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Analysis and Comparison of Some LEFM Parameters

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Abstract

This paper presents and analyses a possible extension of the well-known mean Strain Energy Density approach, proposed and developed by Paolo Lazzarin for the strength characterization and for the structural analysis of sharp notches. The new parameter, that here will be defined and discussed only for the case of a crack subjected to mode I loading conditions, will be shown to be able to characterize the superficial energy per unity of area due to the presence of a crack in a plate. Then it can be considered to be an Intensity Factor, in analogy to the Stress Intensity Factor K_I. For this reason it will be called the Strain Energy Density Intensity Factor (SEDIF). Aim of the introduction of this new approach is to simplify both the characterization of the material and the structural analysis of the components, since the proposed parameter does not depend on the strength of an un-notched specimen taken as reference and does not need the evaluation of the radius R_0 of the area to be considered for the evaluation of SED. Two in some way similar parameters (the J integral and the S factor proposed by Sih) will be discussed and compared to the proposed Strain Energy Density Intensity Factor.

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1. Introduction

The analysis of the strength of a structural component is usually accomplished by comparing the evaluated stresses to the strength of the material, although in general the assumed limiting parameter is the strain energy density (for metallic materials in the Beltrami or in the Von Mises formulation). The evaluated stresses are the nominal stresses

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(as in Classic Mechanics) or the local stresses (as in FEM assisted structural analysis), usually under the hypothesis of linear elastic behaviour of the material.

In Fracture Mechanics it was first developed an energy-based approach (Griffith 1921; Irwin 1948) and then a stress based approach (Irwin 1956). In 1957 Irwin (Irwin 1957), on the basis of the local stress field equations developed by Westergaard (Westergaard 1939), defined the singular stress field around the tip of a crack showing that it could be described by a single parameter that was related to the energy release rate. This approach was based on the $1/r^{0.5}$ singularity of the stress field around the crack tip. The local stress field was then represented by a single parameter, the Stress Intensity Factor K_I. The immediate success of this approach in LEFM could be ascribed to the fact that the SIF could be evaluated as a function of the gross nominal stress, the only stress that could be easily evaluated at that time. This approach is the most applied also today, but, with the today commonly employed Finite Element (FE) analysis, the nominal stress is not useful any more and the SIF is usually evaluated on the basis of the local stress field.

The energy release rate concept found an useful calculation way with the introduction of a path-independent integral, called J integral by Rice, (Eshelby 1956; Sanders 1960; Cherepanov 1967; Rice 1968) on the basis of a very general mathematical theory given by Noether in 1918. Since this parameter could be applied also to nonlinear elastic material behaviour, Rice generalised this approach to elastic-plastic material behaviour, while Hutchinson (Hutchinson 1968) and Rice and Rosengreen (Rice and Rosengren 1968) derived the so called HRR stress field, which is the crack-tip stress field for non-linear materials.

In Notch Mechanics, initially developed by several authors on the basis of the local FE evaluated stresses (Tanaka 1983; Atzori and Tovo 1992; Taylor 1999), the local strain energy density has been introduced as an useful parameter, both as a FE evaluated mean Strain Energy Density (Lazzarin and Zambardi 2001) or an experimentally measured heat energy dissipated per cycle Q (Meneghetti 2007).

All the different approaches above recalled are based, in an explicit or implicit way, on the homothetic stress and strain energy density fields around the crack tip, then they should be quite similar the one to the other, but, due to different reasons, the ways in which they are usually applied are different for each of them (e.g. the bi- or tri-axial state of stress is taken into account only in the assessment of the fracture toughness ΔK_{th} in the SIF approach, while is taken into account, but in different ways, in the J integral and in the SED approaches). Aim of the present paper is to introduce a new intensity factor, based on the mean SED approach proposed by Lazzarin, and to analyse (for the case of linear elastic behaviour of a crack loaded in mode I) the differences between some Strain Energy Density possible approaches.

2. Comparison between approaches based on strain energy density

Although several fracture and fatigue approaches based on the strain energy density concept have been proposed in the literature, for the purposes of the present work, three approaches will be analysed in the following, namely the J integral (Rice 1968) and two based on the strain energy density evaluated locally in a point or in an area around the crack tip (Sih 1974; Lazzarin and Zambardi 2001).

While the first two are field approaches (therefore they are not referred to a specific distance from the crack tip), the third one has been defined and it is generally applied with reference to a properly defined radius R_0. Here also the last one will be converted into a field approach. The obtained parameter, which can be thought as the surface energy per unit area required to cause each critical condition, will no longer be referred to a specific radius R_0, so that it will allow an easier and more general application of the averaged strain energy density concept, at least for the considered crack case.

The three approaches, which will now be briefly recalled considering the crack case under opening (mode I) loading under linear elastic behaviour:

- are based on the well-known Irwin equations to express the stress fields around the crack tip, which under plane stress conditions can be expressed as:

$$\begin{Bmatrix} \sigma_{\theta\theta} \\ \sigma_{rr} \\ \tau_{r\theta} \end{Bmatrix} = \frac{K_I}{\sqrt{2\pi r}} \left[\begin{Bmatrix} \frac{5}{4}\cos\frac{\theta}{2} \\ \frac{3}{4}\cos\frac{\theta}{2} \\ \frac{1}{4}\sin\frac{\theta}{2} \end{Bmatrix} + \begin{Bmatrix} -\frac{1}{4}\cos\frac{3\theta}{2} \\ \frac{1}{4}\cos\frac{3\theta}{2} \\ \frac{1}{4}\sin\frac{3\theta}{2} \end{Bmatrix} \right] = \frac{K_I}{\sqrt{r}} \begin{Bmatrix} \tilde{\sigma}_{\theta\theta}^{(1)} \\ \tilde{\sigma}_{rr}^{(1)} \\ \tilde{\tau}_{r\theta}^{(1)} \end{Bmatrix} \tag{1}$$

- use the total strain energy criterion by Beltrami, for the calculation of the strain energy density, which under plane stress conditions can be expressed as:

$$W(r,\theta) = \frac{1}{2E} \cdot [\sigma_{\theta\theta}{}^2 + \sigma_{rr}{}^2 - 2\nu\sigma_{\theta\theta}\sigma_{rr} + 2(1+\nu)\tau_{r\theta}{}^2] \tag{2}$$

- the approaches based on the strain energy density differ only for taking into account the strain energy densities referred to single points of the field (Sih) or those averaged in a properly defined volume (Lazzarin).

1) J integral: it is the energy parameter commonly employed under elastic-plastic behaviour, but it is increasingly used also under linear elastic behaviour, given the simplicity and speed of calculation by means of finite element methods, which perform an area integration for two-dimensional problems and a volume integration for three-dimensional problems.

In the case of linear elastic behaviour and opening (mode I) loading $J=K_I^2 / E'$, with E'=E for plane stress and $E'=E/(1-\nu^2)$ for plane strain. Since K_I can be expressed as a function of both the nominal stress and of the local stress normal to the crack bisector line, in the case of plane stress it can be derived:

$$J = \frac{\alpha^2\sigma_g^2(\pi a)}{E} = \lim_{x\to 0}\frac{\sigma_y^2(2\pi x)}{E} = 4\pi \cdot W_J \cdot x \qquad \text{being} \quad W_J = \lim_{x\to 0}\frac{\sigma_y^2}{2E} \tag{3}$$

and in the case of plain strain:

$$J = \frac{\alpha^2\sigma_g^2(\pi a)(1-\nu^2)}{E} = \lim_{x\to 0}\frac{\sigma_y^2(2\pi x)(1-\nu^2)}{E} = 4\pi \cdot W_J \cdot x \qquad \text{being} \quad W_J = \lim_{x\to 0}\frac{(1-\nu^2)\sigma_y^2}{2E} \tag{4}$$

2) Strain Energy Density Intensity Factor S: in several contributions starting from 1973, Sih introduced this new energy parameter, given by the product of the strain energy density W_S calculated in a given point by the distance of the same point from the crack tip.

This parameter is very simple and intuitive in the considered crack case, since it represents the natural extension of the stress field criterion (degree of singularity equal to 0.5) to a strain energy density field criterion (degree of singularity equal to 1). It is therefore not a point-wise criterion, as it has sometimes been referred to, but a field criterion, with a greater potential than the criterion based on K_I, such as the possibility of estimating in a simple and natural way not only the critical conditions for crack propagation, but also the direction of its propagation (Sih 1974). Differently from the stress field approach, this strain energy density field approach is practically forgotten today, but the simplicity of its use with the current diffusion of FE codes, recommends its rediscovery. This approach will be expressed as a function of the SIF K_I to allow a rapid comparison with the J integral.

In the case of plane stress, replacing Eqs. (1) in Eq. (2) and considering a generic point on the crack bisector (θ = 0) at a distance r = x from the crack tip, it can be derived:

$$W_s(x,\theta=0) = \frac{1}{x}\cdot\frac{K_I{}^2}{2E}\frac{[1+1-2\nu(1)]}{2\pi} = \frac{1}{x}\cdot\frac{(1-\nu)}{2\pi}\cdot\frac{K_I{}^2}{E} \tag{5}$$

Therefore:

$$S = W_s(x,\theta=0)\cdot x = \frac{1-\nu}{2\pi}\frac{K_I^2}{E} = \frac{1-\nu}{2\pi}J \tag{6}$$

While, in the case of plane strain:

$$W_s(x, \theta = 0) = \frac{1}{x} \cdot \frac{K_I^2}{2E} \frac{\left[1+1+(2\nu)^2-2\nu(1+2\nu+2\nu)\right]}{2\pi} = \frac{1}{x} \cdot \frac{K_I^2}{E} \frac{(1+\nu)(1-2\nu)}{2\pi} \tag{7}$$

Therefore

$$S = W_s(x, \theta = 0) \cdot x = \frac{(1+\nu)(1-2\nu)}{2\pi} \frac{K_I^2}{E} = \frac{(1-2\nu)}{2\pi(1-\nu)} J \tag{8}$$

3) Strain Energy Density Factor L: in several contributions starting from 2001, Lazzarin proposed a new energy parameter, the Strain Energy Density (SED), which has originally been introduced to allow the comparison of the criticality between sharp notches with different opening angles (such those at weld toe and weld root sides of welded joints). Then, it has been extended to successfully deal with several other problems, as summarized in (Berto and Lazzarin 2009) and subsequently, more widely, in (Radaj and Vormwald 2013).

In the considered crack case, the SED represents the average value W_L of the strain energy density calculated in the unit-thickness volume defined by a circumference having radius $x = R_0$ and being centred at the crack tip. Here, this parameter will be transformed into a corresponding field parameter, which we propose to call L, in honor and in memory of Prof. Paolo Lazzarin. This parameter can be derived by multiplying the average strain energy density W_L by the radius R_0: $L = W_L R_0$ (and therefore $W_L = L x^{-1}$). It does not depend on the distance from the crack tip and, contrary to the Sih parameter S, is unique, obviously not depending on a particular direction. In the following, it will be expressed as a function of the SIF K_I to allow a rapid comparison with the J integral.

In the case of plane stress, by employing Eqs. (1) and Eq. (2), it can be derived:

$$W_L = \frac{\int_A W(r,\theta)\, dA}{\int_A dA} = \frac{\int_0^{R_0}\left(\frac{1}{2E}\frac{K_I^2}{r} r\right) dr \int_{-\pi}^{+\pi} \widetilde{W}(\theta) d\theta}{\int_0^{R_0}\int_{-\pi}^{+\pi} r\, dr d\theta} = \frac{\frac{1}{2E} K_I^2 R_0 \cdot I_W(\pi)}{R_0^2 \pi} = \frac{1}{R_0} \cdot \frac{K_I^2}{E} \cdot \frac{(5-3\nu)}{8\pi} \tag{9}$$

therefore

$$L = W_L \cdot R_0 = \frac{5-3\nu}{8\pi} \frac{K_I^2}{E} = \frac{5-3\nu}{8\pi} J \tag{10}$$

while, in the case of plane strain:

$$W_L = \frac{\int_A W(r,\theta)\, dA}{\int_A dA} = \frac{\int_0^{R_0}\left(\frac{1}{2E}\frac{K_I^2}{r} r\right) dr \int_{-\pi}^{+\pi} \widetilde{W}(\theta) d\theta}{\int_0^{R_0}\int_{-\pi}^{+\pi} r\, dr d\theta} = \frac{\frac{1}{2E} K_I^2 R_0 \cdot I_W(\pi)}{R_0^2 \pi} = \frac{1}{R_0} \cdot \frac{K_I^2}{E} \cdot \frac{(1+\nu)(5-8\nu)}{8\pi} \tag{11}$$

therefore

$$L = W_L \cdot R_0 = \frac{(1+\nu)(5-8\nu)}{8\pi} \frac{K_I^2}{E} = \frac{(5-8\nu)}{8\pi(1-\nu)} J \tag{12}$$

Finally, a linear-elastic FE analysis has been performed by using Ansys FE code and by modelling the AISI 304L steel specimen shown in Fig. 1, whose fatigue and crack propagation behaviours have been analysed by some of the present authors in a previous contribution (Meneghetti et al. 2016). The total length a = 18 mm represents the crack length plus the notch depth. Figures 2 and 3 report the numerical results relevant to the stress field components and to the three considered strain energy density parameters, respectively. The asymptotic lines which represent the stress field and the strain energy density fields have been derived from the J integral values, directly calculated by Ansys considering plane stress or plane strain conditions. A very good agreement can be observed between numerical results and the results based on the J-integral value.

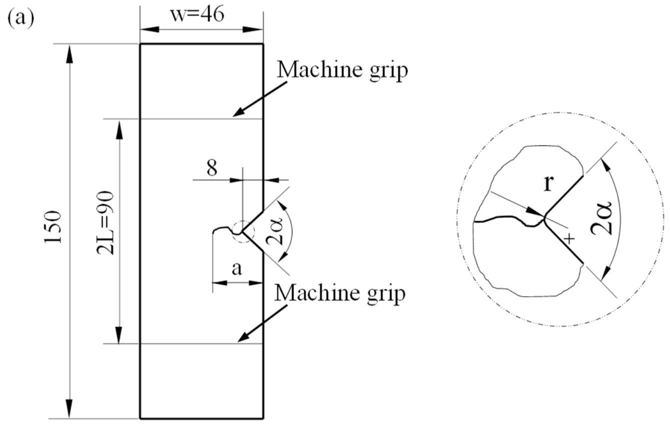

Fig. 1. Specimen geometry (r=0.1 mm, 2α=45°, a=18 mm, thickness is 4 mm; dimensions in mm).

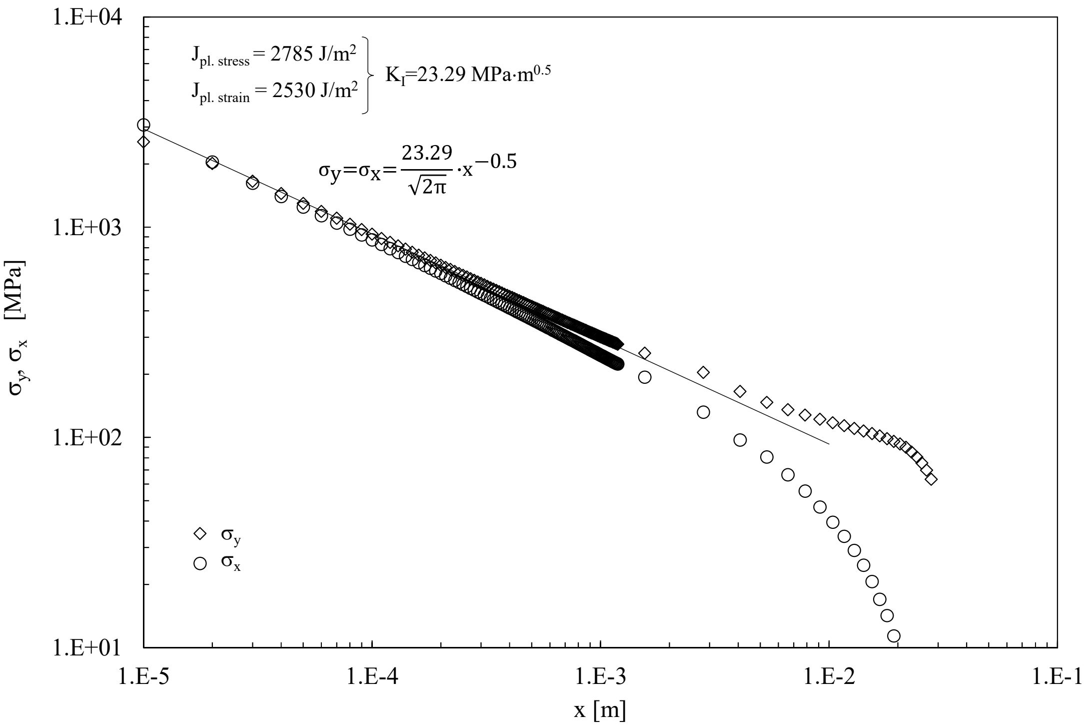

Fig. 2. Local stress rise along the crack bisector line and related parameters obtained by finite element analysis.

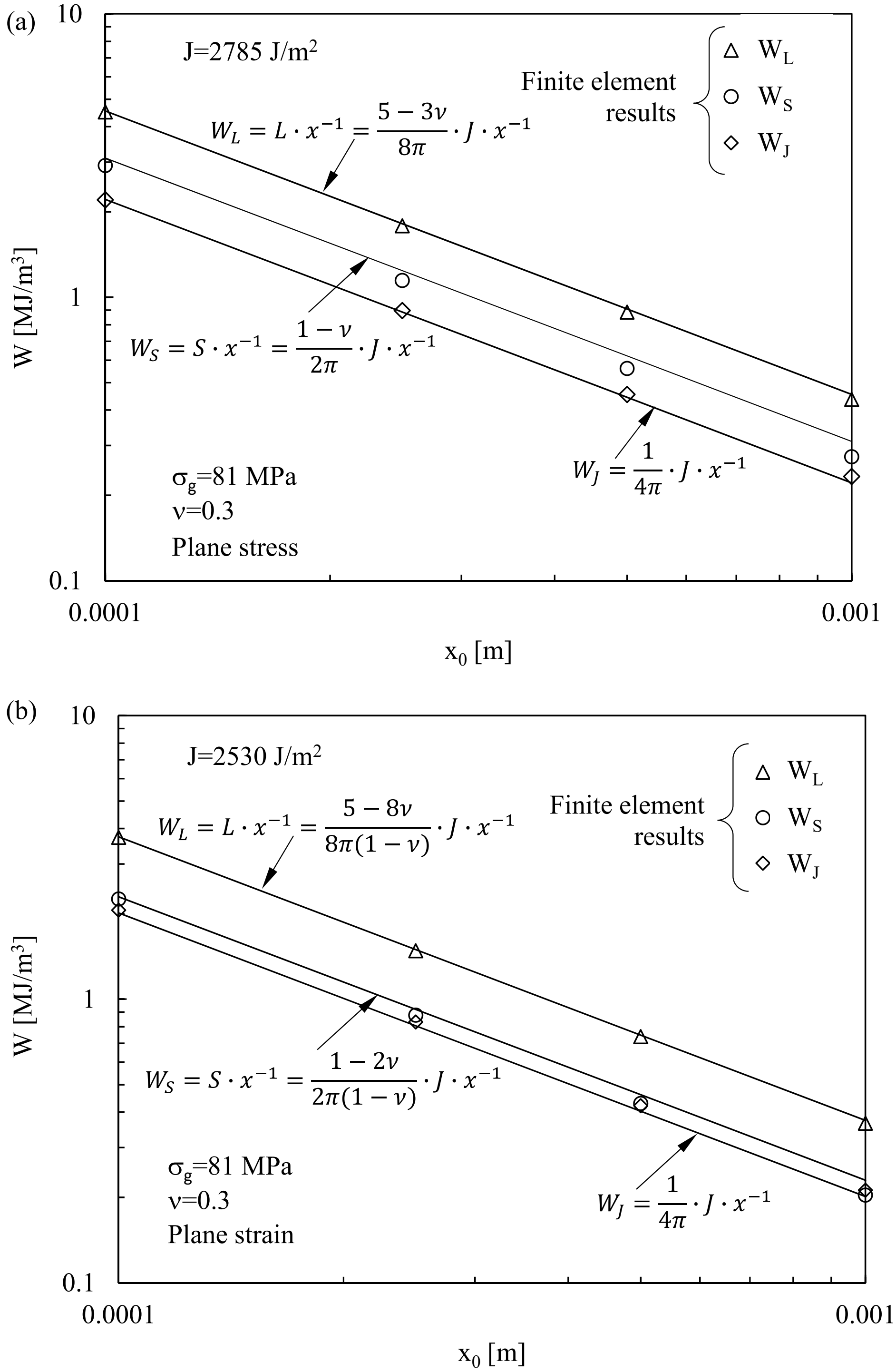

Figure 3. Local Strain Energy Densities (W_L, W_S, W_J) rise along the crack bisector line and related parameters obtained by finite element analysis, assuming (a) plane stress and (b) plane stain conditions.

3. Fatigue strength expressed by strain energy density intensity factors

The fatigue initiation and short propagation life of structural components with cracks or sharp notches could be today analysed as a function of the local stress (Tanaka 1983; Atzori and Tovo 1992; Taylor 1999), or the local Strain Energy Density (Lazzarin and Zambardi 2001), related in various ways to the fatigue limit of an un-notched specimen. Although these approaches have been very useful in the past and quite well known and applied today, in our opinion the Strain Energy Density Intensity Factor (SEDIF) here introduced could be a more general and straightforward parameter to characterize this fatigue behaviour. As a matter of fact, when the fatigue failure starts from a crack of the structural component, the fatigue behaviour is related to the homothetic field of stress, strain and strain energy density around the tip of the crack, not to the uniform conditions of an un-notched specimen, and the matching of the two situations at fatigue threshold could be misleading.

In principle, as far as there is a null notch opening angle and ν is considered to be constant, each one of the discussed Strain Energy Density Intensity Factors could be used (or similar ones, e.g. considering the deviatoric SED, as also proposed by Lazzarin), since each of them has its peculiar advantages and disadvantages, but the discussion of this subject is beyond the aims of this paper. Due to the existing correlation between the different parameters, the fatigue curves corresponding to each of them are very similar, since the inverse slope k is the same for all of them, as shown in Fig. 4 for an 8 mm lateral crack in plane strain. In the same figure the theoretical curves have been compared to the results of a series of experimental fatigue tests on the notched specimens of Fig. 1 (without the crack), fully reported in (Meneghetti et al. 2016), where they have been employed for the evaluation of the ΔK_{th} of the material (AISI 304 L). Since the degree of singularity of the stress field for the 45° open notch is very similar to the one of the 0° notch (i.e. the crack case), the SEDIF for the fatigue data have been evaluated on the equivalent 0° V-notch. Whichever will be the chosen parameter, it is evident from the figure that the characterization of the crack initiation fatigue life of the material will be defined by itself, obviously by different curves depending on the chosen parameter, but in any case directly related to the crack tip conditions, not to the very different conditions of an un-notched specimen as in the usual approaches. As it is well known, in that case the fatigue behaviour of a structural component with a sharp crack is evaluated on the basis of a critical distance correlated, in different ways, to a fictitious "intrinsic crack" a_0 on the un-notched specimen, function of the fatigue limits $\Delta\sigma_0$ of the plain material and ΔK_{th} of the material with a long crack:

$$a_0 = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\Delta\sigma_0}\right)^2 \tag{13}$$

This parameter, which has been used to analyse the "defect sensitivity" (that is the transition between the long crack and the plain material fatigue limits), can be evaluated also for the proposed Strain Energy Density Intensity Factor fatigue characterization. For plane strain we have:

$$\Delta J_0 = \frac{\Delta\sigma_0^2 \pi a_0 \cdot (1-\nu^2)}{E} = \Delta J_{th} = \frac{\Delta K_{th}^2}{E}(1-\nu^2) \tag{14}$$

$$\Delta S_0 = \frac{(1-2\nu)}{2\pi\cdot(1-\nu)}\Delta J_0 = \frac{(1-2\nu)}{2\pi\cdot(1-\nu)}\Delta J_{th} \tag{15}$$

$$\Delta L_0 = \frac{(5-8\nu)}{8\pi\cdot(1-\nu)}\Delta J_0 = \frac{(5-8\nu)}{8\pi\cdot(1-\nu)}\Delta J_{th} \tag{16}$$

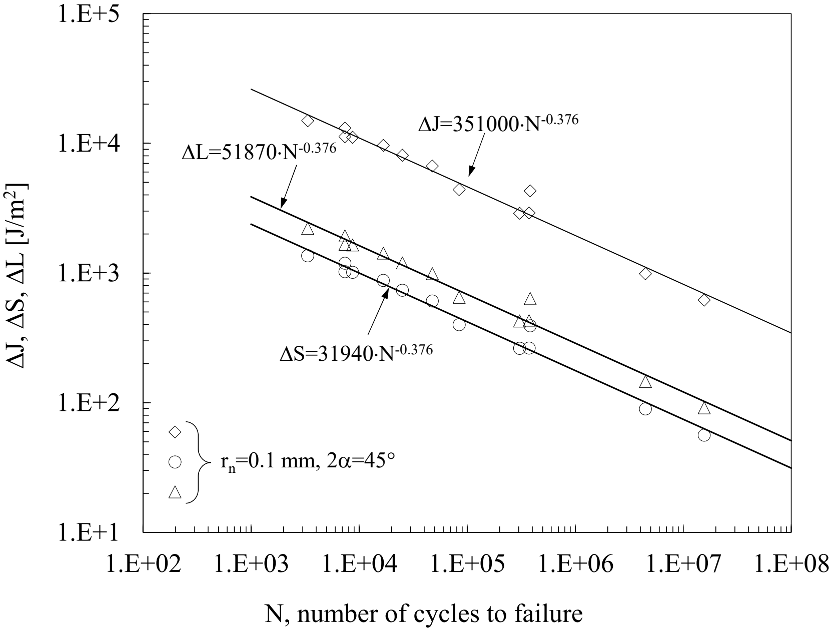

Fig. 4. Fatigue results obtained on the sharp notched specimens of Figure 1 (without the crack) as a function of different Strain Energy Density Intensity Factors evaluated on a crack of the same length.

It is immediate to verify that the parameter $a_0 = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\Delta\sigma_0}\right)^2$ is the same that could be evaluated with the usual Stress Intensity Factor approach and that it does not change when considering a plane stress condition or a different Strain Energy Density approach.

As far as the matching between sharp cracks and plain material fatigue behaviour, although not necessary for the proposed SEDIF approach, it seems to be quite natural to extend to any number of cycles N_i the comparison between notched and un-notched behaviour (as shown in Fig. 5) by introducing a "matching parameter $a_i\,(N_i)$" function of the chosen life:

$$a_i(N_i) = \frac{1}{\pi}\left(\frac{\Delta K_i(N_i)}{\Delta\sigma_i(N_i)}\right)^2 \tag{17}$$

In Fig. 5 the comparison is shown not only for the usual N_{th} ($N_{th} = 93.8\text{x}10^6$ cycles for the analysed material) but also for $N = 0.16\text{x}10^6$ cycles (knee at the fatigue limit of the un-notched specimen) and for $N = 10^3$cycles (usual matching point assumed for blunt notches analysed in nominal net stress). The corresponding values of the matching parameter $a_i\,(N_i)$ change from 0.147 mm to 6.05 mm.

4. Conclusions

An extension of the Strain Energy Density approach, proposed and developed by P. Lazzarin, has been given and discussed. For this purpose a Strain Energy Density Intensity Factor L has been introduced and the correlations with two in same way similar parameters (the J integral and the S factor) have been given. The analysis has evidenced that all the considered parameters can be expressed as a function of J-integral multiplied (for each of them) by a different constant which is function only of the Poisson's coefficient. The constant has different formulation for the case of plane stress and for that of plane strain, differentiating then the Intensity Factor approaches in Stress and in Strain Energy Density. It has been also evidenced that all the considered parameters are suitable to characterize in strain energy density the fatigue strength of a material for a very sharp notch with zero or small opening angle, in a way that is not dependent on the fatigue strength of an un-notched specimen. For the case of the fatigue limits the analysis has shown that the classical correlation between cracked and plain material behaviour, performed through an “intrinsic crack” a_0 for an engineering evaluation of the “defect sensitivity”, is valid also for the energy-based approaches, with the same value of a_0 for all the approaches.

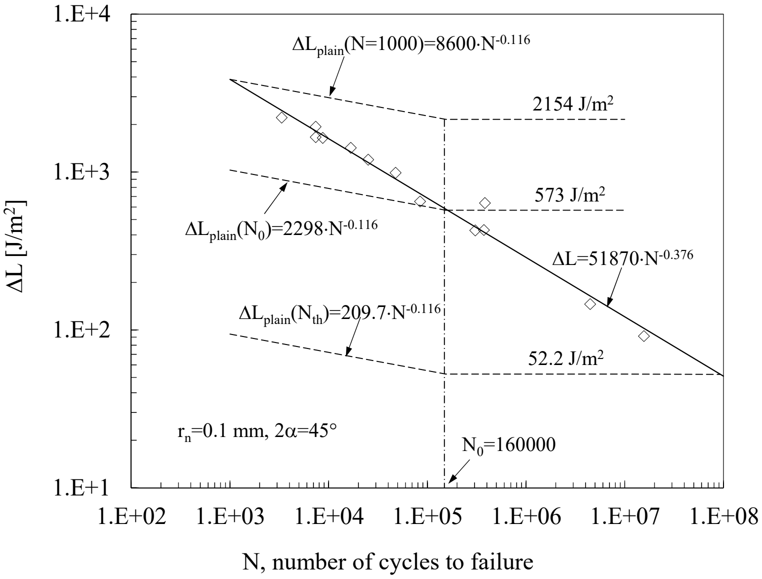

Fig. 5. Illustration of the $a_i(N_i)$ concept for the ΔL-N curve of Figure 4; the choice of N_i will not influence the fatigue curve of the specimen with a crack when the SEDIF is taken as reference.

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