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# Model and Solution Methods for the Mixed-Fleet Multi-Terminal Bus Scheduling Problem

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## Abstract

Public transport services are currently executing or planning a fundamental transition from traditional buses to electric buses. During this transition phase, the public transport offering is fulfilled with a mixed fleet across multiple bus terminals, which poses operational challenges for optimal vehicle scheduling, a problem not yet addressed in literature. As researchers in Transport Engineering and Operational Research at the University of Luxembourg, in collaboration with the Roma Tre University, we support the Ministry of Transport of Luxembourg and Volvo buses by modelling and simulating this transition phase, to help them managing and solving such challenges. In this work we develop a mixed-integer linear programming (MILP) formulation of the problem and implement a time-based decomposition framework, through which we can optimize real-life daily instances. This method is tested on the main urban bus lines that connect Central Station, Luxembourg Airport and ten other major terminals within Luxembourg City, providing (near) optimal solutions that explicitly consider the energy constraints arising from electric bus operations, while establishing an advantageous trade-off between delaying trips, to implement quick-charging of electric buses, and performing the same trip with costlier traditional buses. The results show a consistent decrease of operational costs as the percentage of e-buses in the fleet increases.

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## 1. Introduction

Operating electric bus transit fleets, compared to traditional internal combustion engines, entails several advantages: from the operators’ perspective, electric buses yield lowered operational costs (Lajunen, 2014;

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Mahmoud et al., 2016; Xylia et al., 2017) and can thus be advantageous to maximise revenues, as well as enabling public transport (PT) services in low emission areas. In addition, they are expected to deliver increased travel comfort to passengers thanks to quieter and smoother operations of electric engines. From a policymakers' perspective, electric operations yield lower pollutant emissions, helping thus in meeting goals and objectives related to core worldwide plans, such as the Paris Agreement on Climate Change. However, the introduction of electric buses requires considerable investments, both at the network infrastructure and operators' fleet level, besides operational aspects such as maintenance, handling costs and recharging costs. The transition from traditional bus fleets to completely electrified fleets will however be gradual, electrifying a few lines at a time with small e-bus fleets, and optimal handling of the resulting scheduling problem is extremely important, in order to minimise operational costs and exploit the coexisting technologies to the best possible extent.

The vehicle scheduling problem is already widely addressed in literature as part of transit planning (Ceder, 2016), and multi-depot models have been developed, considering homogeneous fleets comprising either conventional (Löbel, 1998; Ribeiro and Soumis, 1994) or electric buses (Fusco et al., 2013; Häll et al., 2018). Working with homogeneous fleets, Multi-Depot Vehicle Scheduling Problem (MDVSP) models can determine the optimal scheduling assigning available buses to predesigned trip chains.

Electric bus scheduling entails additional complexity with regards to its conventional counterpart, as individual bus availability depends not only on whether a given bus is performing a trip or not, but also to its charging and recharging dynamics: e-buses must always operate above a certain threshold of residual charge, and therefore, given the energy capacity of the current technology, a bus must be recharged multiple times to operate a daily schedule (Baita et al., 2000; Chao and Xiaohong, 2013; Wang et al., 2017; Wen et al., 2016). Recharging operations, even considering the latest developments in terms of fast charging technologies, require the bus to remain at the charging station for an amount of time which is not negligible for scheduling purposes. Such aspects must then be explicitly considered when modelling the problem. When scheduling homogeneous fleets of electric buses, existing MDVSP models can anyway assume that the timetabling designs the trip chains accordingly, and accounts for the time slots required for recharging. Therefore, scheduling is still performed assigning buses to predesigned trip chains.

Our research addresses the scheduling of mixed fleets of buses, to support the aforementioned gradual transition. The main advantage of using a mixed fleet is an increased flexibility and improved timetable adherence, thanks to the fact that during the intermittent unavailability of the electric buses, trip execution can be taken over by a traditional bus, that has no charging constraints but has higher operating costs and higher emissions. Such multi-terminal mixed fleet approach, however, does significantly impact the modelling assumptions, with new complex aspects to address such as mixed fleet trip chaining, where round-trips of a same line might be performed by two different technologies, as we will describe later.

The focus of this work is specifically on the scheduling problem, and therefore we assume all planning factors, like time tabling and sizing of the electrical and conventional fleets, as exogenous predetermined variables. Optimisation of the schedule of a given fleet against a given time table is therefore driven only by the operational costs of executing the trips. All other factors which impact capital expenditures and TCO of the transition to a fully electrified transport, like extending the e-fleet and its related staff or widening the charging infrastructure, must be addressed in the planning phases. The model and methodology we develop in this work can be used to support such planning phases, evaluating optimal scheduling and related operational costs under different scenarios, hence simulating for example the operational impact of additional investments in the bus fleet or in the charging infrastructure.

## 2. Research contribution

This work's key contribution with respect to the existing literature is that of modelling of the mixed-fleet multi-terminal vehicle scheduling problem. The focus on *mixed* fleets of buses does significantly impact the modelling assumptions. Working with a mixed fleet it is in fact possible to cover a given daily trip chain of a target bus line by using different buses, both electric and conventional. In this way we can leverage the advantage of partial transition to electric transport on timetables which are not necessarily designed to accommodate recharging operations of a homogeneous electric fleet. For example, it is possible to schedule an outbound trip of a given bus line using an electric bus and then cover the following inbound trip of the same line with a conventional bus or with a different

electric bus, while the previous e-bus is recharging. This flexibility comes at the expense of a higher complexity, caused by the fact that the problem requires to schedule each trip independently, rather than scheduling entire trip chains. Therefore, besides the dependency on the initial depot, the availability of a given bus to perform a given trip does also have a dependency on time (after completing a given trip, the bus is free to perform a different trip, within the same set of daily operations) and on location (in order to initiate a trip, the bus must either be located at its departure terminal, or move there with a deadheading trip from its current location). To ensure the expected trip comfort, we assume that a single trip is always performed by a single bus, and that recharging operations can be performed only at terminal stations, between different trips, and not at intermediate stations during trip execution. For these reasons, we generalize the multi-depot vehicle scheduling problem to a multi-terminal vehicle scheduling problem, that considers all departure and arrival terminals of all target bus lines. The depots, in this modelling approach, are just specific terminals that are involved only in deadheading trips. Another complex issue to address is the limited charger infrastructure, which requires to schedule EV trips either towards a terminal equipped with charging stations or using appropriate deadheading trips, whose cost and time must be considered in the trade off. Our model addresses all these concerns.

### 3. Methodology

The problem is formulated through a Discrete-Time MILP model, which includes:

- adherence constraints that ensure that trips are executed correctly, within the maximum allowed delay and with no conflict of usage of buses across the mixed-fleet;
- charging constraints for electric buses, that capture battery charging/discharging dynamics, ensure that the battery level does not go below the minimum residual charge and that no conflict arises when using the shared chargers;
- location constraints that control the location status of each bus of the mixed-fleet, ensuring that all operations are consistent and enabling deadheading trips for electric buses.

We developed our formulation incrementally. At first, we targeted single-terminal mixed-fleet applications (Rinaldi et al., 2019), which allowed us to develop and test an initial model without having to deal with location constraints, targeting only circular trips which departed and arrived at the same terminal. In the formulation presented in this work, we improve and extend that model to address multi-terminal schedules featuring deadheading trips. The MILP formulation allows to possibly delay the execution of a trip and to give more flexibility to the optimisation approach in evaluating trade-offs between, for example, recharging an electric bus or performing a trip with a conventional bus. The maximum allowed delay is limited, so that we can restrict the number of time intervals which are relevant for the most complex constraints, thus reducing the computational complexity of the model.

We formulate the problem of dispatching a mixed fleet of  $I = \{1, \dots, i\}$  electric buses and  $H = \{1, \dots, h\}$  hybrid buses to serve a set of scheduled trips  $J = \{1, \dots, j\}$ , each comprising a desired departure time  $D_j = \{1, \dots, d_j\}$  [time steps], duration  $T_j = \{1, \dots, t_j\}$  [time steps] and total energy required  $U_j = \{1, \dots, u_j\}$  [kWh]. Each trip  $j$  departs from terminal  $\alpha_j$  and arrives at terminal  $\beta_j$  both within a given set of bus terminals  $B = \{1, \dots, b\}$ . The set of bus terminals can also include any number of bus depot(s), where buses are stored when not in service. The subset  $\bar{B} \subseteq B$  of bus terminals is equipped with charging stations. We assume, for the sake of simplicity, that each terminal of the  $\bar{B}$  subset is equipped with the same amount of  $m$  chargers. Deadheading trips are possible between any combination of terminals, with a required total energy  $\hat{u}_{b_1, b_2}$ , and duration  $\hat{t}_{b_1, b_2}$ .

We discretise time in consecutive time steps  $\tau = [0, 1, \dots, N]$ , with a discretisation step  $T_s$ . Decision variables  $y_{i,j}^t$  and  $z_{h,j}^t$  control trip execution by electric and hybrid buses respectively, variable  $\omega_{i,b_1, b_2}^t$  controls execution of deadheading trips, and variable  $x_{i,b,m}^t$  captures recharging decisions. We adopt the assumption that full charging of e-buses happens within a single time step. Locations of the electric and hybrid buses are captured by variables  $g_{i,b}^t$  and  $p_{h,b}^t$  respectively.

In this work, we allow deadheading trips for electric buses only. Deadheading is, in fact, critical to optimise usage of electric buses, which have cheaper operational costs, and to optimise their charging dynamics, allowing them to move to terminals equipped with charging stations when needed, while it is not strictly necessary for

optimal dispatching of hybrid/conventional combustion buses. The model could anyway be easily extended to consider deadheading for hybrid buses.

Table 1 introduces the meaning of each variable, as well as its domain.

Table 1. Problem variables

Explanation	Variable	Domain
1 if e-bus $i$ is initiating trip $j$ at time $t$ , 0 otherwise	$y_{i,j}^t$	{0,1}
1 if h-bus $h$ is initiating trip $j$ at time $t$ , 0 otherwise	$z_{h,j}^t$	{0,1}
1 if e-bus $i$ is being recharged at charging station $m$ of bus terminal $b$ at time $t$ , 0 otherwise	$x_{i,b,m}^t$	{0,1}
1 if e-bus $i$ starts a deadheading trip from terminal $b_1$ to terminal $b_2$ at time $t$ , 0 otherwise	$\omega'_{i,b_1,b_2}$	{0,1}
Total energy in kWh that e-bus $i$ has at time $t$	$\varepsilon_i^t$	[0, E]
Slack variable, necessary to ensure that recharges do not cause $\varepsilon_i^t$ to violate its domain	$s_i^t$	[0, E]
1 if e-bus $i$ at time $t$ is located at terminal $b$ , or is performing a previously initiated trip directed to terminal $b$ ; 0 otherwise	$p_{h,b}^t$	{0,1}
1 if h-bus $h$ at time $t$ is located at terminal $b$ , or is performing a previously initiated trip directed to terminal $b$ ; 0 otherwise	$g_{h,b}^t$	{0,1}

Table 2 introduces the meaning of each parameter, as well as its domain.

Table 2. Problem parameters

Explanation	Var./Par.	Domain
Total energy in kWh required to perform trip $j$	$u_j$	$\mathbb{R}^+$
Preferred departure time step for trip $j$	$d_j$	$\mathbb{Z}^+$
Duration of trip $j$ in time steps	$t_j$	$\mathbb{Z}^+$
1 if e-bus $i$ is <b>not</b> available to perform any trip nor recharge at time $t$ , 0 otherwise	$A_i^t$	{0,1}
1 if h-bus $h$ is <b>not</b> available to perform any trip at time $t$ , 0 otherwise	$H_h^t$	{0,1}
Initial energy in kWh that e-bus $i$ has at time 0	$\bar{\varepsilon}_i$	[0, E]
Total battery capacity in kWh for an e-bus	$E$	$\mathbb{R}^+$
Minimum battery charge in percentage for each electric bus	$\mu$	(0,1)
Maximum acceptable delay in time steps	$\theta$	$\mathbb{Z}^+$
Departing bus terminal of trip $j$	$\alpha_j$	$\mathbb{Z}^+$
Arriving bus terminal of trip $j$	$\beta_j$	$\mathbb{Z}^+$
Total energy in kWh required to perform a deadheading trip from terminal $b_1$ to terminal $b_2$	$\hat{u}_{b_1,b_2}$	$\mathbb{R}^+$
Duration of deadheading trip from terminal $b_1$ to terminal $b_2$	$\hat{t}_{b_1,b_2}$	$\mathbb{Z}^+$
Initial location of e-bus $i$	$G_i$	$\mathbb{Z}^+$
Initial location of h-bus $h$	$P_h$	$\mathbb{Z}^+$

The formulation’s objective function, in Equation (1), is that of minimizing the total operational cost:

$$\min \sum_t \sum_i \sum_j c \cdot (1+r \cdot (t-d_j)) \cdot y_{i,j}^t + \sum_t \sum_h \sum_j \hat{c} \cdot (1+r \cdot (t-d_j)) \cdot z_{h,j}^t + \sum_t \sum_i \sum_{b_1} \sum_{b_2} \bar{c} \cdot \omega'_{i,b_1,b_2} + \sum_t \sum_i \sum_b \sum_m q_i^t \cdot x_{i,b,m}^t \quad (1)$$

Cost vectors  $c$ ,  $\hat{c}$  and  $\bar{c}$  are computed as shown in Equation (2), considering average cost rates per kWh of energy components  $\eta_1$ ,  $\eta_2$  and  $\eta_3$  for e-buses, h-buses and deadheading trips respectively:

$$\begin{aligned}
c &= \eta_1 \cdot u_j \\
\hat{c} &= \eta_2 \cdot u_j \\
\bar{c} &= \eta_3 \cdot \hat{u}_{b_1, b_2}
\end{aligned} \tag{2}$$

Energy component  $\eta_2$  includes an adaptation coefficient to correctly consider the difference in consumption rates between e-buses and h-buses. A penalty term  $r$  [EUR] is applied to trips being performed later than their preferred departure time, to evaluate trade-offs between schedule adherence and operational performance. Regarding the cost of recharging, we explicitly take into account the time dependent cost  $q_i^t$  of recharging bus  $i$  at time  $t$  as part of the operational cost needing minimization.

System dynamics are captured by constraints (3-27) as follows:

$$\sum_j y_{i,j}^t + \sum_{b_1, b_2 \in B} \omega_{i, b_1, b_2}^t + \sum_b \sum_m x_{i, b, m}^t \leq 1 - A_i^t \quad \forall i, t \tag{3}$$

$$y_{i,j}^t + \frac{1}{t_j - 1} \sum_{\bar{t}=t+1}^{t+t_j-1} \left( \sum_j y_{i, \bar{j}}^{\bar{t}} + \sum_{b_1, b_2 \in B} \omega_{i, b_1, b_2}^{\bar{t}} + \sum_{b \in \bar{B}} \sum_m x_{i, b, m}^{\bar{t}} \right) \leq 1 \quad \forall i, \forall j : t_j > 1, \forall t : d_j \leq t \leq d_j + \theta \tag{4}$$

$$\omega_{i, b_1, b_2}^t + \frac{1}{\hat{t}_b - 1} \sum_{\bar{t}=t+1}^{t+\hat{t}_b-1} \left( \sum_j y_{i, \bar{j}}^{\bar{t}} + \sum_{b_1, b_2} \omega_{i, b_1, b_2}^{\bar{t}} + \sum_{b \in \bar{B}} \sum_m x_{i, b, m}^{\bar{t}} \right) \leq 1 \quad \forall i, \forall b_1 \in B, \forall b_2 \in B, \forall t : \hat{t}_b > 1 \tag{5}$$

$$\sum_j z_{h,j}^t \leq 1 - H_h^t \quad \forall h, t \tag{6}$$

$$z_{h,j}^t + \frac{1}{t_j - 1} \sum_{\bar{t}=t+1}^{t+t_j-1} \sum_j z_{h, \bar{j}}^{\bar{t}} \leq 1 \quad \forall h, \forall j : t_j > 1, \forall t : d_j \leq t \leq d_j + \theta \tag{7}$$

$$\sum_t \left( \sum_i y_{i,j}^t + \sum_h z_{h,j}^t \right) = 1 \quad \forall j \tag{8}$$

$$\sum_{t < d_j \cup t > d_j + \theta} \left( \sum_i y_{i,j}^t + \sum_h z_{h,j}^t \right) = 0 \quad \forall j \tag{9}$$

$$\sum_i x_{i, b, m}^t \leq 1 \quad \forall m, t, \forall b \in \bar{B} \tag{10}$$

$$y_{i,j}^t - \frac{\mathcal{E}_i^t}{u_j + \min_{\beta_j \in \bar{B}, b_2 \in \bar{B}} (\hat{u}_{\beta_j, b_2}) + \mu E} \leq 0 \quad \forall i, j, \forall t : d_j \leq t \leq d_j + \theta \tag{11}$$

$$\omega_{i, b_1, b_2}^t - \frac{\mathcal{E}_i^t}{\hat{u}_{b_1, b_2} + \mu E} \leq 0 \quad \forall i, \forall t, \forall b_1, \forall b_2 \tag{12}$$

$$\mathcal{E}_i^0 = \bar{\mathcal{E}}_i \quad \forall i \tag{13}$$

$$E \cdot \sum_{b \in \bar{B}} \sum_m x_{i, b, m}^t - \sum_j y_{i,j}^t \cdot u_j - \sum_{b_1, b_2 \in B} \omega_{i, b_1, b_2}^t \cdot \hat{u}_{b_1, b_2} + \mathcal{E}_i^t - s_i^t = \mathcal{E}_i^{t+1} \quad \forall i, t \tag{14}$$

$$\sum_{b \in \bar{B}} \sum_m x_{i, b, m}^t - \frac{s_i^t}{E} \geq 0 \quad \forall i, t \tag{15}$$

$$\frac{1}{E} \cdot s_i^t - \frac{1}{E} \mathcal{E}_i^t \leq 0 \quad \forall i, t \tag{16}$$

$$\sum_m x_{i,b,m}^t - g_{i,b}^t \leq 0 \quad \forall t, i, \forall b \in \bar{B} \quad (17)$$

$$\sum_{j:\alpha_j=b_1} y_{i,j}^t + \sum_{b_2} \omega_{i,b_1,b_2}^t - g_{i,b_1}^t \leq 0 \quad \forall i, b_1, t \quad (18)$$

$$\sum_{j:\beta_j=b_2} y_{i,j}^t + \sum_{b_1} \omega_{i,b_1,b_2}^t - g_{i,b_2}^{t+1} \leq 0 \quad \forall i, b_2, t \quad (19)$$

$$\sum_{j:\beta_j=b_2} y_{i,j}^t + \sum_{b_1} \omega_{i,b_1,b_2}^t - (g_{i,b_2}^{t+1} - g_{i,b_2}^t) \geq 0 \quad \forall i, b_2, t \quad (20)$$

$$\sum_b g_{i,b}^t = 1 \quad \forall i, t \quad (21)$$

$$g_{i,b}^0 = \begin{cases} 1 & \text{if } b = G_i \\ 0 & \text{otherwise} \end{cases} \quad \forall i, b \quad (22)$$

$$\sum_{j:\alpha_j=b} z_{h,j}^t - p_{h,b}^t \leq 0 \quad \forall h, b, t \quad (23)$$

$$\sum_{j:\beta_j=b} z_{h,j}^t - p_{h,b}^{t+1} \leq 0 \quad \forall h, b, t \quad (24)$$

$$\sum_{j:\beta_j=b} z_{h,j}^t - (p_{h,b}^{t+1} - p_{h,b}^t) \geq 0 \quad \forall h, b, t \quad (25)$$

$$\sum_b p_{h,b}^t = 1 \quad \forall h, t \quad (26)$$

$$p_{h,b}^0 = \begin{cases} 1 & \text{if } b = P_h \\ 0 & \text{otherwise} \end{cases} \quad \forall h, b \quad (27)$$

Constraints (3-7) avoid conflicts in the usage of shared resources. Constraint (3) ensures that an e-bus can either initiate at most one scheduled trip or one deadheading trip or recharge in at most one charger at a time, only in those time steps in which the bus is available. Constraints (4) and (5) ensure that an e-bus which initiates a trip, scheduled or deadheading respectively, whose duration in time steps is greater than one, cannot be used to perform any other scheduled or deadheading trip nor be recharged while current trip is ongoing. Constraints (6) and (7) avoid conflicts in usage of hybrid buses, in an equivalent way to what constraints (3) and (4) respectively enforce for e-buses.

Constraints (8-9) model when the scheduled trips should be executed. Constraint (8) guarantees that each trip must (eventually) be performed exactly once, by either kind of bus. Constraint (9) implies that no trip  $j$  can be initiated before its preferred departure time, or after its preferred departure time plus the maximum *acceptable* delay.

Constraints (10-16) control the charging and discharging dynamics and ensure that the trip execution is consistent with battery status. Constraint (10) implies that a charger can charge at most one e-bus at any given time. Constraint (11) guarantees that an e-bus will not perform a scheduled trip unless it has enough energy to do so, i.e. as long as its current battery status allows to perform the trip and then perform a deadheading trip to the nearest bus terminal equipped with chargers, if the arrival terminal has none, without going below the minimum allowed residual charge. Similarly, constraint (12) ensures that an e-bus will not perform a deadheading trip unless it has enough energy to do so. Constraint (13) controls the initial battery status for each electric bus. Constraint (14) captures the discharging/recharging dynamics of bus  $i$  at time  $t$ : if an electric bus is initiating a trip  $j$  at time  $t$ , its available charge at time  $t+1$  will be reduced by  $u_j$ . Similarly, if an electric bus is initiating a deadheading trip from terminal  $b_1$  to terminal  $b_2$  at time  $t$ , its available charge at time  $t+1$  will be reduced by  $\hat{u}_{b_1,b_2}$ . Conversely, if the electric bus is being recharged at time  $t$ , its available charge at time  $t+1$  will be equal to the total battery capacity  $E$ . Constraint (15) ensures that the slack variable  $s_i^t$  can be non-zero only during recharging operations, and constraint (16) ensures that its maximum value can be  $\varepsilon_i^t$ . Note that constraint (14) implies that when a bus is being recharged, slack variable  $s_i^t$  must assume a value at least equal to  $\varepsilon_i^t$  to ensure that  $\varepsilon_i^{t+1}$  does not violate its domain. Therefore, the combination of constraints (14), (15) and (16) ensures that the slack variable  $s_i^t$  is either 0, if bus  $i$  is not recharging at time  $t$ , or exactly  $\varepsilon_i^t$  if the bus is recharging.

Constraints (17-27) control the location dynamics of each bus. Constraint (17) ensures that an e-bus  $i$  can be charged at a charging station of bus terminal  $b$  only if the current location of bus  $i$  is terminal  $b$ . Constraint (18) ensures that an e-bus  $i$  can initiate a scheduled trip or a deadheading trip departing from bus terminal  $b$  only if its location at the time of departure is  $b$ . Constraint (19) ensures that if e-bus  $i$  initiates a scheduled or a deadheading trip, its new location at the time step immediately after departure is updated to the arrival terminal of that trip. Constraint (20) ensures that an e-bus can change its location only by performing a trip, either a scheduled one or a deadheading one: more precisely, the constraint implies that an e-bus can arrive to a new location only performing a trip. Thanks to constraint (21) it is not however necessary to control that an e-bus can leave a location only by performing a trip, as constraint (21) enforces that every e-bus  $i$  has exactly one location at any time interval  $t$ . Constraint (22) sets the initial value of the e-buses locations. Similarly, constraints (23), (24), (25), (26) and (27) entail location dynamics for h-buses, equivalent to what constraints (18), (19), (20), (21) and (22) respectively enforce on e-buses.

A significant implementation challenge we had to solve, when applying this model to real-life instances, was that the problem translates to MILP representations which are too large to store on common hardware. In order to ensure scalability, we therefore defined a time-based decomposition framework that divides the problem into several different MILP problems, each addressing a subset of the overall time span, to which we refer as *time lapse*, to be solved sequentially. The framework is controlled by a set of equations, exogenous to the MILP formulation, that define how the input parameters for the MILP problem addressing time lapse  $f$  are set based on the results of the MILP problem related to time lapse  $f-1$ . This allows to dramatically improve the scalability of the computational complexity, at the price of a slight loss of optimality. For brevity reasons, we will not describe the decomposition framework, its equations and its results here, and we will instead treat it in detail in a separate work.

#### 4. Computational experiments

We implemented the MILP model and the related time-based decomposition framework in Mathworks® Matlab™, employing IBM's ILOG Cplex 12.7 as optimization software. We validated our multi-terminal model against a real-life problem instance in the city of Luxembourg, considering the following bidirectional urban bus lines: line 1 (Bouillion-Kirchberg), line 16 (Gare C.-Aéroport), lines 9 and 14 (Gare C.-Cents), line 10 (Gare C.-Steinsel), line 12 (Bouillon-Dommeldange), line 13 (Gare C.-Centre Hospitalier), line 17 (Bouillon-Monterey), line 27 (Gare C. Bertrange B.E.), line 28 (Gare C.-Bertrange E.E.). Four of the terminals (Gare Centrale, Cents, Bouillon and Bertrange B.E.) are already equipped with two opportunity charging stations each, while the others are not. We validate our model on two different sets of tests: one addressing a subset of bus lines (lines 1, 16, 9 and 14, entailing 536 daily trips across 5 bus terminals, 2 of which are equipped with chargers), and one addressing the entire problem (10 bus lines, entailing 1034 daily trips across 12 terminals, 4 of which are equipped with chargers).

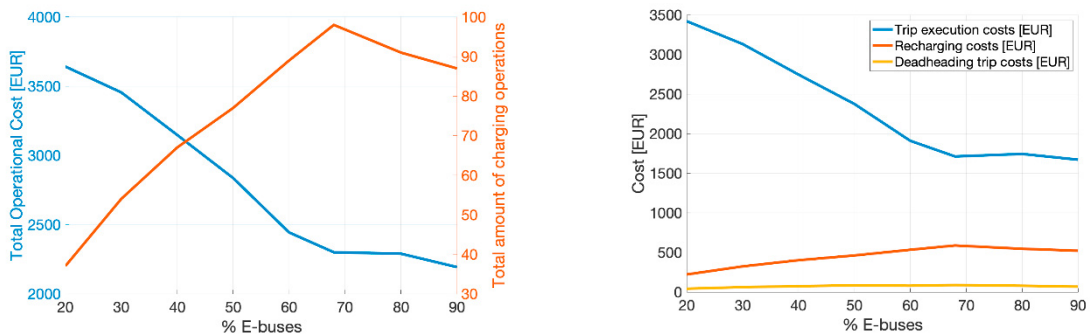


Figure 1: 4 bus lines, 536 trips – Total operational costs and recharge operations (left); distinct cost factors (right)

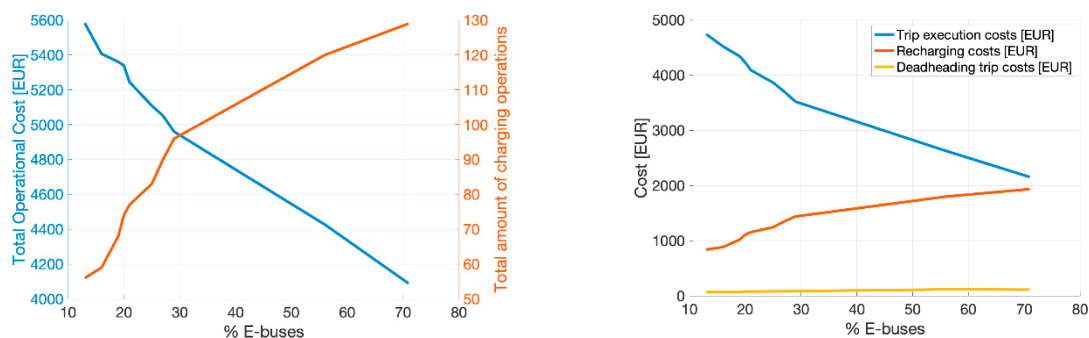


Figure 2: 10 bus lines, 1034 trips – Total operational cost and recharge operations (left); distinct cost factors (right)

The preliminary results shown in Figures 1 and 2 show consistently that as the fleet transitions towards full electrification, the overall operational cost decreases, and the number of total recharges increases accordingly. It is interesting to note that the rate at which operational costs decrease and the total amount of recharging operations increase both exhibit an inflection point: in the set of tests addressing all the 10 lines, the gradient decreases at about 30% of electrified fleet, while in the reduced problem addressing 4 bus lines it becomes actually flat at about 70% of electrified fleet. These results, entirely in line with our previous findings related to single terminal operations, hint at the fact that a diminishing return effect might arise when transitioning towards full electric operations. This effect is far less pronounced for the larger scenario, implying that larger gains might be had for more complex instances.

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