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Procedia Structural Integrity 13 (2018) 1961–1966

Structural Integrity Procedia

www.elsevier.com/locate/procedia

ECF22 - Loading and Environmental effects on Structural Integrity

Correlation among Energy Based Fatigue Curves and Fatigue Design Approaches

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Abstract

In this paper, with reference to the strain controlled fatigue characterization of AISI 304L stainless steel, the correlations between plain material fatigue curves based on different definitions of the strain energy densities, namely the elastic, plastic and elastoplastic strain energy densities evaluated under the cyclic stress-strain curve and the plastic strain hysteresis energy density (per cycle and total at fracture) are investigated. On this basis, a diagram showing the link among the different energy-based fatigue curves is proposed and is applied to find the correlation between plain material strain energy density fatigue curves and some fatigue strength assessment methods for notched structural components, namely the one based on the experimental evaluation of the heat energy dissipated by the material per cycle and the one based on the evaluation of the linear elastic strain energy density, averaged in a properly defined structural volume.

Peer-review under responsibility of the ECF22 organizers.

Keywords: low-cycle fatigue, strain energy density, compatibility equations, Manson-Coffin curves, AISI 304L, fatigue design

1. Introduction

Fatigue characterisation of metallic materials is usually expressed via equations that relate a damage variable with the number of cycles to failure. In classical approaches, fatigue life is given as a function of the stress amplitude or strain amplitude. According to a different approach, the strain energy density is adopted. Energy-based analyses of fatigue damage were first introduced approximately one century ago by Bairstow and have been subsequently formalised in different fashions by several authors. (see Klesnil and Lukas (1992), Ellyin (1997)).

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2452-3216
Peer-review under responsibility of the ECF22 organizers.
10.1016/j.prostr.2018.12.264

Recently, Meneghetti (2007) proposed the specific heat loss dissipated by the material undergoing a fatigue load as fatigue damage index (the Q parameter) and a linear elastic approach, based on the evaluation of the Strain Energy Density (SED) averaged inside a properly defined structural volume, has been proposed by Lazzarin et al (2001).
The aim of this paper is to analyse the correlations among the Strain Energy Density based approaches developed for plain and notched material, on the basis of full compatibility conditions, originally proposed by Morrow (1965). These correlations were validated with reference to a large amount of experimental results on the fatigue behaviour of AISI 304 L stainless steel, published elsewhere by Meneghetti et al (2013), reanalysed in terms of the heat energy dissipated by the material per cycle (Meneghetti 2007) as well as in terms of the linear elastic Strain Energy Density, averaged in a properly defined structural volume, according to Lazzarin and Zambardi (2001).

Nomenclature

a_f = fatigue toughness exponent
a = plastic strain energy per cycle exponent
b = fatigue strength exponent
c = fatigue ductility exponent
E = elastic modulus measured from a static tensile test
K' = cyclic strength coefficient
n' = cyclic strain hardening exponent
N_f = number of cycles to failure (half the number of reversals)
W_{fp} = fatigue toughness (plastic strain hysteresis energy density to fatigue fracture), PSEDF
W_{SCe}, W_{SCp}, W_{SC} = strain energy density evaluated under the cyclic stress-strain curve (elastic, plastic, elastoplastic)
ΔW_p = plastic strain hysteresis energy density per cycle (area of the hysteresis loop), PSEDC
$\Delta\varepsilon$ = total strain range
$\Delta\varepsilon_e$ = elastic strain range
$\Delta\varepsilon_p$ = plastic strain range
$\Delta\sigma$ = cyclic stress range
ε'_f, σ'_f, W'_{fp}, W'_{SC}, $\Delta W'_p$, W'_{SCe}, W'_{SCp},= fatigue coefficients (values at 1 reversal)

2. Hypotheses and basic equations

Fatigue characterisation of metallic materials can be performed by constant amplitude, push-pull, strain controlled fatigue tests carried out on plain specimens. It is hypothesised that the stress-strain behaviour of the material stabilises (in practise, when this circumstance does not occur, the stress-strain behaviour at the half fatigue life is assumed to be characteristic of the applied strain amplitude). Concerning the strain-life equation, the Basquin, Manson and Coffin equations are assumed:

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma'_f}{E}\cdot\left(2N_f\right)^b + \varepsilon'_f\cdot\left(2N_f\right)^c \tag{1}$$

The tips of the stabilised hysteresis loops measured at different strain amplitudes are assumed to be interpolated by the cyclic Ramberg-Osgood stress-strain equation:

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K'}\right)^{\frac{1}{n'}} \tag{2}$$

Taking advantage of the Stabilised cyclic Curve, the fatigue curves in terms of strain can be converted in terms of Strain Energy Density under the curves:

$$W_{SC} = W_{SCe} + W_{SCp} = \frac{\left(\sigma'_f\right)^2}{2E}\cdot\left(2N_f\right)^{2b} + \frac{\sigma'_f\varepsilon'_f}{1+n'}\cdot\left(2N_f\right)^{b+c} = W'_{SCe}\cdot\left(2N_f\right)^{2b} + W'_{SCp}\cdot\left(2N_f\right)^{b+c} \tag{3}$$

Concerning the fatigue life curves based on the experimental values of PSEDC, ΔW_p, the following energy-life equations cab be considered to be applicable, under the hypothesis that the evolution of the material stress-strain behaviour makes ΔW_p reasonably constant throughout the duration of the fatigue test (Morrow 1965):

$$\Delta W_p = \alpha \cdot \Delta\sigma \cdot \Delta\varepsilon_p \tag{4a}$$

$$W_f = \Delta W_p \cdot N_f = W'_{fp} \cdot \left(2N_f\right)^{a_f} \tag{4b}$$

$$\Delta W_p = \Delta W_p^{'} \cdot \left(2N_f\right)^{a} \tag{4c}$$

For a Masing material, the branches of the hysteresis loops for different applied strain amplitudes can be superimposed starting from the lower tip and can be analytically expressed by the cyclic stress-strain curve (Eq. 2) magnified by a factor of two. Under this hypothesis, Halford (1966) derived the following analytical expression to calculate the area of the hysteresis loop:

$$\Delta W_p = \frac{\left(1-n'\right)}{\left(1+n'\right)} \cdot \Delta\sigma \cdot \Delta\varepsilon_p \tag{5}$$

where $\alpha = \left(1-n'\right)/\left(1+n'\right)$ is the material constant appearing in Eq. (4a). It has been shown lately that the analytical evaluation of the plastic strain energy density per cycle gives in general very poor results, because of the variability of the shape of the hysteresis loops (Klesnil and Lukas 1992), suggesting that the value of α should be experimentally evaluated.

Since from the same fatigue tests different data are usually recorded and reported in diagrams, independent analyses of the data will result, in general, in best fitting curves, which are not congruent with each other. This issue can be avoided by introducing appropriate compatibility conditions, as suggested by Morrow (1965). These conditions can be applied in the generalised form that was already introduced to analyse ductile irons (Atzori et al 2014):

- compatibility among the parameters of the strain-life and cyclic stress-strain equations:

$$n' = b/c\ ;\ K' = \sigma_f^{'} \Big/ \left(\varepsilon_f^{'}\right)^{n'} \tag{6}$$

- compatibility among the parameters of the energy-life and strain-life equations:

$$a_f = 1 + a\ ;\ a = b + c\ ;\ \Delta W_p{}' = 4 \cdot \alpha \cdot \sigma_f^{'} \cdot \varepsilon_f^{'} \tag{7a}$$

$$W_{fp}{}' = \Delta W_p^{'} \big/ 2 \tag{7b}$$

3. Fatigue characterisation of AISI 304L stainless steel plain material

Constant amplitude, push-pull, strain and stress controlled fatigue tests were carried out on specimens prepared from 6-mm-thick hot rolled AISI 304L stainless steel sheets (Meneghetti et al. 2013). During the strain controlled fatigue tests, the hysteresis loops (measured by combining the signals acquired from the load cell and the extensometer) were acquired over a fixed number of cycles, with the aim of having at least five acquisitions in the very low cycle fatigue tests ($N_f < 250$ cycles), in such a way to have, for each test, the experimental values were obtained for the energy per cycle and the total energy to failure. Stabilised hysteresis loops were considered to occur at half the fatigue life of the specimen. Moreover, during the fatigue tests, it was noted that - although the measured stress amplitude increased during the tests according to the hardening behaviour and for an applied strain amplitude greater than 0.5%, the stress amplitude continuously increased and did not reach a stabilised value and the area of the loop was slightly varied, as already evidenced by Morrow (1965) for many materials.

The evaluated parameters that characterise the material for the objectives of this work are summarised in Table 1, also indicting if a parameter was assumed to be a principal parameter (obtained by the best fit of the experimental results) or derived parameter (obtained by application of corresponding compatibility equation, as specified in Table 1).

Table 1. Fatigue characterisation of AISI 304L based on energy-life experimental data.

Fatigue curves	Material parameters	Principal parameters*	Derived parameters**	Equations
$W_{fp} = W'_{fp} \cdot (2N_f)^{a_f}$	a_f	a_f=0.494	/	/
	W'_{fp}	W'_{fp}=130 MJ/m^3	/	/
$\Delta W_p = \Delta W'_p \cdot (2N_f)^a$	a	/	a=-0.506	(7a)
	$\Delta W'_p$	/	$\Delta W'_p$=260 MJ/(m^{3}cycle)	(7b)
$\Delta W_p = \alpha \cdot \Delta\sigma \cdot \Delta\varepsilon_p$	α	α=0.666	/	/
$\frac{\Delta\varepsilon_p}{2} = \left(\frac{\Delta\sigma}{2K'}\right)^{\frac{1}{n'}}$	n'	n'=0.336	/	/
	K'	K'=2328 MPa	/	/
$\frac{\Delta\sigma}{2} = \sigma'_f \cdot (2N_f)^b$	b	/	b=-0.127	(6), (7a)
	σ'_f	/	σ'_f = 1048 MPa	(6)
$\frac{\Delta\varepsilon_p}{2} = \varepsilon'_f \cdot (2N_f)^c$	c	/	c=-0.379	(7a)
	ε'_f	/	ε'_f =0.093	(6), (7a)

*: from best fitting of experimental results;**: from compatibility equations reported in the Table

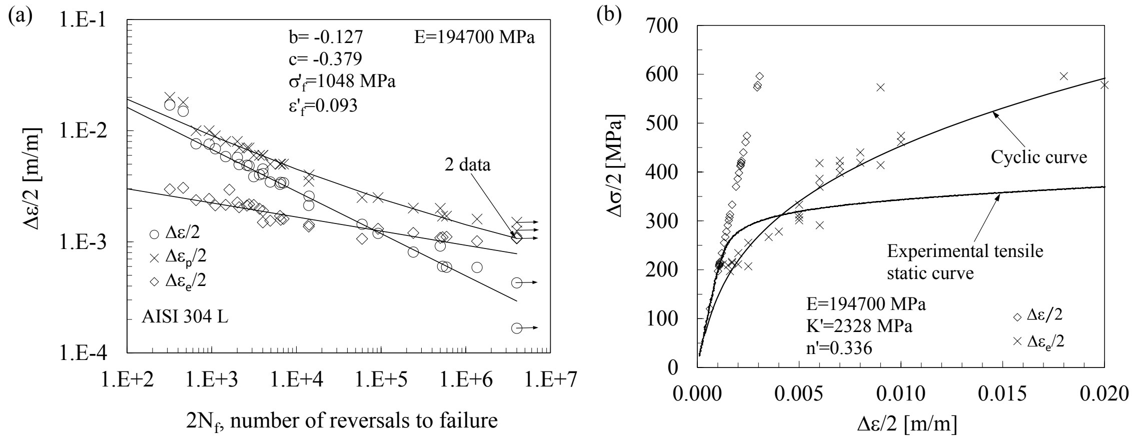

Fig. 1. (a) experimental strain-controlled fatigue results and (b) stress-strain cyclic curve of AISI 304L plain material

Fig. 1a shows the results of the strain controlled fatigue tests in terms of Manson-Coffin curves, and Fig. 1b shows the stabilised Cyclic Curve of the material, which was derived by applying the Ramberg-Osgood model to the same results at the half-life. From this curve (Fig. 1b), the results of Fig. 1a can be converted in terms of the Strain Energy Density under the cyclic stress-strain curve, as shown in Fig. 2a. The analysis of stabilised or half-life hysteresis loops showed that the Masing hypothesis was not satisfied. Therefore, the ΔW_p values were derived by evaluating the area of the experimental hysteresis cycles and α was evaluated by fitting the ΔW_p versus the $\Delta\sigma \cdot \Delta\varepsilon_p$ data at the half-life. It was found that α =0.666 instead of (1-n')/(1+n')=0.497, as evaluated according to Halford (1966). The fatigue curves of the different plastic strain energy densities are summarised in Fig. 2b compared to the experimental results. For the same life, the $\Delta W_p/W_{SCp}$ ratio is equal to 3.56 (290/73.1=3.56, from Fig. 2b). For each fatigue test, the Q parameter was evaluated according to Meneghetti (2007). Fig. 3a shows a comparison between this quantity and ΔW_p measured during the same fatigue tests. It appears that, except for very low cycle tests, the values of the two energies are very similar, because of the very low value of the internal energy per cycle stored within the material (Meneghetti et al 2013).

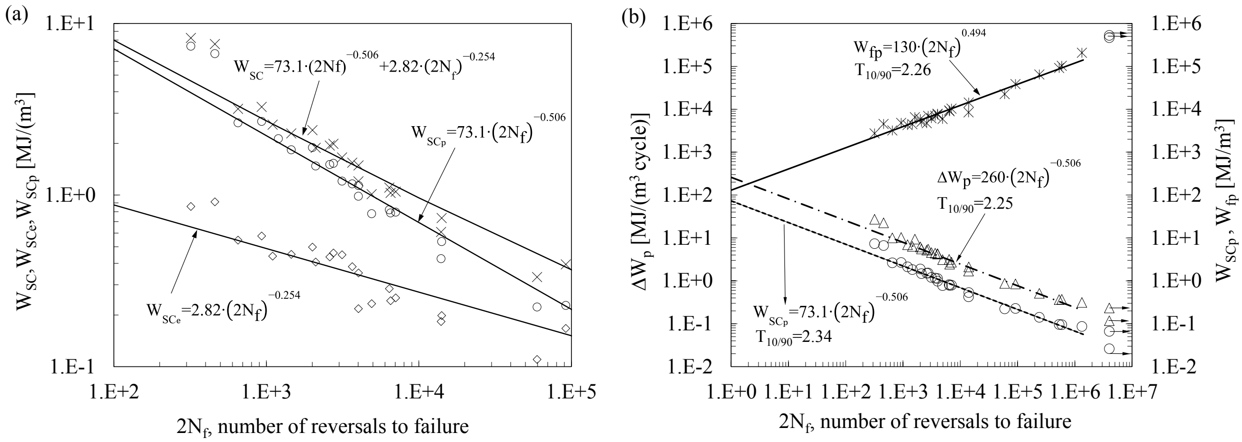

Fig. 2. (a) experimental strain life results converted into Strain Energy Densities under the Stabilised cyclic Curve (elastic, plastic, elastoplastic) and (b) comparison of Plastic Strain Energy Density fatigue curves, as applied to fully reversed axial fatigue tests on AISI 304L stainless steel.

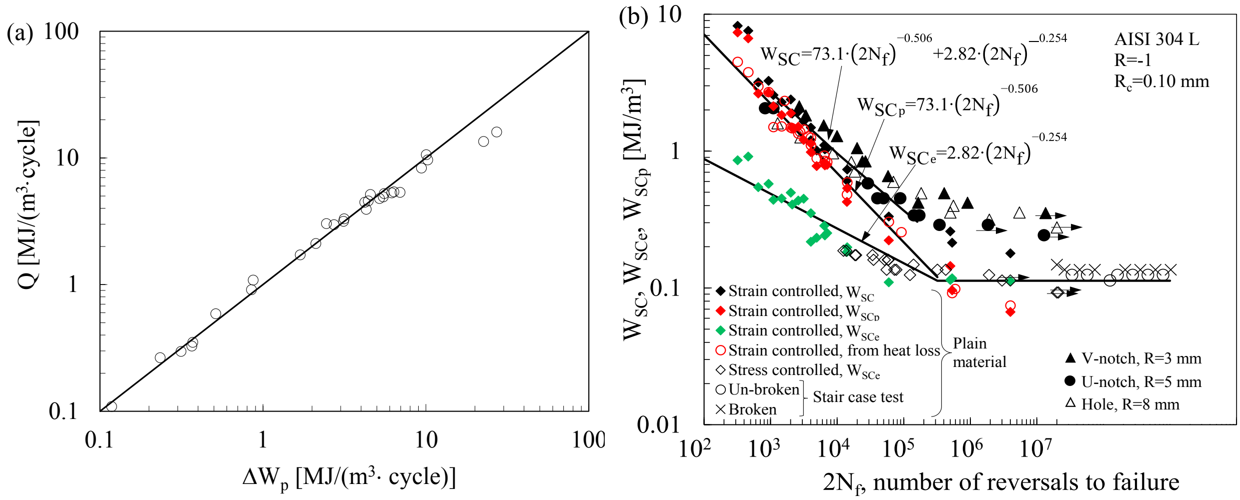

Fig. 3. (a) correlation between Q and ΔW_p and (b) diagram for the comparison of Plastic Strain Energy Density fatigue curves, applied to fully reversed axial fatigue tests on AISI 304L stainless steel.

4. Design approaches

The fatigue design approaches based on the strain energy density were subdivided into three groups: linear elastic-, plastic- and elastic-plastic based methods (Macha and Sonsino 1999) In this paper, the Q-based approach (Meneghetti 2007) and the linear elastic Strain Energy Density averaged inside a properly defined structural volume (SED) approach (Lazzarin and Zambardi 2001) were considered. 6-mm-thick-hot rolled AISI 304L bluntly notched specimens were tested in push pull fatigue with R=-1 and the Q parameter was measured at the notch tip (Meneghetti et al 2013). Due to the practical equivalence of Q and ΔW_p (Fig. 3a), the fatigue curve of the analysed material in terms of ΔW_p (Fig. 2b) should be directly correlated to the experimental fatigue results of notched and un-notched specimens in terms of Q. The SED approach is based on the linear elastic numerical evaluation of the strain density energy averaged in a structural volume, thought as a material property (Lazzarin and Zambardi 2001). Since at the fatigue knee, the material plasticity can be assumed to be localised around the notch tip, Glinka's hypothesis of equivalent energies can be applied for this level of stress (Glinka 1981). Thus, the averaged SED of the notched component can be assumed to be equivalent to the elastoplastic strain energy density evaluated under the Stabilised cyclic Curve of the plain material, and the W_{SC}-life curve (Fig. 2a) should be directly correlated to the experimental fatigue results of the notched specimens in terms of SED. Since the strain energy densities considered by the analysed design approaches are proportional to W_{sc}, the curves shown in Fig. 2a are taken as the basis for validating the experimental and numerical results, as shown in Fig. 3b, in which the experimental results reanalysed in terms of ΔW_p and Q have been divided by 3.56 (according to Fig. 2b) and the numerical SED results have been divided by 2 (because the SED approach, as currently applied, is evaluated for the full range, not for the amplitude of stress and strain). Finally, Fig. 3b shows that at the fatigue knee, the linear elastic SED of plain material is different from that of notched specimens, although the dimension of the critical volume was calculated by equalling the linear elastic SED of plain material to that of a cracked geometry, according to Lazzarin and Zambardi (2001). It was calculated R_c=0.10 mm, having ΔK_{th}=8.69 MPa·m$^{0.5}$ (Meneghetti et al 2017) and the fatigue limit of plain material $\sigma_{A,-1}$=225 MPa (Meneghetti et al 2015). This result is obtained because the SED approach is based on the assumption of linear elastic behaviour of the material at fatigue limit, while the AISI 304L plain material showed plasticity at the fatigue knee (Atzori et al 2018).

5. Conclusions

In this paper, the constant amplitude, fully reversed, strain controlled, axial fatigue behaviour of a AISI 304L stainless steel plain material was investigated in terms of different forms of plastic strain energy densities, namely, the plastic strain energy density per cycle, ΔW_p, the total plastic strain energy density to fatigue fracture,W_{pf}, and the plastic strain energy density evaluated under the cyclic stress-strain curve, W_{SCp}. A diagram showing the correlation among the considered energy-based curves was proposed and applied to the material investigated. The analysis of the data was performed, taking into account the so-called *full-compatibility* conditions, which ensure analytical coherence among the parameters appearing in the fatigue life equations and the material constitutive laws. Next, ΔW_p was shown to be equivalent to the heat energy density dissipated by the material per cycle, Q, that was successfully adopted by the authors to correlate the fatigue strength of plain and notched specimens made of AISI 304L stainless steel. Therefore, taking advantage of the analytical links developed in this paper, the Q-life curve was correlated to the $W_{SC,p}$-life curve. Finally, it was shown that the fatigue curve correlating the experimental results on notched specimens in terms of the averaged SED was homothetic to the W_{SC}-life curve. The ratio, for the analysed testing conditions, was 2.0.

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