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1D simulation and experimental analysis of a turbocharger turbine for automotive engines under steady and unsteady flow conditions

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Abstract

Turbocharging technique is more and more widely employed on compression ignition and spark ignition internal combustion engines, as well, to improve performance and reduce total displacement. Experimental studies, developed on dedicated test facilities, can supply a lot of information to optimize the engine-turbocharger matching, especially if tests can be extended to the typical engine operating conditions (unsteady flow). A specialized components test rig (particularly suited to study automotive turbochargers) has been operating since several years at the University of Genoa. The test facility allows to develop studies under steady or unsteady flow conditions both on single components and subassemblies of engine intake and exhaust circuit.

In the paper the results of an experimental campaign developed on a turbocharger waste-gated turbine for gasoline engine application are presented. Preliminarily, the measurement of the turbine steady flow performance map is carried out. In a second step the same component is tested under unsteady flow conditions. Instantaneous inlet and outlet static pressure, mass flow rate and turbocharger rotational speed are measured, together with average inlet and outlet temperatures.

A numerical procedure, recently developed at the University of Naples, is then utilized to predict the steady turbine performance map, following a 1D approach. The model geometrically schematizes the component basing on few linear and angular dimensions directly measured on the hardware. Then, the 1D steady flow equations are solved within the stationary and rotating channels constituting the device. All the main flow losses are properly taken into account in the model. The procedure is able to provide the sole "wheel-map" and the overall turbine map. After a tuning, the overall turbine map is compared with the experimental one, showing a very good agreement.

Moreover, in order to improve the accuracy of a 1D engine simulation model, the classical map-based approach is suitably corrected with a sequence of pipes that schematizes each component of the device (inlet/outlet ducts, volute and wheel) included upstream and downstream the turbine to account for the wave propagation and accumulation phenomena inside the machine. In this case, the previously computed "wheel-map" is utilized. The turbine pipes dimensions, are automatically provided by the geometrical module of the proposed procedure to correctly reproduce the device volume and the flow path length.

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The unsteady numerical results are finally compared to the experimental findings and a good agreement is demonstrated. Some examples are finally presented to put into evidence the improvements obtained by the employment of the wheel map coupled to properly dimensioned pipes, compared to a classical map-based approach considering the overall turbine map.

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Nomenclature

c, c_w	Absolute velocity, Absolute velocity component along the meanline in the wheel
c_p	Constant pressure specific heat
d	Equivalent diameter
F	Flux terms vector
f	Friction coefficient
h	Enthalpy per unit mass
p	Pressure
q	Specific rate of heat exchange
r	Radius
S	Vector of the source terms
s	Wheel curvilinear abscissa
T	Temperature
u	Tangential blade velocity
x	Stationary ducts abscissa
w	Relative velocity
Z	Number of blades

Greeks

α	Duct area variation term
δ	Radius variation term along a streamline
ρ	Density
Ω	Cross section area
ω	Angular speed

Subscripts

0	Total conditions
h	Hub
s	Spatial derivative for rotating ducts, Shroud
W	Referred to rotating ducts
w	Referred to the relative motion, Wheel

Acronyms

1DTM	Refined GT-Power turbine model, including 1D components and the wheel map
GTP	Standard GT-Power turbine model
TC	Turbocharger
TD	Time delay produced by the virtual pipe

1. Introduction

It is widely recognized that turbocharging is a very useful technique commonly applied in both Spark-Ignition and Compression Ignition engines to improve engine efficiency [1]-[3]. Advanced technologies, such as variable geometry and multi-stage turbocharging, high and low pressure loop EGR systems, assisted turbocharging and

turbocompounding, are emerging to control emissions and maintain fuel economy in both diesel and gasoline engines. Moreover, an effective management of the turbocharging system gives the possibility to achieve a suitable engine torque curve and an excellent vehicle driveability [4].

The engine-turbocharger matching can be successfully carried out by means of one-dimensional (1D) models. The classical quasi-steady approach, based on the employment of compressor and turbine characteristic maps (provided by the manufacturers or directly measured) is usually followed [5]. Anyway, in many cases, a limited operating domain of the turbocharger device is available, and mathematical techniques for the extrapolation of the performance maps are commonly utilized in commercial software products [6], [7]. In addition, due to various phenomena, the actual compressor and turbine behavior may differ from the one expressed by the characteristic map. First of all, performance data refer to a steady operation of the devices, while the turbocharger, due to the pressure pulsations caused by the engine [8], [9], always operates under unsteady conditions [10], [12]. This is particularly true in the case of small SI engines fitted with a reduced number of cylinders, determining a wide mass flow rate and pressure ratio oscillations during each engine cycle. Moreover, heat transfer through the casing and heat radiated by the turbine and the engine may affect the efficiency values derived at the test-bench.

In order to analyze the unsteady behavior, since several years, a specialized components test facility has been operating at the University of Genoa [9], [15]. The test rig is fitted with specific devices which allow for investigations to be performed under steady and unsteady flow operation both on single components and engine subassemblies. Several experimental studies on automotive turbocharger turbines, also fitted with different regulating devices (waste-gate valve and variable geometry systems), have been performed [15]-[19].

From a numerical point of view, a first attempt to better take into account the effects of the pulsating flow, is the use of a “virtual pipe” having the same equivalent length and volume as the actual device, together with characteristic map [20], [21]. Other refined methodologies are proposed in [22]-[24]. It is highlighted that, under a pulsating flow, the turbine wheel still experiences almost steady state conditions, while storage effects mainly occur within the volute.

In the present paper, a small in-series waste-gated turbocharger turbine (IHI RHF3) has been experimentally characterized at the test rig both under steady and unsteady operations. Then, the turbine has been modeled in the GT-Power™ environment following different methods: the classical map-based approach, the map-based approach with a virtual pipe upstream the turbine object and an innovative methodology based on the schematization of each component of the device (inlet and outlet pipes, volute and wheel) by means of pipes of proper dimensions. The latter, recently proposed by the University of Naples [13], [14] is based on the solution of the 1D steady state flow equations within the stationary and rotating channels constituting the turbine, starting from a reduced set of geometrical data. A direct modeling of main phenomena and losses is performed, also referring to correlations from the literature. The previously recalled steady 1D turbine model provides as an output the geometrical dimensions of the turbine sub-components; in addition, after a tuning based on the experimental steady state map, it furnishes the extended characteristic map of the sole wheel. The GT-Power turbine model is then used to predict instantaneous pressure levels in four stations along the test rig and mass flow rate at the turbine inlet. Through the comparison with the experimental findings, the numerical results demonstrate some advantages with respect to the classical map-based approaches.

2. Test rig description

The experimental activity reported in the paper is developed at the turbocharger test facility of the University of Genoa, fully described in previous papers [15]-[19]. The test bench, schematically shown in Figure 1a is a cold air apparatus and consists of the research turbine, an air supply system, a power absorber in the form of a centrifugal compressor, and the data acquisition system. In order to investigate turbine maps over an extended range while keeping the turbocharger compressor coupled, modulation of compressor inlet pressure is performed, thus controlling its power absorption. The turbine and compressor air is supplied by three screw-type compressors, which can deliver up to 0.6 kg/s at a maximum pressure of 8.0 bar. The turbine inlet air is heated by an electric heater with a maximum power of 54 kW to prevent condensation of water vapors particularly at high expansion ratios. The turbine inlet temperature is set to a definite level (about 80 °C in the presented investigations) and automatically regulated by a PID controller. The air leaving the heaters flows through a plenum and a pulse generator device

consisted of rotating valves. The main pressure pulse parameters (amplitude and mean value) can be controlled by properly mixing in a Y-junction two flow components (steady and pulsating one) and adjusting the upstream plenum pressure. The pulsating air then flows through an instrumented section before being expanded through the turbine to atmosphere. The experimental investigations are performed on a small automotive turbocharger matched to a SI engine. The single entry nozzle-less radial flow turbine is fitted with a waste-gate valve.

Instantaneous parameters are measured at different locations upstream and downstream of the turbine. Measurements of pressure, mass flow rate, temperature and rotational speed are performed. Mean and instantaneous pressure levels are measured through strain-gauge transducers characterized by an accuracy of 0.15% of full scale. Platinum resistance thermometers (with an uncertainty of 0.2 ± 0.15 °C) are adopted to record the mean temperature levels, while the instantaneous ones are calculated considering an adiabatic process of an ideal gas. The average turbine mass flow rate is measured through a laminar flow meter, while the instantaneous level at the turbine inlet is detected using a constant temperature hot-wire anemometric system. The fiber film probe is calibrated against the laminar flow meter for three different air temperature levels selected to include unsteady instantaneous conditions. The output probe voltage is then corrected according to the temperature variation during the pulse cycle. The uncertainty in mass flow measurement is about $\pm 2\%$ throughout the calibration range. Turbocharger rotational speed measurement is performed through a variable inductance sensor (full scale frequency precision of 0.009%), able to detect each blade passage. Measurements under steady and unsteady flow conditions are collected by an automatic data acquisition system. Instantaneous parameters are recorded using two synchronized data acquisition cards, using a trigger signal corresponding to a definite position of the rotating valve to start data acquisition on both cards. For each operating conditions (i. e., pulse frequency levels and average inlet pressure) measurements are conducted at constant level of turbine rotational speed factor. For each turbocharger speed level the turbine maps are explored by changing the compressor power absorption both under steady and unsteady flow conditions. All signals reported in the paper are an ensemble average of several consecutive cycles.

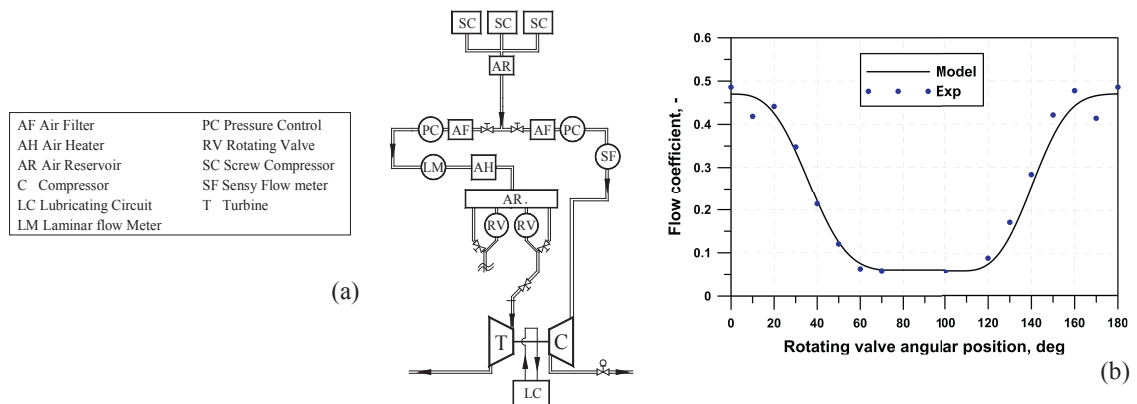


Figure 1. (a) Turbocharger test facility (b) Experimental and numerically fitted flow coefficients of the rotating valve

In order to characterize the pulse generator device, an experimental investigation is performed to evaluate the discharge coefficient of the rotating valve (Figure 1a). This is achieved through steady measurements of pressures, temperatures and mass flow rate across the valve at different angular position. The discharge coefficient is then estimated as the ratio between the measured mass flow rate and the isentropic one. The reference area for isentropic mass flow rate definition is equal to the geometrical flow area in the fully opened valve position.

3. Steady 1D turbine model

The turbine model includes a geometrical module that provides the data required to solve the 1D flow equations inside the different ducts composing the device. In particular, the geometrical schematization includes 5 pipes: the

inlet duct, the volute, the nozzle, the wheel and the outlet duct. Figure 2a shows the 1D schematization of a waste-gated turbine, while Figure 2b highlights the main stations along the fluid path.

3.1. Geometrical module

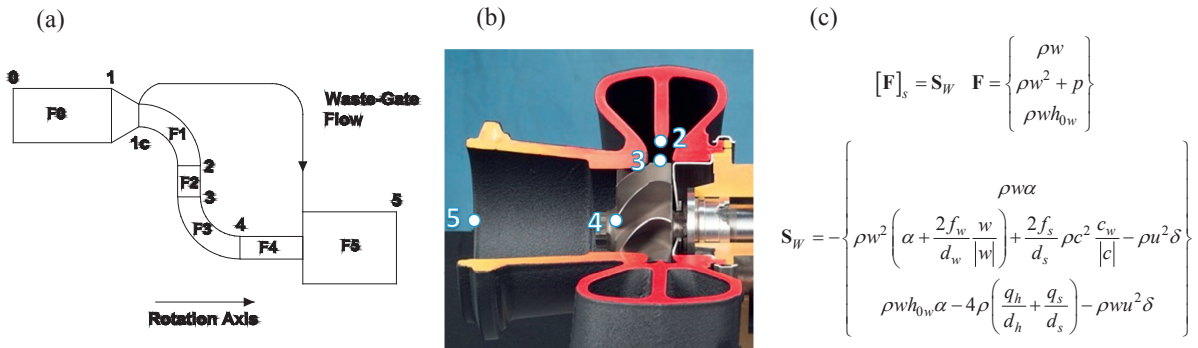


Figure 2. (a) 1D schematization of a waste-gated turbine; (b) main stations of the flow path; (c) 1D Flow equations for the wheel

The turbine is schematized as a sequence of fixed (F0: inlet, F1: volute, F2: wheel gap, F4 and F5: outlet) and rotating (F3) pipes. The latter consists of Z rotating pipes, Z being the number of blades. The profile of the blade-to-blade duct is assigned in terms of blade angles, rotation angles and radii at the inlet and outlet. A geometrical procedure is included to reproduce the 3D wheel geometry basing on the previously listed data. The equivalent 1D profile of blade-to-blade duct (local blade angle, radius, cross section area and wetted perimeter) is automatically provided by the geometrical module, and is employed in 1D flow equations.

3.2. 1D flow model

Stationary ducts flow model – The steady 1D flow in the stationary ducts (i.e. the inlet pipe, the volute, the nozzle and the outlet duct) is described by the standard continuity, momentum and energy equations, including friction, f, area variation, $\alpha = 1/\Omega dQ/dx$, and heat transfer, q, terms.

Wheel flow model – Within the wheel, the balance equations in Figure 2c apply, ρ , c and p, being density, absolute velocity, and static pressure, respectively. w is the relative velocity, $u = \omega r$, is the tangential velocity (blade speed) and $h_{0w} = c_p T_{0w} = c_p T + w^2/2$ is the total enthalpy in the relative motion. The source term S_W includes additional contributions arising as a consequence of pipe rotation. They are related to the centrifugal forces acting on the fluid particle and are computed as a function of the radius variation along a streamline: $\delta = 1/r dr/ds$.

Numerical integration of flow equations – Computation starts with a fixed value of mass flow rate at the turbine inlet and imposing inlet total pressure and temperature derived by experimental reference data. Flow equations are discretized in finite differences and integrated by an iterative procedure.

Boundary conditions – The boundary conditions in the nodes of adjacent ducts are specified by applying a classical quasi-steady pipe-to-pipe junction problem. Mass and energy conservation at the junction are imposed, while the momentum equation is substituted by a total pressure loss relation between adjacent sections of the connected pipes. Formally, the total pressure loss can be specified as a function of a Mach number expressing the total head loss. Loss related Mach number is opportunely specified according to the junction and the considered loss mechanisms. The model accounts for all the main losses occurring in the turbine: incidence, leakage and blade loading in the wheel and volute losses. Due to the limited generality of employed loss correlation, complexity of the phenomena, and intrinsic limitations of the 1D approach, each correlation includes a tuning constant, which must be identified through comparisons with experimental data. Additional details can be found in [13].

4. Model tuning and computation of the extended steady state map

The above summarized steady procedure is completely defined once the whole set of geometrical data and the loss correlations tuning constants have been specified. In [13], a refined tuning methodology was realized with reference to five different turbines, through the employment of an optimization code. The same approach is here followed to tune the model constants for the considered IHI-RHF3 turbine.

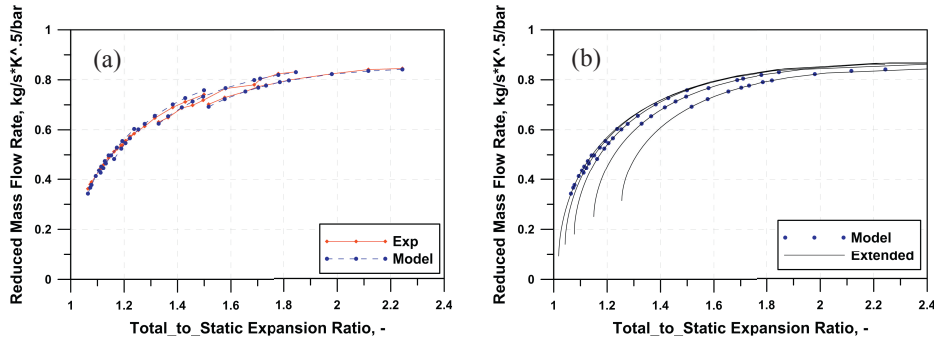


Figure 3. (a) Computed and measured mass flow rate vs. expansion ratio turbine map comparison; (b) Computed extended map

The agreement with the experimental map is very good (Figure 3a); in fact, the averaged percent error between the computed reduced mass flow rate and the experimental one is equal to 1,5 %. Once tuned, the model, as shown in [14], is able to provide a physical extension of the map over the whole operative domain (Figure 3b). In addition, computed results can be post-processed to extract the map of the sole turbine wheel: it includes the computed total-to-static expansion ratio across the wheel and the related mass flow rate, reduced with the total pressure and temperature at the wheel inlet (section 3 in Figure 2).

5. Unsteady operation results

To investigate the unsteady behavior of the turbine, a 1D model is realized in GT-Power environment, including each element of the described test rig, starting from the upstream air reservoir. A particular attention is devoted to the rotating valve implementation: a “user routine” schematizes the valve as a nozzle, and assigns a proper discharge coefficient according to the angular position of the rotating valve. The experimentally measured discharge coefficients, mentioned in section 2, are reproduced in the calculations by using properly tuned analytical functions, as shown in Figure 1b. In this way, a smooth transition is realized between the opening and closing valve phases. The turbine itself is described as a sequence of objects, labeled “1DTM” (Figure 4a), including:

1. the inlet cone, schematized for the analyzed turbine as a constant section pipe;
2. the volute, schematized as a converging pipe ;
3. a virtual pipe having the same equivalent length and volume of the blade-to-blade ducts constituting the entire wheel. Friction and heat exchange contributions are herein cancelled;
4. the turbine wheel, where the extended wheel map is specified. The latter is a zero-volume object utilized to impose the mass flow rate related to the instantaneous rotational speed and pressure ratio across the wheel;
5. the outlet duct, schematized as a constant section pipe.

All the geometrical information required to build up the above described schematization, usually unavailable or hard to measure, are automatically provided by the geometrical module of the steady 1D turbine model. It is not worthless to emphasize that the turbine model tuning, identified under steady conditions as described in the previous section, is adopted in the unsteady version of the methodology without requiring any additional tweak.

In order to highlight the differences with the state of art approaches in the turbine modeling, other two methodologies are implemented: the first, labeled “GTP”, is based on the standard GT-Power management of the turbine maps; the second, labeled “GTP+TD”, is similar to the first one with the introduction of a “virtual pipe”

before the turbine object, adding a “time delay” in the pressure wave propagation through the device (Figure 4b). The “virtual pipe” also takes into account the mass and energy storage within the turbine volumes. The pipe dimensions are assigned basing on the information commonly available for an engine manufacturer, i.e. the wheel and volute inlet diameters. The first is usually provided by the turbocharger manufacturer, while the second can be easily estimated or measured on the hardware. In particular, the “virtual pipe” length is computed as half the wheel inlet circumference, while the diameter is imposed equal to the volute inlet.

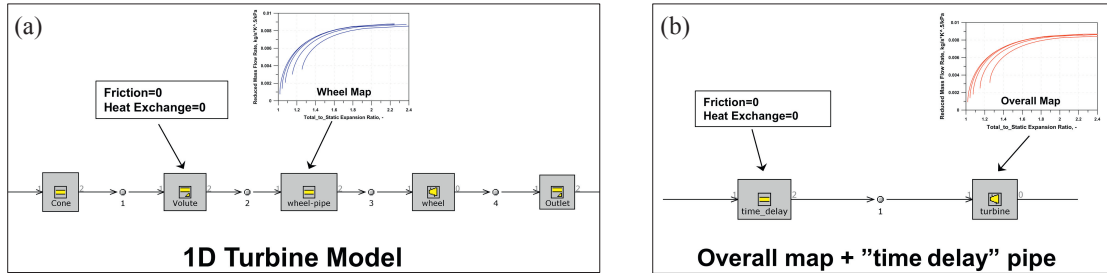


Figure 4. Turbine modeling in GT-Power environment: (a) 1D turbine model; (b) map based approach + “time delay” pipe correction

Table 1 summarizes the experimentally and/or numerically analyzed cases. Firstly, the 1DTM accuracy is tested through comparison with the experimental findings for different turbine speeds and pulse frequencies (cases 1 to 4 in Table 1), than the comparison between the different methodologies is proposed (cases 4 and 5 in Table 1). Boundary conditions for the numerical analyses are provided by the pressure and temperature measured in the upstream air reservoir, and by the measured average turbine speed. The unsteady operation is only induced by the rotating valve that defines the pulse frequency. Figures 5-8 show the pressure profiles in some sections along the circuit: downstream the rotating valve, in the above mentioned Y section, upstream and downstream the turbine.

The numerical results denote a satisfactory agreement with the experimental data for the considered turbine speeds and pulse frequencies. Actually, upstream pressure signals mainly depend on the valve flow characteristic and rotational speed, while the pressure fluctuations downstream the turbine are driven by the natural oscillations of the gas in the outlet circuit. Amplitudes of mass flow rate traces (Figure 9) finally depend on the instantaneous pressure ratio and on the local slope of the turbine iso-speed line. They are correctly captured by the model, as well.

Table 1 – Operating conditions of the tested cases

Case Number	Pulse frequency, Hz	Turbine speed, rpm	Turbine reduced speed, rpm	Exp	1DTM	GTM	GTM+TD
1	40	113000	6000	✓	✓		
2	100	113000	6000	✓	✓		
3	40	75000	4000	✓	✓		
4	100	75000	4000	✓	✓	✓	✓
5	200	75000	4000		✓	✓	✓

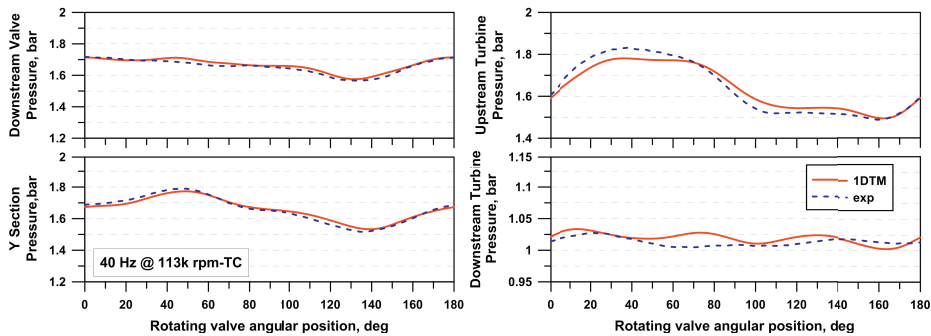


Figure 5. Experimental/numerical comparison of the pressure traces for a pulse frequency of 40 Hz and a rotational speed of 113000 rpm

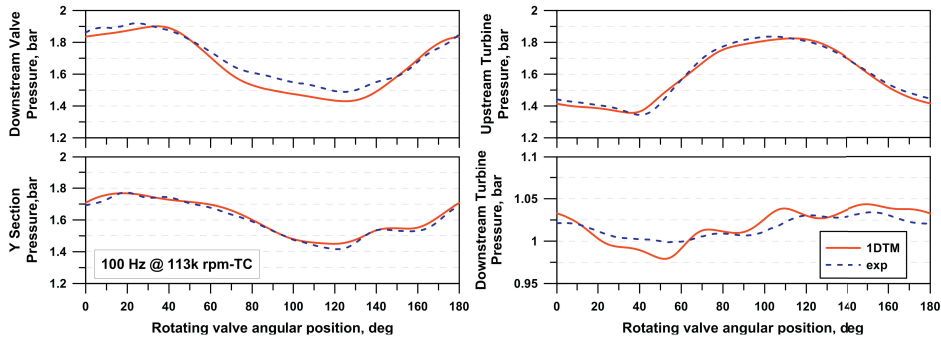


Figure 6. Experimental/numerical comparison of the pressure traces for a pulse frequency of 100 Hz and a rotational speed of 113000 rpm

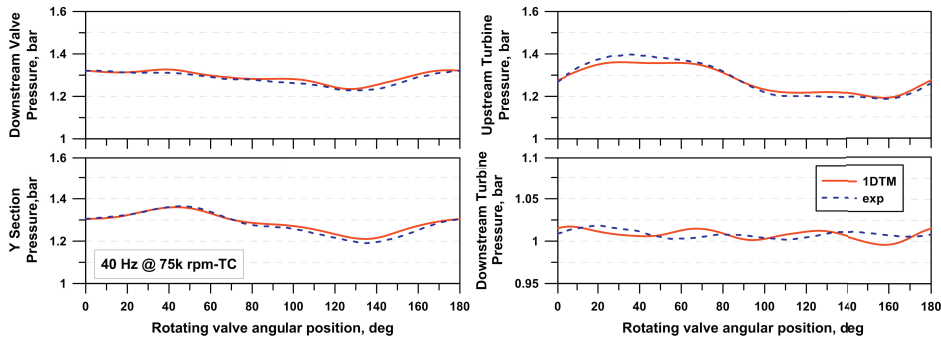


Figure 7. Experimental/numerical comparison of the pressure traces for a pulse frequency of 40 Hz and a rotational speed of 75000 rpm

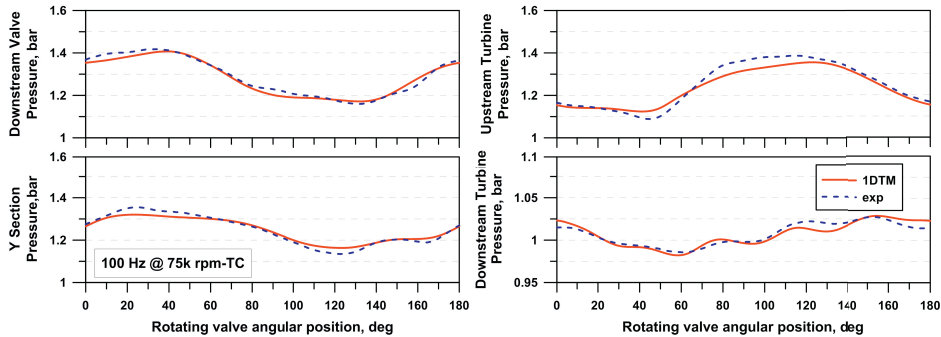


Figure 8. Experimental/numerical comparison of the pressure traces for a pulse frequency of 100 Hz and a rotational speed of 75000 rpm

The plots on the right side of Figure 9 illustrate both numerical and experimental instantaneous operating points on the reduced mass flow rate/pressure ratio plane. The typical loops around the steady state map are correctly predicted by the proposed methodology, especially at 40 Hz. The amplitude of the above loops, increasing as expected with pulse frequency, is related to the mass and energy storage within the device, despite an almost constant rotational speed. Although here not shown, the classical map based quasi-steady approach (GTP) would have involved a trajectory of the operating point almost superimposed to the steady state characteristic.

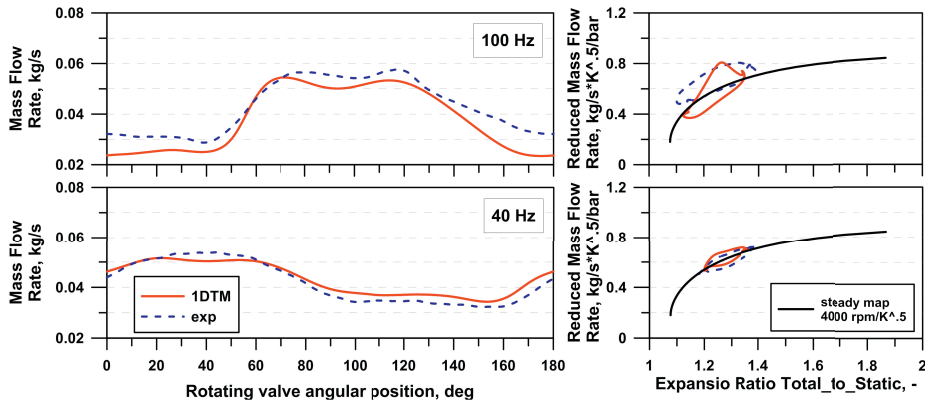


Figure 9. Experimental/numerical comparison of the mass flow rate for a rotational speed of 75000 rpm

Once demonstrated the reliability of the proposed 1DTM, its advantages with respect the above mentioned methodologies, GTP and GTP+TD, are investigated at a single turbine rotational, 75000 rpm, and pulse frequency, 100 Hz. Since the upstream pressure is mainly driven by the rotating valve characteristic, comparison among different approaches are reported in Figure 10a with reference only to the upstream mass flow rate and downstream turbine pressure. It can be observed that the 1DTM, thanks to a more refined definition of the various turbine components and volumes, is able to detect larger oscillations, similar to the experimentally experienced ones. As expected, the standard GTP method strongly underestimates the experimental oscillations. The introduction of the “time delay” duct (GTP+TD approach), significantly improves the results, even if the 1DTM remains the most accurate method. Similar consideration can be drawn looking at pressure traces downstream the turbine: the best pulses phasing with respect to the experimental data is once again attained by the 1DTM. This is mainly due to a proper computation of the flow path length in the turbine.

Although not experimentally tested, an operating condition at 200 Hz is numerically analyzed (Figure 10b), as well. As expected, differences on the downstream pressure are now amplified, since the angular duration of the pressure waves path along the turbine becomes longer and the effects of different equivalent length becomes more relevant. Discrepancies on mass flow rate traces remain similar to the ones detected at 100Hz, but are now located in a different valve angular position.

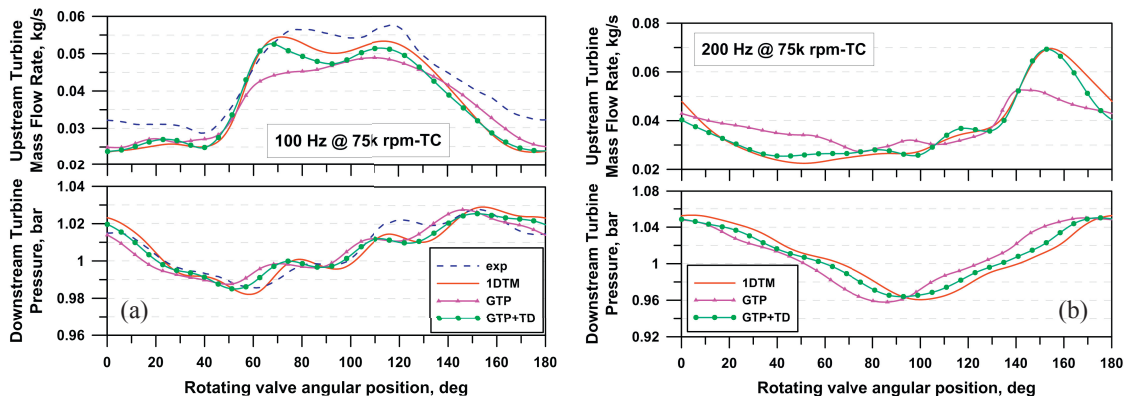


Figure 10. 1D turbine model, GT-Power and GTP with virtual piper comparison for a pulse frequency of 100 Hz (a) and 200 Hz (b) and a rotational speed of 75000 rpm

6. Conclusions

In the present paper, the unsteady behavior of a small engine turbocharger turbine, IHI RHF3, is experimentally analyzed; the employed test rig is provided by a rotating valve that induces unsteady operations. An innovative 1D unsteady turbine model is presented and implemented within the well known GT-Power software.

The model is based on a 1D schematization of the components constituting the device, i.e. inlet and outlet ducts, volute and wheel. The latter is treated as a quasi-steady component, described in terms of a steady map. After a tuning based on the measured map, the sole wheel map is estimated and employed in the unsteady turbine model.

Different approaches are then tested to predict the unsteady turbine behavior, characterized by the measurement of instantaneous pressure data at different stations along the test rig circuit, and the mass flow rate upstream the turbine, as well. With respect to the classical map based approach, a partial improvement is shown if a proper “time delay” pipe is introduced in the schematization. The detailed 1D turbine model based on the sole wheel map indeed produces additional advantages, and reveals the capability to better predict the phasing of the downstream turbine pressure and the amplitude of mass flow rate oscillations. These advantages mainly come out from a more correct computation of the mass and energy storage in the device, realizing a more refined description of pressure waves propagation.

References

- [1] Stokes J, Lake TH, Osborne RJ. A Gasoline Engine Concept for Improved Fuel Economy –The Lean Boost System. SAE 2000-01-2902.
- [2] Leduc P, Dubar B, Ranini A, Monnier G. Downsizing of Gasoline Engines: an Efficient Way to Reduce CO2 Emissions. *Oil and Gas Science and Technology - Rev. IFP*, Vol. 58 (2003), No. 1, pp. 115-127.
- [3] Bandel W, Fraidl GK, Kapus PE, Sikinger H. The Turbocharged GDI Engine: Boosted Synergies for High Fuel Economy Plus Ultra-low Emission. SAE paper 2006-01-1266.
- [4] Petitjean D, Bernardini L, Middlemass C, Shahed SM, Advanced Gasoline Engine Turbocharging Technology for Fuel Economy Improvements. SAE paper 2004-01-0988.
- [5] Martin G, Talon V, Higelin P, Charlet A, Caillol C. Implementing Turbomachinery Physics into Data Map-Based Turbocharger Models. SAE paper 2009-01-0310.
- [6] GT-Power User's Manual and Tutorial, GT Suite Version 7.3. Gamma Technologies Inc., Westmont, IL, USA, 2012
- [7] Boost User's Manual, AVL List GmbH, Graz, Austria, 2013
- [8] Guilain S. Modeling and Measurement of the Transient Response of a Turbocharged SI Engine. SAE paper 2005-01-0691.
- [9] Capobianco M, Marelli S. Turbocharger Turbine Performance under steady and unsteady flow: test bed analysis and correlation criteria. 8th International Conference on Turbochargers and Turbocharging, Inst.Mech.Engrs., London, 2006.
- [10] Bozza F, De Bellis V, Marelli S, Capobianco M. 1D Simulation and Experimental Analysis of a Turbocharger Compressor for Automotive Engines under Unsteady Flow Conditions. *SAE Int. J. Engines*, June 2011 vol. 4 no. 1 1365-1384, doi: 10.4271/2011-01-114.
- [11] Capobianco M, Gambarotta A. Variable geometry and waste-gated automotive turbochargers: measurements and comparison of turbine
- [12] Marelli S, Capobianco M. Steady and pulsating flow efficiency of a waste-gated turbocharger radial flow turbine for automotive application. *Energy*, 2011, vol. 36, n. 1, pag. 459-465. ISSN 0360-5442.
- [13] Bozza F, De Bellis V. Steady Modeling of a Turbocharger Turbine for Automotive Engines, *J Eng Gas Turb Power*, 2013, 136(1), 011701-011701-13, doi:10.1115/1.4025263.
- [14] De Bellis V, Bozza F, Schernus C, Uhlmann T. Advanced Numerical and Experimental Techniques for the Extension of a Turbine Mapping. *SAE Int. J. Engines*, 2013, 6(3):1771-1785, doi:10.4271/2013-24-0119.
- [15] Capobianco M, Marelli S. Experimental Investigation into the Pulsating Flow Performance of a Turbocharger Turbine in the Closed and Open Waste-Gate Region. 9th International Conference on Turbochargers and Turbocharging, London, 2010.
- [16] Capobianco M., Gambarotta A. Variable geometry and waste-gated automotive turbochargers: measurements and comparison of turbine performance. ASME Transactions, Journal of Engineering for Gas Turbines and Power, vol.114, 553-560, 1992.
- [17] Capobianco M, Gambarotta A. Performance of a twin-entry automotive turbocharger turbine. ASME Energy-Sources Technology Conference and Exhibition, paper 93-ICE-2, Houston, 1993.
- [18] Capobianco M, Marelli S. Waste-gate turbocharging control in automotive SI engines: effect on steady and unsteady turbine performance. 14th Asia Pacific Automotive Conference, Hollywood, SAE paper 2007-01-3543.
- [19] Marelli S, Capobianco M. Measurement of instantaneous fluid dynamic parameters in automotive turbocharging circuit, Proceedings of the 9th international conference on engines for automobile, SAE Technical paper 2009-24-0124, doi:10.4271/2009-24-0124, 2009.
- [20] Piscaglia F, Onorati A, Marelli S, Capobianco M. Unsteady Behavior in Turbocharger Turbines: Experimental Analysis and Numerical Simulation. SAE paper 2007-24-0081.
- [21] Aymanns R, Scharf J, Uhlmann T, Lückmann D. A Revision of Quasi Steady Modelling of Turbocharger Turbines in the Simulation of Pulse Charged Engines. ATK, 2011.
- [22] Galindo J, Fajardo P, Navarro R, García-Cuevas LM. Characterization of a radial turbocharger turbine in pulsating flow by means of CFD and its application to engine modeling. *Applied Energy* 103 (2013) 116–127.
- [23] Costall AW, McDavid RM, Martinez-Botas RF, Baines N. Pulse Performance Modeling of a Twin Entry Turbocharger Turbine Under Full and Unequal Admission. *Journal of Turbomachinery*, April 2011, Vol. 133 / 021005-9.
- [24] Bin Mamat AMI., Martinez-Botas RF. Mean Line Flow Model of Steady and Pulsating Flow of a Mixed-Flow Turbine Turbocharger. Proceedings of ASME Turbo Expo 2010: Power for Land, Sea and Air, Glasgow, Scotland, 2010, GT2010-22441.