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Fatigue reliability analysis of a turbine disc under multi-source uncertainties

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Abstract

Life and reliability analysis of hot section components like high pressure turbine (HPT) discs plays an important role for ensuring the engine structural integrity. HPT disc operates under high temperatures to withstand complex loadings, its basic parameters, including the applied loads, material properties and working environments, have shown multi-source uncertainties. The influence of these uncertainties on the structural response of the turbine disc cannot be ignored. According to this, the variations of applied loads and material properties are quantified for fatigue reliability analysis of turbine disc. In particular, material response variability is modeled by using the Chaboche model and Fatemi-Socie damage criterion. Moreover, the inhomogeneity of its constituent material is also considered through combining FE simulation with Latin hypercube sampling. Finally, fatigue reliability analysis of a HPT disc under multi-source uncertainties is conducted for different flight missions.

Keywords: uncertainty; fatigue reliability; turbine disc; FE analysis; Fatemi-Socie criterion

1. Introduction

As one of fatigue critical part of an aero engine, hot section components like high pressure turbine (HPT) discs operate under harsh conditions. Low cycle fatigue (LCF) at high temperature is one of the main failure modes of a HPT disc, and its operational reliability is of critical importance for ensuring the engine structural integrity. In general, LCF analysis of complicated part, such as a turbine disc-blade contact system, is performed using deterministic methods and models based on certain basic variables, which predicts the life with a larger scatter [1-4]. Accordingly, fatigue reliability analysis has been conducted to consider multi-source uncertainties result from these basic variables. Among them, the strain load-strength interference model is developed by Zhao et al. [5], in which the strength coefficient σ_f' and plasticity coefficient ε_f' are assumed to follow normal or Weibull distributions. Gao et al. [6] developed a distributed collaborative response surface method for fatigue reliability analysis of a turbine blade. Due to the complexity of the turbine disc structure, it is difficult to build the relationship between the basic random variables and LCF life by using explicit functions. Therefore, the probability density function (PDF) of LCF life cannot be obtained by using deterministic method based on random characteristics of the basic variables. According to this, various numerical methods, such as the Monte Carlo method, response surface method [7, 8] and improved response surface method [9-11] have been developed for structural fatigue reliability analysis. In this paper, Latin hypercube sampling technique [12] is introduced to obtain the PDFs of basic random variables. In addition, finite element (FE) analysis is conducted to obtain the PDFs of stress-strain response of the turbine disc by using the Chaboche plasticity model and Fatemi-Socie (FS) damage criterion.

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2. Analysis of LCF life

The Coffin-Manson (CM) equation [13, 14] has commonly been used for LCF analysis under uniaxial loadings, which can be expressed by:

$$\varepsilon_a = \frac{\Delta\varepsilon}{2} = \frac{\sigma_f'}{E}(2N_f)^b + \varepsilon_f'(2N_f)^c \tag{1}$$

where ε_a is the total strain amplitude; $\Delta\varepsilon$ is the total strain range; σ_f' is the fatigue strength coefficient; E is the Young's modulus; N_f is the fatigue life; b is the fatigue strength exponent; ε_f' is the fatigue ductility coefficient; c is the fatigue ductility exponent.

In practical application, hot section components like turbine discs are frequently subjected to complex loadings like multiaxial loadings at high temperatures. Through using critical plane approaches, the complex multiaxial stress and strain states can be simplified into equivalent uniaxial ones, and when the cumulative fatigue damage on the material plane reached the damage threshold, fatigue failure occurs. Among them, a well-known Fatemi-Socie multiaxial fatigue criterion [15] is used for estimating fatigue life and locating fracture plane positions. For FS criterion, the critical plane is defined by the equivalent shear strain amplitude $\gamma_{a,eq}$

$$\gamma_{a,eq} = \frac{\Delta\gamma_{max}}{2}\left(1 + k\frac{\sigma_{n,max}}{\sigma_y}\right) = \frac{\tau_f'}{G}(2N_f)^{b_\gamma} + \gamma_f'(2N_f)^{c_\gamma} = f_{FS}(N_f) \tag{2}$$

where $\Delta\gamma_{max}$ and $\sigma_{n,max}$ are the maximum shear strain and the maximum normal stress on the critical plane, respectively; τ_f' and b_γ are the shear fatigue strength coefficient and exponent, respectively; γ_f' and c_γ are the shear fatigue ductility coefficient and exponent; G is the shear modulus; k is a material and life dependent coefficient and generally fitted from uniaxial to torsion fatigue tests [16].

In general, Eq. (2) can be used to predict the LCF life under single level of cyclic loadings. In this paper, LCF life of the turbine disc under multiple levels of cyclic loadings are obtained by using the linear damage rule

$$D_T = \sum_{i=1}^{m} \frac{n_i}{N_{fi}} \tag{3}$$

$$T_f = T_0 \frac{1}{D_T} \tag{4}$$

where m is the number of stress levels, n_i is the number of loading cycles at the ith stress level, N_{fi} is the corresponding life of the ith stress level, D_T is the total damage over a period of time, T_0 is the period time of the load spectrum, and T_f is the total fatigue life.

3. Uncertainty in LCF life

For the turbine disc of this study, its basic random variables can be divided into three categories: material properties, loads and external environmental factors.

3.1. Material response variability

When considering the uncertainty result from material properties, the physical property, such as the density ρ, and the mechanical behavior under monotonic/cyclic loadings should be considered in LCF analysis. When modelling the fatigue mechanical behavior by using Eq. (1), the stress-strain relationship can be described by the Ramberg-Osgood (RO) equation as

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{1/n'} = f_{RO}(\sigma) \tag{5}$$

where ε and σ are the local strain and stress at a given location; K' and n' are the cyclic strength coefficient and cyclic strain hardening exponent, respectively.

A correlation between the elastic and plastic components in the CM equation and that in RO equation can be derived as [17]

$$\sigma_f' = K'(\varepsilon_f')^{n'} \tag{6}$$

$$n' = \frac{b}{c} \tag{7}$$

Using Eq. (6) and Eq. (7), cyclic fatigue behavior of the material can be well described, in which eight parameters are involved, including CM parameters $\{\sigma_f', \varepsilon_f', b, c\}$, RO parameters $\{K', n'\}$, elastic modulus E, and yield strength σ_y obtained by:

$$\sigma_y = K'\left(\varepsilon_{pa}\right)^{n'} \tag{8}$$

Note from [18] that the uncertainty in material response can be well characterized by quantifying the variability of the four parameters $\{\sigma_f', \varepsilon_f', b, c\}$, where the RO parameters $\{K', n'\}$ and yield strength σ_y are treated as intermediate variables. Material physical properties $\{\rho, E\}$ are also considered as random inputs. The coefficients of variation (CV) of $\{\rho, E, \sigma_f', \varepsilon_f'\}$ are obtained according to [19]. To describe the variability of material response, the Chaboche model with three evolution parts ($M = 3$) [20] is used to build a procedure to determine the constitutive coefficients, which provides sufficient variations to calibrate the nonlinear behavior of materials [21, 22]. Based on the Armstrong-Frederick evolution law, the stress range $\Delta\sigma$ can be expressed by a function of plastic strain range $\Delta\varepsilon_p$ as:

$$\frac{\Delta\sigma}{2} = \sigma_y + \sum_{i-1}^{M} \frac{C_i}{\gamma_i} \tanh\left(\gamma_i \frac{\Delta\varepsilon_p}{2}\right) \tag{9}$$

In this study, the uncertainties associated with material variability are quantified by using the derived constitutive equation parameters in Eq. (9). Based on the stochastic material properties, an example plot of stochastic stress-strain response of GH4169 are shown in Fig. 1.

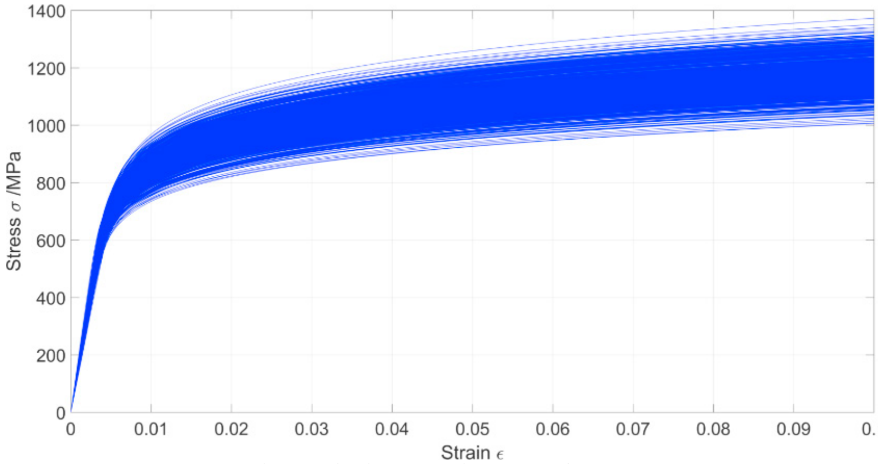

Fig. 1 Stochastic stress-strain response of GH4169

In this paper, both of the uncertainty of material properties and the inhomogeneity of the material are taken into account in this paper. To be more specific, the stochastic material properties are assigned randomly to each FE element. The material properties of each element are randomly distributed in FE simulation, the influence of grid size and density has been investigated. Results shown that the effect of elements density can be ignored when using this method for fatigue reliability analysis.

3.2. Variations in cyclic loads

The load spectrum of a HPT disc varies from different flight missions, and often include multiple load levels [23, 24]. In order to facilitate this analysis, the load spectrum for the given turbine disc during the total working life can be divided into four typical load cases, which are take off, maximum continue, idle and cruise. These four load cases constitute three levels of cyclic loads, which are (take off) 0-maximum continue-0 (take off), idle-maximum continue-idle and cruise-maximum continue-cruise. These four load cases are used as normal inputs with the coefficients of variation 0.01 according to [19].

4. Stochastic stress-strain analysis of a turbine disc

In this analysis, a 1/90 FE model of a HPT disc is built for fatigue reliability analysis by using ANSYS 14.5, whose FE mesh is shown in Fig. 2(a). The load of HPT disc is mainly centrifugal load during the engine operation, which mainly includes the centrifugal loads from the disc-blade contact system. The centrifugal load from the blade is loaded on the surfaces of six tooth of the mortise in the form of pressure

$$P = \frac{mr\omega^2}{S\cos\theta} \tag{10}$$

where P is the pressure, m is the mass of the blade, ω is the rotational speed, S is the total area of the stressed surface, θ is the angle between centrifugal force and bearing surface.

From Eq. (10), P is related to the rotational speed of the disc. Note that the HPT disc works at high temperatures, and the effect of high temperatures on the disc material properties cannot be ignored, the temperature field distribution in service can be simulated in FE analysis. According to the engine field data, the disc temperature is distributed along its radius direction, as shown in Fig. 2(b). Three levels of cyclic loads are introduced to facilitate FE analysis of the turbine disc. Through using basic random variables to produce the turbine disc samples, the stress-strain response at the dangerous region of each turbine disc sample by FE simulations. According to the FE results, the most dangerous region is located at the bottom groove of the turbine disc. Distribution of the maximum plastic strain of the turbine disc is shown in Fig. 2(c).

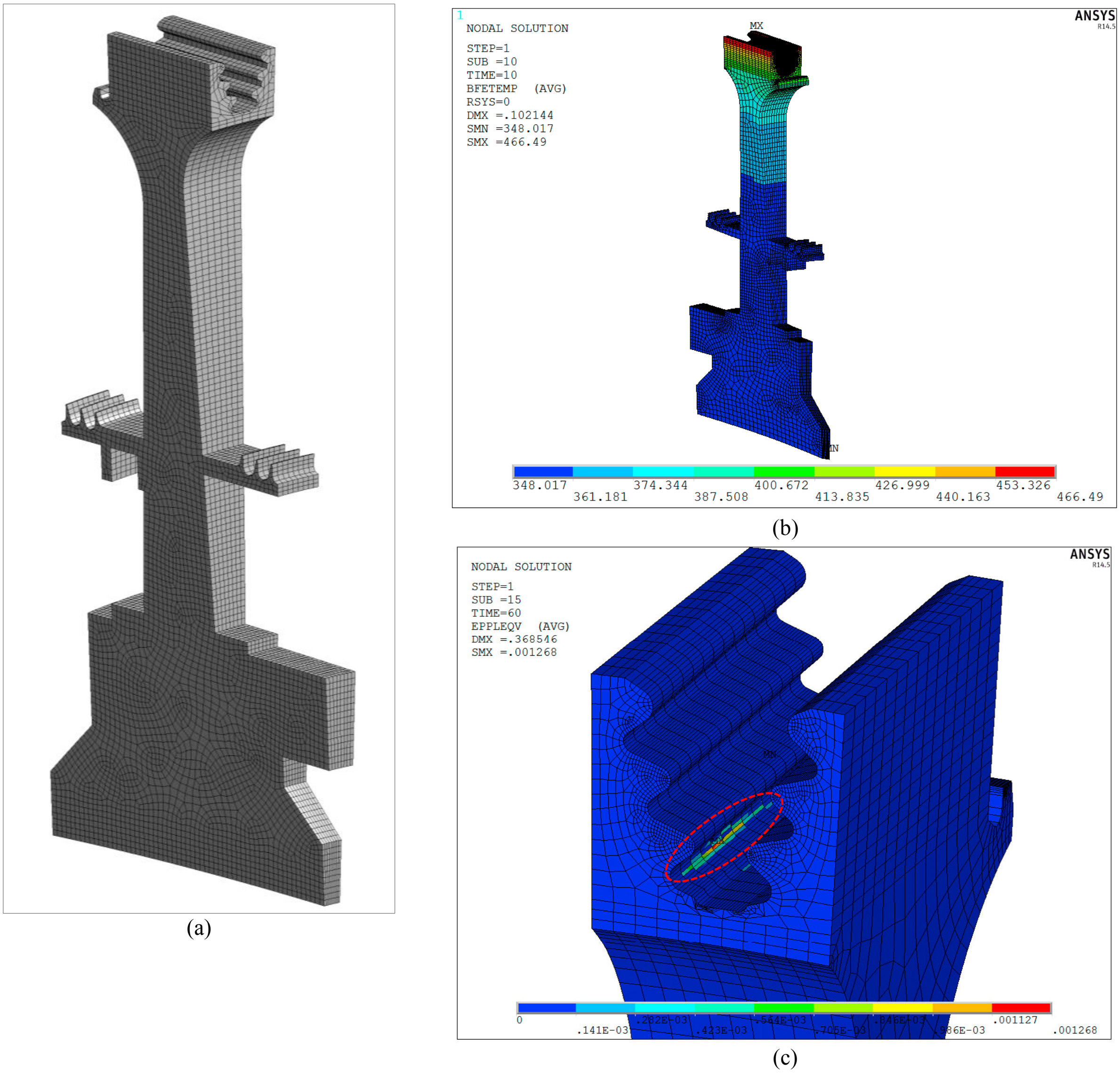

Fig. 2 (a) 1/90 FE mesh; (b) Temperature field distribution; (c) Plastic strain nephograms of the turbine disc

5. Stochastic stress-strain analysis of a turbine disc

5.1. Fatigue life prediction of the turbine disc

From FE analysis of the turbine disc, the stress-strain response of all samples at the most dangerous region under different flight missions can be obtained, then the equivalent shear strain amplitude $\gamma_{a,eq}$ can be calculated by using FS criterion. Using Eq. (2), an example plot on fatigue life distribution of the turbine disc under take off-maximum continue-take off load level can be derived as shown in Fig. 3. According to the load spectrum under T_0 = 750 hours, the total fatigue life T_f of the turbine disc can be obtained by using Eq. (3) and Eq. (4), and its total fatigue life distribution can be shown in Fig. 4.

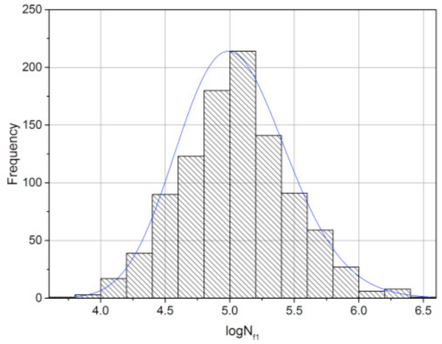

Fig. 3 Example plot of fatigue life distribution of the turbine disc under 0-maximum continue-0

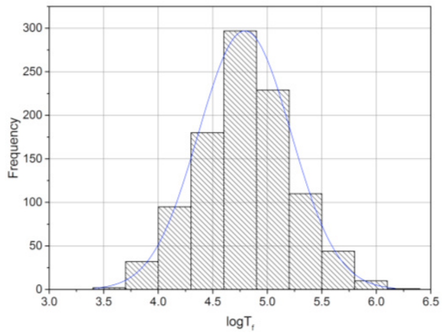

Fig. 4 Total fatigue life distribution of the turbine disc

5.2. Fatigue reliability analysis of the turbine disc

It should be pointed out that the total fatigue life in Fig. 4 roughly follows a lognormal distribution. In order to assess the fatigue reliability of the turbine disc, a limit state equation needs to be established firstly. Assuming that fatigue life follows a lognormal distribution, the limit state equation is given by

$$g = \log\left(\frac{T_f}{T'}\right) = \log T_f - \log T' \quad (11)$$

where T' is the designed life of the turbine disc, and $\log T_f$ follows a normal distribution.

Based on the stress-strength interference theory, failure probability P_f can be calculated by

$$P_f = P(T_f < T') = P(g < 0) = \Phi\left(\frac{\log T' - \mu_{\log T_f}}{\sigma_{\log T_f}}\right) \quad (12)$$

where $\mu_{\log T_f}$ and $\sigma_{\log T_f}$ can be estimated based on the sample data of total fatigue life T_f. The calculated failure probability P_f vs. designed life is shown in Fig. 5.

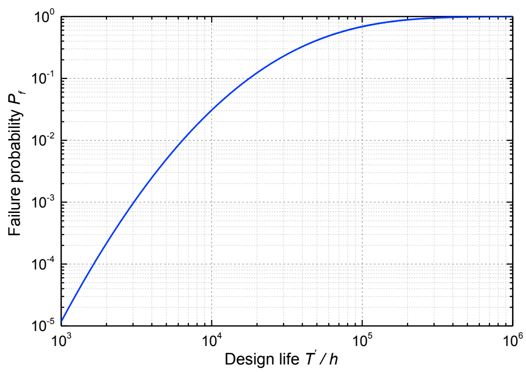

Fig. 5 Failure probability P_f

6. Conclusions

In this paper, a probabilistic framework for fatigue reliability analysis of a HPT disc under multi-source uncertainties is developed by using FE simulations and LHS technique. Through structural FE analysis of the turbine disc under different load spectrums, distributions of stress-strain response at the most dangerous region can be obtained. Then, fatigue life distribution of the turbine disc is estimated by using FS multiaxial fatigue criterion. In addition, its failure probability has been estimated under different flight missions. Compared with traditional methods, more practical factors are considered in this method, which provides reasonably acceptable correlations with its field number of flights.

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