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# Nonlinear H-infinity control for switched reluctance machines

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**Abstract:** The article proposes a nonlinear H-infinity control method for switched reluctance machines. The dynamic model of the switched reluctance machine undergoes approximate linearization round local operating points which are redefined at each iteration of the control algorithm. These temporary equilibria consist of the last value of the reluctance machine's state vector and of the last value of the control signal that was exerted on it. For the approximate linearization of the reluctance machine's dynamics, Taylor series expansion is performed through the computation of the associated Jacobian matrices. The modelling errors are compensated by the robustness of the control algorithm. Next, for the linearized equivalent model of the reluctance machine an H-infinity feedback controller is designed. This requires the solution of an algebraic Riccati equation at each time-step of the control method. It is shown that the control scheme achieves H-infinity tracking performance, which implies maximum robustness to modelling errors and external perturbations. The stability of the control loop is proven through Lyapunov analysis.

**Keywords:** switched reluctance machines, nonlinear H-infinity control, approximate linearization, Taylor series expansion, Jacobian matrices, Riccati equation, robust control, asymptotic stability

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## 1 Introduction

Switched reluctance machines (SRM) exhibit specific advantages comparing to other electric machines. Unlike electric machines having windings at the rotor (such as electromagnetically excited synchronous or wound rotor asynchronous AC machines as well as DC motors), the manufacturing of SRMs is easier and the appearance of faults is less frequent [1–4]. Moreover, such machines exhibit improved torque characteristics thus becoming suitable for several electromechanical actuation systems. Additionally, comparing to other electric machines the SRM power consumption is reduced thus making them suitable for use in industry [5–8]. Control of SRMs is a nontrivial problem because their dynamic model is a highly nonlinear one [9–14]. Stability and robustness are also features of primary importance in the development of SRM control schemes [15–20]. It is noteworthy that the use of SRMs in traction of electric vehicles is gaining ground [21–26]. Actually such motors are less costly, less prone to failure and more convenient to install and maintain than Permanent Magnet motors [27–31]. However, it should not be overlooked that due to the nonlinearities in SRMs electric dynamics induced by the simultaneous activation of several stator phases, the control of these motors remains an open and challenging problem [32–36]. It is remarkable, that despite the raise of research production on this topic few new results have been given on globally stable control methods [37–39]. It is also noted that in an aim to implement control of SRMs under model uncertainty learning-based and robust control methods have been developed [40–42]. In particular adaptive control addressed to complex nonlinear dynamical systems can be also considered for the control problem of SRMs under an imprecise or even unknown model [43–45].

In this article an H-infinity control method is developed for switched reluctance machines. The dynamic model of the SRM undergoes approximate linearization around local operating points (equilibria) which are recomputed at each iteration of the control algorithm [46, 47]. These equilibria are defined by the present value of the SMR's state vector and the last value of the control input vector that was exerted on it. The linearization procedure

makes use of Taylor series expansion [48–51]. This relies on the computation of the Jacobian matrices of the SRM's state-space model. The modelling error which is due to truncation of higher order terms in the Taylor series expansion is considered as perturbation to the system's dynamic model which is finally compensated by the robustness of the control method. For the linearized model of the SRM an H-infinity feedback controller is computed.

The H-infinity controller for the SRM stands for the solution of the nonlinear optimal control problem for this machine under model uncertainty and external perturbations. It actually represents the solution of a mini-max differential game in which the control inputs try to minimize a quadratic cost functional of the SRM's state vector error while the model uncertainty and perturbations' inputs try to maximize it. The feedback gain of the H-infinity controller is obtained from the solution of an algebraic Riccati equation, taking place at each iteration of the control method [52–56]. The stability properties of the control scheme are confirmed through Lyapunov analysis. First, it is shown that the control loop satisfies the H-infinity tracking performance criterion. This signifies elevated robustness against model uncertainty and exogenous disturbances. Moreover, under moderate conditions it is proven that the control loop is globally asymptotically stable. Finally, to implement the proposed H-infinity control method using only output feedback, state estimation for the SRM with the H-infinity Kalman Filter is proposed [57, 58].

The structure of the article is as follows: in Section 2 the dynamic model of the switched reluctance machine is analyzed and its state-space model is obtained. In Section 3 the approximately linearized model of the SRM is developed through Taylor series expansion and the computation of the system's Jacobian matrices. In Section 4 the linearized description of the SRM's dynamics is used to develop an H-infinity feedback controller. In Section 5 the stability of the H-infinity feedback control scheme is proven through Lyapunov analysis. In Section 6 robust state estimation for the SRM's model is developed using the H-infinity Kalman Filter. This allows for the implementation of state-estimation based feedback control through the processing of measurements from a limited number of sensors. In Section 7 the efficiency of the proposed control method for the SRM model is confirmed through simulation experiments. Finally, in Section 8 concluding remarks are stated.

## 2 Dynamic model of the Switched Reluctance machine

It is considered that the switched reluctance machine (SRM) comprises  $m$  phases  $j = 1, 2, \dots, m$  as shown in Fig. 1. Then by applying Kirchhoff's law at the  $j$ -th phase one has [6]

$$v_j = Ri_j + \frac{d\psi_j}{dt} \quad j = 1, 2, \dots, m \quad (1)$$

The aggregate electric torque is the sum of the torques generated by the individual phases of the machine [6]

$$T_e(\theta, i_1, i_2, \dots, i_m) = \sum_{j=1}^m T_j(\theta, i_j) \quad (2)$$

where  $T_j$  is defined using the co-energy function

$$T_j(\theta, i_j) = \frac{\partial W'_j}{\partial \theta} \quad j = 1, 2, \dots, m \quad (3)$$

where the co-energy function is given by

$$W'_j(\theta, i_j) = \int_0^{i_j} \psi_j(\theta, i_j) di_j \quad (4)$$

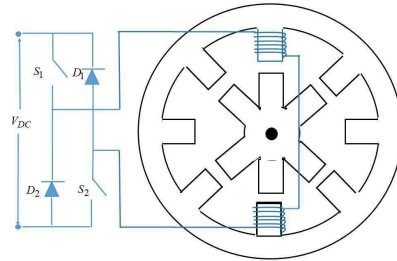


Fig. 1: Switched reluctance machine and its control circuit

The term  $\psi_j(\theta, i_j)$  denotes the flux linkage and is given by [6]

$$\psi_j(\theta, i_j) = \psi_s(1 - e^{-i_j f_j(\theta)}) \quad (5)$$

$j = 1, 2, \dots, m$ .  $\psi_s$  is magnetic flux at saturation, while  $f_j(\theta)$  is initially considered to be given by the Fourier series expansion

$$f_j(\theta) = a + \sum_{n=1}^{\infty} \{b_n \sin \left[ nN_r \theta - (j-1) \frac{2\pi}{m} \right] + c_n \cos \left[ nN_r \theta - (j-1) \frac{2\pi}{m} \right]\} \quad (6)$$

where  $N_r$  is the number of rotor poles, and finally by truncating higher order terms in this expansion one gets

$$f_j(\theta) = a + b \sin \left[ N_r \theta - (j-1) \frac{2\pi}{m} \right] \quad (7)$$

Using the previous relations, the electric torque of the machine due to the  $j$ -th phase is shown to be given by [6]

$$T_j(\theta, i_j) = \frac{\psi_s}{f_j^2(\theta)} \frac{\partial f_j(\theta)}{\partial \theta} \{1 - [1 + i_j f_j(\theta)] e^{-i_j f_j(\theta)}\} \quad (8)$$

where  $j = 1, 2, \dots, m$ . In the case of a switched reluctance machine with  $m$  phases, the machine's state vector comprises the following state variables  $[\theta, \omega, i_1, i_2, \dots, i_m]$ . The state-space equations of the machine are given by [6, 11]

$$\frac{d\theta}{dt} = \omega \quad (9)$$

$$\frac{d\omega}{dt} = \frac{1}{J} \left\{ \sum_{j=1}^m T_j(\theta, i_j) - T_l(\theta, \omega) \right\} \quad (10)$$

$$\frac{di_j}{dt} = - \left( \frac{\partial \psi_j}{\partial i_j} \right)^{-1} \left( Ri_j + \frac{\partial \psi_j}{\partial \theta} \omega \right) + \left( \frac{\partial \psi_j}{\partial i_j} \right)^{-1} u_j \quad (11)$$

Next, without loss of generality the case of a switched reluctance machine with  $m = 4$  phases is considered. The machine's state vector is  $[\theta, \omega, i_1, i_2, i_3, i_4] = [x_1, x_2, x_3, x_4, x_5, x_6]$ . The state-space equations of the machine are

$$\dot{x}_1 = x_2 \quad (12)$$

$$\begin{aligned} \dot{x}_2 &= \frac{1}{J} [T_1(\theta, x_3) + T_2(\theta, x_4) + T_3(\theta, x_5) + T_4(\theta, x_6) \\ &\quad - T_l(x_1, x_2)] \Rightarrow \\ \dot{x}_2 &= \frac{1}{J} \left[ \frac{\psi_s}{f_1^2(x_1)} \frac{\partial f_1(x_1)}{\partial x_1} N_r \{1 - [1 + x_3 f_1(x_1)] e^{-x_3 f_1(x_1)}\} \right. \\ &\quad + \frac{\psi_s}{f_2^2(x_1)} \frac{\partial f_2(x_1)}{\partial x_1} N_r \{1 - [1 + x_4 f_2(x_1)] e^{-x_4 f_2(x_1)}\} \\ &\quad + \frac{\psi_s}{f_3^2(x_1)} \frac{\partial f_3(x_1)}{\partial x_1} N_r \{1 - [1 + x_5 f_3(x_1)] e^{-x_5 f_3(x_1)}\} \\ &\quad + \frac{\psi_s}{f_4^2(x_1)} \frac{\partial f_4(x_1)}{\partial x_1} N_r \{1 - [1 + x_6 f_4(x_1)] e^{-x_6 f_4(x_1)}\} \\ &\quad \left. - Bx_2 - mgl \sin(x_1) \right] \quad (13) \end{aligned}$$

$$\begin{aligned} \dot{x}_3 &= [-\psi_s e^{-x_3 f_1(x_1)} f_1(x_1)]^{-1} [Rx_3 + (\psi_s e^{-x_3 f_1(x_1)}) \\ &\quad \cdot (x_3 \cdot \frac{\partial f_1(x_1)}{\partial x_1}) x_2] + [\psi_s e^{-x_3 f_1(x_1)} f_1(x_1)]^{-1} u_1 \quad (14) \end{aligned}$$

$$\dot{x}_4 = [-\psi_s e^{-x_4 f_2(x_1)} f_2(x_1)]^{-1} [Rx_4 + (\psi_s e^{-x_4 f_2(x_1)})$$

$$\cdot (x_4 \cdot \frac{\partial f_2(x_1)}{\partial x_1})] x_2 + [\psi_s e^{-x_4 f_2(x_1)} f_2(x_1)]^{-1} u_2 \quad (15)$$

$$\begin{aligned} \dot{x}_5 &= [-\psi_s e^{-x_5 f_3(x_1)} f_3(x_1)]^{-1} [Rx_5 + (\psi_s e^{-x_5 f_3(x_1)}) \\ &\quad \cdot (x_5 \cdot \frac{\partial f_3(x_1)}{\partial x_1}) x_2] + [\psi_s e^{-x_5 f_3(x_1)} f_3(x_1)]^{-1} u_3 \quad (16) \end{aligned}$$

$$\begin{aligned} \dot{x}_6 &= [-\psi_s e^{-x_6 f_4(x_1)} f_4(x_1)]^{-1} [Rx_6 + (\psi_s e^{-x_6 f_4(x_1)}) \\ &\quad \cdot (x_6 \cdot \frac{\partial f_4(x_1)}{\partial x_1}) x_2] + [\psi_s e^{-x_6 f_4(x_1)} f_4(x_1)]^{-1} u_4 \quad (17) \end{aligned}$$

where

$$\frac{\partial f_j}{\partial x_1} = b N_s \cos(N_s x_1 - (j-1) \frac{2\pi}{m}) \quad j = 1, 2, \dots, m \quad (18)$$

It is noted that in the above state-space description  $Bx_2$  is a damping term that opposes to the rotational motion of the machine, while  $mgl \sin(x_1)$  is the mechanical load torque, for instance in the case that the SRM lifts a rod of length  $l$  with a mass  $m$  attached to its end.

### 3 Linearization for the switched reluctance machine

For the SRM's rotor dynamics it has been shown to hold

$$\dot{x}_1 = x_2 \quad (19)$$

$$\begin{aligned} \dot{x}_2 &= \frac{1}{J} [T_1(\theta, x_3) + T_2(\theta, x_4) + T_3(\theta, x_5) + T_4(\theta, x_6) \\ &\quad - T_l(x_1, x_2)] \Rightarrow \\ \dot{x}_2 &= \frac{1}{J} \left[ \frac{\psi_s}{f_1^2(x_1)} \frac{\partial f_1(x_1)}{\partial x_1} N_r \{1 - [1 + x_3 f_1(x_1)] e^{-x_3 f_1(x_1)}\} \right. \\ &\quad + \frac{\psi_s}{f_2^2(x_1)} \frac{\partial f_2(x_1)}{\partial x_1} N_r \{1 - [1 + x_4 f_2(x_1)] e^{-x_4 f_2(x_1)}\} \\ &\quad + \frac{\psi_s}{f_3^2(x_1)} \frac{\partial f_3(x_1)}{\partial x_1} N_r \{1 - [1 + x_5 f_3(x_1)] e^{-x_5 f_3(x_1)}\} \\ &\quad + \frac{\psi_s}{f_4^2(x_1)} \frac{\partial f_4(x_1)}{\partial x_1} N_r \{1 - [1 + x_6 f_4(x_1)] e^{-x_6 f_4(x_1)}\} \\ &\quad \left. - Bx_2 - mgl \sin(x_1) \right] \quad (20) \end{aligned}$$

Eq. (20) can be also written in the form

$$\begin{aligned} \dot{x}_2 &= \frac{1}{J} \left\{ \frac{\psi_s}{f_1^2(x_1)} \frac{\partial f_1(x_1)}{\partial x_1} N_r [1 - e^{-x_3 f_1(x_1)}] \right. \\ &\quad + \frac{\psi_s}{f_1^2(x_1)} \frac{\partial f_1(x_1)}{\partial x_1} N_r [-f_1(x_1) e^{-x_3 f_1(x_1)}] x_3 \} \\ &\quad + \frac{1}{J} \left\{ \frac{\psi_s}{f_2^2(x_1)} \frac{\partial f_2(x_1)}{\partial x_1} N_r [1 - e^{-x_4 f_2(x_1)}] \right. \end{aligned}$$

$$\begin{aligned}
& + \frac{\psi_s}{f_2^2(x_1)} \frac{\partial f_2(x_1)}{\partial x_1} N_r [-f_2(x_1) e^{-x_4 f_2(x_1)}] x_4 \} \\
& + \frac{1}{J} \left\{ \frac{\psi_s}{f_3^2(x_1)} \frac{\partial f_3(x_1)}{\partial x_1} N_r [1 - e^{-x_5 f_3(x_1)}] \right. \\
& + \frac{\psi_s}{f_3^2(x_1)} \frac{\partial f_3(x_1)}{\partial x_1} N_r [-f_3(x_1) e^{-x_5 f_3(x_1)}] x_5 \} \\
& + \frac{1}{J} \left\{ \frac{\psi_s}{f_4^2(x_1)} \frac{\partial f_4(x_1)}{\partial x_1} N_r [1 - e^{-x_6 f_4(x_1)}] \right. \\
& + \frac{\psi_s}{f_4^2(x_1)} \frac{\partial f_4(x_1)}{\partial x_1} N_r [-f_4(x_1) e^{-x_6 f_4(x_1)}] x_6 \} \\
& - \frac{B}{J} x_2 - \frac{mgl}{J} \sin(x_1)
\end{aligned} \quad (21)$$

By defining the auxiliary functions

$$\begin{aligned}
f_a(x) &= \frac{1}{J} \left\{ \frac{\psi_s}{f_1^2(x_1)} \frac{\partial f_1(x_1)}{\partial x_1} N_r [1 - e^{-x_3 f_1(x_1)}] \right\} \\
g_a(x) &= \frac{1}{J} \left\{ \frac{\psi_s}{f_1^2(x_1)} \frac{\partial f_1(x_1)}{\partial x_1} N_r [-f_1(x_1) e^{-x_3 f_1(x_1)}] \right\} \\
f_b(x) &= \frac{1}{J} \left\{ \frac{\psi_s}{f_2^2(x_1)} \frac{\partial f_2(x_1)}{\partial x_1} N_r [1 - e^{-x_4 f_2(x_1)}] \right\} \\
g_a(x) &= \frac{1}{J} \left\{ \frac{\psi_s}{f_2^2(x_1)} \frac{\partial f_2(x_1)}{\partial x_1} N_r [-f_2(x_1) e^{-x_4 f_2(x_1)}] \right\} \\
f_c(x) &= \frac{1}{J} \left\{ \frac{\psi_s}{f_3^2(x_1)} \frac{\partial f_3(x_1)}{\partial x_1} N_r [1 - e^{-x_5 f_3(x_1)}] \right\} \\
g_c(x) &= \frac{1}{J} \left\{ \frac{\psi_s}{f_3^2(x_1)} \frac{\partial f_3(x_1)}{\partial x_1} N_r [-f_3(x_1) e^{-x_5 f_3(x_1)}] \right\} \\
f_d(x) &= \frac{1}{J} \left\{ \frac{\psi_s}{f_4^2(x_1)} \frac{\partial f_4(x_1)}{\partial x_1} N_r [1 - e^{-x_6 f_4(x_1)}] \right\} \\
g_d(x) &= \frac{1}{J} \left\{ \frac{\psi_s}{f_4^2(x_1)} \frac{\partial f_4(x_1)}{\partial x_1} N_r [-f_4(x_1) e^{-x_6 f_4(x_1)}] \right\}
\end{aligned} \quad (22)$$

Eq. (21) can be also written in the form

$$\begin{aligned}
\dot{x}_2 &= [f_a(x) + g_a(x)x_3] + [f_b(x) + g_b(x)x_4] \\
& + [f_c(x) + g_c(x)x_5] + [f_d(x) + g_d(x)x_6] \\
& - \frac{B}{J} x_2 - \frac{mgl}{J} \sin(x_1)
\end{aligned} \quad (23)$$

By differentiating Eq. (23) with respect to time one gets

$$\begin{aligned}
\ddot{x}_2 &= [\dot{f}_a(x) + \dot{g}_a(x)x_3 + g_a(x)\dot{x}_3] \\
& + [\dot{f}_b(x) + \dot{g}_b(x)x_4 + g_b(x)\dot{x}_4] \\
& + [\dot{f}_c(x) + \dot{g}_c(x)x_5 + g_c(x)\dot{x}_5] \\
& + [\dot{f}_d(x) + \dot{g}_d(x)x_6 + g_d(x)\dot{x}_6] \\
& - \frac{B}{J} \dot{x}_2 - \frac{mgl}{J} \cos(x_1)\dot{x}_1
\end{aligned} \quad (24)$$

Additionally Eq. (14) to Eq. (17) are used. By defining the auxiliary functions

$$p_a(x) = [-\psi_s e^{-x_3 f_1(x_1)} f_1(x_1)]^{-1} [R x_3 + (\psi_s e^{-x_3 f_1(x_1)})$$

$$\begin{aligned}
& \cdot (x_3 \cdot \frac{\partial f_1(x_1)}{\partial x_1}) x_2] \\
q_a(x) &= [\psi_s e^{-x_3 f_1(x_1)} f_1(x_1)]^{-1} \\
p_b(x) &= [-\psi_s e^{-x_4 f_2(x_1)} f_2(x_1)]^{-1} [R x_4 + (\psi_s e^{-x_4 f_2(x_1)}) \\
& \cdot (x_4 \cdot \frac{\partial f_2(x_1)}{\partial x_1}) x_2] \\
q_b(x) &= [\psi_s e^{-x_4 f_2(x_1)} f_2(x_1)]^{-1} \\
p_c(x) &= [-\psi_s e^{-x_5 f_3(x_1)} f_3(x_1)]^{-1} [R x_5 + (\psi_s e^{-x_5 f_3(x_1)}) \\
& \cdot (x_5 \cdot \frac{\partial f_3(x_1)}{\partial x_1}) x_2] \\
q_c(x) &= [\psi_s e^{-x_5 f_3(x_1)} f_3(x_1)]^{-1} \\
p_d(x) &= [-\psi_s e^{-x_6 f_4(x_1)} f_4(x_1)]^{-1} [R x_6 + (\psi_s e^{-x_6 f_4(x_1)}) \\
& \cdot (x_6 \cdot \frac{\partial f_4(x_1)}{\partial x_1}) x_2] \\
q_d(x) &= [\psi_s e^{-x_6 f_4(x_1)} f_4(x_1)]^{-1}
\end{aligned} \quad (25)$$

Using Eq. (25), one can write Eq. (14) to Eq. (17) in the form

$$\begin{aligned}
\dot{x}_3 &= p_a(x) + q_a(x)u_1 \\
\dot{x}_4 &= p_b(x) + q_b(x)u_2 \\
\dot{x}_5 &= p_c(x) + q_c(x)u_3 \\
\dot{x}_6 &= p_d(x) + q_d(x)u_4
\end{aligned} \quad (26)$$

By substituting Eq. (26) into Eq. (23) one gets

$$\begin{aligned}
\ddot{x}_2 &= \dot{f}_a(x) + \dot{g}_a(x)x_3 + g_a(x)p_a(x) + g_a(x)q_a(x)u_1 + \\
& \dot{f}_b(x) + \dot{g}_b(x)x_4 + g_b(x)p_b(x) + g_b(x)q_b(x)u_2 + \dot{f}_c(x) \\
& + \dot{g}_c(x)x_5 + g_c(x)p_c(x) + g_c(x)q_c(x)u_3 + \dot{f}_d(x) + \dot{g}_d(x)x_6 \\
& + g_d(x)p_d(x) + g_d(x)q_d(x)u_4 - \frac{B}{J} \dot{x}_2 - \frac{mgl}{J} \cos(x_1)\dot{x}_1
\end{aligned} \quad (27)$$

Considering that input voltages  $u_j$ ,  $j = 1, 2, 3, 4$  are generated by a commutation scheme (Fig. 1), that is  $u_j = k_j u$ ,  $j = 1, 2, 3, 4$  where  $k_j$  can take values equal to 0 or 1, Eq. (27) is written as

$$\begin{aligned}
\ddot{x}_2 &= [\dot{f}_a(x) + \dot{g}_a(x)x_3 + g_a(x)p_a(x) + \dot{f}_b(x) + \dot{g}_b(x)x_4 \\
& + g_b(x)p_b(x) + \dot{f}_c(x) + \dot{g}_c(x)x_5 + g_c(x)p_c(x) + \dot{f}_d(x) \\
& + \dot{g}_d(x)x_6 + g_d(x)p_d(x)] + [g_a(x)q_a(x)k_1 + g_b(x)q_b(x)k_2 \\
& + g_c(x)q_c(x)k_3 + g_d(x)q_d(x)k_4]u - \frac{B}{J} \dot{x}_2 - \frac{mgl}{J} \cos(x_1)\dot{x}_1
\end{aligned} \quad (28)$$

Next, by defining functions  $F(x) = [\dot{f}_a(x) + \dot{g}_a(x)x_3 + g_a(x)p_a(x) + \dot{f}_b(x) + \dot{g}_b(x)x_4 + g_b(x)p_b(x) + \dot{f}_c(x) + \dot{g}_c(x)x_5 + g_c(x)p_c(x) + \dot{f}_d(x) + \dot{g}_d(x)x_6 + g_d(x)p_d(x)]$  and  $G(x) = [g_a(x)q_a(x)k_1 + g_b(x)q_b(x)k_2 + g_c(x)q_c(x)k_3 + g_d(x)q_d(x)k_4]$  one can write Eq. (28) in the form

$$\ddot{x}_2 = F(x) + G(x)u - \frac{B}{J} \dot{x}_2 - \frac{mgl}{J} \cos(x_1)\dot{x}_1 \quad (29)$$

The new control input  $v = F(x) + G(x)u$  is defined. Moreover, the new state vector  $x = [x_1, \dot{x}_1, \ddot{x}_1]^T$  is now introduced. Thus now  $x_1 = \theta$ ,  $x_2 = \omega$  and  $x_3 = \dot{\omega}$ . This allows to obtain the following state-space description for the SRM dynamics

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= -\frac{B}{J}x_3 - \frac{mgl}{J}\cos(x_1)x_2 + v\end{aligned}\quad (30)$$

or equivalently, in matrix form

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} x_2 \\ x_3 \\ -\frac{B}{J}x_3 - \frac{mgl}{J}\cos(x_1)x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} v \quad (31)$$

and by defining the state vector  $z = [z_1, z_2, z_3]^T$  and the vector fields

$$\tilde{F}(x) = \begin{pmatrix} x_2 \\ x_3 \\ -\frac{B}{J}x_3 - \frac{mgl}{J}\cos(x_1)x_2 \end{pmatrix} \quad \tilde{G}(x) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (32)$$

one finally arrives at the following description of the SRM dynamics

$$\dot{x} = \tilde{F}(x) + \tilde{G}(x)v \quad (33)$$

The system of Eq. (33) undergoes approximate linearization around a temporary equilibrium which is defined by the present value of the system's state vector  $x^*$  and the last value of the control inputs vector  $v^*$  exerted on it. The approximate linearization is based on Taylor series expansion and on the computation of the associated Jacobian matrices. The locally linearized description of the SRM is given by

$$\dot{x} = Ax + Bv + \tilde{d} \quad (34)$$

where  $\tilde{d}$  is the modelling error term and

$$\begin{aligned}A &= \nabla_x[\tilde{F}(x) + \tilde{G}(x)v]_{(x^*, v^*)} \\ B &= \nabla_u[\tilde{F}(x) + \tilde{G}(x)v]_{(x^*, v^*)} \\ A &= \nabla_x \tilde{F}(x) \quad B = G(x)\end{aligned}\quad (35)$$

Thus one obtains the following matrices  $A$  and  $B$  in the linearized description of the switched reluctance machine

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{mgl}{J}\sin(x_1)x_2 & -\frac{mgl}{J}\cos(x_1) & -\frac{B}{J} \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (36)$$

## 4 Design of an H-infinity nonlinear feedback controller

### 4.1 Equivalent linearized dynamics of the SRM

After linearization around its current operating point, the dynamic model of the switched reluctance machine (SRM) is written as

$$\dot{x} = Ax + Bu + d_1 \quad (37)$$

Parameter  $d_1$  stands for the linearization error in the SRM's dynamic model appearing in Eq. (37). The reference setpoints for the SRM's state vector are denoted by  $\mathbf{x}_d = [x_1^d, x_2^d, x_3^d]^T$ . Tracking of this trajectory is succeeded after applying the control input  $u^*$ . At every time instant the control input  $u^*$  is assumed to differ from the control input  $u$  appearing in Eq. (37) by an amount equal to  $\Delta u$ , that is  $u^* = u + \Delta u$

$$\dot{x}_d = Ax_d + Bu^* + d_2 \quad (38)$$

The dynamics of the controlled system described in Eq. (37) can be also written as

$$\dot{x} = Ax + Bu + Bu^* - Bu^* + d_1 \quad (39)$$

and by denoting  $d_3 = -Bu^* + d_1$  as an aggregate disturbance term one obtains

$$\dot{x} = Ax + Bu + Bu^* + d_3 \quad (40)$$

By subtracting Eq. (38) from Eq. (40) one has

$$\dot{x} - \dot{x}_d = A(x - x_d) + Bu + d_3 - d_2 \quad (41)$$

By denoting the tracking error as  $e = x - x_d$  and the aggregate disturbance term as  $\tilde{d} = d_3 - d_2$ , the tracking error dynamics becomes

$$\dot{e} = Ae + Bu + \tilde{d} \quad (42)$$

The above linearized form of the SRM's model can be efficiently controlled after applying an H-infinity feedback control scheme.

### 4.2 The nonlinear H-infinity control

The initial nonlinear model of the switched reluctance machine is in the form

$$\dot{x} = \tilde{f}(x, u) \quad x \in \mathbb{R}^n, \quad u \in \mathbb{R}^m \quad (43)$$

Linearization of the system (switched reluctance machine) is performed at each iteration of the control algorithm round its present operating point  $(x^*, u^*) = (x(t), u(t-T_s))$ , where  $T_s$  is the sampling period. The linearized equivalent model of the system is described by

$$\dot{x} = Ax + Bu + L\tilde{d} \quad x \in R^n, u \in R^m, \tilde{d} \in R^q \quad (44)$$

where matrices  $A$  and  $B$  are obtained from the computation of the Jacobians

$$A = \begin{pmatrix} \frac{\partial \tilde{f}_1}{\partial x_1} & \frac{\partial \tilde{f}_1}{\partial x_2} & \dots & \frac{\partial \tilde{f}_1}{\partial x_n} \\ \frac{\partial \tilde{f}_2}{\partial x_1} & \frac{\partial \tilde{f}_2}{\partial x_2} & \dots & \frac{\partial \tilde{f}_2}{\partial x_n} \\ \dots & \dots & \dots & \dots \\ \frac{\partial \tilde{f}_n}{\partial x_1} & \frac{\partial \tilde{f}_n}{\partial x_2} & \dots & \frac{\partial \tilde{f}_n}{\partial x_n} \end{pmatrix} \Big|_{(x^*, u^*)}$$

$$B = \begin{pmatrix} \frac{\partial \tilde{f}_1}{\partial u_1} & \frac{\partial \tilde{f}_1}{\partial u_2} & \dots & \frac{\partial \tilde{f}_1}{\partial u_m} \\ \frac{\partial \tilde{f}_2}{\partial u_1} & \frac{\partial \tilde{f}_2}{\partial u_2} & \dots & \frac{\partial \tilde{f}_2}{\partial u_m} \\ \dots & \dots & \dots & \dots \\ \frac{\partial \tilde{f}_n}{\partial u_1} & \frac{\partial \tilde{f}_n}{\partial u_2} & \dots & \frac{\partial \tilde{f}_n}{\partial u_m} \end{pmatrix} \Big|_{(x^*, u^*)} \quad (45)$$

and vector  $\tilde{d}$  denotes disturbance terms due to linearization errors. The problem of disturbance rejection for the linearized model that is described by

$$\dot{x} = Ax + Bu + L\tilde{d} \quad y = Cx \quad (46)$$

where  $x \in R^n$ ,  $u \in R^m$ ,  $\tilde{d} \in R^q$  and  $y \in R^p$ , cannot be handled efficiently if the classical LQR control scheme is applied. This is because of the existence of the perturbation term  $\tilde{d}$ . The disturbance term  $\tilde{d}$  apart from modeling (parametric) uncertainty and external perturbation terms can also represent noise terms of any distribution.

In the  $H_\infty$  control approach, a feedback control scheme is designed for trajectory tracking by the system's state vector and simultaneous disturbance rejection, considering that the disturbance affects the system in the worst possible manner. The disturbances' effects are incorporated in the following quadratic cost function:

$$J(t) = \frac{1}{2} \int_0^T [y^T(t)y(t) + ru^T(t)u(t) - \rho^2 \tilde{d}^T(t)\tilde{d}(t)] dt, \quad r, \rho > 0 \quad (47)$$

The significance of the negative sign in the cost function's term that is associated with the perturbation variable  $\tilde{d}(t)$  is that the disturbance tries to maximize the cost function  $J(t)$  while the control signal  $u(t)$  tries to minimize it. The physical meaning of the relation given above is that the control signal and the disturbances compete to each other within a min-max differential game. This problem of mini-max optimization can be written as

$$\min_u \max_{\tilde{d}} J(u, \tilde{d}) \quad (48)$$

The objective of the optimization procedure is to compute a control signal  $u(t)$  which can compensate for the worst possible disturbance, that is externally imposed to the system. However, the solution to the mini-max optimization problem is directly related to the value of the parameter  $\rho$ . This means that there is an upper bound in the disturbances magnitude that can be annihilated by the control signal.

### 4.3 Computation of the feedback control gains

For the linearized system given by Eq. (46) the cost function of Eq. (47) is defined, where the coefficient  $r$  determines the penalization of the control input and the weight coefficient  $\rho$  determines the reward of the disturbances' effects.

It is assumed that (i) The energy that is transferred from the disturbances signal  $\tilde{d}(t)$  is bounded, that is  $\int_0^\infty \tilde{d}^T(t)\tilde{d}(t)dt < \infty$ , (ii) matrices  $[A, B]$  and  $[A, L]$  are stabilizable, (iii) matrix  $[A, C]$  is detectable. Then, the optimal feedback control law is given by

$$u(t) = -Kx(t) \quad (49)$$

with

$$K = \frac{1}{r} B^T P \quad (50)$$

where  $P$  is a positive definite symmetric matrix which is obtained from the solution of the Riccati equation

$$A^T P + PA + Q - P \left( \frac{1}{r} B B^T - \frac{1}{2\rho^2} L L^T \right) P = 0 \quad (51)$$

where  $Q$  is also a positive semi-definite symmetric matrix. The worst case disturbance is given by

$$\tilde{d}(t) = \frac{1}{\rho^2} L^T P x(t) \quad (52)$$

The diagram of the considered control loop is depicted in Fig. 2.

### 4.4 The role of Riccati equation coefficients in $H_\infty$ control robustness

Parameter  $\rho$  in Eq. (47), is an indication of the closed-loop system robustness. If the values of  $\rho > 0$  are excessively decreased with respect to  $r$ , then the solution of the Riccati equation is no longer a positive definite matrix. Consequently there is a lower bound  $\rho_{min}$  of  $\rho$  for which the  $H_\infty$  control problem has a solution. The acceptable values

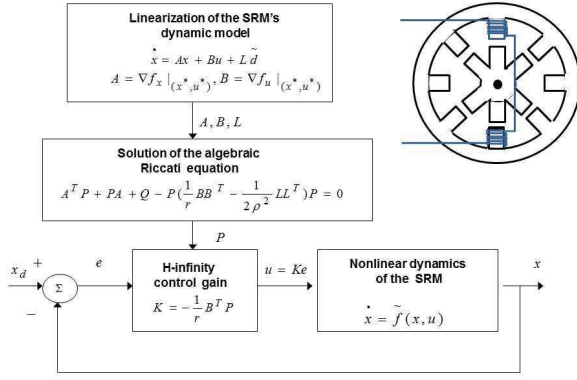


Fig. 2: Diagram of the control scheme for the switched reluctance machine

of  $\rho$  lie in the interval  $[\rho_{min}, \infty)$ . If  $\rho_{min}$  is found and used in the design of the  $H_\infty$  controller, then the closed-loop system will have increased robustness. Unlike this, if a value  $\rho > \rho_{min}$  is used, then an admissible stabilizing  $H_\infty$  controller will be derived but it will be a suboptimal one. The Hamiltonian matrix

$$H = \begin{pmatrix} A & -(\frac{1}{r}BB^T - \frac{1}{\rho^2}LL^T) \\ -Q & -A^T \end{pmatrix} \quad (53)$$

provides a criterion for the existence of a solution of the Riccati equation Eq. (51). A necessary condition for the solution of the algebraic Riccati equation to be a positive definite symmetric matrix is that  $H$  has no imaginary eigenvalues [52].

## 5 Lyapunov stability analysis

Through Lyapunov stability analysis it will be shown that the proposed nonlinear control scheme assures  $H_\infty$  tracking performance for the switched reluctance machine, and that in case of bounded disturbance terms asymptotic convergence to the reference setpoints is succeeded.

The tracking error dynamics for the switched reluctance machine is written in the form

$$\dot{e} = Ae + Bu + L\tilde{d} \quad (54)$$

where in the SRM's case  $L = I \in R^3$  with  $I$  being the identity matrix. Variable  $\tilde{d}$  denotes model uncertainties and external disturbances of the SRM's model. The following Lyapunov equation is considered

$$V = \frac{1}{2}e^T Pe \quad (55)$$

where  $e = x - x_d$  is the tracking error. By differentiating with respect to time one obtains

$$\begin{aligned} \dot{V} &= \frac{1}{2}\dot{e}^T Pe + \frac{1}{2}e^T \dot{P}e \Rightarrow \\ \dot{V} &= \frac{1}{2}[Ae + Bu + L\tilde{d}]^T Pe + \frac{1}{2}e^T P[Ae + Bu + L\tilde{d}] \Rightarrow \quad (56) \end{aligned}$$

$$\dot{V} = \frac{1}{2}[e^T A^T + u^T B^T + \tilde{d}^T L^T]Pe + \frac{1}{2}e^T P[Ae + Bu + L\tilde{d}] \Rightarrow \quad (57)$$

$$\begin{aligned} \dot{V} &= \frac{1}{2}e^T A^T Pe + \frac{1}{2}u^T B^T Pe + \frac{1}{2}\tilde{d}^T L^T Pe \\ &+ \frac{1}{2}e^T PAe + \frac{1}{2}e^T PBu + \frac{1}{2}e^T PL\tilde{d} \quad (58) \end{aligned}$$

The previous equation is rewritten as

$$\begin{aligned} \dot{V} &= \frac{1}{2}e^T (A^T P + PA)e + (\frac{1}{2}u^T B^T Pe + \frac{1}{2}e^T PBu) \\ &+ (\frac{1}{2}\tilde{d}^T L^T Pe + \frac{1}{2}e^T PL\tilde{d}) \quad (59) \end{aligned}$$

*Assumption:* For given positive definite matrix  $Q$  and coefficients  $r$  and  $\rho$  there exists a positive definite matrix  $P$ , which is the solution of the following matrix equation

$$A^T P + PA = -Q + P(\frac{2}{r}BB^T - \frac{1}{\rho^2}LL^T)P \quad (60)$$

Moreover, the following feedback control law is applied to the system

$$u = -\frac{1}{r}B^T Pe \quad (61)$$

By substituting Eq. (60) and Eq. (61) one obtains

$$\begin{aligned} \dot{V} &= \frac{1}{2}e^T [-Q + P(\frac{2}{r}BB^T - \frac{1}{\rho^2}LL^T)P]e + e^T PB(-\frac{1}{r}B^T Pe) \\ &+ e^T PL\tilde{d} \Rightarrow \quad (62) \end{aligned}$$

$$\begin{aligned} \dot{V} &= -\frac{1}{2}e^T Qe + \frac{1}{r}e^T PBB^T Pe - \frac{1}{2\rho^2}e^T PLL^T Pe \\ &- \frac{1}{r}e^T PBB^T Pe + e^T PL\tilde{d} \quad (63) \end{aligned}$$

which after intermediate operations gives

$$\dot{V} = -\frac{1}{2}e^T Qe - \frac{1}{2\rho^2}e^T PLL^T Pe + e^T PL\tilde{d} \quad (64)$$

or, equivalently

$$\dot{V} = -\frac{1}{2}e^T Qe - \frac{1}{2\rho^2}e^T PLL^T Pe + \frac{1}{2}e^T PL\tilde{d} + \frac{1}{2}\tilde{d}^T L^T Pe \quad (65)$$

*Lemma:* The following inequality holds

$$\frac{1}{2}e^T PL\tilde{d} + \frac{1}{2}\tilde{d}^T L^T Pe - \frac{1}{2\rho^2}e^T PLL^T Pe \leq \frac{1}{2}\rho^2 \tilde{d}^T \tilde{d} \quad (66)$$

*Proof:* The binomial  $(\rho a - \frac{1}{\rho} b)^2$  is considered. Expanding the left part of the above inequality one gets

$$\begin{aligned} \rho^2 a^2 + \frac{1}{\rho^2} b^2 - 2ab &\geq 0 \Rightarrow \frac{1}{2}\rho^2 a^2 + \frac{1}{2\rho^2} b^2 \\ -ab &\geq 0 \Rightarrow ab - \frac{1}{2\rho^2} b^2 \leq \frac{1}{2}\rho^2 a^2 \Rightarrow \frac{1}{2}ab + \frac{1}{2}ab \\ -\frac{1}{2\rho^2} b^2 &\leq \frac{1}{2}\rho^2 a^2 \end{aligned} \quad (67)$$

The following substitutions are carried out:  $a = \tilde{d}$  and  $b = e^T PL$  and the previous relation becomes

$$\frac{1}{2}\tilde{d}^T L^T P e + \frac{1}{2}e^T PL \tilde{d} - \frac{1}{2\rho^2}e^T PLL^T P e \leq \frac{1}{2}\rho^2 \tilde{d}^T \tilde{d} \quad (68)$$

Eq. (68) is substituted in Eq. (65) and the inequality is enforced, thus giving

$$\dot{V} \leq -\frac{1}{2}e^T Q e + \frac{1}{2}\rho^2 \tilde{d}^T \tilde{d} \quad (69)$$

Eq. (69) shows that the  $H_\infty$  tracking performance criterion is satisfied. The integration of  $\dot{V}$  from 0 to  $T$  gives

$$\begin{aligned} \int_0^T \dot{V}(t) dt &\leq -\frac{1}{2} \int_0^T \|e\|_Q^2 dt + \frac{1}{2}\rho^2 \int_0^T \|\tilde{d}\|^2 dt \Rightarrow 2V(T) \\ &+ \int_0^T \|e\|_Q^2 dt \leq 2V(0) + \rho^2 \int_0^T \|\tilde{d}\|^2 dt \end{aligned} \quad (70)$$

Moreover, if there exists a positive constant  $M_d > 0$  such that

$$\int_0^\infty \|\tilde{d}\|^2 dt \leq M_d \quad (71)$$

then one gets

$$\int_0^\infty \|e\|_Q^2 dt \leq 2V(0) + \rho^2 M_d \quad (72)$$

Thus, the integral  $\int_0^\infty \|e\|_Q^2 dt$  is bounded. Moreover,  $V(T)$  is bounded and from the definition of the Lyapunov function  $V$  in Eq. (55) it becomes clear that  $e(t)$  will be also bounded since  $e(t) \in \Omega_e = \{e | e^T P e \leq 2V(0) + \rho^2 M_d\}$ . According to the above and with the use of Barbalat's Lemma one obtains  $\lim_{t \rightarrow \infty} e(t) = 0$ .

Elaborating on the above, it can be noted that the proof of global asymptotic stability for the control loop of the switched reluctance machine is based on Eq. (69) and on the application of Barbalat's Lemma. It uses the condition of Eq. (71) about the boundedness of the square of the aggregate disturbance and modelling error term  $\tilde{d}$  that affects the model. However, as explained above the proof of

global asymptotic stability is not restricted by this condition. By selecting the attenuation coefficient  $\rho$  to be sufficiently small and in particular to satisfy  $\rho^2 < \|e\|_Q^2 / \|\tilde{d}\|^2$  one has that the first derivative of the Lyapunov function is upper bounded by 0. Therefore for the  $i$ -th time interval it is proven that the Lyapunov function defined in Eq. (55) is a decreasing one. This also assures the Lyapunov function of the system defined in Eq. (29) will always have a negative first-order derivative.

Because of the complexity of the associated state-space model, most of the results that appear in the relevant bibliography are related with heuristics-based PID control techniques. The control gains in such methods are chosen empirically and the functioning of the related control loop remains reliable only around local operating points. Change of the operating conditions, as well as external perturbations, are likely to make the PID control loops be unstable. On the other side, the article's technical approach offers one of the very few results on feedback control of Switched Reluctance Machines that assure the global stability of the control loop. Besides, the article's method pursues optimality. This signifies that the functioning of the electric machine reaches the given specifications under minimal energy consumption. The design stages of the article's control scheme are clearly defined, while the method's computational complexity is moderate.

## 6 Robust state estimation with the use of the $H_\infty$ Kalman Filter

The control loop has to be implemented with the use of information provided by a small number of sensors and by processing only a small number of state variables. To reconstruct the missing information about the state vector of the switched reluctance machine it is proposed to use a filtering scheme and based on it to apply state estimation-based control [54, 57, 58]. The recursion of the  $H_\infty$  Kalman Filter, for the model of the SRM, can be formulated in terms of a *measurement update* and a *time update* part

*Measurement update:*

$$\begin{aligned} D(k) &= [I - \theta W(k)P^-(k) + C^T(k)R(k)^{-1}C(k)P^-(k)]^{-1} \\ K(k) &= P^-(k)D(k)C^T(k)R(k)^{-1} \\ \hat{x}(k) &= \hat{x}^-(k) + K(k)[y(k) - C\hat{x}^-(k)] \end{aligned} \quad (73)$$



Time update:

$$\begin{aligned}\hat{x}^-(k+1) &= A(k)x(k) + B(k)u(k) \\ P^-(k+1) &= A(k)P^-(k)D(k)A^T(k) + Q(k)\end{aligned}\quad (74)$$

where it is assumed that parameter  $\theta$  is sufficiently small to assure that the covariance matrix  $P^-(k)^{-1} - \theta W(k) + C^T(k)R(k)^{-1}C(k)$  will be positive definite. When  $\theta = 0$  the  $H_\infty$  Kalman Filter becomes equivalent to the standard Kalman Filter. One can measure only the output  $x_1 = \theta$  of the SRM's state vector, and can estimate through filtering the rest of the state vector elements.

## 7 Simulation tests

The performance of the proposed nonlinear H-infinity control method for switched reluctance machines has been tested through simulation experiments. The obtained results confirm the previous theoretical findings. The computation of the feedback control gain was based on the solution of the algebraic Riccati equation given in Eq. (60), through a procedure that was repeated at each iteration of the control method. The obtained results are depicted in Fig. 3 to Fig. 7. It can be confirmed that fast and accurate convergence of the state variables of the SRM to the reference setpoints was achieved. Moreover, it can be seen that the variation of the control inputs remained smooth and within moderate ranges. Despite nonlinearities, the control method's performance was satisfactory and precise tracking of the reference setpoints was achieved. It is also noted that in practice the proposed control method can be finally implemented by applying PWM.

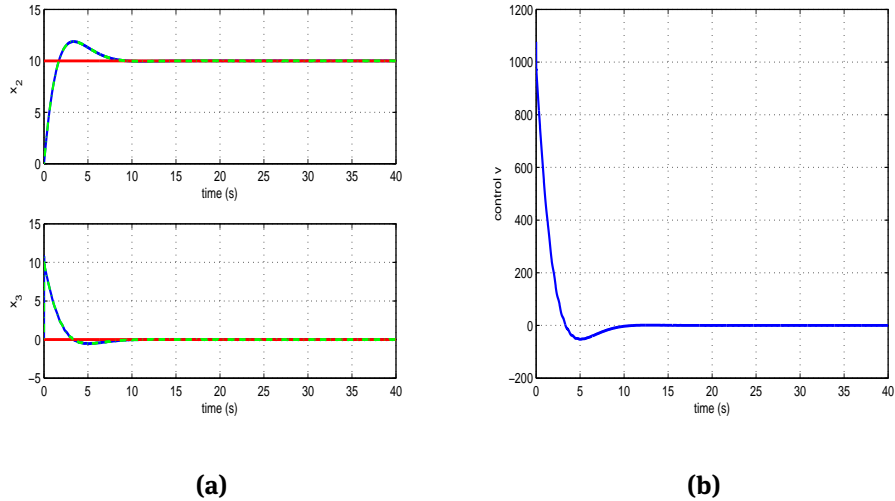
In the presented simulation experiments state estimation-based control has been implemented. Out of the 3 state variables of the electric machine only output  $x_1 = \theta$  was considered to be measurable. The rest of the state variables of the SRM were indirectly estimated with the use of the H-infinity Kalman Filter. The real value of each state variable has been plotted in blue, the estimated value has been plotted in green, while the associated reference setpoint has been plotted in red. It can be noticed that despite model uncertainty the H-infinity Kalman Filter achieved accurate estimation of the real values of the state vector elements. In this manner the robustness of the state estimation-based H-infinity control scheme was also improved.

*Remark 1:* The proposed nonlinear optimal control method is suitable for a wide class of electric machines and power electronics [55]. It can result in the optimized functioning of various types of power generators used in the electricity

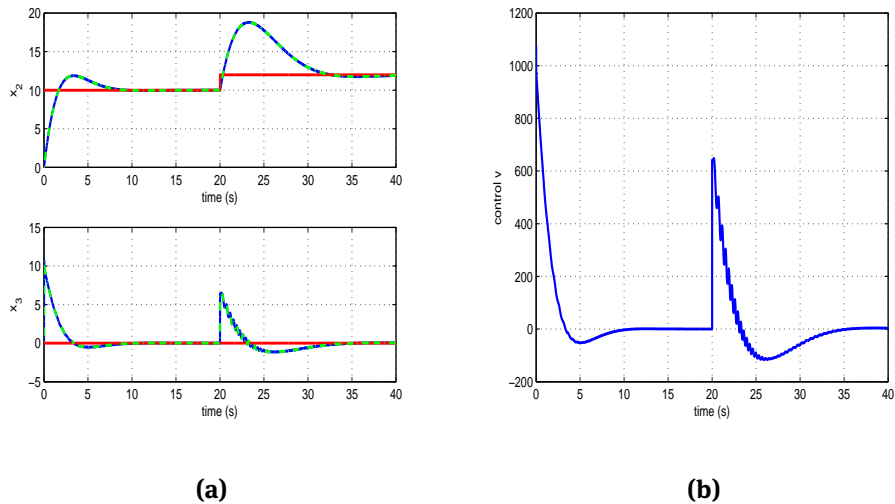
grid, and of electric motors in the traction of electric vehicles. One can consider the application of the proposed control method to (a) Control for Synchronous and Permanent Magnet Synchronous Generators, Doubly-Fed Induction Generators, Synchronous reluctance generators, and Doubly-fed reluctance generators used in power generation (b) Control for DC motors, Switched reluctance motors, Permanent Magnet Synchronous Motors, Induction motors, Synchronous reluctance motors, Doubly-fed reluctance motors and Multi-phase electric motors used in traction and propulsion of transportation means.

*Remark 2:* The proposed nonlinear optimal control method is a generic one and its application is not dependent on a specific form or structure of the dynamic model of the electric machine under control. For instance, it is known that backstepping control cannot be applied to dynamical systems which are not in the triangular form. Moreover, it is known that the application of sliding-mode control is hindered by the selection of the associated sliding surface, and that it is not straightforward to define such a surface if the system cannot be transformed into a canonical form. On the other side, the proposed nonlinear optimal control method can be applied to a wide class of dynamic models of electric motors, even to those which are not written in an affine in the input form. Besides, by proving through the article's nonlinear stability analysis that the control method satisfies the H-infinity tracking performance criterion of Eq. (60), it is confirmed that it exhibits elevated robustness levels. Therefore, even if specific parameters or terms in the dynamic model of the switched reluctance motor are not precisely known it can be ensured that the functioning of the control loop will remain reliable. This covers also the case about uncertainty in the modelling of the flux linkage of the motor.

*Remark 3:* The comparison between the article's nonlinear optimal control method and other optimal control approaches for industrial systems is outlined as follows: MPC is deemed to be unsuitable for the model of the switched reluctance motor because this control approach is primarily addressed to linear dynamical systems, whereas in the case of the switched reluctance motor it lacks stability. Besides, the NMPC, standing for the nonlinear variant of MPC may also be of questionable performance because its iterative search for an optimum is dependent on initial parametrization while its convergence to the optimum cannot be assured either. On the other side, the proposed nonlinear optimal control method retains the advantages of typical optimal control, that is fast and accurate convergence to the reference setpoints while keeping moderate the variations of the control inputs.



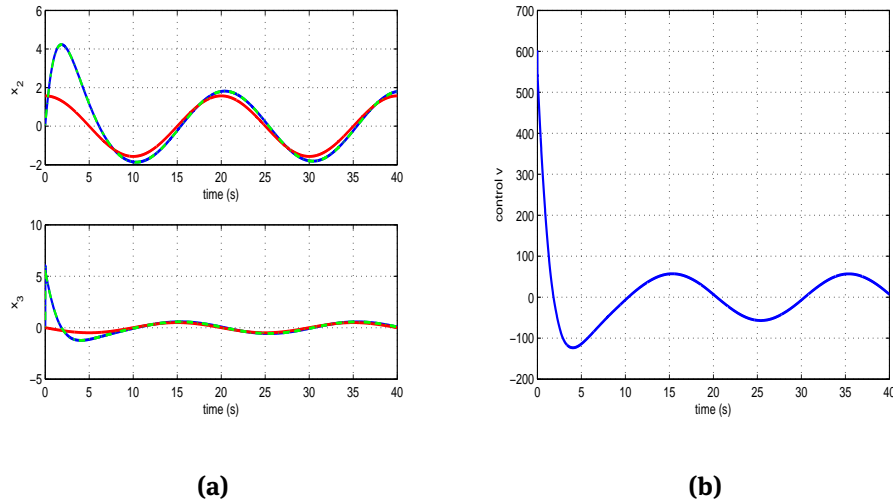
**Fig. 3:** Setpoint 1: (a) Convergence of the state variables of the SRM  $x_2 = \omega$ ,  $x_3 = \dot{\omega}$  (blue line: real value, green line: estimate value) by the H-infinity filter) to the associated reference values (red line). (b) Control input  $v$  applied to the control system of the SRM



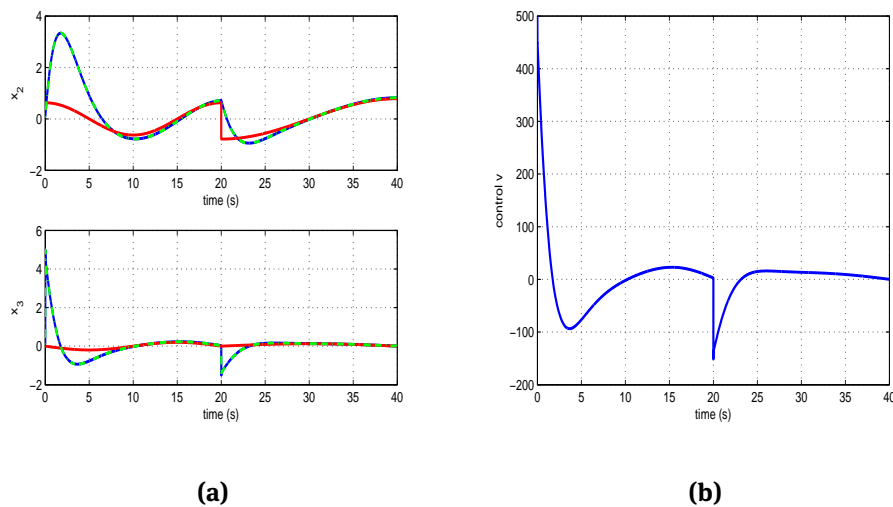
**Fig. 4:** Setpoint 2: (a) Convergence of the state variables of the SRM  $x_2 = \omega$ ,  $x_3 = \dot{\omega}$  (blue line: real value, green line: estimate value) by the H-infinity filter) to the associated reference values (red line). (b) Control input  $v$  applied to the control system of the SRM

*Remark 4:* The article offers one of the few approaches to the control of Switched Reluctance Machines (SRMs) which are of global asymptotic stability. This is meaningful and significant for many practical applications of SRMs, as for instance in the case of electric vehicles' traction. In such cases one cannot rely on empirical controller tuning (as for example PID controllers) because the vehicles' functioning takes place under variable operating conditions and is subject to several perturbations. Control schemes which are not of proven global asymptotic stability may become of questionable performance. Another

advantage of the proposed control algorithm is that it offers solution to the nonlinear optimal control problem for SRMs. This is important for reducing energy consumption of electric vehicles and for achieving a satisfactory performance of the vehicle's traction system without the need for frequent battery recharging. Under nonlinear optimal control all technical characteristics of the traction system of electric vehicles are significantly improved, for instance the motor's torque and traction force, as well as acceleration features.



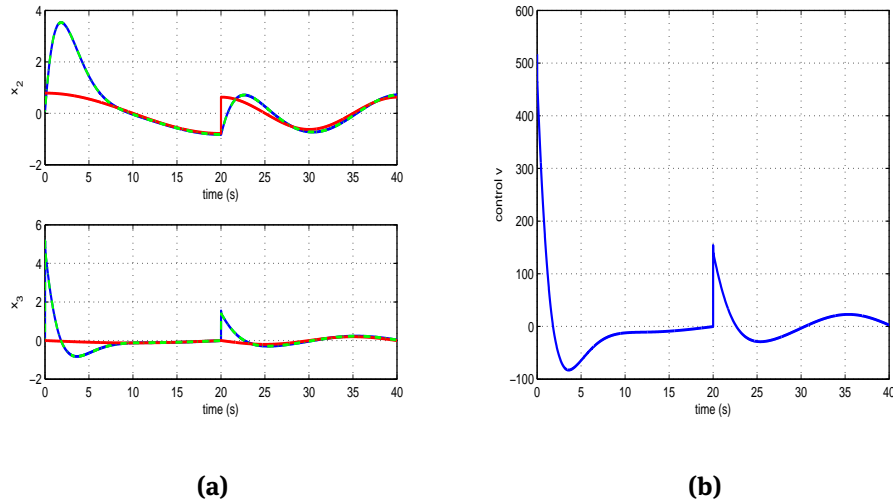
**Fig. 5:** Setpoint 3: (a) Convergence of the state variables of the SRM  $x_2 = \omega$ ,  $x_3 = \dot{\omega}$  (blue line: real value, green line: estimate value) by the H-infinity filter) to the associated reference values (red line). (b) Control input  $v$  applied to the control system of the SRM



**Fig. 6:** Setpoint 4: (a) Convergence of the state variables of the SRM  $x_2 = \omega$ ,  $x_3 = \dot{\omega}$  (blue line: real value, green line: estimate value) by the H-infinity filter) to the associated reference values (red line). (b) Control input  $v$  applied to the control system of the SRM

*Remark 5:* To implement state estimation-based feedback control for the SRM it suffices to measure only the turn angle of the rotor, while the rest of the aforementioned variables can be estimated through a filtering procedure, which eliminates the effects of the measurement noise. State-estimation-based control for Switched Reluctance Motors can contribute to improving the functioning of such machines. It is clear that not all-state vector elements of these electric machines can be measured through sensors, whereas such sensors are failure prone and consequently their measurements can be unreliable. The lat-

ter hold particularly in a major application field for SRMs which is electric vehicles traction. Under the harsh operating conditions of Switched Reluctance Motors it is anticipated that several sensors will exhibit malfunctioning. State estimation for SRMs through filtering techniques enables to avoid this degradation in the sensors' performance and allows to robustify the control loop for such electric machines.



**Fig. 7:** Setpoint 5: (a) Convergence of the state variables of the SRM  $x_2 = \omega$ ,  $x_3 = \dot{\omega}$  (blue line: real value, green line: estimate value) by the H-infinity filter) to the associated reference values (red line). (b) Control input  $v$  applied to the control system of the SRM

## 8 Conclusions

A nonlinear H-infinity control method has been developed for the dynamic model of switched reluctance machines. The method allows control of proven stability and of elevated accuracy for the aforementioned type of electric machines, and has good potential for several industrial applications (for instance actuation in robotic and mechatronic systems as well as traction in electric vehicles). The nonlinear dynamic model of the SRM has undergone approximate linearization around a temporary operating point (equilibrium) which was recomputed at each iteration of the control algorithm. The equilibrium was defined by the value of the system's state vector at each time instant and by the last value of the control input vector that was applied to the SRM prior to that instant. The linearization procedure made use of Taylor series expansion and required the computation of the Jacobian matrices of the SRM state-space model.

For the approximately linearized model of the SRM an H-infinity feedback controller was designed. The computation of the controller's feedback gains was based on the repetitive solution of an algebraic Riccati equation, taking place at each iteration of the control algorithm. The stability properties of the control scheme were confirmed through Lyapunov analysis. First, it was proven that the control method satisfied the H-infinity tracking performance criterion, which signified elevated robustness against modelling errors and exogenous disturbances. Moreover, under moderate conditions the global

asymptotic stability of the control method was proven. The excellent tracking performance of the control algorithm and its fast convergence to reference setpoints was further demonstrated through simulation experiments.

The advantages of the proposed nonlinear optimal control method for Switched Reluctance Machines are outlined as follows: (i) unlike global linearization-based control schemes, the proposed nonlinear optimal control method does not require changes of variables (diffeomorphisms) and application of complicated transformations of the system's state-space model (ii) the new control approach retains the advantages of typical optimal control, that is fast and accurate tracking of the reference setpoints, under moderate variations of the control, inputs, (iii) unlike NMPC approaches the proposed control method is of proven convergence and stability, (iv) unlike PID control the new nonlinear optimal control method does not rely on empirical parameters tuning and is of global stability (v) unlike backstepping control approaches the proposed control method does not require the system to be written in a specific (triangular) state-space form.

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