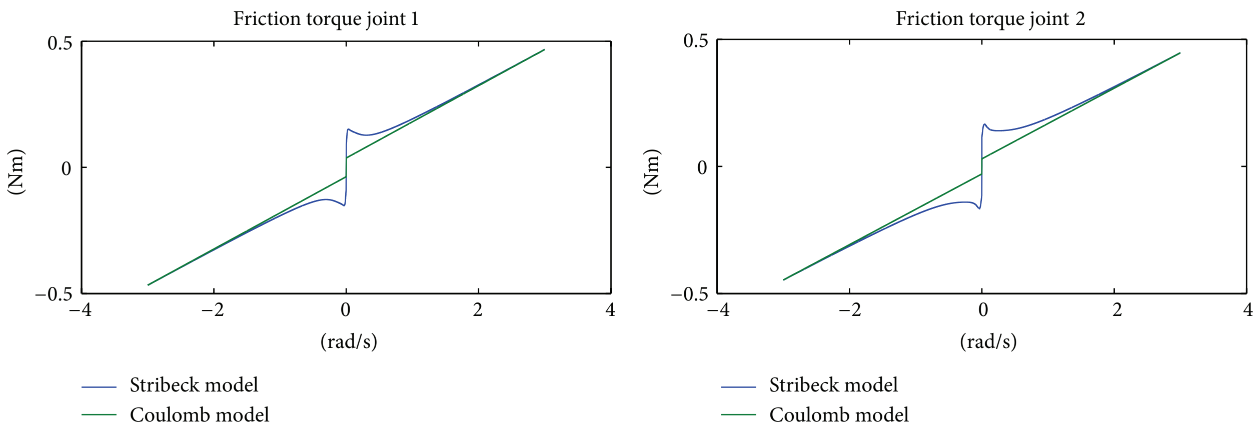

FIGURE 3: Complete (Stribeck) friction model [22] and Coulomb friction model.

The computed body parameters are given by

$$\begin{gathered} m_1 = 21\,\text{kg} \qquad m_2 = 10\,\text{kg} \\ \mathbf{c}_1 = \begin{bmatrix} -5 & 0 & 0 \end{bmatrix}^T \text{kg} \cdot \text{m} \\ \mathbf{c}_2 = \begin{bmatrix} -2.5 & 0 & 0 \end{bmatrix}^T \text{kg} \cdot \text{m} \\ \mathbf{I}_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1.25 & 0 \\ 0 & 0 & 13.8167 \end{bmatrix} \text{kg} \cdot \text{m}^2 \\ \mathbf{I}_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.625 & 0 \\ 0 & 0 & 0.8583 \end{bmatrix} \text{kg} \cdot \text{m}^2. \end{gathered} \tag{45}$$

The PD control proposed in [3] has been used to control robot motion in the free space. Excitation trajectories based on fifth order B-splines, as in [23], have been used as reference trajectory for each joint.

Joint angular positions have been sampled at 1 kHz and quantized to simulate a real scenario, by assuming to have a high resolution encoder (1250000 counts/round). Data have been filtered through a Butterworth IIR low-pass filter of fifth-order and cut-off frequency of 20 Hz. Motor and joint angular velocities and accelerations have been computed through a first-order numerical differentiation. Additionally, the effect of sensor noise on the performance of the proposed identification procedure has been evaluated. To this purpose, a Gaussian noise with zero mean value has been added to motor and joint position, velocity, and acceleration and the error percentage between real and estimated dynamic parameters has been measured for five different levels of signal-to-noise ratio (i.e., SN = 20, SN = 70, SN = 80, SN = 100, and SN = 200), in addition to absence of noise. Motion variables have been obtained by means of the robot forward dynamics also accounting for the friction model in (43). In Figure 4 motor torques applied to the simulated manipulator are shown, with the corresponding motion in the joint space. As shown in Figure 5, the maximum difference between link and motor position is 0.39° and 0.10° for first and second joint, respectively.

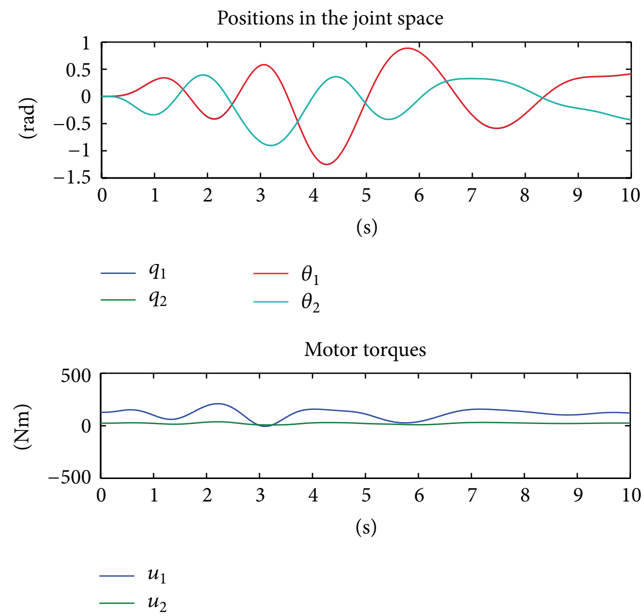

FIGURE 4: Actuation torques for the 2-dof planar manipulator have been generated with the PD control in [3]; corresponding robot motion in the joint space is shown.

A total of 8900 measurements have been performed, thus constructing observation matrix $\mathbf{W}$ and torque vector $\mathbf{T}$ in (36). In order to reject nonlinear friction effects, measurements where joint velocity was lower than a threshold value of 0.5 rad/s have been discarded (a total of 1334 measurements have been discarded). A systematic approach to define this threshold value is presented in [24]. By inspecting matrix $\mathbf{W}$, vector $\mathbf{\Pi}^*$ of 14 identifiable parameters (or linear combination of parameters) has been extracted by means of relation (41), and matrix $\mathbf{L}$ has been computed. The obtained values of parameters in $\mathbf{\Pi}^*$ in absence of measurement noise are reported in Table 1, where $\mathbf{c}_i = [c_{x,i} \quad c_{y,i} \quad c_{z,i}]^T$ (in SI units).

As one can easily observe in Table 1, the error between the obtained estimated parameters and the real ones is very low. The normalised mean error is 0.103; it becomes 0.00862

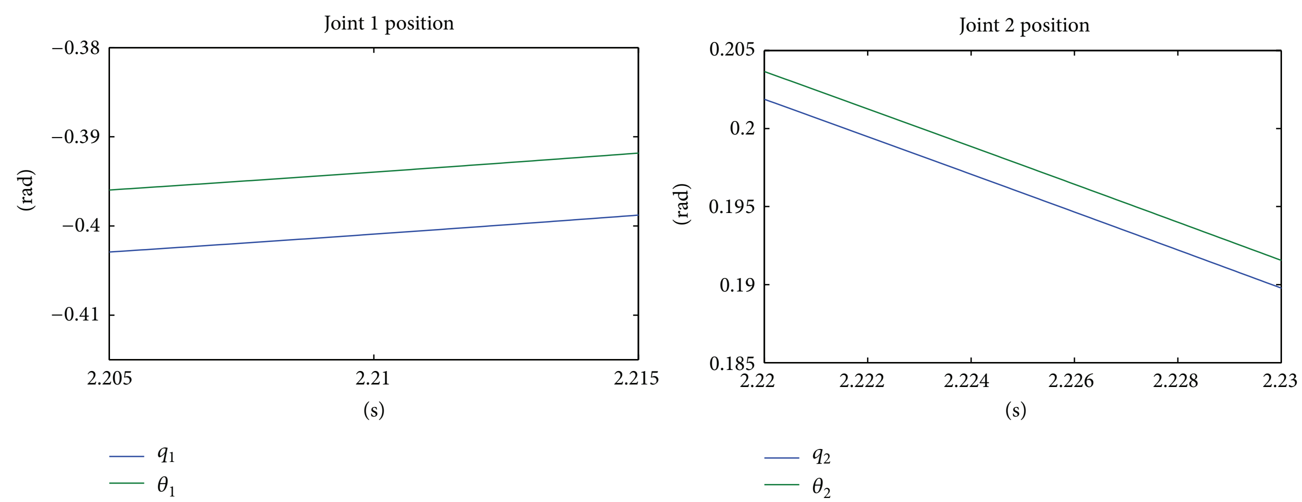

FIGURE 5: Zoom on the main differences between motor and link positions for the 2-dof planar manipulator.

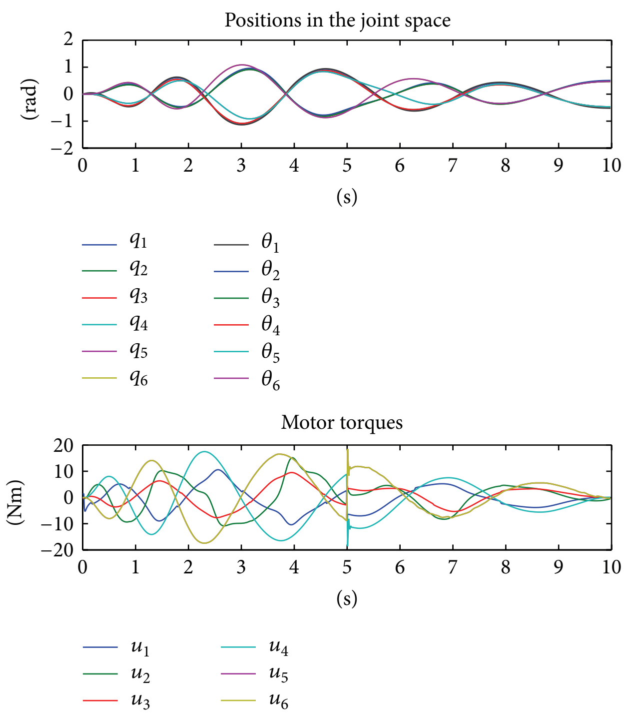

FIGURE 6: Actuation torques for the 6-dof manipulator have been generated with the PD control in [3]; corresponding robot motion in the joint space is shown.

TABLE 1: Results of the identification procedure.

Parameter	Real value	Estimated value
$\pi_1^* = K_1$	3000.00 N·m/rad	3000.79 N·m/rad
$\pi_2^* = K_2$	1800.00 N·m/rad	1799.79 N·m/rad
$\pi_3^* = m_1 - 4 \cdot I_{zz,1} + 4 \cdot I_{zz,2}$	−30.8333 kg	−30.8352 kg
$\pi_4^* = m_2 - 4 \cdot I_{zz,2}$	6.5667 kg	6.5670 kg
$\pi_5^* = c_{x,1} + 2 \cdot I_{zz,1}$	22.6333 kg·m	22.6345 kg·m
$\pi_6^* = c_{y,1}$	0.0000 kg·m	$-1.04E^{-5}$ kg·m
$\pi_7^* = c_{x,2} + 2 \cdot I_{zz,2}$	−0.7833 kg·m	−0.7835 kg·m
$\pi_8^* = c_{y,2}$	0.0000 kg·m	$3.18E^{-6}$ kg·m
$\pi_9^* = \Upsilon_1$	21.1800 kg·m^2	21.1446 kg·m^2
$\pi_{10}^* = \Upsilon_2$	12.1000 kg·m^2	12.1036 kg·m^2
$\pi_{11}^* = f_{s,1}$	0.3302 N·m	0.1016 N·m
$\pi_{12}^* = f_{s,2}$	0.3576 N·m	0.1249 N·m
$\pi_{13}^* = f_{v,1}$	0.1434 N·m·s/rad	0.1288 N·m·s/rad
$\pi_{14}^* = f_{v,2}$	0.1391 N·m·s/rad	0.1389 N·m·s/rad

TABLE 2: Variation of the estimation error with signal-to-noise ratio.

Signal-to-noise ratio	Normalised mean error
SN = 20	$3.84E^{+3}$
SN = 70	44.50
SN = 80	4.73
SN = 100	0.157
SN = 200	0.111

(i.e., 0.862 per cent as percentage error) without the static friction, thus pointing out that probably the neglected nonlinear terms of the adopted model may cause an increase of the error. Furthermore, Table 2 reports the normalised mean error between the estimated parameters and the real ones in the case of noisy measurements of position, velocity, and acceleration, for five different levels of signal-to-noise ratio (i.e., SN = 20, SN = 70, SN = 80, SN = 100, and SN = 200).

One can observe that for high values of signal-to-noise ratio, up to SN = 80, the error still remains small and comparable to the absence of noise. For lower values of signal-to-noise ratio, the error becomes very high.

As aforementioned, the proposed identification procedure of dynamic parameters has also been applied to a 6-dof robot manipulator with elastic joints. It has been modeled as a PUMA 560 with elastic joints. Robot dynamic parameters

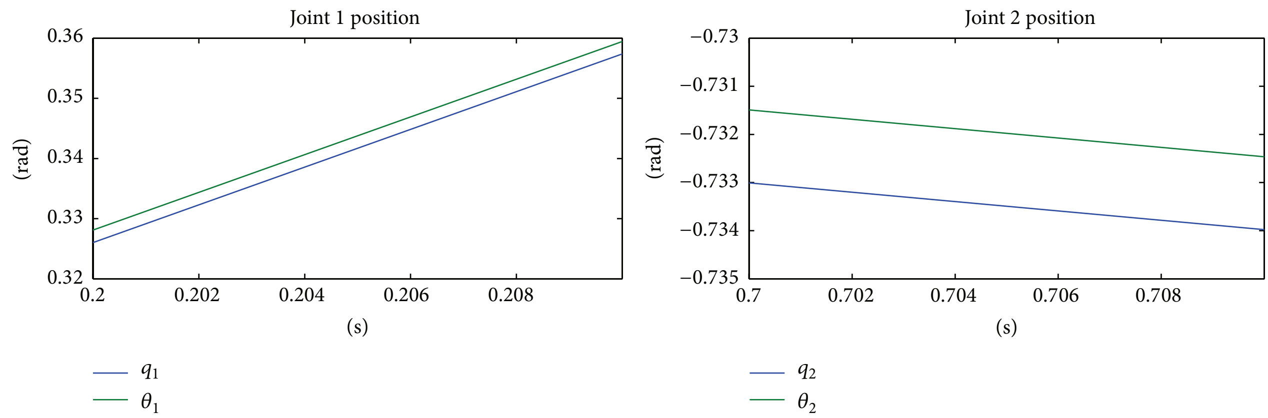

FIGURE 7: Zoom on the main differences between motor and link positions for the 6-dof manipulator.

(except for joint elasticity) are reported in [25]. On the other hand, transmission elasticity for each joint has been assumed to be as $K_i = 2000\,\mathrm{N}\cdot\mathrm{m/rad}$ for $i = 1, \ldots, 6$.

By analogy with the planar case, excitation trajectories based on fifth-order B-splines have been planned in the joint space and the PD control in [3] has been used to regulate robot position. Motor torques applied to the simulated manipulator and resulting motion in the joint space are reported in Figure 6. For brevity, only the maximum difference between link and motor position for joints 1 and 2 is shown in Figure 7. However, the mean value over the six joints resulted to be 0.09 rad.

As for the planar case, observation matrix $\mathbf{W}$ and torque vector $\mathbf{T}$ in (36) have been constructed. The complete vector of 84 dynamic parameters (i.e., Π) has been estimated through the proposed procedure. The obtained parameters are reported in Tables 3, 4, 5, and 6 together with the values of the real dynamic parameters related to joint elasticity, masses, centers of mass, inertia tensors, motor inertias, and friction. Afterwards, vector $\mathbf{\Pi}^*$ of 62 identifiable parameters (or linear combination of parameters) and matrix $\mathbf{L}$ have been extracted. The normalised mean error between estimated and real parameters resulted to be 0.123, that is, very closed to the value obtained in the planar case. The provided simulation tests prove the correctness of the proposed methodology. As future work, the performance on a real robot will be measured.

7. Conclusions

A novel procedure for identifying dynamic parameters for robots with elastic joints has been proposed. The proposed procedure is based on the Lagrangian formulation of the dynamic model of the manipulator, accounting for robot elastic energy, in addition to kinetic and gravitational energy contributions. This feature represents the actual main novelty of this work, since no systematic approaches for parameter identification have been reported in the literature for robots with elastic joints.

Each term of the dynamic model has been analyzed in order to linearize it and express the vector of external and motor torques as the product of a regressor matrix by a vector of dynamic parameters to identify. Special attention has been paid to the extraction of the regressor matrix, in order to facilitate parametric identification. Also a method is proposed to identify the category (unidentifiable, linearly dependent, or else independent) to which a parameter belongs. Two robotic structures with elastic joints of different complexity have been simulated in order to validate the procedure of parametric identification, that is, a 2-dof planar manipulator and a 6-dof manipulator. The PD control in [3] has been used to move the robot in the free space; the simulation of the forward dynamics of the manipulator has permitted to collect joint and motor positions, velocities, and accelerations during motion and consequently apply the identification procedure with and without measurement noise.

The obtained results have shown that the proposed procedure leads to a correct identification of the manipulator dynamic parameters with a very low error; the mean normalised error between actual parameters and estimated ones is 0.103 for the planar manipulator and 0.123 for the 6-dof manipulator. The percentage error decreases to a value around 0.862 percent if the static friction is excluded (which is more affected from the neglected nonlinearities of the adopted friction model). In presence of measurement noise, the error is still low for signal-to-noise ratios higher than 80. The correctness of the analytical formulation of the regressor matrix and, consequently, of the parametric identification procedure is thus assessed and the application on a real robot with elastic joints can be envisaged as future activity.

Appendices

A. Robot Kinetic Energy

This Appendix extends the formulation of kinetic energy in [20] to robots with elastic joints. Before presenting it, it is useful to introduce following notation.

(i) If not differently specified, positions are defined in the robot base frame, that is, the "0" frame, apart from motor axis vector $\mathbf{z}_{m_i}$, which is defined in frame

Table 3: Real and estimated parameters related to joint elasticity and masses for the 6-dof manipulator.

	π_1	π_2	π_3	π_4	π_5	π_6	π_7	π_8	π_9	π_{10}	π_{11}	π_{12}
Real	2000	2000	2000	2000	2000	2000	0.10	17.4	4.80	0.82	0.34	0.09
Estimated	1999.3	1999.8	1999.6	1999.9	1999.3	2000.1	0.0985	17.41	4.7988	0.819	0.339	0.0889

Table 4: Real and estimated parameters related to the centers of mass for the 6-dof manipulator.

	π_{13}	π_{14}	π_{15}	π_{16}	π_{17}	π_{18}	π_{19}	π_{20}	π_{21}
Real	2000	0	0	−6.3301	0.1044	3.9585	−0.0974	−0.0677	0.336
Estimated	$-0.25E^{-4}$	$-0.22E^{-4}$	$-0.15E^{-5}$	−6.33087	0.10425	3.95835	−0.09769	−0.0679	0.33587
	π_{22}	π_{23}	π_{24}	π_{25}	π_{26}	π_{27}	π_{28}	π_{29}	π_{30}
Real	0	0.0156	0	0	0	0	0	0	0.0029
Estimated	−0.00015	0.015	$-0.12E^{-4}$	$-0.21E^{-4}$	$-0.15E^{-4}$	$-0.25E^{-4}$	$-0.18E^{-4}$	$-0.25E^{-4}$	0.0026

$i-1$ and vector $\mathbf{c}_i$, which is defined in frame i, as introduced in Section 2.

(ii) Inertia tensor is defined as ${}^a\mathbf{I}_c^b$; it expresses the inertia tensor of body c relatively to point b and defined in reference frame a. Symbol $\mathbf{I}_c$ indicates the inertia tensor when $a=b=c$.

Moreover, the following relations hold.

Velocity Composition. If points $\mathbf{p}_i$ and $\mathbf{p}_j$ are rigidly connected, their velocities are related as

$$\dot{\mathbf{p}}_i = \dot{\mathbf{p}}_j + \boldsymbol{\omega}_i \times \left(\mathbf{p}_i - \mathbf{p}_j\right), \tag{A.1}$$

where $\boldsymbol{\omega}_i$ is the angular velocity.

Steiner's Theorem for Tensors. Steiner's theorem relates inertia tensor ${}^a\mathbf{I}_b^{\mathbf{p}_i}$ of body b relative to point $\mathbf{p}_i$ to inertia tensor ${}^a\mathbf{I}_b^{\mathbf{p}_j}$ relative to point $\mathbf{p}_j$ (defined in the same frame a) as follows:

$${}^a\mathbf{I}_b^{p_i} = {}^a\mathbf{I}_b^{p_j} + m_b\left(\mathbf{p}_i - \mathbf{p}_j\right)_{[x]}^T\left(\mathbf{p}_i - \mathbf{p}_j\right)_{[x]}, \tag{A.2}$$

where m_b is the mass of body b.

Tensor Reference Frame. Tensor reference frame can be easily moved from frame i to frame j with the following relation:

$${}^i\mathbf{I}_b^c = {}^i\mathbf{R}_j {}^j\mathbf{I}_b^c {}^i\mathbf{R}_j^T. \tag{A.3}$$

Kinetic energy of a robot with elastic joints is given by

$$T = \sum_{i=1}^{n}\left(T_{l_i} + T_{m_i}\right), \tag{A.4}$$

where T_{l_i} is the energy of link i and T_{m_i} is the energy of rotor of motor i. Stator of motor $i+1$ is supposed to be rigidly connected to link i; therefore, its contribution to kinetic energy is included in T_{l_i}.

Energy of link i is given by

$$T_{l_i} = \frac{1}{2}m_{l_i}\dot{\mathbf{p}}_{l_i}^T\dot{\mathbf{p}}_{l_i} + \frac{1}{2}\boldsymbol{\omega}_i^T {}^0\mathbf{I}_{l_i}^{l_i}\boldsymbol{\omega}_i, \tag{A.5}$$

where $\mathbf{p}_{l_i}$ is the position of the center of mass of link i, and ${}^0\mathbf{I}_{l_i}^{l_i}$ is its inertia tensor.

Indicating with $\mathbf{p}_{C_i}$ the position of the center of mass of body i and exploiting relations (A.1) and (A.2), kinetic energy can be expressed as

$$T_{l_i} = \frac{1}{2}m_{l_i}\dot{\mathbf{p}}_{C_i}^T\dot{\mathbf{p}}_{C_i} + m_{l_i}\dot{\mathbf{p}}_{C_i}^T\boldsymbol{\omega}_{i_{[x]}}\left(\mathbf{p}_{l_i} - \mathbf{p}_{C_i}\right) + \frac{1}{2}\boldsymbol{\omega}_i^T {}^0\mathbf{I}_{l_i}^{C_i}\boldsymbol{\omega}_i. \tag{A.6}$$

Energy of motor $m+1$, rigidly connected to link i, is given by

$$T_{m_{i+1}} = \frac{1}{2}m_{m_{i+1}}\dot{\mathbf{p}}_{m_{i+1}}^T\dot{\mathbf{p}}_{m_{i+1}} + \frac{1}{2}\boldsymbol{\omega}_{m_{i+1}}^T {}^0\mathbf{I}_{m_{i+1}}^{m_{i+1}}\boldsymbol{\omega}_{m_{i+1}}. \tag{A.7}$$

For computing motor kinetic energy, it should be considered that rotor of motor $m+1$ rotates with angular velocity

$$\boldsymbol{\omega}_{m_{i+1}} = \boldsymbol{\omega}_i + k_{m_{i+1}} {}^0\mathbf{z}_{m_{i+1}}\dot{\theta}_{i+1} \tag{A.8}$$

and that rotor inertia tensor of motor $m+1$ relatively to its center of mass and defined in a reference frame having its $\mathbf{z}_{m_{i+1}}$ axis parallel to the rotation axis is given by

$${}^{m_{i+1}}\mathbf{I}_{m_{i+1}}^{m_{i+1}} = \begin{bmatrix} I_{\rho_{i+1}} & 0 & 0 \\ 0 & I_{\rho_{i+1}} & 0 \\ 0 & 0 & \Upsilon_{i+1} \end{bmatrix}, \tag{A.9}$$

being the rotor regarded as a rigid body rotating around axis $\mathbf{z}_{m_{i+1}}$.

Further, according to (A.3)

$$\begin{aligned} {}^0\mathbf{I}_{m_{i+1}}^{m_{i+1}} {}^0\mathbf{z}_{m_{i+1}} &= \left(\mathbf{R}_{m_{i+1}} {}^{m_{i+1}}\mathbf{I}_{m_{i+1}}^{m_{i+1}}\mathbf{R}_{m_{i+1}}^T\right)\left(\mathbf{R}_{m_{i+1}}\left[0\ \ 0\ \ 1\right]^T\right) \\ &= \mathbf{R}_{m_{i+1}} {}^{m_{i+1}}\mathbf{I}_{m_{i+1}}^{m_{i+1}}\left[0\ \ 0\ \ 1\right]^T \\ &= \mathbf{R}_{m_{i+1}}\left[0\ \ 0\ \ \Upsilon_{i+1}\right]^T \\ &= \left[{}^0\mathbf{x}_{m_{i+1}}\ \ {}^0\mathbf{y}_{m_{i+1}}\ \ {}^0\mathbf{z}_{m_{i+1}}\right]\left[0\ \ 0\ \ \Upsilon_{i+1}\right]^T \\ &= \Upsilon_{i+1} {}^0\mathbf{z}_{m_{i+1}} = \Upsilon_{i+1}\mathbf{R}_i\mathbf{z}_{m_{i+1}}, \end{aligned} \tag{A.10}$$

Table 5: Real and estimated parameters related to the inertia tensors for the 6-dof manipulator.

	π_{31}	π_{32}	π_{33}	π_{34}	π_{35}	π_{36}	π_{37}	π_{38}	π_{39}	π_{40}	π_{41}	π_{42}
Real	0	0.35	$0.2E^{-4}$	0	0	0	1.031	3.727	2.842	0.037	1.440	−0.023
Estimated	$-0.15E^{-5}$	0.34	−0.0015	$-0.1E^{-4}$	$-0.1E^{-5}$	$-0.9E^{-5}$	1.016	3.714	2.838	0.033	1.435	−0.038
	π_{43}	π_{44}	π_{45}	π_{46}	π_{47}	π_{48}	π_{49}	π_{50}	π_{51}	π_{52}	π_{53}	π_{54}
Real	0.090	0.111	0.015	−0.0013	0.0068	0.0047	0.0021	0.0013	0.0021	0	0	0
Estimated	0.077	0.098	$0.5E^{-3}$	−0.0026	0.0055	0.0034	0.0008	0	0.0008	−0.0013	−0.0013	−0.0013
	π_{55}	π_{56}	π_{57}	π_{58}	π_{59}	π_{60}	π_{61}	π_{62}	π_{63}	π_{64}	π_{65}	π_{66}
Real	$0.3E^{-4}$	$0.4E^{-4}$	$0.3E^{-4}$	0	0	0	0.0002	0.0002	$4E^{-05}$	0	0	0
Estimated	$0.1E^{-4}$	$0.9E^{-4}$	$0.9E^{-4}$	$-0.1E^{-4}$	$-0.1E^{-4}$	$-0.1E^{-5}$	−0.0011	−0.0011	$0.12E^{-4}$	$0.13E^{-4}$	$-0.13E^{-4}$	$-0.11E^{-4}$

Table 6: Real and estimated parameters related to motor inertias and friction for the 6-dof manipulator.

	π_{67}	π_{68}	π_{69}	π_{70}	π_{71}	π_{72}	π_{73}	π_{74}	π_{75}
Real	0.0002	0.0002	0.0002	$3.3E^{-5}$	$3.3E^{-5}$	$3.0E^{-5}$	0.395	0.126	0.132
Estimated	0.0252	0.0252	0.0252	$0.25E^{-4}$	$0.25E^{-4}$	$0.15E^{-4}$	0.396	0.127	0.147
	π_{76}	π_{77}	π_{78}	π_{79}	π_{80}	π_{81}	π_{82}	π_{83}	π_{84}
Real	0.0112	0.0093	0.0039	0.0014	0.0008	0.0014	$7.12E^{-5}$	$8.26E^{-5}$	$3.67E^{-5}$
Estimated	0.0118	0.0140	0.0040	0.0006	0.0005	0.0017	$2.01E^{-5}$	$5.95E^{-5}$	$1.42E^{-5}$

and, due to symmetry of inertia tensor,

$$\begin{aligned} {}^0\mathbf{z}_{m_{i+1}}^T\,{}^0\mathbf{I}_{m_{i+1}}^{m_{i+1}} &= \left({}^0\mathbf{I}_{m_{i+1}}^{m_{i+1}}\,{}^0\mathbf{z}_{m_{i+1}}\right)^T \\ &= \left(\Upsilon_{i+1}\mathbf{R}_i\mathbf{z}_{m_{i+1}}\right)^T = \Upsilon_{i+1}\mathbf{z}_{m_{i+1}}^T\mathbf{R}_i^T. \end{aligned} \tag{A.11}$$

Thus, according to (A.1), (A.2), (A.10), and (A.11), kinetic energy of motor $m+1$ is expressed as

$$\begin{aligned} T_{m_{i+1}} = {} & \frac{1}{2}m_{m_{i+1}}\dot{\mathbf{p}}_{C_i}^T\dot{\mathbf{p}}_{C_i} \\ & + m_{m_{i+1}}\dot{\mathbf{p}}_{C_i}^T\boldsymbol{\omega}_{i_{[x]}}\left(\mathbf{p}_{m_{i+1}} - \mathbf{p}_{C_i}\right) \\ & + \frac{1}{2}\boldsymbol{\omega}_i^{T\,0}\mathbf{I}_{m_{i+1}}^{C_i}\boldsymbol{\omega}_i + \Upsilon_{i+1}k_{m_{i+1}}\dot{\theta}_{i+1}\mathbf{z}_{m_{i+1}}^T\mathbf{R}_i^T\boldsymbol{\omega}_i \\ & + \frac{1}{2}\Upsilon_{i+1}k_{m_{i+1}}^2\dot{\theta}_{i+1}^2. \end{aligned} \tag{A.12}$$

Thus, considering that

(i) total inertia of body i can be expressed as ${}^0\mathbf{I}_i^{C_i} = {}^0\mathbf{I}_{l_i}^{C_i} + {}^0\mathbf{I}_{m_{i+1}}^{C_i}$,

(ii) total mass of body i can be expressed as $m_i = m_{l_i} + m_{m_{i+1}}$,

(iii) center of mass of body i related to centers of mass of link i and motor $i+1$ as $m_i\mathbf{p}_{C_i} = m_{l_i}\mathbf{p}_{l_i} + m_{m_{i+1}}\mathbf{p}_{m_{i+1}}$,

(iv) position of center of mass of body i can be expressed in the ith reference frame as $\mathbf{p}_{C_i} = \mathbf{R}_i{}^i\mathbf{P}_{C_i} + \mathbf{p}_i$,

(v) total inertia of body i can be expressed in the ith reference frame, through (A.3) as ${}^0\mathbf{I}_i^i = \mathbf{R}_i{}^i\mathbf{I}_i^i\mathbf{R}_i^T$,

then the robot total kinetic energy in (A.4) can be expressed as

$$\begin{aligned} T = \sum_{i=1}^{n}\Bigg(& \frac{1}{2}m_i\dot{\mathbf{p}}_i^T\dot{\mathbf{p}}_i + \dot{\mathbf{p}}_i^T\boldsymbol{\omega}_{i_{[x]}}\mathbf{R}_i\mathbf{c}_i + \frac{1}{2}\boldsymbol{\omega}_i^T\mathbf{R}_i\mathbf{I}_i\mathbf{R}_i^T\boldsymbol{\omega}_i \\ & + \Upsilon_i k_{m_i}\dot{\theta}_i\mathbf{z}_{m_i}^T\mathbf{R}_{i-1}^T\boldsymbol{\omega}_{i-1} + \frac{1}{2}\Upsilon_i k_{m_i}^2\dot{\theta}_i^2\Bigg), \end{aligned} \tag{A.13}$$

where $\mathbf{I}_i = {}^i\mathbf{I}_i^i$ and $\mathbf{c}_i = m_i{}^i\mathbf{P}_{C_i}$ and body parameters m_i, $\mathbf{c}_i$, and $\mathbf{I}_i$ are related to link and motor parameters as follows

$$\begin{aligned} m_i &= m_{l_i} + m_{m_{i+1}} \\ \mathbf{c}_i &= m_{l_i}\mathbf{p}_{l_i} + m_{m_{i+1}}\mathbf{p}_{m_{i+1}} \\ \mathbf{I}_i &= {}^i\mathbf{I}_i^i = {}^i\mathbf{I}_{l_i}^i + {}^i\mathbf{I}_{m_{i+1}}^i \\ &= {}^i\mathbf{I}_{m_{i+1}}^{m_{i+1}} + m_{m_{i+1}}\left({}^i\mathbf{p}_{m_{i+1}}\right)_{[x]}^T\left({}^i\mathbf{p}_{m_{i+1}}\right)_{[x]} \\ &\quad + {}^i\mathbf{I}_{l_i}^{l_i} + m_{l_i}\left({}^i\mathbf{p}_{l_i}\right)_{[x]}^T\left({}^i\mathbf{p}_{l_i}\right)_{[x]}. \end{aligned} \tag{A.14}$$

A.1. Formulation of Matrix $\mathbf{B}(\mathbf{q})$. Kinetic energy expressed as in (A.13) can be decomposed into five terms as $T = T_m + T_c + T_I + T_{\Upsilon 1} + T_{\Upsilon 2}$, where T_m linearly depends on masses, T_c linearly depends on products of masses and centers of gravity, T_I linearly depends on inertia tensors, and $T_{\Upsilon 1}$ and $T_{\Upsilon 2}$ linearly depend on rotor inertias.

Correspondingly, four blocks of inertia matrix $\mathbf{B}(\mathbf{q})$ can be defined.

Dependence on Masses. T_m can be written as

$$\begin{aligned} T_m &= \sum_{i=1}^{n} \frac{1}{2} m_i \dot{\mathbf{p}}_i^T \dot{\mathbf{p}}_i = \sum_{i=1}^{n} \frac{1}{2} m_i \dot{\mathbf{q}}^T \mathbf{J}_P^{(i)^T} \mathbf{J}_P^{(i)} \dot{\mathbf{q}} \\ &= \frac{1}{2} \dot{\mathbf{q}}^T \underbrace{\left(\sum_{i=1}^{n} m_i \mathbf{J}_P^{(i)^T} \mathbf{J}_P^{(i)} \right)}_{\mathbf{B}_m(\mathbf{q})} \dot{\mathbf{q}}. \end{aligned} \tag{A.15}$$

Dependence on Products of Masses and Centers of Gravity. T_c can be written as

$$\begin{aligned} T_c &= \sum_{i=1}^{n} \dot{\mathbf{p}}_i^T \boldsymbol{\omega}_{i_{[x]}} \mathbf{R}_i \mathbf{c}_i = \sum_{i=1}^{n} \dot{\mathbf{p}}_i^T \left(\boldsymbol{\omega}_i \times (\mathbf{R}_i \mathbf{c}_i) \right) \\ &= -\sum_{i=1}^{n} \dot{\mathbf{q}}^T \mathbf{J}_P^{(i)^T} \left((\mathbf{R}_i \mathbf{c}_i) \times \mathbf{J}_O^{(i)} \dot{\mathbf{q}} \right) \\ &= -\frac{1}{2} \dot{\mathbf{q}}^T \sum_{i=1}^{n} \left(\mathbf{J}_P^{(i)^T} (\mathbf{R}_i \mathbf{c}_i)_{[x]} \mathbf{J}_O^{(i)} + \mathbf{J}_O^{(i)^T} (\mathbf{R}_i \mathbf{c}_i)_{[x]}^T \mathbf{J}_P^{(i)} \right) \dot{\mathbf{q}} \\ &= \frac{1}{2} \dot{\mathbf{q}}^T \underbrace{\sum_{i=1}^{n} \left(\mathbf{J}_O^{(i)^T} (\mathbf{R}_i \mathbf{c}_i)_{[x]} \mathbf{J}_P^{(i)} - \mathbf{J}_P^{(i)^T} (\mathbf{R}_i \mathbf{c}_i)_{[x]} \mathbf{J}_O^{(i)} \right)}_{\mathbf{B}_c(\mathbf{q})} \dot{\mathbf{q}}. \end{aligned} \tag{A.16}$$

Dependence on Inertia Tensors. One can write

$$\begin{aligned} T_I &= \sum_{i=1}^{n} \frac{1}{2} \boldsymbol{\omega}_i^T \mathbf{R}_i \mathbf{I}_i \mathbf{R}_i^T \boldsymbol{\omega}_i = \sum_{i=1}^{n} \frac{1}{2} \dot{\mathbf{q}}^T \mathbf{J}_O^{(i)^T} \mathbf{R}_i \mathbf{I}_i \mathbf{R}_i^T \mathbf{J}_O^{(i)} \dot{\mathbf{q}} \\ &= \frac{1}{2} \dot{\mathbf{q}}^T \underbrace{\left(\sum_{i=1}^{n} \mathbf{J}_O^{(i)^T} \mathbf{R}_i \mathbf{I}_i \mathbf{R}_i^T \mathbf{J}_O^{(i)} \right)}_{\mathbf{B}_I(\mathbf{q})} \dot{\mathbf{q}}. \end{aligned} \tag{A.17}$$

Dependence on Rotor Inertias. In (A.13) two terms can be identified, the first one is linearly varying and the latter one is quadratically varying with reduction gear k_{m_i}. In particular, the first term is given by

$$\begin{aligned} T_{\Upsilon 1} &= \sum_{i=1}^{n} \Upsilon_i k_{m_i} \dot{\theta}_i \mathbf{z}_{m_i}^T \mathbf{R}_{i-1}^T \boldsymbol{\omega}_{i-1} \\ &= \sum_{i=1}^{n} \left(\Upsilon_i k_{m_i} \dot{\theta}_i \mathbf{z}_{m_i}^T \mathbf{R}_{i-1}^T \mathbf{J}_O^{(i-1)} \dot{\mathbf{q}} \right) \\ &= \frac{1}{2} \dot{\mathbf{q}}^T \underbrace{\left[\Upsilon_1 k_{m_1} \mathbf{J}_O^{(0)^T} \mathbf{R}_0 \mathbf{z}_{m_1} \cdots \Upsilon_n k_{m_n} \mathbf{J}_O^{(n-1)^T} \mathbf{R}_{n-1} \mathbf{z}_{m_n} \right]}_{\mathbf{B}_d(\mathbf{q})} \dot{\boldsymbol{\theta}} \\ &\quad + \frac{1}{2} \dot{\boldsymbol{\theta}}^T \underbrace{\left[\Upsilon_1 k_{m_1} \mathbf{J}_O^{(0)^T} \mathbf{R}_0 \mathbf{z}_{m_1} \cdots \Upsilon_n k_{m_n} \mathbf{J}_O^{(n-1)^T} \mathbf{R}_{n-1} \mathbf{z}_{m_n} \right]^T}_{\mathbf{B}_d^T(\mathbf{q})} \dot{\mathbf{q}}. \end{aligned} \tag{A.18}$$

The latter one can be written as

$$T_{\Upsilon 2} = \frac{1}{2} \Upsilon_i k_{m_i}^2 \dot{\theta}_i^2 = \frac{1}{2} \dot{\boldsymbol{\theta}}^T \underbrace{\operatorname{diag}\left\{ \Upsilon_1 k_{m_1}^2, \ldots, \Upsilon_n k_{m_n}^2 \right\}}_{\mathbf{B}_\Upsilon} \dot{\boldsymbol{\theta}}. \tag{A.19}$$

Thus, (A.15), (A.16), (A.17), (A.18), and (A.19) allow expressing total kinetic energy in (A.13) as

$$\begin{aligned} T &= \frac{1}{2} \dot{\mathbf{q}}^T \mathbf{B}_m \dot{\mathbf{q}} + \frac{1}{2} \dot{\mathbf{q}}^T \mathbf{B}_c \dot{\mathbf{q}} + \frac{1}{2} \dot{\mathbf{q}}^T \mathbf{B}_I \dot{\mathbf{q}} \\ &\quad + \frac{1}{2} \dot{\mathbf{q}}^T \mathbf{B}_d \dot{\boldsymbol{\theta}} + \frac{1}{2} \dot{\boldsymbol{\theta}}^T \mathbf{B}_d^T \dot{\mathbf{q}} + \frac{\mathbf{1}}{\mathbf{2}} \dot{\boldsymbol{\theta}}^{\mathbf{T}} \mathbf{B}_\Upsilon \dot{\boldsymbol{\theta}} \\ &= \frac{1}{2} \dot{\bar{\mathbf{q}}}^T \mathbf{B}(\mathbf{q}) \dot{\bar{\mathbf{q}}}, \end{aligned} \tag{A.20}$$

where inertia matrix $\mathbf{B}(\mathbf{q})$ is given by

$$\mathbf{B}(\mathbf{q}) = \begin{bmatrix} \mathbf{B}_m(\mathbf{q}) + \mathbf{B}_c(\mathbf{q}) + \mathbf{B}_I(\mathbf{q}) & \mathbf{B}_d(\mathbf{q}) \\ \mathbf{B}_d(\mathbf{q})^T & \mathbf{B}_\Upsilon \end{bmatrix}. \tag{A.21}$$

Note that matrix $\mathbf{B}(\mathbf{q})$ depends only on $\mathbf{q}$.

B. Robot Centrifugal and Coriolis Torque Matrix

Elements c_{ij} of matrix $\mathbf{C}(\mathbf{q}, \dot{\bar{\mathbf{q}}})$ are related to elements of $\mathbf{B}(\mathbf{q})$ as

$$c_{ij} = \sum_{k=1}^{n} c_{ijk} \dot{\bar{q}}_k, \quad c_{ijk} = \frac{1}{2} \left(\frac{\partial b_{ij}}{\partial \bar{q}_k} - \frac{\partial b_{jk}}{\partial \bar{q}_i} + \frac{\partial b_{ik}}{\partial \bar{q}_j} \right), \tag{B.1}$$

being c_{ijk} the Christoffel symbols.

Thus, three matrices $\mathbf{C}'(\mathbf{q}, \dot{\mathbf{q}})$, $\mathbf{C}''(\mathbf{q}, \dot{\bar{\mathbf{q}}})$, and $\mathbf{C}'''(\mathbf{q}, \dot{\bar{\mathbf{q}}})$ can be introduced, so that

$$\mathbf{C}(\mathbf{q}, \dot{\bar{\mathbf{q}}}) = \mathbf{C}'(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{C}''(\mathbf{q}, \dot{\bar{\mathbf{q}}}) + \mathbf{C}'''(\mathbf{q}, \dot{\bar{\mathbf{q}}}). \tag{B.2}$$

Coefficients of matrices $\mathbf{C}'$, $\mathbf{C}''$, $\mathbf{C}'''$ are given by

$$c'_{ij} = \frac{1}{2} \sum_{k=1}^{2n} \frac{\partial b_{ij}}{\partial \bar{q}_k} \dot{\bar{q}}_k, \tag{B.3}$$

$$c''_{ij} = -\frac{1}{2} \sum_{k=1}^{2n} \frac{\partial b_{jk}}{\partial \bar{q}_i} \dot{\bar{q}}_k, \tag{B.4}$$

$$c'''_{ij} = \frac{1}{2} \sum_{k=1}^{2n} \frac{\partial b_{ik}}{\partial \bar{q}_j} \dot{\bar{q}}_k. \tag{B.5}$$

Moreover, n matrices $\mathbf{H}^{(k)}$ can be defined as

$$\mathbf{H}^{(k)} = \frac{\partial}{\partial \bar{q}_k} \mathbf{B}(\mathbf{q}) = \frac{\partial}{\partial q_k} \mathbf{B}(\mathbf{q}) \quad k = 1 \ldots n, \tag{B.6}$$

being $\mathbf{B}(\mathbf{q})$ dependent only on $\mathbf{q}$.

It can be easily observed from (B.3) that

$$c'_{ij} = \frac{1}{2} \sum_{k=1}^{2n} h_{ij}^{(k)} \dot{\bar{q}}_k = \frac{1}{2} \sum_{k=1}^{n} h_{ij}^{(k)} \dot{q}_k, \tag{B.7}$$

and, consequently,

$$\mathbf{C}'(\mathbf{q},\dot{\mathbf{q}}) = \frac{1}{2}\sum_{k=1}^{n}\dot{q}_k\mathbf{H}^{(k)}. \quad \text{(B.8)}$$

From (B.4), exploiting the symmetry property of matrix $\mathbf{B}(\mathbf{q})$ (i.e., $b_{jk} = b_{kj}$), one can write

$$c''_{ij} = -\frac{1}{2}\sum_{k=1}^{2n}\frac{\partial b_{kj}}{\partial \bar{q}_i}\dot{\bar{q}}_k = -\frac{1}{2}\sum_{k=1}^{2n}h_{kj}^{(i)}\dot{\bar{q}}_k. \quad \text{(B.9)}$$

Hence,

$$\mathbf{C}''(\mathbf{q},\dot{\bar{\mathbf{q}}}) = -\frac{1}{2}\begin{bmatrix}\dot{\bar{\mathbf{q}}}^T\mathbf{H}^{(1)}\\ \vdots \\ \dot{\bar{\mathbf{q}}}^T\mathbf{H}^{(2n)}\end{bmatrix} = -\frac{1}{2}\begin{bmatrix}\dot{\bar{\mathbf{q}}}^T\mathbf{H}^{(1)}\\ \vdots \\ \dot{\bar{\mathbf{q}}}^T\mathbf{H}^{(n)}\\ \text{zeros}\,(n,2n)\end{bmatrix}. \quad \text{(B.10)}$$

Eventually, from (B.5), one can write

$$c'''_{ij} = \frac{1}{2}\sum_{k=1}^{2n}h_{ik}^{(j)}\dot{\bar{q}}_k, \quad \text{(B.11)}$$

$$\begin{aligned}\mathbf{C}'''(\mathbf{q},\dot{\bar{\mathbf{q}}}) &= \frac{1}{2}\left[\mathbf{H}^{(1)^T}\dot{\mathbf{q}}\cdots\mathbf{H}^{(2n)^T}\dot{\mathbf{q}}\right]\\ &= \frac{1}{2}\left[\mathbf{H}^{(1)^T}\dot{\mathbf{q}}\cdots\mathbf{H}^{(n)^T}\dot{\mathbf{q}}\,\text{zeros}\,(2n,n)\right]\\ &= -\mathbf{C}''(\mathbf{q},\dot{\bar{\mathbf{q}}})^T.\end{aligned} \quad \text{(B.12)}$$

Thus, (B.2) yields

$$\begin{aligned}\mathbf{C}(\mathbf{q},\dot{\bar{\mathbf{q}}}) &= \mathbf{C}'(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{C}''(\mathbf{q},\dot{\bar{\mathbf{q}}}) - \mathbf{C}''(\mathbf{q},\dot{\bar{\mathbf{q}}})^T\\ &= \mathbf{C}'(\mathbf{q},\dot{\mathbf{q}}) + 2\cdot\text{asym}\left(\mathbf{C}''(\mathbf{q},\dot{\bar{\mathbf{q}}})\right),\end{aligned} \quad \text{(B.13)}$$

where the operator asym($\mathbf{M}$) is defined as asym($\mathbf{M}$) = $(\mathbf{M}-\mathbf{M}^T)/2$ and $\mathbf{H}^{(k)}(\mathbf{q}) = \partial\mathbf{B}/\partial q_k$.

B.1. Regressor Matrices $\mathbf{Y}_\mathbf{C}(\mathbf{q},\dot{\mathbf{q}},\dot{\boldsymbol{\theta}})$ *and* $\mathbf{Y}_\mathbf{H}(\mathbf{q},\dot{\mathbf{q}},\dot{\boldsymbol{\theta}})$. Linear regressor $\mathbf{Y}_\mathbf{C}(\mathbf{q},\dot{\mathbf{q}},\dot{\boldsymbol{\theta}})$ allows expressing matrix $\mathbf{C}(\mathbf{q},\dot{\bar{\mathbf{q}}})$ as follows:

$$\begin{aligned}\mathbf{C}(\mathbf{q},\dot{\bar{\mathbf{q}}})\dot{\bar{\mathbf{q}}} &= \mathbf{C}'(\mathbf{q},\dot{\mathbf{q}})\dot{\bar{\mathbf{q}}} + \mathbf{C}''(\mathbf{q},\dot{\bar{\mathbf{q}}})\dot{\bar{\mathbf{q}}} + \mathbf{C}'''(\mathbf{q},\dot{\bar{\mathbf{q}}})\dot{\bar{\mathbf{q}}}\\ &= \mathbf{Y}_\mathbf{C}(\mathbf{q},\dot{\mathbf{q}},\dot{\boldsymbol{\theta}})\boldsymbol{\Pi}.\end{aligned} \quad \text{(B.14)}$$

Given regressor matrix $\mathbf{Y}_{\mathbf{H}^{(k)}}$, (B.8) and (B.10) allow writing

$$\begin{aligned}\mathbf{C}'(\mathbf{q},\dot{\mathbf{q}})\dot{\bar{\mathbf{q}}} &= \frac{1}{2}\sum_{k=1}^{n}\left(\dot{q}_k\left(\mathbf{H}^{(k)}\dot{\bar{\mathbf{q}}}\right)\right)\\ &= \frac{1}{2}\sum_{k=1}^{n}\left(\dot{q}_k\mathbf{Y}_{\mathbf{H}^{(k)}}\right)\boldsymbol{\Pi},\end{aligned} \quad \text{(B.15)}$$

$$\mathbf{C}''(\mathbf{q},\dot{\bar{\mathbf{q}}})\dot{\bar{\mathbf{q}}} = -\frac{1}{2}\begin{bmatrix}\dot{\bar{\mathbf{q}}}^T\mathbf{H}^{(1)}\dot{\bar{\mathbf{q}}}\\ \vdots \\ \dot{\bar{\mathbf{q}}}^T\mathbf{H}^{(n)}\dot{\bar{\mathbf{q}}}\\ \text{zeros}\,(n,2n)\end{bmatrix} = -\frac{1}{2}\begin{bmatrix}\dot{\bar{\mathbf{q}}}^T\mathbf{Y}_{\mathbf{H}^{(1)}}\\ \vdots \\ \dot{\bar{\mathbf{q}}}^T\mathbf{Y}_{\mathbf{H}^{(n)}}\\ \text{zeros}\,(n,2n)\end{bmatrix}\boldsymbol{\Pi} \quad \text{(B.16)}$$

and, from (B.12),

$$\mathbf{C}'''(\mathbf{q},\dot{\bar{\mathbf{q}}})\dot{\bar{\mathbf{q}}} = \frac{1}{2}\left[\mathbf{Y}_{\mathbf{H}^{(1)}}\boldsymbol{\Pi}\cdots\mathbf{Y}_{\mathbf{H}^{(n)}}\boldsymbol{\Pi}\,\text{zeros}\,(2n,n)\right]\dot{\mathbf{q}}. \quad \text{(B.17)}$$

Element i of vector $\mathbf{C}'''(\mathbf{q},\dot{\bar{\mathbf{q}}})\dot{\bar{\mathbf{q}}}$ is given by

$$\begin{aligned}\left(\mathbf{C}'''(\mathbf{q},\dot{\bar{\mathbf{q}}})\dot{\bar{\mathbf{q}}}\right)_i &= \sum_{j=1}^{2n}c'''_{ij}\dot{\bar{q}}_j = \sum_{j=1}^{n}\left(\frac{1}{2}\sum_{k=1}^{14n}y_{ik}^{(j)}\pi_k\right)\dot{\bar{q}}_j\\ &= \frac{1}{2}\sum_{k=1}^{14n}\left(\sum_{j=1}^{2n}y_{ik}^{(j)}\dot{\bar{q}}_j\right)\pi_k\\ &= \frac{1}{2}\left(\sum_{j=1}^{2n}\mathbf{y}_{i:}^{(j)}\dot{\bar{q}}_j\right)\boldsymbol{\Pi},\end{aligned} \quad \text{(B.18)}$$

where $y_{ik}^{(j)}$ is the element (ik) of matrix $\mathbf{Y}_{\mathbf{H}^{(j)}}$, and $\mathbf{y}_{i:}^{(j)}$ is the row (i) of matrix $\mathbf{Y}_{\mathbf{H}^{(j)}}$. Hence, vector $\mathbf{C}'''(\mathbf{q},\dot{\bar{\mathbf{q}}})\dot{\mathbf{q}}$ is given by

$$\mathbf{C}'''(\mathbf{q},\dot{\bar{\mathbf{q}}})\dot{\mathbf{q}} = \frac{1}{2}\sum_{j=1}^{2n}\left(\dot{\bar{q}}_j\mathbf{Y}_{\mathbf{H}^{(j)}}\right)\boldsymbol{\Pi} = \frac{1}{2}\sum_{j=1}^{n}\left(\dot{q}_j\mathbf{Y}_{\mathbf{H}^{(j)}}\right)\boldsymbol{\Pi} \quad \text{(B.19)}$$

which is equal to vector $\mathbf{C}'(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}$ as defined in (B.15).

Consequently, regressor $\mathbf{Y}_\mathbf{C}(\mathbf{q},\dot{\mathbf{q}},\dot{\boldsymbol{\theta}})$ can be defined as

$$\mathbf{Y}_\mathbf{C}(\mathbf{q},\dot{\bar{\mathbf{q}}}) = \sum_{k=1}^{n}\left(\dot{q}_k\mathbf{Y}_{\mathbf{H}^{(k)}}\right) - \frac{1}{2}\begin{bmatrix}\dot{\bar{\mathbf{q}}}^T\mathbf{Y}_{\mathbf{H}^{(1)}}\\ \vdots \\ \dot{\bar{\mathbf{q}}}^T\mathbf{Y}_{\mathbf{H}^{(n)}}\\ \text{zeros}\,(n,2n)\end{bmatrix}. \quad \text{(B.20)}$$

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Authors' Contribution

Loredana Zollo and Edoardo Lopez contributed equally to this paper.

Endnotes

1. In (18) and (26) operator $(\mathbf{a})_{[\triangleright\triangleleft]}$ has been introduced, which generates a matrix (3×6) from a (3×1) vector $\mathbf{a}$. It is defined as

$$(\mathbf{a})_{[\triangleright\triangleleft]} = \begin{bmatrix} a_1 & 0 & 0 & a_2 & a_3 & 0 \\ 0 & a_2 & 0 & a_1 & 0 & a_3 \\ 0 & 0 & a_3 & 0 & a_1 & a_2 \end{bmatrix}. \qquad (*)$$

Given a (3×3) symmetric matrix $\mathbf{S}$ of elements s_{ij}, operator $(\mathbf{a})_{[\triangleright\triangleleft]}$ allows expressing product $\mathbf{S}\mathbf{a}$ as

$$\mathbf{S}\mathbf{a} = (\mathbf{a})_{[\triangleright\triangleleft]} \begin{bmatrix} s_{11} & s_{22} & s_{33} & s_{12} & s_{13} & s_{23} \end{bmatrix}^T. \qquad (**)$$

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