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Higher-order moments of eigenvalue and eigenvector distributions for the nonlinear stochastic dynamics of cable networks

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Abstract

Cross-ties are employed on cable-stayed bridges to reduce wind-induced stay vibration. The occurrence of a number of failures has motivated investigations on the nonlinear dynamics of these systems. The non-linear behavior is possible at incipient failure of the cross-tie. This paper combines the features of the Equivalent Linearization Method, recently derived to study non-linear behavior in the cross-ties at moderate stay vibration amplitudes, with implementation of the Stochastic Approximation (SA) to account for the presence of parameter variability during aeroelastic vibration. The problem becomes an equivalent random eigenvalue problem with eigenvalues (equivalent frequencies) depending on a random vibration amplitude parameter. Novel implementations of the SA are considered to evaluate higher-order statistical moments of eigenvalue distributions; the consequent randomness in the eigenvectors is also examined.

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Keywords: Cable-stayed bridges; Stays; Nonlinear vibration; Wind load effects; Stochastic Approximation; Monte Carlo method.

1. Introduction

Wind-induced stay vibration on cable-stayed [1]and related cable-deck issues [2] can be reduced by deploying cross-ties, which are the building blocks of an in-plane cable network. Non-linear dynamics behaviour has been

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observed in cable networks at incipient failure of the cross-tie. This interesting behaviour can be simulated by appropriate models of the restoring force transferred by the cross-tie[3]. Nevertheless, very few studies have addressed the relevant question of cable network performance under the influence of non-linear dynamics. In recent years, the authors have proposed the use of an Equivalent Linearization Method (ELM) to examine and approximately solve the nonlinear free vibrations of these cable systems[4].The ELM derives a set of equivalent natural frequencies (eigenvalues) and mode functions (eigenvectors), which depend on vibration amplitude. The main parameter, controlling the dynamics, is a vibration amplitude parameter, λ. Since the value of the initial vibration amplitude is affected by parameter variability, the problem becomes stochastic. Randomness reflects the uncertainty in the frequency estimation, which is a consequence of the inability to describe various aeroelastic mechanisms[1,5].

The resulting nonlinear random eigenvalue problem is subsequently solved by using implementations of the Stochastic Approximation (SA) method and the Robbins-Monro (RM) theorem [6,7], which are adaptive numerical techniques designed for stochastic environments. Adequate convergence properties of the SA are usually achieved in a number of applications [8]. Even though several methods are available to solve random eigenvalue problems, the SA has been selected because of its efficiency and simplicity. This paper examines a new application of the SA to calculate higher-order statistical moments of the eigenvalue distribution, following recent work [9]. The study makes use of a recently-developed numerical procedure, designated as "layered" SA algorithm to estimate the mean, standard deviation and skewness of the eigenvalue distribution. Numerical estimation of higher statistical moments is needed since the distribution of the random frequencies is often skewed. Furthermore, the method is employed to examine the tolerance bounds of the "eigenvector clouds"[9]. A benchmark three-stay cable network with variable cross-tie geometry is used in the numerical computations. The SA method is computationally efficient and adequately reproduces the main features of the random frequency distributions.

2. Background on nonlinear cable – network dynamics

The benchmark configuration is a three-cable system, installed on the Fred Hartman Bridge and labeled as BSL network in Fig. 1a ("B-line", "South" tower, "Lower" network). The system is composed of three stays (BS13, BS14 and BS15) of the "B-line" on the south tower of the bridge and one cross-tie (Fig. 1b).

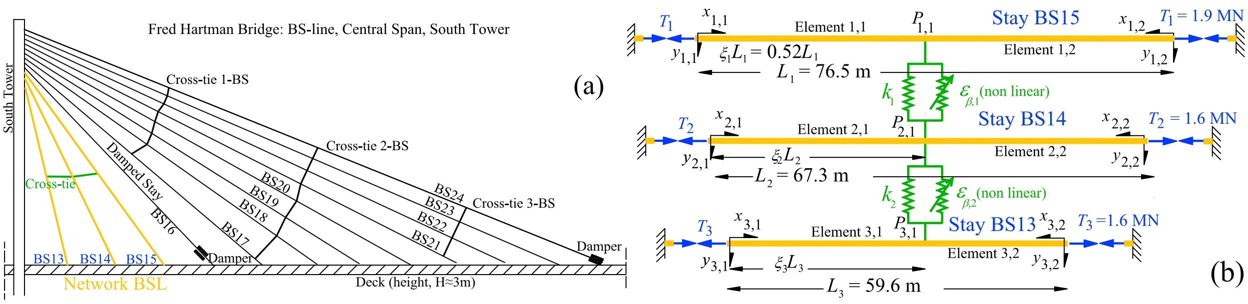

Fig. 1.Case study:(a) prototype three-cable networks (refer to "Network BSL") installed on the BS-line, South Tower, of the Fred Hartman Bridge, Houston, Texas, USA; (b) cable network model with nonlinear cross-tie.

In the case of cable-cross-tie systems, flexural stiffness and nonlinear sag effects in the primary stays can be neglected [10], even though geometric non-linearity may influence the dynamics of "stand-alone" cables [11,12]. The properties of the stays (j={1,2,3}) are: tension T_j, length L_j and mass per unit length μ_j. The reference stay is BS15 with index j=1 (Fig. 1b). Description of the stay properties may be found in Table 1 of [13].Each segment of the stays is simulated as a continuous taut string, with transverse oscillation $y_{j,p}$ at a generic location $x_{j,p}$ along each segment with j={1,2,3} and p={1,2} in Fig. 1b. The dynamic model employs the wave equation (time t):

$$T_j\,\partial^2 y_{j,p}\big/\partial x_{j,p}^2 = \mu_j\,\partial^2 y_{j,p}\big/\partial t^2 \tag{1}$$

The boundary conditions enforced at each end include nonlinearity. Nonlinearity simulates incipient failure in the cross-ties through the internal restoring force in the cross-tie. The internal force transferred between stays j and j+1 at $P_{j,1}$ and $P_{j+1,1}$ is: $F_{j,j+1} = -[k_j(y_{j,1} - y_{j+1,1}) + \varepsilon_{\beta,j} \mid y_{j,1} - y_{j+1,1} \mid^{\beta} \text{sign}(y_{j,1} - y_{j+1,1})]$;$y_{j,1}$ and $y_{j,1+1}$ are the transverse vibrations of the main cables at $P_{j,1}$ and $P_{j+1,1}$. In the expression for $F_{j,j+1}$, justified by a series of experimental results [10], a linear spring of stiffness k_j is used in parallel with a nonlinear element of "stiffness" $\varepsilon_{\beta,j}$. The quantity $2 \le \beta \le 3$ is a positive integer number (power-law coefficient);$\varepsilon_{\beta,j}>0$ simulates an increment of stiffness, a performance increase after initial imperfect installation of the wire, whereas $\varepsilon_{\beta,j}<0$ simulates degradation in the cross-tie properties[14]. Pre-stressing in the cross-ties may readily be included [13] but is not considered in this study.

The ELM [15]is used to examine the free-vibration dynamics of the system in Fig. 1b;the following properties are considered: restrainer position $x_{1,1}=\xi_{1,1}L_1$; linear stiffness properties in both cross-tie segments with $d_k=d_{k,1}=d_{k,2}$ given by $d_{k,1}=T_1/(k_1L_1)$, $d_{k,2}=T_2/(k_2L_2)$; the quantity $d_{k,j}=T_j/(k_jL_j)$ ($j=\{1,2\}$) is a dimensionless stiffness parameter [14].

Application of the ELM between anchorage points $P_{j,1}$ and $P_{j+1,1}$ in Fig. 1b leads to the linearization of $F_{j,j+1}$ and:

$$1/d_{kELM,j} = \left[T_j / \left(k_{ELM,j} L_j\right)\right]^{-1} = 2\nu_{\beta,j} \lambda L_1 (\beta+1)^{-1} \left|S_{j,1} - \delta_j S_{j+1,1}\right|^{(\beta-1)},\ \nu_{\beta,j} = \varepsilon_{\beta,j} L_j^{\beta} / T_j \tag{2, 3}$$

In Eqs. (2,3) $d_{kELM,j}$ and $k_{ELM,j}$ are dimensionless and dimensional equivalent (linearized) stiffness parameters; $\nu_{\beta,j}$is a dimensionless nonlinear stiffness coefficient measuring the contribution provided by the nonlinear component of the cross-tie. Equation (2) also depends on the dimensionless amplitude parameter $\lambda \ge 0$ and the local displacement field at nodes $P_{j,1}$ and$P_{j+1,1}$, designated by quantities $S_{j,1}$ and$S_{j+1,1}$ and the "modal amplitude ratio δ_j" [14].The quantity λ is the ratio between anticipated vibration amplitude in the reference stay (j=1 or BS15), mode by mode, and the length L_1;in the present setting, this approximation is acceptable at moderate amplitudes[15].

A total equivalent stiffness parameter $d_{kEj}$of the parallel spring model for the cross-tie segment between $P_{j,1}$ and$P_{j+1,1}$is obtained by combining Eqs. (2,3) with $d_{k,j}$. After full linearization [4]and Eq. (3), the dynamic free-vibration wave equations[Eq. (1)] are converted to a system of homogeneous algebraic equations; this is an equivalent eigenvalue/eigenvector problem in terms of "equivalent frequency" α_E and harmonic "modes". Despite the linearization, harmonic oscillation still predominantly occurs. The quantity α_E is usually normalized to $\omega_{01} = \pi / L_1 \left(T_1 / \mu_1\right)^{0.5}$, the fundamental "native-cable" pulsation of the reference stay (j=1, BS15 in Fig. 1b). The mode shapes $Y_{j,p}$ are real trigonometric functions. The ELM also requires linearization of initial free-vibration conditions, imposed through the definition of λ. For reference, the fundamental native cable frequencies of the stays in Fig. 1b are: 1.30 Hz [BS15, corresponding to $\omega_{01}/(2\pi)$], 1.36 (BS14) and 1.89 Hz (BS13).

3. Stochastic dynamics and layered algorithms for SA

In this section, the standard SA method and its application [15] are initially reviewed; the layered SA algorithm is subsequently introduced. The SA [6,7]is an iterative method for finding the root of a function, even if its exact formula is unknown because of the presence of statistical noise. The SA formulation has been used to solve many engineering and physics problems [8,16].

As outlined in the previous sections, the quantity λ is a random variable to account for the inadequate knowledge of the various stay vibration mechanisms, which are related to irregular wind load features and aeroelasticity. One plausible assumption for describing this modeling uncertainty is to consider λ as uniformly distributed in the interval $0 \le \lambda \le \lambda_u$; the lower limit λ=0 corresponds to the linear solution whereas $\lambda=\lambda_u$ can be inferred from the anticipated level of wind-induced aeroelastic vibration. The problem is consequently mapped into a random eigenvalue problem in terms of α_E, which can be solved using the SA. The nature of the stochastic free vibration can be represented through the random sequence $\{\lambda_1,\lambda_2,...,\lambda_q,...,\lambda_n\}$, which generates a corresponding sequence of equivalent frequencies$\{\alpha_{E,1},\alpha_{E,2},...,\alpha_{E,q},...,\alpha_{E,n}\}$. These roots are found by ELM from the roots of the characteristic polynomial associated with the equivalent eigenvalue/eigenvector problem, designated as $Q_{\lambda q}(\alpha_{E,q})=0$.

The true average value of the average frequency $\overline{\alpha}_E$ can be found from the roots of this sequence by SA (mode by mode) using a recursive approach [7] (with q the generic step of the algorithm):

$$\overline{\alpha}_{E,q+1} = \overline{\alpha}_{E,q} - \zeta a_q Q_{\lambda_q}(\overline{\alpha}_{E,q}) \tag{4}$$

Eq. (4) estimates the root of a function, which corresponds to the true value of the ensemble function of the characteristic polynomial, by avoiding expensive root finding for each $Q_{\lambda q}$ from $Q_{\lambda q}(\alpha_{E,q})=0$. The approximate solution is iteratively found through Eq. (4), in which $a_q=a/(q+1)^{\delta_{SA}}$ is a damping parameter with a and δ_{SA} two arbitrary constants. In ordinary conditions [15], the values a=0.25 and δ_{SA}=0.95 are employed for convergence. Equation (4) also includes a "relaxation parameter" $0<\zeta<1$, which rescales a_q to facilitate numerical convergence in special situations (e.g., closely-spaced frequencies [15]).

In the present problem setting, efficient estimation of the standard deviation and higher-order statistical moments of α_E is needed since the probability distributions of α_E are asymmetrical. For this purpose, the layered SA algorithm is introduced as a direct derivation of the standard SA algorithm. The idea is to replace the continuous output distribution of α_E by an approximate discrete equivalent random eigen-value variable ("DE", $\alpha_E^{(DE)}$). The probability mass function (PMF) of the discrete output variable is approximately evaluated. If the nonlinear relationship between random λ and frequency is written as $f_\alpha=|\alpha_E-Q_{\lambda q}(\alpha_{E,q})|$ (zeros of the characteristic polynomial), the true functional relationship between input and output variables (λ and α_E) is a transformation of random variables.

In the layered SA setting, this relationship is replaced by an approximated f_α of the stratified discrete equivalent variable $\alpha_E^{(DE)}$. Furthermore, λ is sampled from m equal-probability independent intervals, Λ_r, or "sets" with r=1,..,m. In each Λ_r the frequency converges to a representative discrete point $\alpha_{E,r}^{(DE)}$. Each discrete point $\alpha_{E,r}^{(DE)}$ can be evaluated as the expected value of the continuous variable, with λ being exclusively sampled inside the set Λ_r:

$$\alpha_{E,r}^{(\mathrm{DE})} = E\left[\alpha_E \mid \lambda \in \Lambda_r\right] \tag{5}$$

Eq. (5) is evaluated by SA and Eq. (4), i.e. the SA is applied m times by "layering" the sampling inside each set.

In previous work [9], mean and standard deviations of α_E have been estimated. In this work, the layered SA is extended to higher statistical moments(order g>2). The expressions for the mean, standard deviation and other moments of the continuous α_E are approximately found from the moments of the discrete variable [Eq. (5)]:

$$E\left[\alpha_E\right] \approx \sum_{r=1}^{m} \alpha_{E,r}^{(\mathrm{DE})}(\mathrm{PMF}), \quad E\left[\left(\alpha_E\right)^2\right] \approx \sum_{r=1}^{m} \left[\alpha_{E,r}^{(\mathrm{DE})}\right]^2(\mathrm{PMF}), \quad E\left[\left(\alpha_E\right)^g\right] \approx \sum_{r=1}^{m} \left[\alpha_{E,r}^{(\mathrm{DE})}\right]^g(\mathrm{PMF}) \tag{6,7,8}$$

With PMF=1/m. The Fisher-Pearson coefficient of skewness [18]is determined from the first three moments (g=3).

4. Numerical computations by Monte Carlo sampling

The panels of Fig. 2 illustrate examples of empirical histograms of both input random amplitude parameter λ(uniformly distributed, generated synthetically) and two output equivalent frequencies α_E of the third cable network mode (Mode III) evaluated for variable d_k. The graphs are determined by Brute Force Method (BFM) via Monte Carlo sampling.

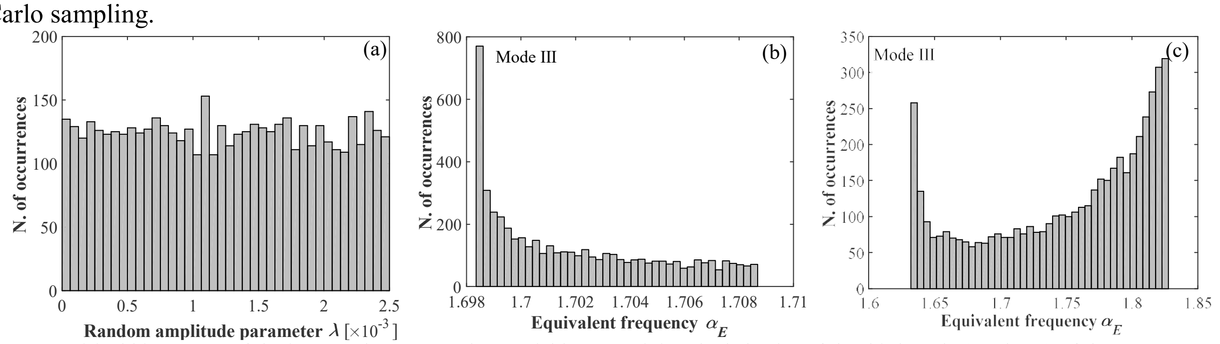

Fig. 2.Empirical histograms of the input and output random variables, examining the behavior of the third mode (Mode III) of the BSL network with cubic-stiffness cross tie (β=3) at $x_{1,1}$=0.66L_1, positive nonlinearity ($\nu_{\beta,1}=\nu_{\beta,2}$=+250) and uniform stochastic vibration amplitude parameter $0<\lambda<(1/400)$: (a) synthetically generated sample of the random input λ (uniform), (b) sample of random output corresponding to d_k=0.40, (c) sample of random output corresponding to d_k=0.90 (by BFM, Monte Carlo).

Comparison of Fig. 2b and 2c suggests that the histogram of α_E depends on the stiffness parameter d_k; there is a clear asymmetry in the histograms, which confirms the need to further study the skewness of α_E. The skewness coefficients of α_E, found by BFM, are 0.59 and -0.56 in Figs. 2b and 2c. Fig. 3 examines of random eigenvectors (mode shape functions, Y_r), generated from the random sample of α_E in Fig. 2bby BFM and Monte Carlo sampling;

the sample is labeled as "eigenvector cloud" [15]. The eigenvector cloud (50 realizations) is compared against the linear solution that neglects input randomness (λ=0)in Fig. 3a, and the "mean" eigenvector, obtained from the singular system matrix with the mean value of α_E found by SA (Robbins-Monro or RM Eigenvector) in Fig. 3b.In Section 5 the new algorithm is used (instead of BFM) to find the skewness of α_E for various cross-tie configurations.

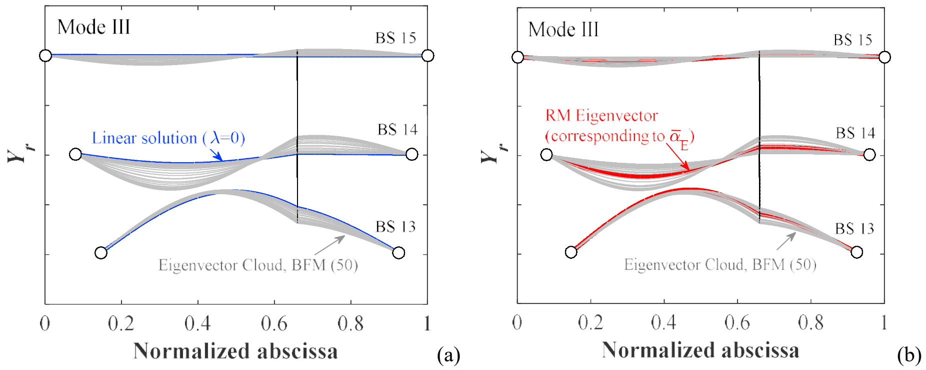

Fig. 3.Examination of the random eigenvector cloud, generated by BFM Monte Carlo, relative to the third mode (Mode III) of BSL network with cubic-stiffness cross tie (β=3) at $x_{1,1}$=0.66L_1, positive nonlinearity ($v_{\beta,1}$=$v_{\beta,2}$=+250) and random vibration amplitude parameter 0<λ<(1/400): (a) eigenvector cloud vs. linear solution (λ=0), (b) eigenvector cloud vs. mean RM eigenvector, derived from the mean value of α_E by SA algorithm.

5. Numerical results by layered SA algorithm

This section briefly summarizes the results of a study exploring the applicability of the layered SA algorithm to a wider class of stochastic problems in cable network dynamics. In particular, a parametric investigation is conducted to examine the influence of the cross-tie location on the skewness of the output α_E distribution by varying the quantity $x_{1,1}$ in the following set $x_{1,1}$={0.25L_1, 0.52L_1,0.66L_1}.Figs. 4 and 5 illustrate numerical results(Mode III).

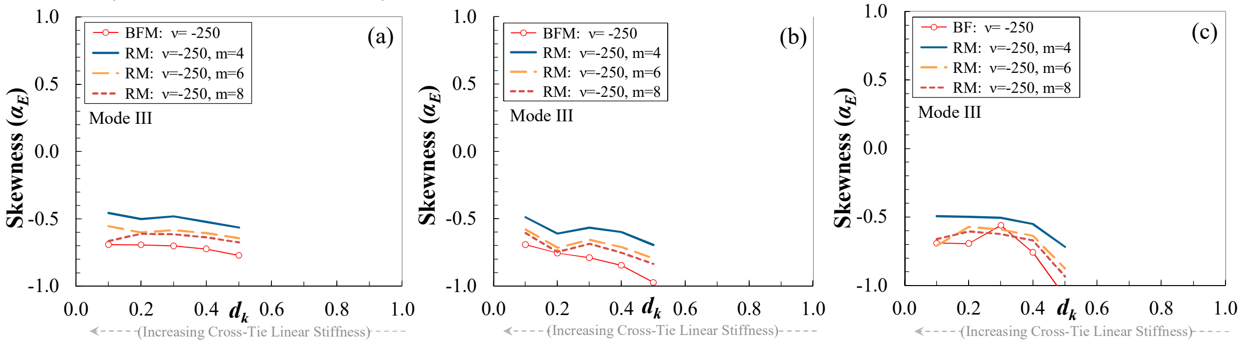

Fig. 4.Skewness coefficient of the random α_E associated with the third mode (Mode III) of the BSL network with cubic-stiffness cross tie (β=3), negative nonlinearity ($v_{\beta,1}$=$v_{\beta,2}$=-250) and random vibration amplitude parameter 0 ≤ λ ≤ (1/400): (a)$x_{1,1}$=0.25L_1, (b)$x_{1,1}$=0.52L_1, (c) $x_{1,1}$=0.66L_1.

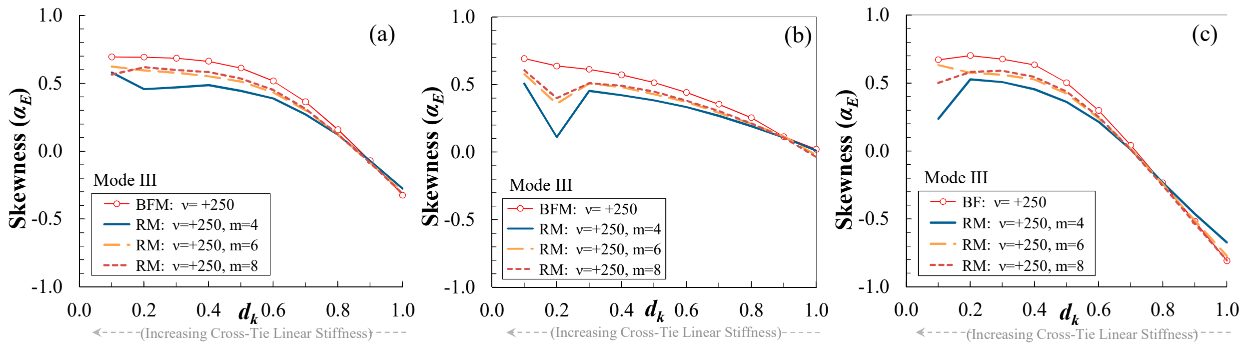

Fig. 5.Skewness coefficient of the random α_E associated with the third mode (Mode III)of the BSL network with cubic-stiffness cross tie (β=3), positive nonlinearity ($v_{\beta,1}$=$v_{\beta,2}$=+250) and random vibration amplitude parameter 0 ≤ λ ≤ (1/400): (a)$x_{1,1}$=0.25L_1, (b)$x_{1,1}$=0.52L_1, (c)$x_{1,1}$=0.66L_1.

In Figs. 4and 5 the results consider a cross-tie with both negative and positive non-linearity. In the figures the skewness of the frequency is presented as a function of the stiffness parameter d_k. Frequency estimations, based on the layered SA algorithm and the RM theorem, are found by varying the number of the intervals Λ_r (m=4,6,8).

The accuracy of the proposed method increases with the number of intervals m. Comparison between BFM and RM results suggests that already with m=6 or m=8 the layered SA algorithm tends to the "exact" BFM curve. Figures 4 and 5 also suggest that the precision deteriorates (in both of cases) with the decrement of the stiffness coefficient d_k. This fact can be explained by noting that the skewness coefficient is normalized to the variance of the distribution, which tends to zero as d_k tends to zero.

6. Discussion and conclusions

The main contribution of this study is the demonstration that the skewness coefficient of the eigen-frequency distribution can be adequately evaluated using a layered SA algorithm. Clearly, the method provides new efficient avenues for the computation of additional statistical descriptors, in particular higher statistical moments, in an attempt to better examine the probability distribution of the random eigenvalues of stochastic cable-network free vibrations. In addition, the concept of eigenvector cloud is introduced to study the stochastic variability in the mode shapes. More research is needed to understand current limitations of the layered SA algorithm, for example by improving its convergence in the presence of a negative-stiffness nonlinear restoring force in the cross-ties.

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