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Procedia Structural Integrity 8 (2018) 390–398

Structural Integrity Procedia

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AIAS 2017 International Conference on Stress Analysis, AIAS 2017, 6-9 September 2017, Pisa, Italy

Correction formula approach to evaluate fatigue damage induced by non-Gaussian stress state

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Abstract

In the present paper the authors define an original analytical expression of a corrective coefficient to evaluate fatigue damage induced by a non-Gaussian stress state affected by high Kurtosis (values higher than 5) and by zero Skewness. This approach starts from a previous activity in which the authors solved an analogous problem but for light non-Gaussian stress states (Kurtosis value less than 5).

The proposed procedure assumes to know the fatigue damage induced by Gaussian equivalent stress state time domain process. This characteristic allows the proposed procedure to be easily adopted inside the so-called Frequency Domain Fatigue Methods but in parallel with the statistical analysis of the system time domain response (Kurtosis and Skewness evaluation).

Interesting considerations about its applicability will be proposed as concerns the non-Gaussianity and non-Stationarity of the inputs when the system is a flexible component excited in its frequency range.

Peer-review under responsibility of the Scientific Committee of AIAS 2017 International Conference on Stress Analysis

Keywords: Random Fatigue, Kurtosis, non-Gaussianity, Spectral methods

1. Introduction

From several years, the fatigue damage is generally computed with a frequency domain approach. Even if the rainflow counting method [1] is considered as the most reliable in terms of results, it requires long computational time

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2452-3216
Peer-review under responsibility of the Scientific Committee of AIAS 2017 International Conference on Stress Analysis
10.1016/j.prostr.2017.12.039

and for this, engineer's community generally compute the fatigue damage of structures subjected to random loads through spectral methods [2]. A random process is generally synthetizes as a Power Spectral Density (PSD) and it is used to estimate both the distribution of the rainflow cycles and the fatigue life [3-4].

The theory usually assumes Gaussian loads since this is a simplifying hypothesis, which allows simulating representative time histories from the load PSD, as well as computing the cycle distribution and the fatigue damage.

However, many real loads, e.g. ocean waves [5], road irregularities [6] or pressure fluctuation [7] show considerable non-Gaussian features such as a skewed probability distribution or high kurtosis. In these situations, the use of frequency methods for damage calculation may lead to serious consequences. In fact, the fatigue life predictions may be far different if compared to the measured one [8-10]. Due to the high relevance of non-Gaussianity in industrial applications, the attention of several researchers has been focused on the evaluation of the real influence of non-Gaussianity in fatigue life. Rizzi et al. [11] and Kihm et al. [12] investigated the influences of kurtosis and skewness on the fatigue life of linear and non-linear system. They found out that in case of linear system non-Gaussian loads produce Gaussian responses due to the respect of the central limit theorem (CLT), while in case of non-linear system all responses are non-Gaussian**.** Wolfsteiner [13] instead, developed a new methodology for the decomposition of a non-stationary random vibration signal into a combination of stationary Gaussian signals allowing to perform accurate frequency domain analysis also in case of non-Gaussian loads.

Benasciutti et al. [14] investigated the possibility to use their own method (TB method [15]) and the well-known narrow band method [16] presented in "non-Gaussian" version certifying how the TB method is able to take into account the non-normality and the bandwidth of the input loads while the narrow-band method may lead to uncorrected results.

By the way, all the approaches herein presented can be easily avoided by the use of a correction approach [17]. Indeed, by evaluating a correction coefficient it is possible to compute the fatigue damage directly with spectral methods, non-considering the non-Gaussianity of the stress state, and then by correcting the fatigue damage it is possible to obtain an accurate results in terms of durability.

In previous activity [18-19] the correction coefficient proposed by Braccesi et al. [17] was used to correct the estimated fatigue damage computed for non-Gaussian stress states, and it was found that for high-kurtosis stress the correction coefficient overestimate the fatigue damage while in case of light non-Gaussian stress states, the correction coefficient allows to correctly estimates the fatigue damage. The same result was obtained by Niesłony et al. [20], where for low kurtosis stress states, a comparison between the obtained fatigue lives in time and in frequency domain shows a good agreement.

The activity herein presented starts from these considerations and it is voted to improve the correction formula previously obtained by Braccesi et al. [17] aimed to guarantying a convergence of the results between experimental and numerical ones in case of high kurtosis and zero skewed stress time histories.

To this goal a set of stationary Gaussian and non-Gaussian signal, representing equivalent uni-axial stress time histories, have been generated and used to compute the fatigue damage for different Wöhler curve slope by the use of the well-known rainflow counting method [1].

The ratio between them allowed to evaluate the real value of the correction coefficient for all the considered cases. An interpolation of these results allowed to formulate the correction coefficient for every kurtosis stress state and for different Wöhler curve slopes.

The paper is organized as follow: firstly a short description of the formula proposed by Braccesi et al. [17] and the results previously obtained is presented. Then, the approach used to formulate the proposed coefficient is given, and at the end, the evaluation of the effectiveness of such formulation is presented.

2. The correction formula approach

The evaluation of the fatigue damage in case of non-Gaussian stress states by the correction coefficient method can be considered one of the most valid approach due to its high simplicity and effectiveness. Indeed, it is possible to not considering the non-Gaussianity of the stress response of the system and computing the fatigue damage with the spectral methods [15,21-22], and then by a correction it is possible to obtain accurate results in terms of fatigue damage. A flowchart of the correction coefficient approach is shown in Fig. (1).

Different correction coefficients are available in literature and whichever is used, it is always defined as a function of the Wöhler curve slope (*i.e.* of the material) and of the kurtosis and the skewness of the stress state. The kurtosis and the skewness are two statistical parameters that allow to completely define the non-Gaussianity of the process. They are formulated as follow:

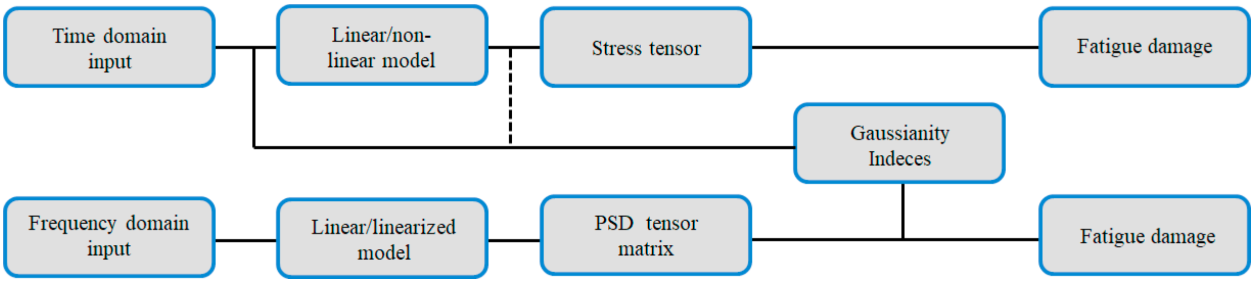

Fig. 1. Flowchart of the correction coefficient approach

$$K_u = \frac{M_4}{M_2^2} = \frac{M_4}{\sigma^2} \qquad S_k = \frac{M_4}{M_2^{3/2}} = \frac{M_4}{\sigma^3} \tag{1}$$

The kurtosis is the normalized central moment of fourth order of the probability distribution $p(x)$ while the skewness is the normalized central moment of the third order of the probability distribution $p(x)$. The kurtosis is 3 while the skewness is 0 for a Gaussian process [3-4].

It is clear how the evaluation of the correction coefficient needs the assessment of the kurtosis and the skewness of the stress response. As shown in Fig. (1) it can be done in three different ways. The first one is to compute these two parameters once the stress tensor is known, so meaning that at least one short time domain analysis must be performed in order to evaluate the kurtosis and the skewness of the stress state. The second and third option are much more interesting because they allows to evaluate the kurtosis and the skewness or directly from the input loadings or by the assessment of the time history of the Lagrangian coordinates [23-24], that requires short computational time. As easy to understand, the evaluation of kurtosis and skewness from the input loads it is possible and realistic only if we are dealing with rigid systems in which the dynamics does not change the probability distribution of the output response.

In such a way, by the assessment of kurtosis and skewness of the loads time history, the computation of the fatigue damage can be performed directly with spectral methods, maintaining all the advantages that such approach guarantees such as the short computational time and the reliability of the results.

Different correction coefficients are available in literature [17]. One of the most used is that proposed by Braccesi et al. [17], defined as function only of kurtosis and skewness of the stress response and on the Wöhler curve slope m. The formulation that they proposed is shown in Eq. (2).

$$\lambda_{ng} = exp\left(\frac{m^{3/2}}{\pi}\left(\frac{K_u - 3}{5} - \frac{S_k^2}{4}\right)\right) \tag{2}$$

The formulation of Eq. (2) was obtained by interpolating a set of experimental data, affected by low kurtosis K_u (i.e. smaller than 5) and skewness S_k, for different Wöhler curve slope m. The formula that they obtained is very easy and compact and it only needs the assessment of kurtosis and skewness of the stress state and the material.

The trend of the Braccesi et al. [17] correction formula is shown in Fig. (2) for two different Wöhler curve slope.

Due to its user-friendliness and the reliability, the correction coefficient of Eq. (2) has been used in different activities [18-20], where a linear flexible system was subjected to several non-stationary non-Gaussian signals with high kurtosis and zero skewness.

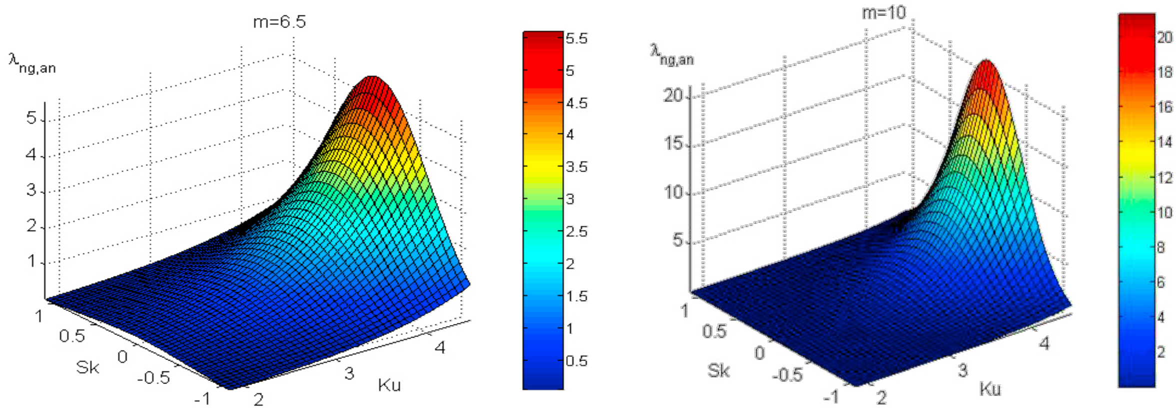

Fig. 2. Braccesi et al. [17] correction formula for two different Wöhler curve slope

In that activity, it was found that in case of stationary non-Gaussian stress states the spectral methods can be used neglecting the non-Gaussianity of the stress response. Instead, in case of non-stationary non-Gaussian stress, the fatigue life obtained with the Dirlik method [21], is much different if compared to that obtained with the rainflow counting method [1].

In Fig. (3) a comparison between the fatigue life obtained in time and in frequency domain in a previous activity [19] is shown. Indeed, it is clear how in case of stationary Gaussian and non-Gaussian stress states there is a good agreement between the two approaches.

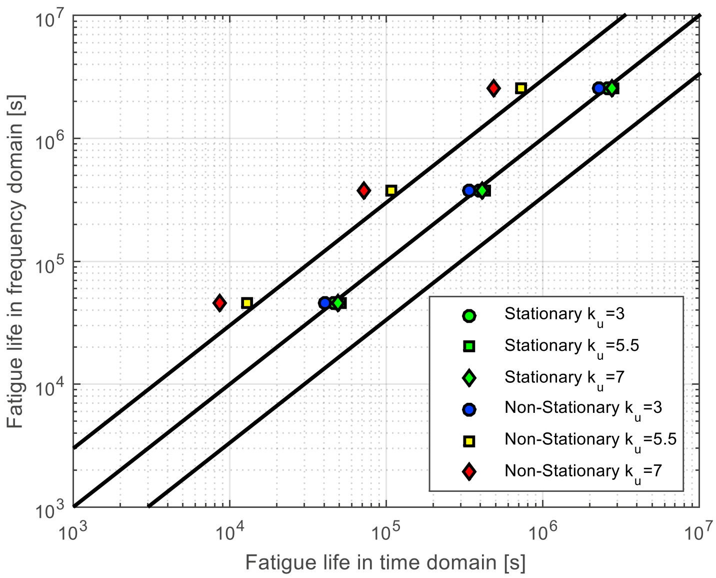

Fig. 3. Comparison between the fatigue life obtained in time and in frequency domain in a previous activity [19]

However, it is possible to attest that in case of non-stationary non-Gaussian stress histories there is a large difference between the results obtained in frequency domain if compared to that obtained with the rainflow counting method [1]. In fact, the yellow and the red point should fall within the confidence interval in order to attest the convergence of the results. Capponi et al. [25] moreover demonstrated the high influence of non-stationary non-Gaussian stress states in fatigue life for different levels of non-stationarity.

Such problem was partially solved with the use of the correction coefficient of Eq. (2). The results obtained by correcting the fatigue damage computed in frequency domain are shown in Fig. (4).

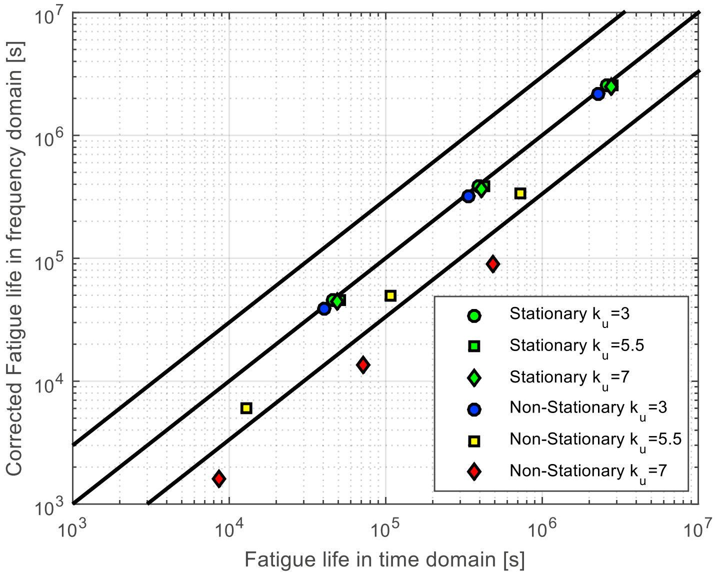

Fig. 4. Comparison between the fatigue life obtained in time and the corrected life obtained in frequency domain in a previous activity [19]

In Fig. (4) a comparison between the corrected fatigue life obtained in frequency domain and that obtained with the rainflow counting is shown. The obtained results show that for the case of non-stationary non-Gaussian stress states with low kurtosis (i.e. yellow point with kurtosis 5), the corrected results are in good agreement with the time domain method, certifying the effectiveness of the Braccesi et al. [17] correction coefficient. Instead, it was found that in case of non-stationary non-Gaussian stress states the correction coefficient overestimates the fatigue life due to its formulation. In fact, being an exponential relation, in case of high kurtosis the formulation of Eq. (2) assumes to high values that it is reflected on an overestimation of the computed damage.

3. The proposed correction coefficient

The presented activity start from these results, indeed it is voted to an improvement of the formula proposed by Braccesi et al. [1], trying to re-arrange the correction coefficient to high kurtosis stress states. The importance to extend the formula of Eq. (2) to high kurtosis stress states resides in the fact that in several industrial application such as road irregularities [6] or pressure fluctuation [7] it is common to deal with mechanical components subjected to high kurtosis stress.

The procedure to obtain the correction coefficient starts form the generation of a set of stationary Gaussian and non-Gaussian signals form two given input Power Spectral Densities (PSD). The generated signals have to be considered as uni-axial equivalent stress time histories from which, by imposing the assessment of the Wöhler curve slope m, it is possible to compute the fatigue damage with the rainflow counting method [1] and the Palmgren-Miner rules [26]. The ratio between the fatigue damage obtained under Gaussian and non-Gaussian equivalent stresses supplies the real value of the correction coefficient for each Wöhler curve slope.

To this aim, starting from two different Power Spectral Densities (PSD), shown in Fig. (5), a set of stationary Gaussian and non-Gaussian signals were generated.

As shown in Fig. (5), a mono-modal and bi-modal PSD with maximum frequency equal to $3000\ Hz$ and with two different RMS were considered in order to avoid the influence of the input PSD and of the RMS.

The generated stationary Gaussian and non-Gaussian signals were zero-skewed and covered a kurtosis range from 2,5 to 10. In such a way, it was possible to certify the correctness of the proposed correction coefficient from low to

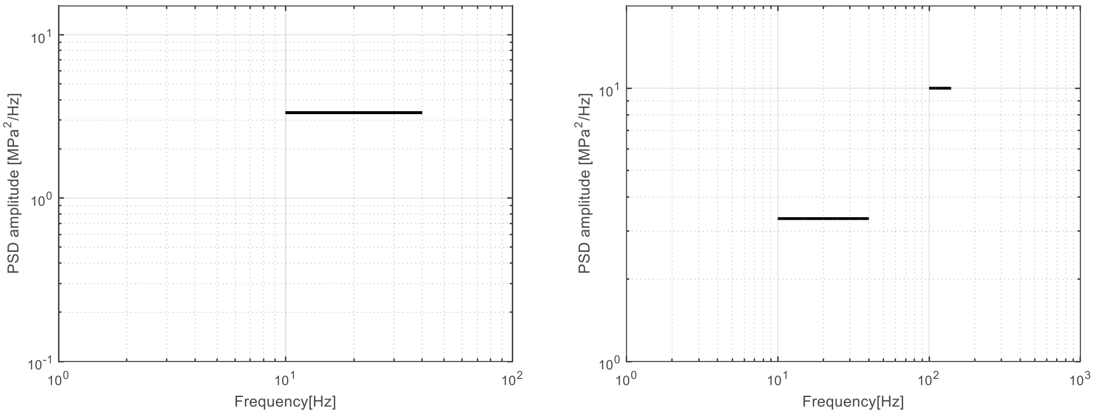

Fig. 5. Input Power spectral Densities. Left: mono-modal PSD. Right: Bi-modal PSD

high kurtosis values. An example of a generated stationary non-Gaussian signals and its histogram is shown in Fig. (6). Once all the input equivalent uni-axial stress histories were generated with kurtosis from 2,5 to 10, it was possible to compute the fatigue damage for a large set of Wöhler curve slope, in particular the considered slope covered a range from 3 to 10 and imposing an intercept equal to 1000 MPa. It worth to state that the fatigue damage was averaged over 100 stress time histories for each combination of kurtosis and Wöhler curve slope. In such way, a steady statistical reliability was achieved.

As stated before, the ratio between the fatigue damage obtained under Gaussian and non-Gaussian conditions for each Wöhler curve slope, led to the assessment of the real value of the correction coefficient.

Interpolating all the results it was possible to formulate the expression, which represents the correction coefficient. The final formulation is shown in Eq. (3).

$$\lambda_{ng} = exp\left(\frac{m^{3/2}}{(0.156 + 0.416K_u)\pi}\left(\frac{K_u - 3}{5}\right)\right) \tag{3}$$

In Fig. (7) a 3-D plot of Eq. (3) for a range of kurtosis from 2 to 12 and for a range Wöhler curve slope from 3 to 10 is presented.

The formulation herein presented and shown in Eq. (3) maintains the same properties of the formula proposed by Braccesi et al. [17] for light non-Gaussian stress states. Instead, in case of high kurtosis stress response the proposed

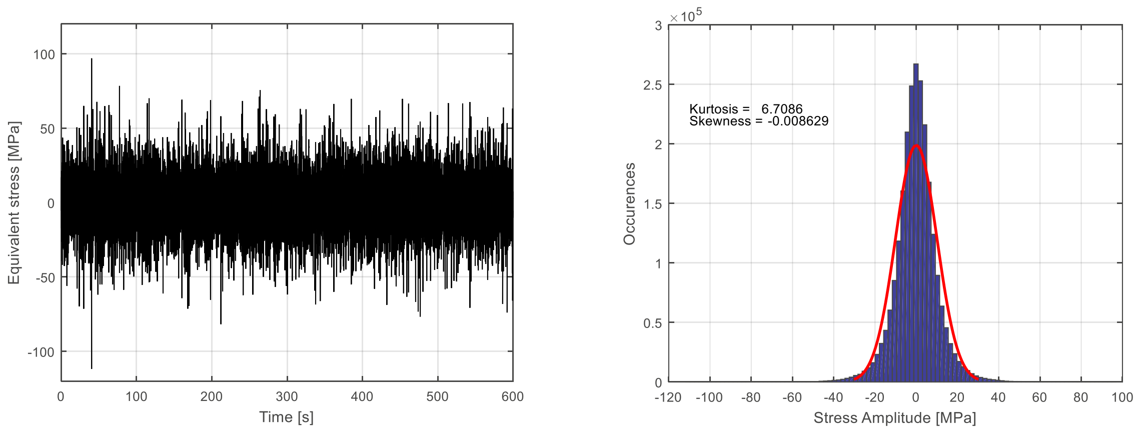

Fig. 6. An example of generated stationary non-Gaussian signal with kurtosis = 6.7 and skewness = 0

formulation allows to better evaluate the fatigue damage computed with the spectral methods, since it has been obtained considering a larger set of kurtosis and Wöhler curve slope.

The effectiveness of the proposed formula can be easily proved just by correcting the results obtained in a previous activity [18] and shown in Fig. (3) with the new coefficient. In Fig. (8) the corrected values of Fig. (3) are shown.

Comparing the results shown in Fig. (4), where the corrected fatigue lives with the previous approach for non-

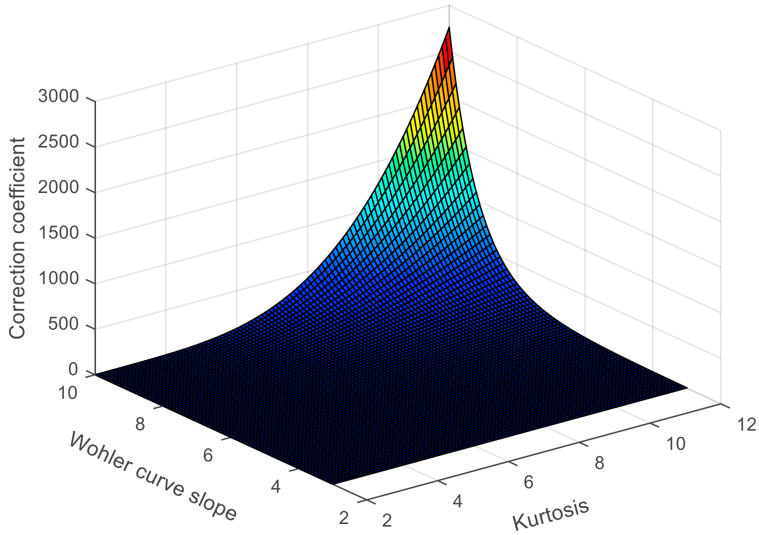

Fig. 7. 3-D plot of the proposed formula for all the considered kurtosis and Wöhler curve slope

stationary non-Gaussian signal felt outside the confidence interval, with the results shown Fig. (8) it is easy to see how the proposed coefficient allows to better correct the fatigue damage obtained with the spectral methods also for strongly non-Gaussian stress states. Indeed, all the points representative of non-stationary non-Gaussian conditions fall within the confidence bands, certifying the effectiveness of the proposed formula.

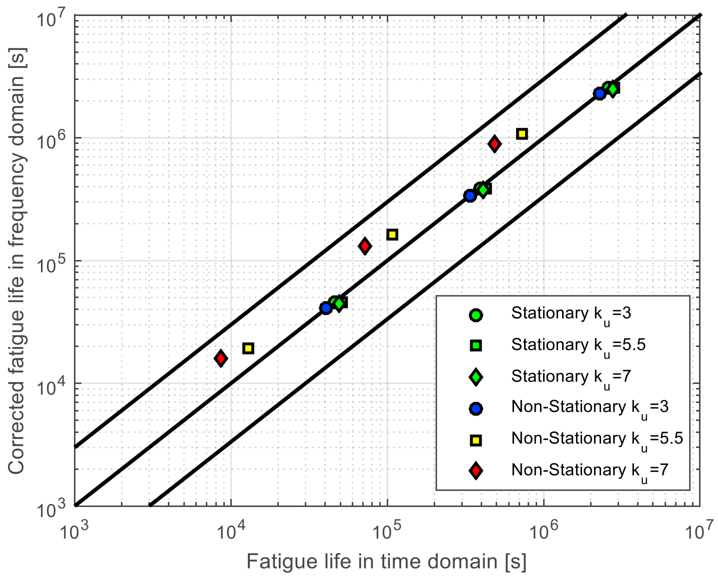

Fig.8. Comparison of the results obtained in previous activity [18] corrected with the new correction coefficient

From Fig. (8) it is possible to affirm that the effectiveness of the Braccesi et al. [17] coefficient for low kurtosis stress states has not be changed. In fact, the position of all the points referred to stationary and non-stationary stress states with low kurtosis has not be modified.

4. Conclusion

In the presented activity, a new correction coefficient was carried out in order to correctly estimate the fatigue life of a mechanical component subjected to strongly non-Gaussian stress state with the standard frequency domain approach. It has been demonstrated how the available correction coefficient allows to correctly estimate the fatigue damage only in case of stress time histories affected by low kurtosis value, while in case of strongly non-Gaussian stress state the corrected fatigue damage results overestimated. For this reasons, an implementation of a new correction coefficient were needed.

Considering a large set of material (*i.e.* different Wöhler curve slope) the fatigue damage was calculated with the rainflow counting and with the Palmgren-Miner rule for a large set of stress states affected by different kurtosis and zero skewness. The ratio between the fatigue damage obtained under Gaussian and non-Gaussian conditions for each considered Wöhler curve slope guarantying the knowledge of the exact value of the correction coefficient. An interpolation of all the obtained results led to the definition of the explicit formulation. By re-arranging the results obtained in a previous activity with the proposed formula it has been possible to demonstrate the effectiveness of the proposed formula when dealing with the evaluation of the fatigue damage with spectral methods for low and high kurtosis stress states.

Beside the good formulation herein presented, it is important to do not forget that the presented formula does not take into account the influence of skewness in the estimation of fatigue life. For this reason, the presented coefficient needs a further development step aimed to examine the influence of skewness in fatigue life calculation and its impact on the correction coefficient formula.

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