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## Development of a simplified model for the vibration analysis of lawn mowers

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### Abstract

The vibrational behavior of vehicles is a crucial issue for the comfort, especially for the professional vehicles. This paper presents a simplified modelling approach for studying the vibrational behavior of a lawn tractor. The vibrational response of a real vehicle is analyzed by an extensive experimental modal analysis and Finite Element model (FE) simulating the modal behavior of the whole tractor. The FEM was then validated by the comparison with the experimental results and then used for identifying the components and connections effectively driving the modal response. Based on these results, a simplified Multi-Body (MB) model, able to reproduce the vibrational response of the studied lawn mower, was then setup, showing good correspondences with experimental results. General guidelines for defining effective vehicles Multi-Body modal models were also derived.

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*Keywords:* Experimental modal analysis; Finite Element modal model; Multi-Body modal model; lawn mower.

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### 1. Introduction

The vibration behavior of vehicles is important for comfort analysis, especially if the vehicle is intended for long-term human operation, such as professional lawn tractors. Unfortunately, experimental analyses are time consuming, expensive and usually provide a partial description of the vehicle behavior. Moreover, the correction of vibration

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phenomena on a prototype is expensive and has a severe impact on the layout. For these reasons, it is useful to have computational tools able to predict the machine behavior. However, the development of detailed models of the vehicle is not affordable in this industrial field and the setup of simplified models should be preferred.

The presented research activity originated in the framework of a collaboration between University of Pisa and Global Garden Products Italy S.p.A. (GGP, Castelfranco Veneto, TV, Italy), with the aim of developing a simplified model for determining the vibration behavior of a lawn mower. Two numerical models of a real lawn mower prototype were developed. The first model exploited the Finite Element (FE) method, considering all the tractor's parts as deformable bodies. This more sophisticated model provided a full description of the vehicle modal response in a wider frequency range and was indeed used to tune the simpler MB model. The second one was a simplified semi-flexible MultiBody (MB) modal model: the main structural parts were modelled as flexible bodies, the tires were modelled as vertical linear springs and all the other parts were modelled as rigid added masses. The main advantage of this simplified model consists in relatively short computational time, while the main drawback is represented by a limited capability of the model to describe the structure's behavior corresponding to high frequency modes.

The FE model was firstly validated by comparison with the experimental modal analysis of the lawn mower; then, the main masses, inertia and compliances to be included in the MB model were identified with a step-by-step procedure: a comparison between the modal responses furnished by the two numerical models was performed to set the parameters until a good agreement was found. The activity allowed to derive some guidelines for developing simplified and accurate semi-flexible multibody models, without the need to repeat the FE modelling.

## 2. Material and methods

### 2.1. Lawn mower description

The vehicles of the present study are professional lawn mowers as shown in Fig. 1. More precisely, the studied tractor is a MTR 122 Proto 1, whose main characteristics are reported in Table 1.

Table 1. Lawn mower main dimensions.

Quantity	Value	Unit
Overall length	1710	mm
Overall width	1255	mm
Overall height	1150	mm
Wheelbase	1250	mm
Total weight	200	kg

The vehicle is powered by a single cylinder 4-stroke air cooled engine, which exert a gross torque of 41 Nm (at 3600 rpm). The mower is also equipped with a cutting deck composed of two counter-rotating blades having a diameter of 1220 mm. The parts of the vehicle can be schematically divided in two groups: structural components and additional parts. The main structural part is the base frame, which is composed of two rectangular tube rails, supporting the seating and the steering frames, and a cross plate supporting the engine. The rails are connected by a U-shape beam at the front end (which supports the front axle) and by an L-shape beam at the rear end. All the connections are obtained through welded joint. The seating frame is composed by beams of various sections (L-shape, and rectangular) two C-shape plates and a folded plate supporting the driver's seat. The connections are obtained through continuous or intermittent welded joints and bolted joints. The steering frame is composed by several components: L-shape and rectangular beams, several folded plates supporting the dashboard frame, two multi-holed plates supporting the fuel tank and four cylindrical spacers. The connections are obtained through intermittent welded joints and bolted joints. Finally, the components supporting the box system are a main folded plate, two brackets and four threaded bars, which are connected through bolted joints.

The main structural parts are assembled to host the additional parts, which may be characterized by their inertia properties, and are mainly represented by: the engine (Briggs & Stratton INTEK 5-210 AVS, dry weight of 29.5 kg,

479×411×327 mm), the cutting deck (1220 mm, weight of 42 kg), the hydrostatic transmission (Tuff Torq's K46, weight of 15 kg), the front tires (WANDA 16×6.50 – 8 NHS, weight of 6.0 kg) and rear tires (WANDA 20×10.00 – 10 NHS, weight of 10.5 kg).

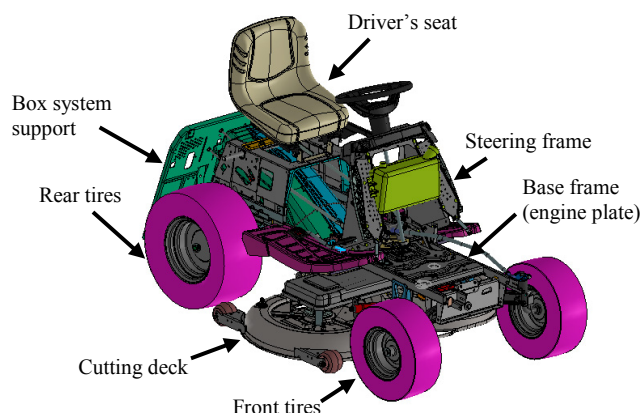


Fig. 1. Lawn mower main parts.

## 2.2. FE modal model

Some basic assumptions were made to simplify the modelling of the assembly. The parts driving the tractor modal response were: the main frame of the vehicle, steering and seating frames, the cutting box supporting system, the cutting desk system, the engine motor, the transmission and the tires. All these parts were included in the model. In particular, all the added masses as described in the section above were modelled as concentrated structural masses. The masses of the transmission and of the cutting desk were obtained from the component specifications, while their inertia tensors and their center of gravity coordinates were obtained via CAD. The tires mass and inertia and center of gravity were also obtained via CAD, while the mass and the radial stiffness were experimentally measured. The engine weight and center of gravity were provided by the manufacturer. Since the complete CAD file was not available, the inertia tensor was estimated by modeling the engine as a box having the same engine dimensions and mass density resulting in the same weight and center of mass position of the actual component. On the other hand, all the structural components were modeled as homogeneous, isotropic and linear elastic material having a density  $\rho=7850 \text{ kg/m}^3$ , a Young's modulus  $E = 210 \text{ GPa}$  and a Poisson ratio  $\nu = 0.29$ . All these parts were modelled as solid bodies, after a CAD model elaboration (cleaning of chamfers, fillets and small details). Structural solid elements (3D, 20-Node) were chosen for all the components. Figure 2 shows an overview of the solid bodies as modeled in the FE analysis. A great effort was spent in determining all the connections between the huge number of components of the assembly. In particular, all the bolted joints were modeled as spot welds whose welded areas were coincident with the compressed flanges mating surfaces. All the contact region were implemented as contact problems using 3D 8-Node *surface-to-surface* contact elements and always setting a bonded contact status. The welding among members were also treated as contact problems by implementing fillet welded joints geometry (bonded contact status). The FE model was implemented and solved using the general purpose software ANSYS® Academic Research Mechanical, Release 17.2. The modal analysis was set to determine the natural modes of the component up to 1000 Hz. An example of numerical mode shape (bending mode) is reported in Fig. 6(a).

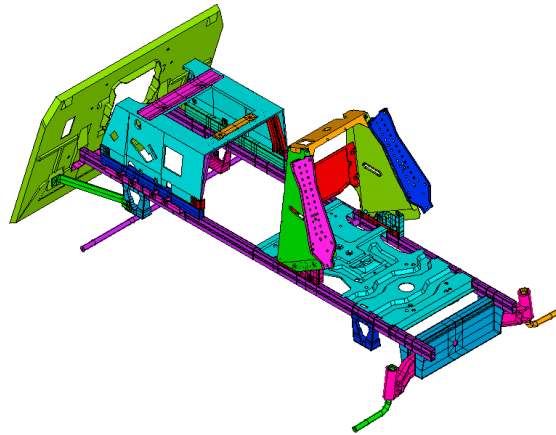


Fig. 2. Axonometric view of the lawn tractor frame implemented in the FEM.

### 2.3. Multi-Body modal model

In order to reduce the degrees of freedom of the model, an alternative approach was considered. The model was developed in MSC Adams. View environment and included rigid parts, lumped stiffness elements and distributed stiffness parts. Before describing the model implementation, it is suitable to split the lawn tractor structure in several sub-structures, as shown in Fig. 3.

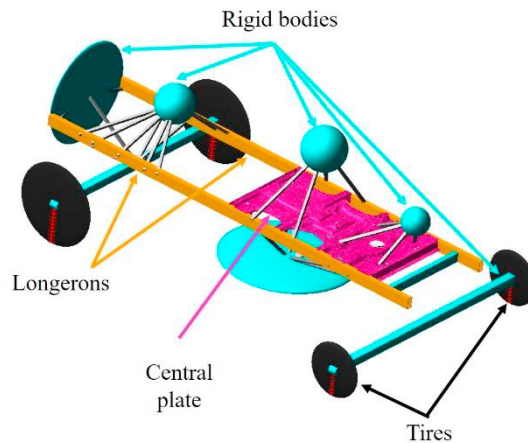


Fig. 3. MB model main parts.

In particular, eight sub-structures were considered as rigid, i.e. the front and rear axles, the cutting deck, the rear plate, the seat structure, the steering structure, the engine and the steering frame structure, while seven sub-structures were identified as flexible, i.e. the four tires, the two longerons and the central plate. Some connecting rods with negligible mass and inertia properties were interposed between the sub-structures, to reproduce the lawn tractor integrity. The rigid sub-structures are represented in Fig. 3 as spheres, plates and beams just to keep simple the model graphics; actually, the mass and inertia properties of these parts were imported from CAD. The flexible sub-structures were implemented differently depending on their shape and role, using lumped stiffness springs, Timoshenko beam elements and Craig-Bampton modal synthesis (Craig and Bampton (1968)).

The tires were modeled as rigid bodies, having mass and inertia properties imported from CAD, which are related to the lawn tractor axles through a revolute joint along the wheel spin axis. Since the aim of the lawn tractor model used in this analysis was to identify the eigenmodes related to a stationary vehicle, only the radial properties of the tires were considered, while no lateral and longitudinal actions were implemented. The center of each wheel was constrained to not move in lateral direction, due to the high stiffness of the tires sidewalls. Also, the rotation about the axis normal to the ground was fixed. This assumption is related to the static friction between the tire and the ground which, at zero speed, allows the tire steering only if the steering torque is greater than the friction limit torque (about 50 Nm or more) as described in Sharp and Granger (2003). For this reason, if the lawn tractor is not moving and no steering torque is applied to the steering wheel, the friction between the ground and the tire can be considered actually a constraint which prevent the tire steering.

The radial stiffness was modeled connecting the wheel center to the ground through a linear spring and damper. The stiffness of the spring was derived from the experimental tests of the tires, while the damping was assumed equal to 10% of the critical wheel damping (see Pacejka (2005)).

The longerons are the chassis elements which connect almost all the vehicle parts and they can be modeled as one dimensional elements, since the length is much greater than the cross section dimensions. In particular, in the multibody model, the longerons were modeled as discrete Timoshenko beams. Each beam was divided in 48 elements, each one measuring about 33 mm in length, which were interconnected through elastic forces and moments on the basis of the Timoshenko beam theory. The elements size was chosen in order to do not have more than one edge of the connecting rods connected to a beam element. The beam section parameters were assumed to be constant within the beam element.

The central plate was imported from FE model using the Craig-Bampton technique. As described in Section 2.2, the imported plate was previously modeled using 3D brick structural 20 node element and 20 connecting regions were identified on the plate. In particular, 16 connecting regions were related to the welding areas between the central plate and the longerons, while 4 connecting regions represented the bolted connection between the plate and the engine.

In order to reproduce the lowest frequency natural modes of the lawn tractor, some simplifying hypotheses were assumed. For each connection region, the centroid was computed and the nearest plate node was identified as the interface point. A modal neutral file was created using the Ansys command *ADAMS*, which combines the constrained normal modes and the constraint modes of the structure on the basis of a slightly modified version of the Craig-Bampton component mode synthesis method implemented in MSC.Adams View.

The connection between the plate interface nodes and the longerons elements was implemented in the multibody model through fixed joints. Differently, the connection between the longerons elements and the rigid bodies connected to these (i.e. the rear axles, the cutting deck, the rear plate, the seat structure, the steering structure and the steering frame structure) was implemented using spherical joints. This modeling choice was assumed after a model tuning phase which highlighted that using fixed constraints between longerons elements and rigid bodies conferred too much stiffness to the longerons, while the use of spherical joints was preferable to reproduce the actual behavior of the system as obtained by the FE model.

### 3. Experimental analysis

Both FEM and MB models were validated through Experimental Modal Analysis. A dedicated test bench was adopted in order to experimentally evaluate natural frequencies and shapes of the modes of interest: a robotic station for automatic modal analysis was developed at University of Pisa along with its control software, and a complete validation of the system was obtained through bladed wheels analysis (Bertini et al. (2014) and Bertini et al. (2017)). The equipment proved to guarantee high fidelity results, having a good correspondence with FEM models both in terms of natural frequencies and of modal shapes. An overview of the test setup is reported in Fig. 4: an ABB anthropomorphic robotic arm was used to handle a Laser Doppler Vibrometer sensor head, so that high precision positioning and orientation, along with high repeatability, could be achieved. A Polytec LDV sensor was used, allowing remote autofocusing. The LMS-Siemens hardware and software were used during the test (8 input and 2 output SCADAS and Test.Lab 2011). Finally, a TiraVib electrodynamic shaker was used to apply the load to the structure. A dedicated Visual Basic software was developed at University of Pisa in order to coordinate all the described hardware and software, so that a fully automated test procedure could be achieved.

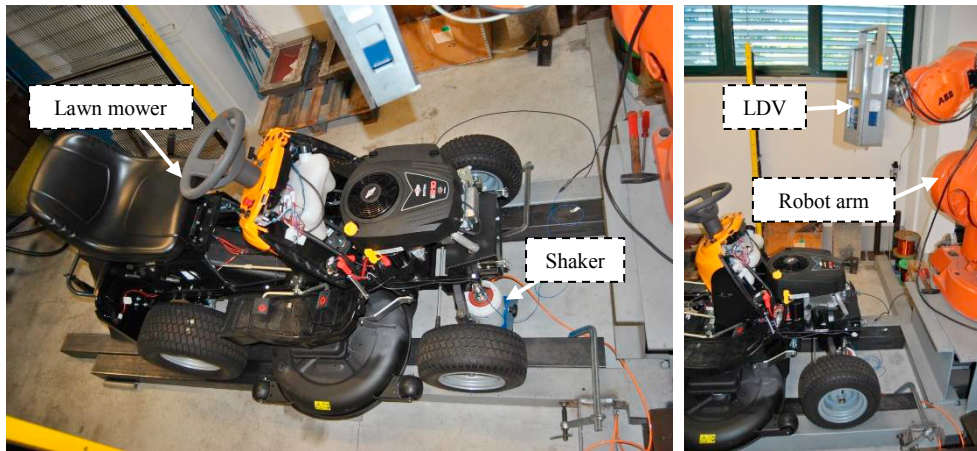


Fig. 4. Overview of the test equipment.

Six measurement locations were chosen on the tractor chassis. The locations choice was made as a compromise between the need of information to describe the mode shapes and the visibility of the parts for the LDV sensor. Reflective tape was glued on the black chassis to increase the laser signal power. Figure 5 highlights the measurement locations: Fig. 5(a) shows a top view of the lawn mower and Fig. 5(b) shows a schematic view of the measurement locations.

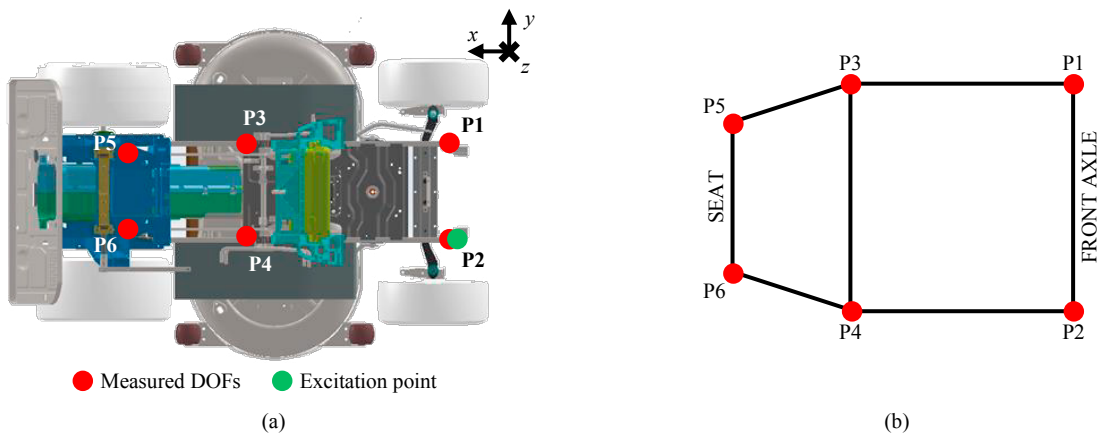


Fig. 5. Overview of the test equipment.

The Single-Input/Multiple-output approach was followed during the test by exciting the structure always at the same location (corresponding to P2) and subsequently measuring locations from P1 to P6 by moving the LDV through the robotic arm. Since the motor was a four strokes engine running at 2900 rpm, and the transmission to the cutting deck was about 1:1, the first load harmonic during operation was about 23-24 Hz (a small speed variation was found when the blades were running). Moreover, the measured Frequency Response Function (FRF) showed a high noise level at frequency greater than 100 Hz. Thus, the range 0-50 Hz was considered for the modal analysis, which covers the firsts two harmonics of the load and contains the rigid body modes and the firsts deformation modes. A complete list of the natural modes in the investigated frequency range is reported in Table 2, along with the comparison with FEM and MB results.

#### 4. Results comparison and discussion

The modal analysis was repeated exploiting the FE model, the MB model and the experimental setup. The characterization was performed in terms of natural frequencies and mode shapes, in the range 0-50 Hz. Table 2 summarizes the comparison between the detected modes frequencies.

Table 2. Natural frequencies comparison.

Mode	FE (Hz)	MB (Hz)	EXP (Hz)
Roll	7.8	7.5	8.0
Heave	8.1	8.0	8.3
Pitch	10.1	9.6	10.0
Bending	18.9	17.0	18.5
Torsion 1	27	25	29
Torsion 2	37	39	35

The comparison highlights a good correspondence between the three mode sets with a relative difference lower than 15%, which was considered acceptable considering the complexity of the actual assembly. Figure 6 shows an example of mode shape comparison, referred to the Bending mode (18.5 Hz). Figure 6(a), (b) and (c) report the mode shape corresponding to the FE model, the MB model and the experimental measurement respectively.

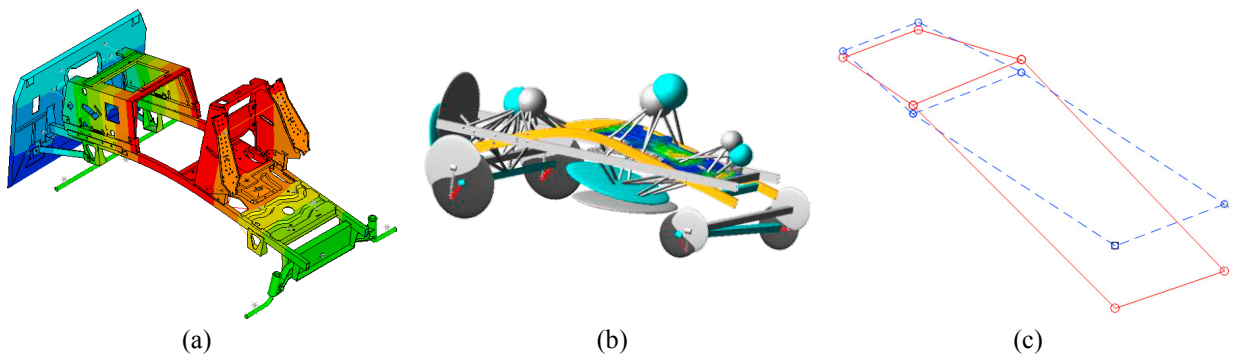


Fig. 6. Mode shape comparison for the first bending mode (18.5 Hz): (a) FE model, (b) MB model, and (c) experimental measurement.

The comparison confirmed a good correspondence between the models' predictions, and also a satisfying validation through the comparison with experimental analysis. It is worth noting that the mode description achievable through the numerical models is indeed much more complete than the available experimental results, due to the low number of measurement points which were accessible during the test. The provided comparison allowed to validate both the FE and the MB models, confirming the robustness of the hypotheses which led to the MB simplified model. The presented results allowed to conclude that the choice of rigid and deformable components in the MB model was coherent with the physical behaviour of the tractor, validating the proposed procedure. Since the results obtained with the MB model were really similar to the FEM predictions, the MB modelling proved to be a more valuable design tool, since it is much easier to be implemented and also requires really faster simulation time. Nevertheless, the FE model was useful to set the MB model for this vehicle class, and would not be needed for designing a new (slightly) different lawn mower.

#### 5. Conclusions

A multilevel approach to the analysis of the vibrational behavior of a lawn mower has been presented in this paper. The aim of the activity was to develop and validate a simplified MB model, including rigid bodies, lumped stiffness

and deformable bodies, able to reproduce the actual vibrational response of a lawn mower. For this reason, an accurate FE model was initially developed, including the reproduction of many vehicle junction details (e.g. welding connections between plate and longerons). Then the vibrational response of the real vehicle was analyzed by an extensive experimental modal analysis. The results of this analysis were compared with the FE modal analysis and confirmed the accuracy of the complete FE model. Finally, a reduction process was performed through the development of the MB model, which was calibrated on the basis of the FE model results. The MB model was able to fairly well reproduce the first 6 natural modes of the vehicle, both in terms of frequency and mode shape, requiring a very low computational effort (if compared to the FE model).

The results of the activity allow the designer to be aware of the model detail level requested to simulate, in a fast and lean way, the vibrational behavior of such vehicles and to potentially extend the use of this technique to other kind of vehicles or structures.

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