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On the mechanical response of multilayered pentamode lattices equipped with hinged and rigid nodes

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Abstract

Purpose – This paper aims to review recent literature results on the mechanical response of confined pentamode structures behaving either in the stretching-dominated or the bending-dominated regimes.

Design/methodology/approach – The analyzed structures consist of multilayer systems formed by pentamode lattices alternated with stiffening plates and are equipped with rigid or hinged connections.

Findings – It is shown that such structures are able to carry unidirectional compressive loads with sufficiently high stiffness, while showing markedly low stiffness against shear loads. In particular, their shear stiffness may approach zero in the stretching-dominated regime.

Originality/value – The presented results highlight the high engineering potential of laminated pentamode metamaterials as novel isolation devices to be used for the protection of buildings against shear waves.

Keywords Bending-dominated regime, Layered systems, Pentamode lattices

Paper type Review paper

1. Introduction

The peculiar mechanical behavior of confined pentamode lattices, which allows such structures to carry unidirectional compressive loads with sufficiently high stiffness, while showing markedly low stiffness against shear loads, has been illustrated in a series of recent studies available in the literature (Amendola *et al.*, 2016a, 2016b, 2016c, 2017). While many cell unconfined pentamode lattices feature zero Young modulus in the stretch-dominated limit (Milton and Cherkaev, 1995; Norris, 2014), the research presented in (Amendola *et al.*, 2016a, 2016b, 2016c, 2017) has shown that single- and multi-layer structures formed by the alternating pentamode lattices and stiffening plates are able to oppose a noticeable degree of rigidity to

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unidirectional compression loads in the bending-dominated regime because of the confinement effect provided by the stiffening plates. Such a feature is essential when developing mechanical metamaterials that need to carry significantly large loads perpendicular to their outer surface while exhibiting low (theoretically zero) rigidity against transverse shear forces (Amendola *et al.*, 2016a, 2016b, 2016c).

The present work reviews the theoretical and numerical results presented in (Amendola *et al.*, 2016a, 2016b, 2016c; Fraternali *et al.*, 2015a, 2015b, 2015c) on the mechanical response of layered pentamode lattices equipped with rigid and hinged connections. We first present a collection of numerical results on the response of pentamode metamaterials in the bending-dominated regime induced by the presence of rigid connections between the bars and the plates forming the structure (Amendola *et al.*, 2016a, 2016b, 2016c). In correspondence with such a regime, we observe considerably high ratios between the effective compression and shear rigidities because of the presence of the stiffening plates and to the nonzero bending rigidity of nodes and rods (Amendola *et al.*, 2016a, 2016b, 2016c).

We continue by reviewing available analytic formulae for the vertical and bending stiffness properties of layered pentamode systems equipped with hinged connections (Fraternali and Amendola, 2017), by studying the variation of such quantities with the lattice constant, the solid volume fraction, the cross-section area of the rods and the layer thickness (Amendola *et al.*, 2016a, 2016b, 2016c).

The section of concluding remarks discusses potential engineering applications of confined pentamode lattices as new-generation anti-seismic devices. The examined applications are aimed to contribute to the diffusion of engineered pentamode lattices into the broad field of structural engineering, which at present makes limited use of mechanical metamaterials.

2. Layered pentamode lattices

Throughout the manuscript, we examine laminated structures built by stacking layers of pentamode lattices and stiffening plates in the vertical direction.

2.1 Face-centered-cubic lattices

We first consider lattices equipped with the typical extended face-centered-cubic (FCC) unit cell of pentamode lattices [Figure 1(a)]. The latter is made of four primitive unit cells, each one formed by four rods meeting at a point.

An example of the analyzed structures is illustrated in Figure 1. We examine FCC lattices equipped with rigid connections, by assuming that the rods of such systems exhibit the biconical shape shown in Figure 1 (Schittny *et al.*, 2013). In particular, we assume that all the members of the examined FCC structures exhibit uniform properties across the thickness.

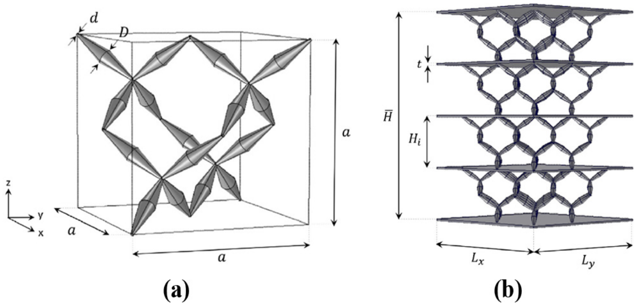

Figure 1.
(a) Extended FCC cell formed by rods with variable cross-section; (b) multilayer system obtained by alternating pentamode lattices and confinement plates

We refer the geometry the layered FCC structure to a (*x, y, z*) Cartesian frame, such that the *x*-, *y*-axes lie in the horizontal plane [Figure 1(b)].

We let D denote the diameter of the generic rod forming the system at the middle span and d denote the diameter of the rod at the extremities; *a* represents the characteristic length of the unit cell [Figure 1(a)]. We refer to the edge lengths of the stiffening plates with the symbols L_x, L_y and denote the number of the unit cells placed along the *x*-, *y*- and *z*-axes by n_x, n_y and n_z, respectively. Moreover, we denote the height of the generic pentamode layer by H_i, and we compute the total height of the pentamode layer as $H = n_z H_i$. The total height of the laminated structure is denoted by $\overline{H}$, which includes the summation of the thicknesses *t* of the stiffening plates. We assume that the rods of the pentamode lattices are formed by a homogeneous and linearly elastic material with Young modulus E_0.

2.2 SFCC lattices

A second class of confined pentamode systems examined in the present work considers lattices formed by the repetition in the 3D space of a suitable sub-lattice of the FCC unit cell, which consists of one half of the FCC cell [SFCC unit cell, cf. Figure 2(a)]. Layered structures based on SFCC cells are illustrated in Figure 2(b). These structures are supposed to be endowed with perfectly hinged connections, so as to respond in the pure stretching regime (Amendola *et al.*, 2016a, 2016b, 2016c; Fraternali *et al.*, 2015a, 2015b, 2015c). Hinged connections may consist, e.g. of the hollow ball joints commonly used in structural space grids (Chilton, 2000). Also, in the case of SFCC systems, as well as in the case of FCC structures, we assume uniform properties of the pentamode lattices and the stiffening plates through the thickness.

The rods forming the SFCC structures are assumed to be cylinders with cross-section area *s*, which are made of a homogeneous and linearly elastic material with Young modulus E_0. The stiffening plates forming SFCC systems are hereafter modeled as 2D rigid bodies.

3. Stiffness properties of multi-layer pentamode lattices

3.1 Bending-dominated regime

We first analyze the elastic response of the laminated FCC systems described in Section 2.1. We focus our study on pentamode lattices formed by a single unit cell across the vertical direction ($n_z = 1$) and 2×2 unit cells in the horizontal plane ($n_x = n_y = 2$). Amendola *et al.* (2016a, 2016b, 2016c) present parametric study on the variation of the elastic moduli of FCC systems with suitable design parameters, which consist of the *d/a* ratio (microscopic aspect ratio) and the *H/a* ratio (macroscopic aspect ratio). The quantity *H/a* gives the number of layers forming the laminated structure because of the assumption $n_z = 1$ in each layer.

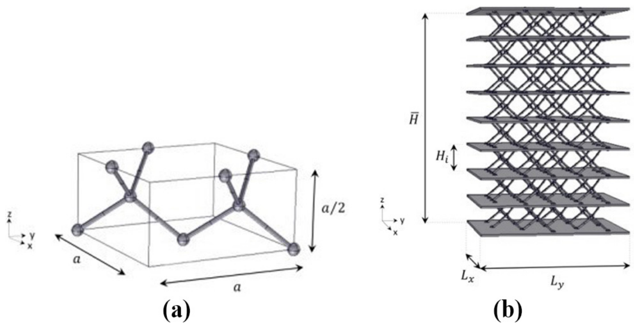

Figure 2.
(a) Sub-lattice of the FCC unit cell formed by two primitive unit cells (SFCC cell); (b) multilayered structure obtained by alternating SFCC lattices and stiffening plates

Let us now examine the effective shear modulus G_c and the effective compression modulus E_c of a laminated pentamode structure, and let us compare such quantities to the Young modulus E_r and the shear modulus G_r of a rubber material typically used for the manufacturing of rubber bearings (Amendola *et al.*, 2016a, 2016b, 2016c).

Figure 4 of Amendola *et al.* (2016a) shows the distributions of the E_c/E_r and G_c/G_r ratios with H/a and d/a, which have been numerically obtained in such a study by making use of finite element simulations and progressively increasing the number of the layers forming the structure.

The results in Figure 4 of Amendola *et al.* (2016a) highlight that the E_c/E_r and G_c/G_rratios significantly increase with decreasing values of the $H = a$ aspect ratio (that is, in "thick" systems), especially in presence of large $d = a$ ratios (large size nodal junctions). For $H/a = 1$, we observe that it results $E_c = 0.071E_r$ and $G_c \approx G_r/1{,}000$ for $d/a = 0.002$; $E_c = 0.92E_r$ $G_c = 0.67G_r$ for $d/a = 0.015$; $E_c = 70.17E_r$ and $G_c = 85.26G_r$ for $d/a = 0.009$. As the elastic moduli of many cells, unconfined pentamode lattices are independent of the H/a ratio and are such that the Young modulus is approximately equal to the shear modulus (Amendola *et al.*, 2016a, 2016b, 2016c), we deduce that the above "stiffening" effects of $E_c$and G_c are because of the confinement effect played by the terminal plates against the deformation of the pentamode lattice. For what specifically concerns the compression modulus E_c, we note that such a property is almost always larger than E_r, with exception to cases with $d/a \leq 0.015$. When it results $H/a \geq 3$, E_c asymptotically tends to a constant value, for $d/a < 0.07$ (small size nodal junctions) or a local minimum at $H/a = 5$ ($E_c \approx 8 \div 16E_r$), for $d/a \geq 0.07$. The effective shear modulus G_c always monotonically decreases with increasing values of $H = a$. When $H/a = 4$, it results $G_c \approx 2/10000G_r$ for $d/a = 0.002$, and $G_c = 7.15G_r$ for $d/a = 0.09$.

Figure 5 of Amendola *et al.* (2016a) analyzes the distribution of the E_c/G_c ratio with H/a and d/a. The results illustrated in this figure show that E_c/G_c ratio significantly grows with the number of layers (H/a) for any analyzed value of d/a.

3.2 Stretching-dominated regime

We now pass to study the effective elastic properties of the multilayer SFCC systems defined in Section 2.2, by reviewing the analytic results presented in (Fraternali and Amendola, 2017). Such a reference obtains the following analytic formulae for the effective compression and bending stiffness properties of SFCC systems, on assuming that the layers forming the laminated structure are connected in series:

$$K_v = \frac{1}{\sum_{i=1}^{n_z} \frac{1}{K_{v_i}}}, K_\varphi = \frac{1}{\sum_{i=1}^{n_z} \frac{1}{K_{\varphi_i}}}, \tag{1}$$

Here, K_{v_i} and K_{φ_i} denote the vertical stiffness and the bending stiffness (about either the x-or the y-axis) of the i-th layer. On assuming that K_{v_i} and K_{φ_i} are constant from layer to layer, we obtain the effective compression modulus of the generic layer (E_{c_i}) as follows:

$$E_c = \frac{K_v H}{A} = \frac{H}{A} \frac{1}{\frac{n_z}{K_{v_i}}} = \frac{K_{v_i} H_i}{A} = E_{c_i} = \frac{4E_o s}{3\sqrt{3}a^2} \tag{2}$$

where:

$$A = n_x n_y a^2 \tag{3}$$

denotes the area of the stiffening plates covered by the pentamode lattices ("load area"). Equation (2) shows that the compression modulus of a laminated SFCC system is equal to that of each individual layer, under the above assumptions. It is worth noting that SFCC systems theoretically exhibit $G_c = 0$ (Fraternali and Amendola, 2017).

We now focus our attention on a square multilayer system ($n_x = n_y = n_a$) such that it results:

$$a = \frac{L}{n_a} = \frac{2H}{n_z} \tag{4}$$

where $L = L_x = L_y$ denotes the edge-length of the load area. By using equation (4) into equation (2), the following expressions of the vertical stiffness K_v and the effective compression modulus E_c of the multilayer system under consideration are obtained (Fraternali and Amendola, 2017):

$$K_v = \frac{2}{3\sqrt{3}} \frac{E_0 sL}{H^2} n_a n_z \tag{5}$$

$$E_c = \frac{2}{3\sqrt{3}} \frac{E_0 s}{LH} n_a n_z \tag{6}$$

When fixing values of L, H, E_0 and s, equation (6) shows that the effective compression modulus of such a system scale linearly with the number of unit cells in the horizontal plane and the number of layers (cf. Figure 2). We can easily notice that K_v and E_c get four times larger when doubling the number of cells in the horizontal plane and the number of layers. It is worth noting that when doubling n_z and keeping H fixed, equation (4) implies that one needs to halve the lattice constant a. The same equation also implies that, in the same conditions, one simultaneously needs to double n_a, to keep also L constant.

4. Concluding remarks

We have reviewed recent studies on the elastic response of multilayered structures obtained by alternating pentamode lattices and stiffening plates. In particular, we have evaluated the effective stiffness properties of confined pentamode lattices in the stretching and bending-dominated regimes. We studied the variation of these values with the number of the unit cells in the horizontal and vertical direction. The analyzed results highlight that layered pentamode lattices exhibit high elastic stiffness against compression loads and, contemporarily, very low or nearly zero rigidity against shear and twisting loads. In particular, it has been shown that the ratio between the compression modulus and the shear modulus of FCC systems increases with the number of layers stacked in the vertical direction. These results allow us to conclude that such laminated metamaterials may be considered as innovative anti-seismic devices (Fabbrocino *et al.*, 2016; Fraternali *et al.*, 2015a, 2015b, 2015c), offering several advantages over other the available structural bearings (Kelly, 1993; Benzoni and Casarotti, 2009). Consider, indeed, that the European Standard EN 15129, as well as other international seismic engineering standards, defines a seismic isolator as a "device possessing the characteristics needed for seismic isolation, namely, the ability to support a gravity load of superstructure, and the ability to accommodate lateral displacements".

One of the key advantages derives from the possibility to design such systems as tension-capable and performance-based systems, whose mechanical properties are driven largely by the geometry of the lattice microstructure (Titirla *et al.*, 2017; Blesgen *et al.*, 2012), rather than the chemical composition of the material. Moreover, the choice of the material offers additional design opportunities, both in terms of the elastic response and the energy dissipation properties of the system. Future directions of the present work will be aimed at exploring a variety of alternative design solutions for pentamode bearings, on considering different solutions in terms of the unit cell geometry (anisotropic design) and the component materials (Fraternali and Feo, 2000; Barretta *et al.*, 2017; Ascione *et al.*, 1992; Daraio *et al.*, 2010; El Sayed *et al.*, 2009; Fabbrocino *et al.*, 2015; Farina *et al.*, 2016; Fraternali and Bilotti, 1997; Fraternali *et al.*, 2012; Fraternali *et al.*, 2011; Fraternali and Reddy, 1993; Schmidt and Fraternali, 2012).

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