

# First-order correction to counter the effect of eccentricity on the hole-drilling integral method with strain-gage rosettes

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## Abstract

The offset between the hole and the centre of the strain-gage rosette is unavoidable, although usually small, in the hole-drilling technique for residual stress evaluation. In this article, we revised the integral method described in the ASTM E837 standard and we recalculated the calibration coefficients. The integral method was then extended by taking into account the two eccentricity components, and a more general procedure was proposed including the first-order correction. A numerical validation analysis was used to consolidate the procedure and evaluate the residual error after implementing the correction. The values of this error resulted limited to a few percentage points, even for eccentricities larger than the usual experimental values. The narrow eccentricity limit claimed by the standard, to keep the maximum error lower than 10%, can now be considered extended by approximately a factor of 10, after implementing the proposed correcting procedure, proving that the effect of the eccentricity is mainly linear within a relatively large range.

## Keywords

Residual stresses, hole-drilling method, integral method, eccentricity correction, finite element analysis

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## Introduction

Residual stresses can significantly influence the strength of mechanical components, especially under cyclic fatigue loading.<sup>1</sup> There are several experimental techniques for measuring residual stresses.<sup>2</sup> These include mechanical methods (destructive or semi-destructive) based on cuttings or local material removals to relieve the embedded residual stresses, such as the contour method,<sup>3</sup> the ring core method<sup>4–6</sup> and other specific approaches adapted to the geometry of the components investigated,<sup>7–11</sup> or also combining the hole-drilling method with the indentation.<sup>12</sup> After the material has been removed, a back calculation is required to obtain the residual stress distribution that has been relaxed.<sup>13,14</sup> The hole-drilling method is the most popular and widely investigated.<sup>15,16</sup> Its hardware is easily implemented, it is relatively inexpensive and it directly provides the stresses at a point; specifically, it averages the stress on the small volume of the removed material. Hole-drilling can be performed by measuring the entire field of the relaxed deformation with optical methods<sup>17</sup> both on the isotropic elastic materials, namely, steel or

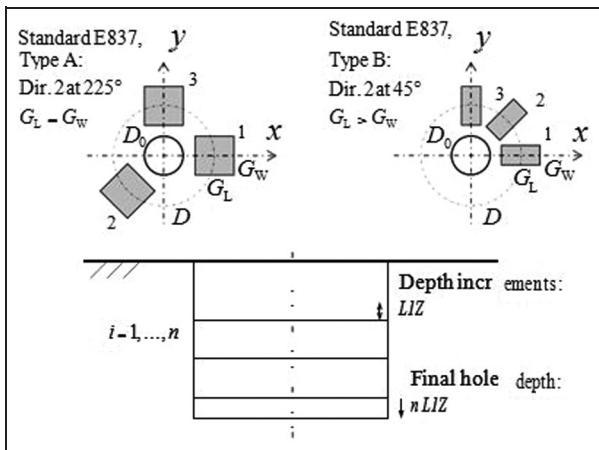
other metallic alloys, and on the orthotropic materials such as composite plates.<sup>18</sup> However, the hole-drilling technique is usually performed just with a strain-gage rosette to measure the relaxed strains in the near area of the drilled hole. There are many application examples of this technique such as large components showing a flat (or almost flat) surface,<sup>19</sup> deep rolled or shot peened flat specimens,<sup>20,21</sup> and recently even on coated surfaces.<sup>22–25</sup> The usual rosette for hole-drilling has three grids angularly placed at 0°, 225°, 90°. Alternatively, the 0°, 45°, 90° grid pattern can be used without making any difference, for the angular periodicity of the stresses, unless a hole eccentricity is introduced. Figure 1 shows the hole-drilling set-up with type A or type B three grid

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**Figure 1.** Hole-drilling method with type A and type B rosettes according to the CCW numbering system.

rosettes according to the American Society for Testing and Materials (ASTM) standard. Besides the direction 2 at  $45^\circ$ , rather than at  $225^\circ$ , type B has a smaller grid width than grid length to prevent any geometry interference.

After having measured and recorded the relaxed strains, the *integral method* is usually considered for the calculation of the residual stress distributions. The integral method was initially proposed by Schajer<sup>26,27</sup> and recently has been widely used and also conformed to specific cases, such as medium thick plates<sup>28</sup> and thin plates,<sup>29</sup> as well as being extended to optical methods.<sup>30</sup> The ASTM E837-13a standard<sup>31</sup> codifies the hole-drilling integral method, with a strain-gage rosette. The calculation proposed takes into account that a variable distribution can be obtained, where the stresses are assumed to be piecewise constant over each hole depth increment. Then, the standard reports the calibration coefficients (dimensionless and in matrix format) which depend only on the geometry, thus enabling the residual stresses from the measured relaxed strains to be calculated.

Error analysis is crucial for any residual stress experimental technique; indeed, we recently proposed a bending test rig for providing a validation concurrently with the measure itself.<sup>32,33</sup> Many investigations can be cited from the literature considering different sources of errors for the hole-drilling technique. The strain measure uncertainty, which is the most important,<sup>34</sup> can be reduced to some extent by following the regularization procedure proposed by Schajer<sup>35</sup> and then implemented in the ASTM E837 starting from the 2008 issue. Another source of error affecting the hole-drilling measures is the plasticity induced by the drilling, for which we proposed a correction in a previous study.<sup>36,37</sup> The shape, radius and the position of the hole are also the reasons for experimental uncertainty. The flatness of the bottom surface was questioned by Scafidi et al.<sup>38</sup> and then by Nau and Scholtes,<sup>39</sup> while the problem of eccentricity was initially investigated

analytically by Ajovalasit<sup>40</sup> for a thin workpiece where a through-hole (plane stress) assumption can be used. More recently, Beghini et al.<sup>41,42</sup> introduced the influence function approach, for a blind hole in a thick workpiece. The strain field was computed starting from a database of numerical solutions, implementing a specific geometric configuration in which the components of eccentricity are merely introduced as the geometry parameters rather than being considered as a source of error. The integral method according to the ASTM E837 standard enables the residual stress calculation to be easily implemented, but the presence of hole eccentricity is a limitation. The standard requires a nearly perfect concentricity between the hole and the rosette. The prescribed maximum allowable eccentricity value is  $0.004D$  so that the maximum eccentricity is 0.02 mm for the usual rosettes with  $D = 5.13$  mm. The current positioning accuracy is usually in the order of a few hundredths of millimetres; thus, the eccentricity values are sometimes higher than this standard limitation. The numerical simulations, as reported below, showed that an eccentricity in the order of these values can introduce some percentage points of error on the residual stress components. In order to improve the integral method, this article proposes a correction procedure based on the first-order linearization of the calibration matrices, which considerably extends the tolerable eccentricity.

## Integral method

Within the concentricity assumption, the axisymmetric geometry allows a decoupling between the components of stress with respect to the corresponding relaxed strains. The residual stresses are related to the relaxed strains according to the general relationship introduced by Schajer<sup>26,27</sup>

$$\varepsilon_r(\vartheta) = A(\sigma_{\max} + \sigma_{\min}) + B(\sigma_{\max} - \sigma_{\min}) \cos(2\vartheta) \quad (1)$$

where  $\sigma_{\max}$  and  $\sigma_{\min}$  are the principal residual stresses,  $\vartheta$  is the angle of the generic rosette grid with respect to the direction of  $\sigma_{\max}$  and  $A$  and  $B$  are the two elastic constants. After introducing a three-grid rosette with a generic orientation with respect to the principal directions, the following matrix equation holds

$$\begin{bmatrix} (A+B) & (A-B) & 0 \\ A & A & 2B \\ (A-B) & (A+B) & 0 \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix} \quad (2)$$

in which 1, 2 and 3 are the three grid directions:  $0^\circ, 225^\circ, 90^\circ$  or  $0^\circ, 45^\circ, 90^\circ$ . Given that directions 1 and 3 are orthogonal and numbered as counter-clockwise (CCW), a reference system for the problem is inherently introduced, and the axes  $x$  and  $y$  for the residual stress components can be oriented according to the directions 1 and 3, respectively (Figure 1). Alternatively, a clockwise (CW) orientation could be introduced (in agreement with the ASTM E837 standard); however, the

orientation of grid 3 would be opposite to the direction of the  $y$ -axis. The form of equation (2) suggests combining the variables according to an equibiaxial stress, and the related strain, plus two components of pure shear stress and the relative strains

$$\begin{aligned} P &= \frac{(\sigma_y + \sigma_x)}{2}, Q = \frac{(\sigma_y - \sigma_x)}{2}, T = \tau_{xy} \\ p &= \frac{(\varepsilon_3 + \varepsilon_1)}{2}, q = \frac{(\varepsilon_3 - \varepsilon_1)}{2}, \\ t &= \frac{2\varepsilon_2 - (\varepsilon_3 + \varepsilon_1)}{2} = \varepsilon_2 - p \end{aligned} \quad (3)$$

In the second of equation (3), the component of strain  $t$  has been intentionally defined with the opposite sign with respect to the ASTM standard in order to have sign consistency between the shear stress  $\tau_{xy}$  and the relaxed strain  $\varepsilon_2$ . For example, when the equibiaxial stress component  $P$  is positive, then the strain  $p$  is negative; indeed, the coefficients  $A$  and  $B$  are both negative since the measured strains are the *relaxed* deformations. Similarly, when a positive shear stress  $\tau_{xy}$  is introduced, the principal stress along the  $45^\circ$  angle is tensile; therefore,  $\varepsilon_2$  turns out to be negative and thus  $t$  also has to be negative. Having the grid direction 2 at  $225^\circ$  makes no difference and the last of equation (3) is still valid. The opposite definition for the  $t$  strain component would be consistent for a CCW rosette with the grid 2 direction at  $135^\circ$ , or alternatively at  $-45^\circ$ . However, this is never the case for the type of rosettes usually available, although the possible use of this scheme is mentioned by Nau and Scholtes.<sup>39</sup> On the other hand, introducing the CW grid numbering system as shown in the ASTM standard, the second grid is either in the second quadrant (type A) or in the fourth quadrant (type B), and consequently, the opposite definition for  $t$  is correct, as reported in the standard. The numbering system CCW was considered preferential for this study since it simplifies the sign definition of the eccentricity components with both the two orthogonal grids 1 and 3 oriented along the positive sign of the  $x$ - and  $y$ -axes. After substituting the definitions of equation (3) into equation (2), the three variables are decoupled; thus, the matrix relating stresses and strains are diagonal containing the two coefficients  $A$  and  $B$

$$\begin{bmatrix} 2A & 0 & 0 \\ 0 & 2B & 0 \\ 0 & 0 & 2B \end{bmatrix} \begin{bmatrix} P \\ Q \\ T \end{bmatrix} = \begin{bmatrix} p \\ q \\ t \end{bmatrix} \quad (4)$$

This linear dependence between the residual stresses and the relaxed strains is a consequence of the elasticity behaviour of the material. Therefore, equation (4) can be rewritten with the explicit dependence of Young's modulus  $E$  and also the negative sign of the two coefficients can be emphasized

$$-\frac{1}{E} \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \beta \end{bmatrix} \begin{bmatrix} P \\ Q \\ T \end{bmatrix} = \begin{bmatrix} p \\ q \\ t \end{bmatrix} \quad (5)$$

where  $\alpha$  and  $\beta$  depend on the Poisson's ratio  $\nu$  and on the ratios between the hole and the rosette dimensions, but they do not depend on Young's modulus.

The stress-strain relation of equation (5) only considers a single value of the stress state, which is assumed to be uniform along the depth. In the specific case of small thickness, plane stress, this relationship can be further developed by making explicit the dependence on the Poisson's ratio  $\nu$  which only applies on the equibiaxial stress component, thus obtaining equation (6)

$$-\frac{1}{E} \begin{bmatrix} a(1+\nu) & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & b \end{bmatrix} \begin{bmatrix} P \\ Q \\ T \end{bmatrix} = \begin{bmatrix} p \\ q \\ t \end{bmatrix} \quad (6)$$

where, according to equation (5),  $\alpha = a(1+\nu)$ ,  $\beta = b$ . Now the coefficients  $a$  and  $b$  are scalar, positive, dimensionless and only depend on the geometry ratios, while they do not depend on the size. When the method is applied to a surface of a bulk component (blind hole), and the state of stress can be assumed to be uniform up to the final depth of the hole, the dependence on the Poisson's ratio according to equation (6) only has an approximate validity. However, this effect can be neglected, since it produces a perturbation of the values in the order of a few percentage points, as discussed below. Thus, the dependency on  $\nu$  can also be kept for the blind hole problem, as proposed in equation (6) which tolerates some inaccuracy; otherwise, a recalculation of the coefficients is required for any value of  $\nu$ .

A major development of the method is the introduction of a possible non-uniform distribution, in which the components of stress are introduced as uniform stepwise. There are thus three independent components for each increment of the hole. In this case, the scalars  $p, q, t$  and  $P, Q, T$  are replaced with vectors and the coefficients  $a, b$  are replaced with the calibration coefficient matrices

$$-\frac{1+\nu}{E} \bar{\mathbf{a}} \mathbf{P} = \mathbf{p}, \quad -\frac{1}{E} \bar{\mathbf{b}} \mathbf{Q} = \mathbf{q}, \quad -\frac{1}{E} \bar{\mathbf{b}} \mathbf{T} = \mathbf{t} \quad (7)$$

Vectors  $\mathbf{P} = (P^{(1)}, P^{(2)}, \dots, P^{(n)})^T$ ,  $\mathbf{Q}, \mathbf{T}$  and similarly  $\mathbf{p} = (p^{(1)}, p^{(2)}, \dots, p^{(n)})^T$ ,  $\mathbf{q}, \mathbf{t}$  represent the stresses and strains following the logic of decoupling, introduced previously, and  $i = 1, 2, \dots, n$  are the depth positions along the increments of the hole. The matrices  $\bar{\mathbf{a}}$  and  $\bar{\mathbf{b}}$  are lower triangular since the residual stress of the material removed in the previous steps has a (linear) cumulative effect on the relaxed strains, while the residual stress of the material below, which still has to be removed, has no effect on the relaxed strains. Therefore, the coefficients  $a_{ij}$  and  $b_{ij}$  of the matrices  $\bar{\mathbf{a}}$  and  $\bar{\mathbf{b}}$  are all positive for  $i \geq j$ , while they are 0 for  $i < j$ . For example, just with three drilling steps, the coefficient matrices are

$$\bar{\mathbf{a}} = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \quad \bar{\mathbf{b}} = \begin{bmatrix} b_{11} & 0 & 0 \\ b_{21} & b_{22} & 0 \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \quad (8)$$

The integral method with variable stress distribution, which is summarized in equation (7), just requires the axial symmetry of the removed material shape and the orientation of the measurement grids according to the 0°, 225°(45°), 90° scheme. Thus, the method can be extended to different problems that share these geometric characteristics, such as the ring core method where material removal is annular and the grids are superimposed on the central volume according to the same angular pattern.

**Extension of the integral method to the eccentricity**

When an eccentricity between the drilled hole and the rosette is introduced, even if small, equation (1) is no longer valid. Therefore, it is not possible to decouple the  $p, q, t$  components and a new formulation is required. By following a more general approach, a linear relationship between all the components of stress and strain can still be proposed, equation (9)

$$-\frac{1}{E}\bar{\mathbf{A}}\mathbf{S} = \mathbf{e} \tag{9}$$

where  $\mathbf{S} = (\sigma_x^{(1)}, \sigma_y^{(1)}, \tau_{xy}^{(1)}, \dots, \sigma_x^{(n)}, \sigma_y^{(n)}, \tau_{xy}^{(n)})^T$  is the vector of all the components of residual stresses, collected in blocks of three terms, for different values of depth, and similarly  $\mathbf{e} = (\varepsilon_1^{(1)}, \varepsilon_2^{(1)}, \varepsilon_3^{(1)}, \dots, \varepsilon_1^{(n)}, \varepsilon_2^{(n)}, \varepsilon_3^{(n)})^T$  is the vector of the relaxed strains. Since the three components of stresses and strains are arranged in blocks,  $\bar{\mathbf{A}}$  is a lower triangular 3×3 block matrix. An example with three depth increments is reported below

$$\bar{\mathbf{A}} = \begin{bmatrix} A_{11}^{(11)} & A_{12}^{(11)} & A_{13}^{(11)} & 0 & 0 & 0 & 0 & 0 & 0 \\ A_{21}^{(11)} & A_{22}^{(11)} & A_{23}^{(11)} & 0 & 0 & 0 & 0 & 0 & 0 \\ A_{31}^{(11)} & A_{32}^{(11)} & A_{33}^{(11)} & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline A_{11}^{(21)} & A_{12}^{(21)} & A_{13}^{(21)} & A^{(22)} & A^{(22)} & A^{(22)} & 0 & 0 & 0 \\ A_{21}^{(21)} & A_{22}^{(21)} & A_{23}^{(21)} & A^{(22)} & A^{(22)} & A^{(22)} & 0 & 0 & 0 \\ A_{31}^{(21)} & A_{32}^{(21)} & A_{33}^{(21)} & A^{(22)} & A^{(22)} & A^{(22)} & 0 & 0 & 0 \\ \hline A_{11}^{(31)} & A_{12}^{(31)} & A_{13}^{(31)} & A^{(32)} & A^{(32)} & A^{(32)} & A^{(33)} & A^{(33)} & A^{(33)} \\ A_{21}^{(31)} & A_{22}^{(31)} & A_{23}^{(31)} & A^{(32)} & A^{(32)} & A^{(32)} & A^{(33)} & A^{(33)} & A^{(33)} \\ A_{31}^{(31)} & A_{32}^{(31)} & A_{33}^{(31)} & A^{(32)} & A^{(32)} & A^{(32)} & A^{(33)} & A^{(33)} & A^{(33)} \end{bmatrix} \tag{10}$$

The coefficients  $A_{hk}^{(ij)}$  have the indexes:  $h, k = 1, 2, 3$  and  $i = 1, \dots, n, j = 1, \dots, i$  where, similarly to above,  $n$  is the total number of drilling steps. Again these coefficients do not depend on Young’s modulus, but do dependent on the Poisson’s ratio, and also on the geometry ratios. More specifically, each of these coefficients also depends on the eccentricity. Therefore, the power series expansion can be applied in terms of the two eccentricity components along directions 1 and 3

$$A_{hk}^{(ij)} = A_{0,hk}^{(ij)} + \frac{\partial}{\partial e_1} A_{hk}^{(ij)} e_1 + \frac{\partial}{\partial e_3} A_{hk}^{(ij)} e_3 + \frac{1}{2} \frac{\partial^2}{\partial e_1^2} A_{hk}^{(ij)} e_1^2 + \frac{1}{2} \frac{\partial^2}{\partial e_3^2} A_{hk}^{(ij)} e_3^2 + \frac{\partial^2}{\partial e_1 \partial e_3} A_{hk}^{(ij)} e_1 e_3 + \dots \tag{11}$$

In equation (11), the two eccentricity components  $e_1$  and  $e_3$  are introduced. Their definition intended here is the offset of the hole with respect to the rosette, which is positive according to the  $x, y$ -axes. Provided that  $x$  is rightwards and  $y$  upwards (Figure 1), when the hole centre is right-shifted, with respect to the grid centre,  $e_1$  is positive; similarly,  $e_3$  is positive when the hole centre is up-shifted. The opposite definition of the eccentricity components, or the use of the CW numbering system, would be equally possible but involving a revision of the derivative coefficient signs. As expected, and confirmed by the numerical analysis reported below, the higher order contributions are negligible, especially for relatively small eccentricity values; thus, it suffices to focus only on the first two linear terms. Furthermore, as matrix  $\bar{\mathbf{A}}$  is dimensionless, the derivative terms can also be put in a dimensionless form just by multiplying by a characteristic length of the problem, for example, the average diameter  $D$  of the strain-gage rosette, equation (12)

$$A_{hk}^{(ij)} = A_{0,hk}^{(ij)} + \alpha_{1,hk}^{(ij)} \eta_1 + \alpha_{3,hk}^{(ij)} \eta_3 \tag{12}$$

where the dimensionless derivative coefficients and the eccentricity components are

$$\alpha_{1,hk}^{(ij)} = D \frac{\partial}{\partial e_1} A_{hk}^{(ij)}, \eta_1 = \frac{e_1}{D} \tag{13}$$

$$\alpha_{3,hk}^{(ij)} = D \frac{\partial}{\partial e_3} A_{hk}^{(ij)}, \eta_3 = \frac{e_3}{D}$$

The coefficients  $A_{0,hk}^{(ij)}, \alpha_{1,hk}^{(ij)}, \alpha_{3,hk}^{(ij)}$  can be collected into block triangular matrices

$$\bar{\mathbf{A}}_0 = [A_{0,hk}^{(ij)}], \bar{\alpha}_{e1} = [\alpha_{1,hk}^{(ij)}], \bar{\alpha}_{e3} = [\alpha_{3,hk}^{(ij)}] \tag{14}$$

and finally, matrix  $\bar{\mathbf{A}}$  can be reconstructed, and approximated to the first order, as

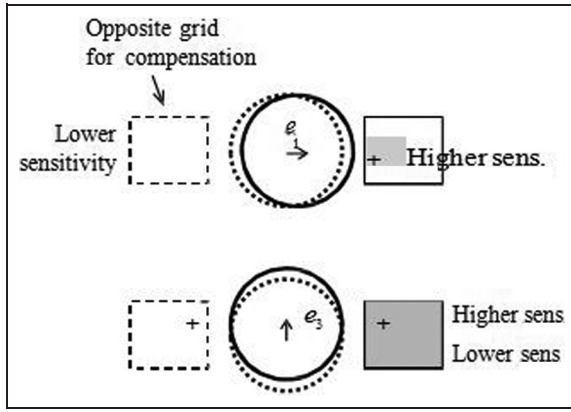
$$\bar{\mathbf{A}} = \bar{\mathbf{A}}_0 + \bar{\alpha}_{e1} \eta_1 + \bar{\alpha}_{e3} \eta_3 \tag{15}$$

Clearly, the availability of matrix  $\bar{\mathbf{A}}$  allows to solve the *inverse* problem, that is, to determine the profile of the residual stresses from the measured relaxed strains, just by calculating the inverse matrix. Given the rather low size of the matrix, the inversion of  $\bar{\mathbf{A}}$  does not involve numerical difficulties and the stress components  $\mathbf{S}$  can be easily obtained from the measured strains  $\mathbf{e}$

$$\mathbf{S} = -E\bar{\mathbf{A}}^{-1}\mathbf{e} \tag{16}$$

Being the stress component decoupling not used, a three-grid rosette with 0°, 225°(45°), 90° pattern is no longer strictly required. A different angular pattern could be proposed or even a higher number of grids could be considered to increase the strain-gage sensitivity. For example, with four grids instead of just three, the matrix would be 4×3 block and the inverse problem solved with the pseudoinverse matrix rather than the square inverse. An example of a rosette with four grids is type D reported by Schajer,<sup>15</sup> which has a





**Figure 2.** Grid sensitivity to longitudinal and transversal eccentricity components.

specific applicability when a correction of the plasticity effect is required.<sup>36</sup>

**Symmetry properties**

As shown in Figure 2, the hole eccentricity affects the sensitivity of the grids. When the eccentricity is along the direction of the grid, the sensitivity is higher if the hole is closer to the grid; thus, the absolute value of the measured relaxed strain is larger. On the other hand, if the eccentricity displacement is transversal, a portion of the grid has a higher sensitivity, while the other side has a lower sensitivity and this implies, for symmetry, that the first-order derivative is 0. In fact, some of the coefficients of the matrices  $\bar{\alpha}_{e1}, \bar{\alpha}_{e3}$  turn out to be null. If an opposite grid is placed at the other side of the hole and connected in series (Figure 2), another feature of symmetry is introduced. Consequently, even the eccentricity first-order sensitivity along the grid direction is reduced to 0, and the derivative matrices are both entirely null. This situation is clearly of interest. In fact, the first order is systematically cancelled, and procedure  $p, q, t$  can be followed without any notable effect caused by the eccentricity. A commercially available rosette of six grids (two grids for each of the three directions) was tested by Beghini et al.<sup>43</sup> and the reduced sensitivity to the eccentricity was experimentally verified. However, introducing another grid for each measurement direction makes the rosette more complicated to manage, and also more expensive. Thus, this kind of rosette is not commonly used despite its inherent advantage. Nau and Scholtes<sup>39</sup> reported a rosette with up to eight grids and discussed how to connect them in different configurations and also highlighted the reduced sensitivity on the hole eccentricity. The rosette type C reported in the ASTM standard also has an opposite grid for each of the three directions; however, one side is radial and the other side is along the hoop direction. For this reason, the interaction is opposed to the compensation since the strain

measured on one side has the opposite sign with respect to the other side; indeed, each couple of strain-gages has to be connected as half-bridge. On the contrary, the ring core method is another interesting example of compensating eccentricity. The three grids are attached to the inside volume; thus, the eccentricity produces a smaller distance from the annular cut in one region and a higher distance in the opposite region of the grid. Therefore, the sensitivity of each grid is compensated for and the first-order matrices  $\bar{\alpha}_{e1}, \bar{\alpha}_{e3}$  are 0. This means that the procedure  $p, q, t$  with decoupled components can be followed regardless of even relatively large eccentricities.

For common hole-drilling, without any compensating extra grid, the problem reformulated according to equation (9) seems to require many terms. In reality, the symmetry properties reduce the number of *free* parameters of the matrix  $\bar{\mathbf{A}}$  and its derivatives, thus making the approach more accessible. Matrix  $\bar{\mathbf{A}}_0$ , which is matrix  $\bar{\mathbf{A}}$  in the case of no eccentricity, is a combination of the coefficients of the two matrices  $\bar{\mathbf{a}}$  and  $\bar{\mathbf{b}}$  previously introduced. It is thus an ‘inflated’ reissue (since it has more coefficients) of  $\bar{\mathbf{a}}$  and  $\bar{\mathbf{b}}$ . but offers a framework for the extension to the eccentricity problem. The following relationship (which is equivalent to equation (2)) offers the easiest way to calculate the coefficients of this matrix

$$\begin{bmatrix} A_{0,11}^{(ij)} & A_{0,12}^{(ij)} & A_{0,13}^{(ij)} \\ A_{0,21}^{(ij)} & A_{0,22}^{(ij)} & A_{0,23}^{(ij)} \\ A_{0,31}^{(ij)} & A_{0,32}^{(ij)} & A_{0,33}^{(ij)} \end{bmatrix} = \begin{bmatrix} \frac{(1+\nu)a_{ij} + b_{ij}}{2} & \frac{(1+\nu)a_{ij} - b_{ij}}{2} & 0 \\ \frac{(1+\nu)a_{ij}}{2} & \frac{(1+\nu)a_{ij}}{2} & b_{ij} \\ \frac{(1+\nu)a_{ij} - b_{ij}}{2} & \frac{(1+\nu)a_{ij} + b_{ij}}{2} & 0 \end{bmatrix} \quad (17)$$

Equation (17) only shows a single block of the matrix  $\bar{\mathbf{A}}_0$ ; anyway, this reconstruction is valid for all the blocks on the diagonal  $i = j$  and for any other block below the diagonal  $i > j$ . A symmetry property also relates matrices  $\bar{\alpha}_{e1}$  and  $\bar{\alpha}_{e3}$ . Given the angular pattern  $0^\circ, 225^\circ(45^\circ), 90^\circ$  of the grids, the derivation terms with respect to the components of eccentricity are the same but in different positions. Thus, one of the two can be obtained by the other just through simple permutations of the coefficients. For example, if  $\bar{\alpha}_{e1}$  is already available,  $\bar{\alpha}_{e3}$  is obtained by the following matrix equation (this relationship is only valid for a CCW system of directions 1 and 3)

$$\bar{\alpha}_{e3} = \bar{\mathbf{P}}_L \bar{\alpha}_{e1} \bar{\mathbf{P}}_R \quad (18)$$

in which  $\bar{\mathbf{P}}_L$  and  $\bar{\mathbf{P}}_R$  are the two permutation *left* and *right* matrices, respectively, and they are  $3 \times 3$  block diagonal containing only 0 and 1 in specific positions. The following is an example with three blocks of these permutation matrices

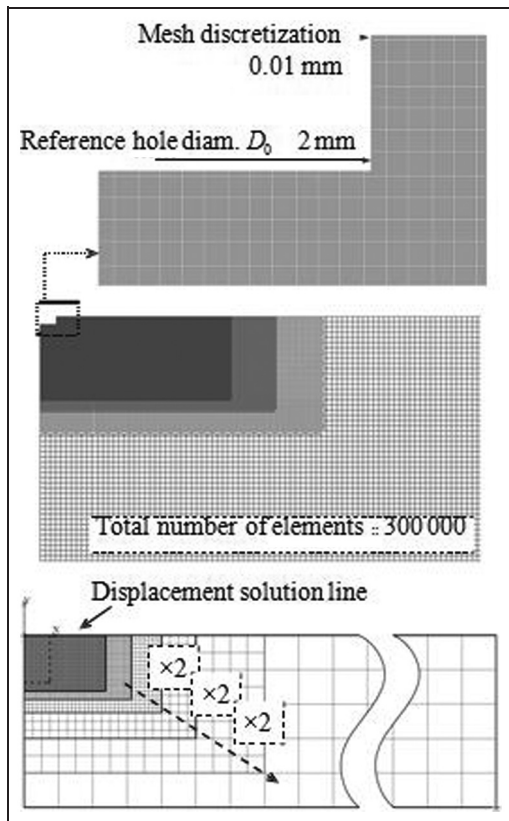


Figure 3. Plane FE model with axial-symmetric harmonic elements and multiple nested refinements.

$$\begin{aligned}
 \bar{\mathbf{P}}_L &= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \\
 \bar{\mathbf{P}}_R &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (19)
 \end{aligned}$$

Finite element modelling

A parametric finite element (FE) model was implemented to determine the coefficients of the matrices introduced above. The axial-symmetric geometry enabled structural plane harmonic elements (ANSYS Plane25) to be used. A full three-dimensional model, as proposed by Aoh and Wei<sup>44</sup> and Xiao and Rong,<sup>45</sup> was

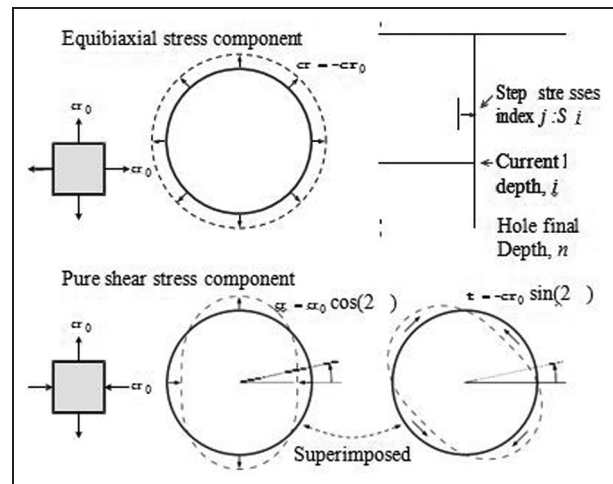
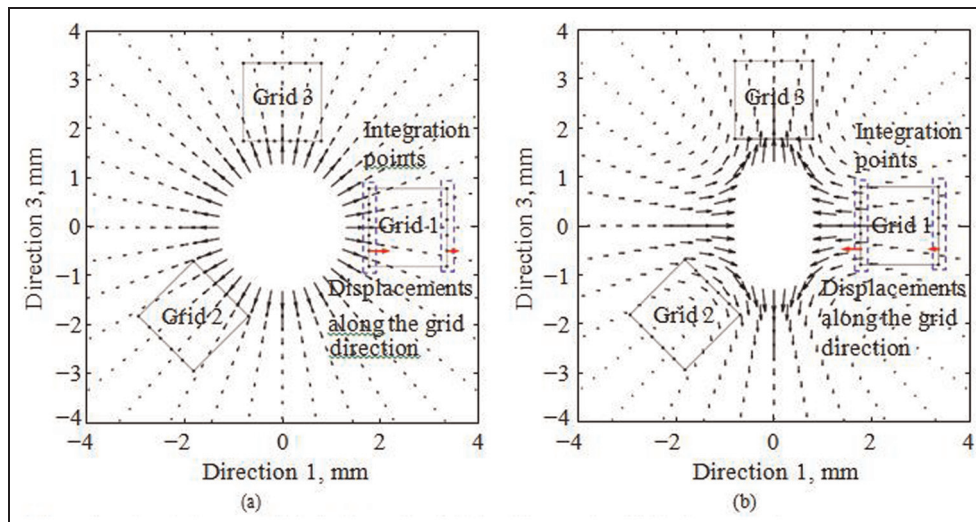


Figure 4. Simulation of material removal for the equibiaxial and the pure shear stress load types.

avoided in order to produce a very high spatial resolution in the region of interest with multiple nested refinements (Figure 3). The mesh division at the hole region was as small as one-hundredth of the radius, which was assumed equal to 1 mm as reference. The accuracy of the FE model was estimated to be approximately 1%, by comparing the numerical results with the analytical solution of the Kirsch equations, after reducing (just for benchmarking) the model height to a single row of elements and simulating a plane stress problem.

Two types of load were imposed on the axial-symmetric harmonic model: equibiaxial and pure shear. As reported in Figure 4, the equibiaxial was obtained with the harmonic order zero, while the pure shear was obtained with the harmonic order 2, which is the tensorial dependence. To obtain the first load type, the removal of material with residual stress embedded was simulated as the application of pressure on the free cylindrical surface of the hole. Then, the pure shear was obtained as pressure (symmetric with respect to  $\vartheta = 0$  angular coordinate) superimposed on a distribution of tangential traction with a relative phase equal to  $45^\circ$  (anti-symmetric). These two load conditions, applied as uniform steps at different hole increments, enabled all the calibration coefficients to be obtained. Initially, without eccentricity, the procedure for  $a_{ij}$  and  $b_{ij}$  coefficients required the evaluation of  $p$  and  $q$  strain components, respectively, given by the combination of directions 1 and 3 alone. On the other hand, all the grids were considered for the more general case with eccentricity, and the stresses  $\sigma_x$  and  $\sigma_y$  were obtained as the subtraction and sum of the equibiaxial and shear components, respectively.

According to equations (7) and (9), the calibration coefficients can be interpreted as the relaxed strains induced by unitary residual stresses. The strains measured by the grids were numerically simulated from the FE analysis displacements by considering the angular  $\vartheta$  dependence. The displacement fields on the upper



**Figure 5.** Type A rosette, displacement fields for (a) the equibiaxial and (b) the pure shear load components.

surface are shown in Figure 5 for the two load types. The displacements along the grid direction, which is slightly different from the radial direction due to the grid width, were calculated and averaged at several integration points along the two opposite sides of the grid. The strain was obtained by computing the difference of the averages at the two sides and finally by dividing by the grid length. This calculation is easier and faster and still equal to computing the average strain over the whole grid area, and for more details see, in particular, Nau and Scholtes.<sup>39</sup> Finally, this numerical analysis was also performed with eccentricity, following the same procedure, after updating the actual position of the grids with respect to the hole centre. A square array of eccentricity component values has been considered in order to obtain the (numerical) derivatives of the calibration coefficients.

## Calibration coefficient matrices

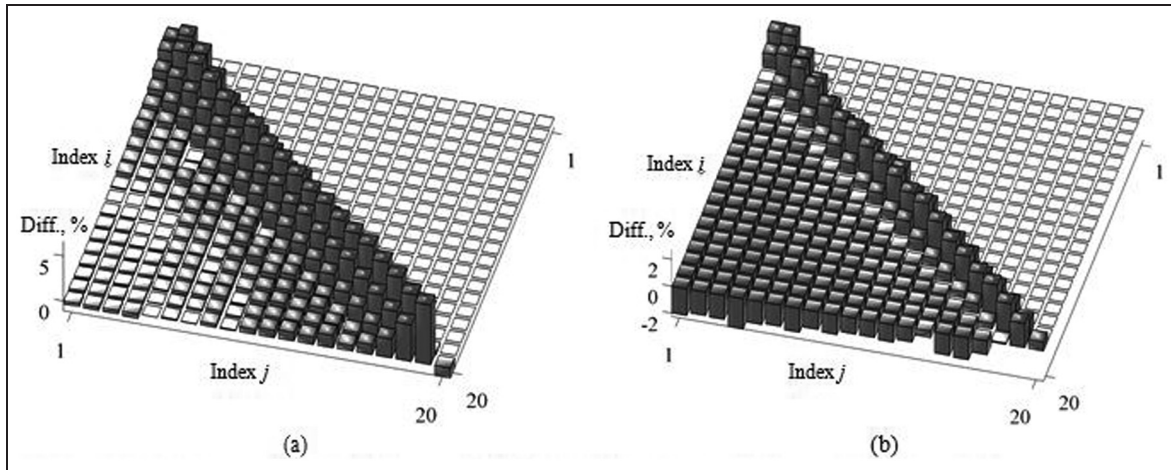
### Standard calibration coefficients with no eccentricity

Although already available on the ASTM E837 standard,<sup>31</sup> the calibration matrices  $\bar{\mathbf{a}}$  and  $\bar{\mathbf{b}}$  were recalculated as a benchmark of the numerical procedure, and also to provide an update of the coefficients with the very accurate FE model shown above. The exact combination of rosette dimensions, hole diameter and hole depth step is required to validate any single set of  $a_{ij}$  and  $b_{ij}$  calibration coefficients. The choice of other dimension ratios involves different coefficients, which need the calculation to be repeated. In fact, the use of an automated algorithm is recommended whereby any geometry configuration can be inputted to generate these coefficients.<sup>41,42</sup> The specific dimensional combinations reported in the ASTM standard are considered here, and the calibration coefficients are provided in Tables 1–4 of the online Appendix (available at: <http://sdj.sagepub.com/>), for type A and type B rosettes. The ASTM standard suggests the use of either millimetres

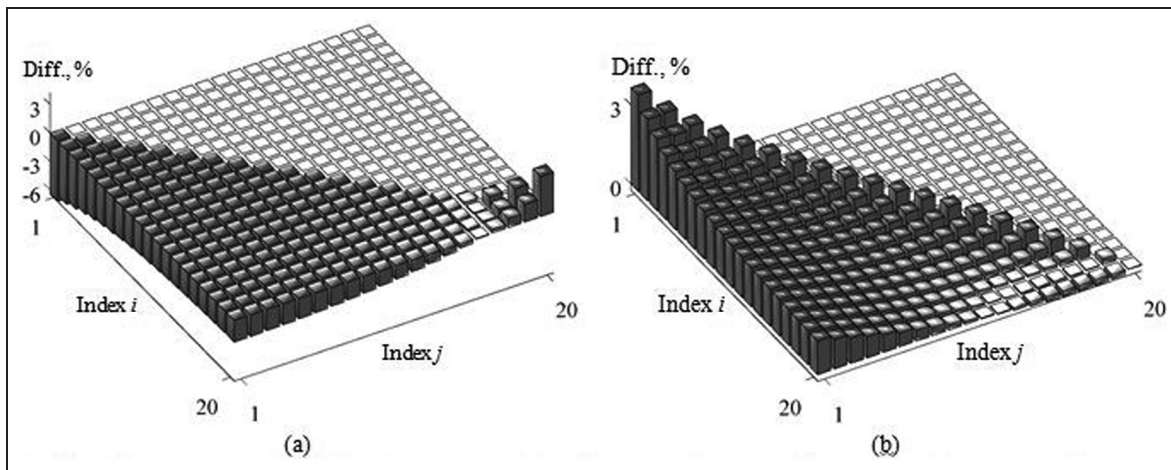
or inches as length units, however, with a simplified rounding. The coefficients according to both the standard millimetres and inch dimensions were calculated and can be found in the tables in the online Appendix. The effect of the approximated conversion was quantified and the coefficient differences are in the order of a few percentage points up to 5%. Using inches, a comparative analysis between the calculated coefficients with this model and the values reported in the ASTM standard is provided in Figure 6 for type A. The histograms highlight that the differences are limited to a few percentage points mainly at, and near to, the matrix diagonal. Very similar comparison results were also obtained for type B.

The role of the Poisson's ratio  $\nu$  as reported in equation (7) is valid, in principle, only for the plane stress problem, which is a very accurate approximation of the through-thickness hole on a thin plate. As discussed above, the calibration coefficients actually have a more general dependence with respect to the Poisson's ratio and the plane stress form is only approximated. In order to quantify the error introduced by assuming no dependence of the coefficients on the Poisson's ratio, the FE analysis was repeated with  $\nu = 0.35$  and the coefficients were recalculated accordingly. This comparison analysis returned similar coefficient values, and again two histograms with percentage differences are reported in Figure 7. These differences are still in the order of a few percentage points, and the highest discrepancies are for the initial depths. Similar results were also obtained by Nau et al.<sup>46</sup> confirming the validity of this analysis. The usual structural metal alloys never show very large Poisson's ratio differences from the reference  $\nu = 0.3$  value. Thus, the form of equation (7) can be considered as satisfactorily accurate, after substituting the appropriate value of  $\nu$  for the specific material, and using the calibration coefficients derived with  $\nu = 0.3$ . In fact, all the coefficients in the tables in the online Appendix are reported for reference  $\nu = 0.3$  alone.





**Figure 6.** Percentage differences obtained with respect to the ASTM type A,  $a_{ij}$  and  $b_{ij}$  in (a) and (b), respectively.



**Figure 7.** Percentage differences for the Poisson's ratio,  $a_{ij}$  and  $b_{ij}$  in (a) and (b), respectively, comparing  $\nu = 0.35$  coefficients with the reference  $\nu = 0.3$ .

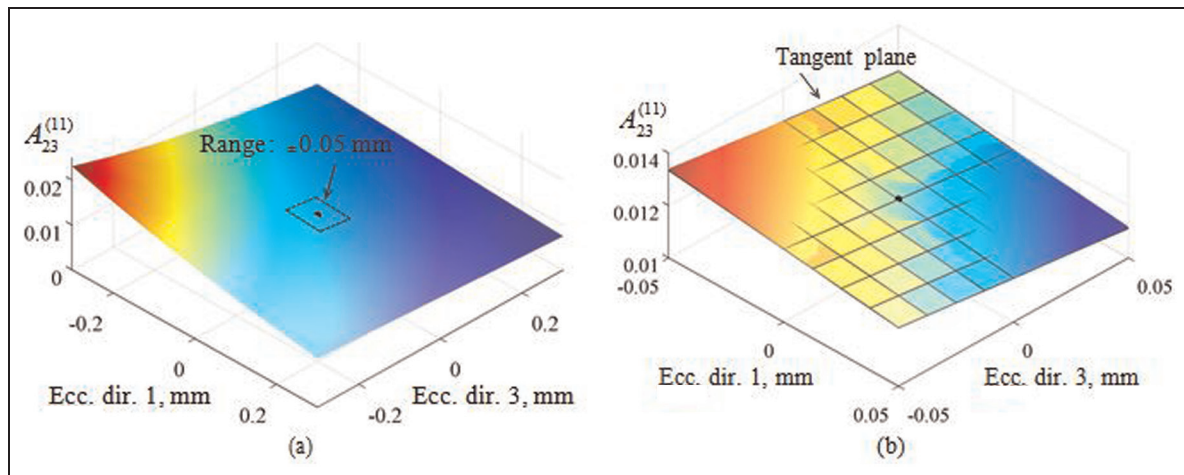
### First-order correction for eccentricity

All the coefficients in matrix  $\bar{\mathbf{A}}$  were calculated for an array of eccentricities using directions 1 and 3 as reference system. Initially, a large eccentricity range  $\pm 0.3$  mm was considered for the rosette diameter  $D = 5.13$  mm. As an example, Figure 8(a) shows the dependence of a single coefficient of the matrix, and similar trends were obtained for all the other coefficients. The dependency of the coefficients on the eccentricity is essentially linear thus, according to equation (11), the higher order terms introduce a further contribution which turns out to be minor, especially near the zero eccentricity origin. By reducing the investigated eccentricity range to  $\pm 0.05$  mm, which is still more than twice the eccentricity allowed by the ASTM standard, Figure 8(b) highlights that the linear dependency is an extremely accurate model and the residual difference is negligible. On the other hand, if a second compensating grid is introduced for each direction, or another self-compensating problem is investigated such as the ring core method, the tangent plane at the origin is perfectly

horizontal as the first-order partial derivatives are 0. In these cases, the higher order terms still play a marginal role, thus the  $p, q, t$  procedure is accurate and the proposed generalization is unnecessary. The matrix  $\bar{\alpha}_{e1}$  was calculated by (numerically) determining the partial derivatives on the small range of Figure 8(b) and just by considering a single eccentricity component, specifically direction 1. The coefficients of the matrix  $\bar{\alpha}_{e1}$  are listed in Tables 5 and 6 of the online Appendix (available at: <http://sdj.sagepub.com/>), for the ASTM standard type A and type B rosettes, respectively. The other derivative matrix  $\bar{\alpha}_{e3}$  can be easily calculated from equation (18), and finally the matrix  $\bar{\mathbf{A}}$  is obtained with equation (15). When a generic couple of eccentricity components is introduced, the matrix  $\bar{\mathbf{A}}$  has no more 0 values at or below the block diagonal; hence, the proposed method is effectively exploited.

The Poisson's ratio dependence of matrix  $\bar{\mathbf{A}}_0$  is given by equation (17). The appropriate value of  $\nu$  can be introduced in the calculation of each block along with the coefficients of the matrices  $\bar{\mathbf{a}}$  and  $\bar{\mathbf{b}}$ . However, the





**Figure 8.** Eccentricity dependence of a single coefficient of matrix  $\bar{A}$ : (a) trend on a large range and (b) tangent plane on a small range.

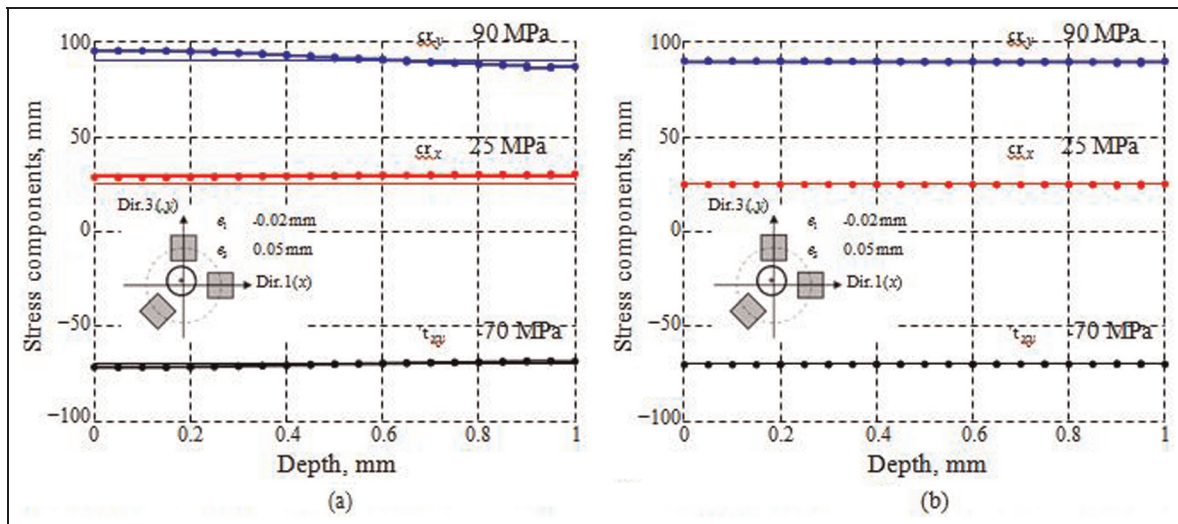
derivative matrix coefficient dependence on the Poisson's ratio is more complicated since the equibiaxial and the shear stress components are coupled. Thus, there is no simple equation to take into account the Poisson's ratio for the derivative matrices, and the coefficients reported in Tables 5 and 6 of the online Appendix (available at: <http://sdj.sagepub.com/>), are just defined for  $\nu = 0.3$ . A recalculation would be needed for any different values of  $\nu$ . However, the Poisson's ratio is usually very close to  $\nu = 0.3$ , at least for metals, as already discussed above. In addition, the derivative matrices only provide a correction for matrix  $\bar{A}$ ; thus, the effect on the final result of a small variation in the Poisson's ratio is marginal.

### Validation analysis

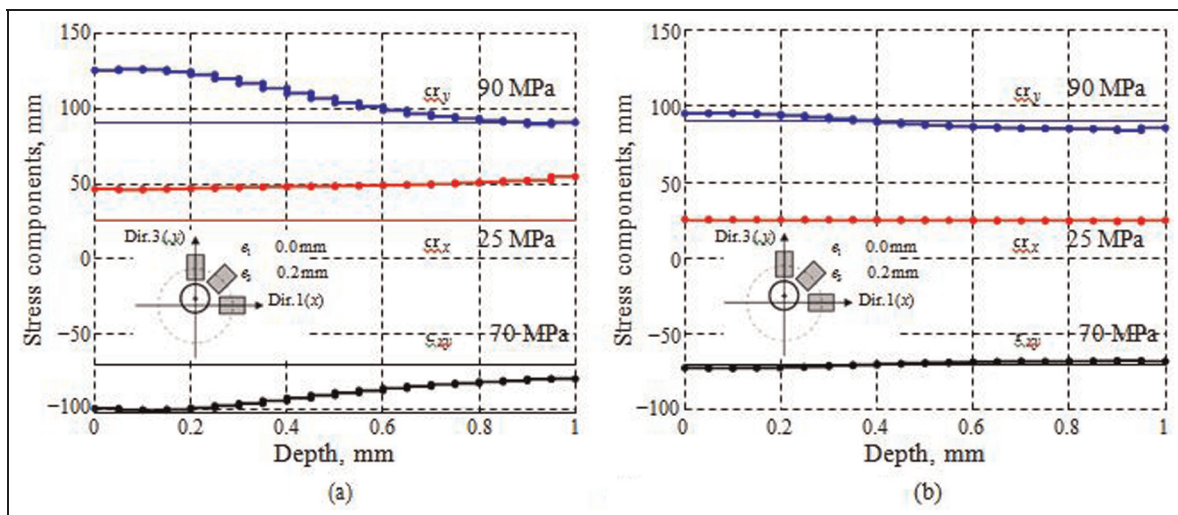
Numerical tests are proposed in this section to validate the introduced eccentricity correction procedure. Using the algorithm developed by Beghini et al.<sup>41,42</sup> based on influence functions, any residual stress distribution can be simulated solving the *direct* problem and the relaxed strains along the depth predicted. The residual stress distribution, which is assumed as being uniform for the sake of simplicity, was initially back-calculated with the integral method according to the standard, applying the usual  $p, q, t$  procedure and neglecting the eccentricity. The calculation was then repeated by introducing the linear correction for matrix  $\bar{A}$  and solving the inverse problem (equation (16)). Initially, a calculation example was performed with both the eccentricity components being not 0 and not equal. The eccentricities introduced were in the order of a few hundredths of millimetres, similar to the values in real tests with  $D = 5.13$  mm rosettes.<sup>32,47</sup> The first calculation was implemented with the type A rosette and eccentricity components  $e_1 = -0.02$  mm and  $e_3 = 0.05$  mm; thus, the overall eccentricity is larger than the limit prescribed by the standard:  $\sqrt{e_1^2 + e_3^2} > 0.02$  mm. The eccentricity introduced a perceptible error with respect to the reference

stresses (Figure 9(a)); then this error was almost completely eliminated after introducing the linear correction (Figure 9(b)), therefore confirming the effectiveness of the proposed procedure and the accuracy of the coefficients provided. The second calculation was repeated with the same reference residual stresses but with type B rosette and just a single component of eccentricity, which, however, was 10 times larger than the limit allowed by the standard. Maximum stress error in the order of 40% was obtained as the result of the  $p, q, t$  procedure, thus meaning a considerable effect due to the large eccentricity (Figure 10(a)). As highlighted in Figure 8, the linear component represents the actual trend of the coefficients very well even for relatively large eccentricities. Thus, the stress reconstruction with the linear correction was still quite accurate, returning a marginal residual error similar to the first example without any correction (Figure 10(b)).

A wider analysis was then performed to test the effect of eccentricity and verify the correction introduced by the procedure, covering all the combinations of eccentricity values and stresses. Six levels of eccentricity were considered:  $e_1, e_3 = 0.02, 0.05, 0.07, 0.10, 0.15, 0.20$  mm and with 16 angular positions for each:  $0.0^\circ, 22.5^\circ, \dots, 337.5^\circ$ . For all the combinations of eccentricity radius and angle, three tests were separately simulated for the stresses  $\sigma_x, \sigma_y, \tau_{xy} = 100$  MPa. The highest percentage difference was found along the depth coordinate and saved for all the eccentricity and stress combinations for both type A and type B rosettes. This calculation was then repeated for the same combinations of eccentricity and stresses applying the correction procedure. Figure 11 highlights that the maximum eccentricity errors increased linearly with the eccentricity, while the correction reduced the residual error by limiting it to a few percentage points even for an eccentricity as large as  $0.1 - 0.15$  mm, which is quite high with respect to the usual best practice in the experiments. The larger grid width of type A rosette averages the strain distribution better; thus, type B rosette experiences a higher eccentricity sensitivity when the



**Figure 9.** Validation example with type A rosette: (a) error caused by the eccentricity and (b) almost perfect result with the first-order correction.



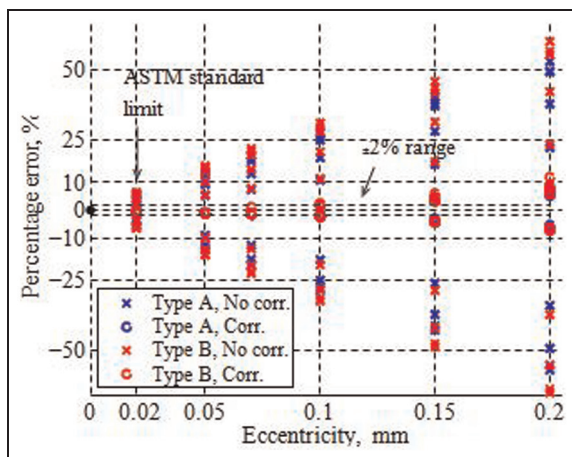
**Figure 10.** Validation example with type B rosette: (a) error caused by a large eccentricity and (b) accurate result with the first-order correction.

standard integral method is applied. With the proposed correction, the higher sensitivity of type B rosette is only evident for large eccentricity values, while in the range where the linearity is predominant, the small residual error is very similar for the two rosette types.

## Conclusion

We have described a generalization of the hole-drilling integral method, which includes a correction for the eccentricity of the hole with respect to the strain-gage rosette. The eccentricity impairs the axial symmetry of the problem; thus, the decoupling of the stress components in an equibiaxial plus two shear stresses is no longer allowed. Consequently, a single matrix is needed to linearly relate the relaxed strains to the residual stress components. After having

grouped the strains and the stresses in vectors along the depth, this matrix is a lower triangular  $3 \times 3$  block, and each of its coefficients can be expressed as a power series of the eccentricity components. The two linear terms alone already proved to be a very accurate model for reproducing the eccentricity effect. The matrix was then expressed as the zero eccentricity term plus a linear correction of the two eccentricity components by introducing the derivative matrices. In addition, the  $0^\circ, 225^\circ(45^\circ), 90^\circ$  grid pattern only requires the knowledge of a single derivative matrix, while the other can be obtained as a permutation of the coefficients, for symmetry reasons. On the other hand, if a compensating grid is applied for each direction, the first-order derivatives are 0 (similarly to the ring core method) and the problem can still be formulated with the usual combined stresses and strains.



**Figure 11.** Maximum percentage errors for several combinations of eccentricity positions and stresses, and corrections obtained with the proposed procedure.

A very refined plane and axial-symmetric harmonic FE model was implemented and the calibration coefficients were calculated. Initially, a revision of the ASTM standard was proposed, according to the combined stresses and strains. The derivative matrices were then calculated and provided for both type A and type B strain-gage rosettes. Finally, an extensive numerical analysis was proposed both to validate the procedure and to show the accuracy of the correction. When the eccentricity is in the order of the small allowed limit prescribed by the standard, the reconstruction of the stress components with the eccentricity correction is very accurate. However, if the eccentricity is small, its measure uncertainty can be of the same entity of the eccentricity itself; thus, the present correction is not recommended. On the other hand, quite accurate results were still obtained with larger eccentricities, approximately up to 10 times the standard limit, since the linear first order is a fine model even for relatively high eccentricity values. In conclusion, the proposed procedure can significantly increase the eccentricity allowed and, even when the hole is produced with a large offset for any experimental reason, the measure can still be useful instead of being discarded or repeated.

#### Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship and/or publication of this article.

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## Appendix I

### Notation

$\bar{A}$	calibration coefficient $3 \times 3$ block matrix
$a, b$	calibration coefficients relating the $P, Q, T$ residual stresses to the $p, q, t$ relaxed strains
$\bar{a}, \bar{b}$	calibration coefficient matrices relating the combined residual stresses and the relaxed strains
$A, B$	general elastic constants relating the residual stresses to the relaxed strains
$D$	average diameter of the strain-gage rosette
$e$	vectors containing the blocks of the three grid relaxed strains

$E, \nu$	Young's modulus and Poisson's ratio of the isotropic linear elastic material	$\Delta Z$	hole depth incremental step
$h, k$	block indexes, for the matrix $\bar{\mathbf{A}}$ , ranging from 1 to 3	$\varepsilon_r$	relaxed strain measured by a generic grid
$i, j$	calibration matrix depth indexes, $i = 1, \dots, n$ and $j = 1, \dots, i$	$\varepsilon_1, \varepsilon_2, \varepsilon_3$	relaxed strains measured by the $0^\circ, 225^\circ(45^\circ), 90^\circ$ grids
$n$	number of hole increments at the final depth	$\eta_1, \eta_3$	dimensionless eccentricity components
$p, q, t$	combined relaxed strains according to the $P, Q, T$ stresses	$\sigma_{\max}, \sigma_{\min}$	principal maximum and minimum residual stresses
$\mathbf{p}, \mathbf{q}, \mathbf{t}$	vectors containing the combined strains along the depth arranged in blocks	$\sigma_x, \sigma_y, \tau_{xy}$	residual stress components according to the rosette reference frame
$P, Q, T$	equibiaxial and shear combined stresses	$\vartheta$	generic grid orientation with respect to the maximum principal stress direction
$\mathbf{P}, \mathbf{Q}, \mathbf{T}$	vectors containing the combined stresses along the depth arranged in blocks		
$\mathbf{S}$	vectors containing the blocks of the three uncombined residual stress components		
<b>Greek symbols</b>			
$\bar{\alpha}_{e1}, \bar{\alpha}_{e3}$	derivative matrices for the calibration coefficient correction of 1 and 3 directions, respectively	<b>Subscripts</b>	
		$\bar{\mathbf{A}}_0$	calibration coefficient matrix $\bar{\mathbf{A}}$ with zero eccentricity
		$D_0$	drilled hole diameter
		$e_1, e_3$	eccentricity components along the directions 1 ( $x$ ) and 3 ( $y$ )
		$G_L, G_W$	strain-gage rosette grid length and grid width
		$\bar{\mathbf{P}}_L, \bar{\mathbf{P}}_R$	left and right $3 \times 3$ block permutation matrices