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Evaluation of pressure drop and particle sphericity for an air-rock bed thermal energy storage system

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Abstract

The pressure drop of a packed bed thermal energy storage system with irregular shaped solid pellets and tank-to-particle diameter ratio of 10.4 is investigated. The bed height to diameter ratio is 2. The particle sphericity is calculated and used to compare pressure drop correlations to the measured values in the particle Reynolds number range of $353 \leq Re_p \leq 5206$.

Keywords: packed bed; pressure drop; tank-to-particle ratio; particle sphericity

1. Introduction

A packed bed thermocline system is a promising thermal energy storage (TES) concept due to its single tank design and employment of cheap and abundant storage media such as sand and rock. Laboratory-scale and pilot-scale packed bed systems have been tested and used to develop empirical correlations and validate numerical models that describe fluid flow and heat transfer within these systems. The models provide an avenue by which the parameters of a packed bed and their influence on performance and thermal behaviour can be investigated.

Within packed bed systems, pressure drop pumping losses can be significant. Thus many studies focus on developing pressure drop correlations which are based on key parameters that affect the transport properties of a system. These parameters must be optimally chosen such that they minimize pressure losses without compromising heat transfer and efficiency.

Nomenclature

A_p surface area of particle, m^2

A_{sp} surface area of sphere, m^2

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d	diameter of packed bed tank, m
D_{eq}	equivalent particle diameter defined by Li and Ma [4], m
D_p	particle diameter, m
D_{sd}	Sauter diameter - equivalent particle diameter by specific surface, m
D_v	equivalent particle diameter by volume, m
f	dimensionless friction factor
L	height of packed bed, m
Re_h	hydraulic particle Reynolds number
Re_p	particle Reynolds number
U	superficial bed velocity, viz. average velocity in an empty tank, m/s
V_p	volume of particle, m^3

Greek letters

ε	void fraction
ΔP	pressure drop of packed bed, Pa
μ	dynamic viscosity of the fluid, kg/(m-s)
ρ	density of the fluid, kg/m^3
ψ	particle sphericity

Of the various pressure drop correlations that have been presented, the Ergun equation [1] is one of the most widely adopted:

$$\frac{\Delta P}{L} = A\frac{(1-\varepsilon)^2}{\varepsilon^3}\frac{\mu U}{D_p^2} + B\frac{1-\varepsilon}{\varepsilon^3}\frac{\rho U^2}{D_p} \tag{1}$$

where the coefficient *A* is 150 and *B* is 1.75.

In Ergun's seminal publication, previous theories and equations on pressure loss through a bed were utilized in conjunction with experimental data to establish the above relationship. The first term on the right-hand side represents viscous energy losses that dominate during laminar flow and the second term accounts for kinetic losses that govern in the turbulent regime. Experiments used in the development of the correlation included particles of various shapes such as spheres, cylinders, tablets, and crushed solids. The only factors considered in the analysis were fluid flow rate, particle diameter, fluid viscosity and density, and fractional void volume. The correlation should be valid for hydraulic particle Reynolds numbers between 1 and 3000. The hydraulic particle Reynolds number differs from the particle Reynolds number that is typically used, in that it has a dependence on the void fraction. The hydraulic particle Reynolds number and particle Reynolds number are defined respectively as [2]:

$$Re_h = \frac{\rho U D_p}{\mu(1-\varepsilon)}, \tag{2}$$

$$Re_p = \frac{\rho U D_p}{\mu} \tag{3}$$

Ergun's equation has been successfully employed to predict the pressure drop of packed beds filled with regular-shaped spherical particles [3-5]. Numerous correlations have also been proposed, some of which simply alter the constants A and B, or modify the other bed parameters in order to develop a more accurate prediction of pressure drop for particle shapes that deviate from spherical. Modification has also been made by incorporating the shape factor, or particle sphericity, which is a measure of the degree to which a particle's shape approaches the shape of a sphere. It does not necessarily define the shape, but it describes the effect that the shape has on the hydrodynamic behaviour [6]. The sphericity is defined as [7]

$$\psi = \frac{\text{surface of sphere of equal volume to the particle}}{\text{surface area of the particle}} = \frac{\pi^{1/3}(6V_p)^{2/3}}{A_p} . \tag{4}$$

In Ergun's publication there is little mention of the tank-to-particle diameter ratio, d/D_p, except that scatter in some of the data was likely due to a d/D_p ratio less than 10. It is speculated that packed beds with low tank-to-particle ratios are strongly influenced by the "wall-effect" which is caused by increased void fraction and viscous friction at a rigid tank wall [8]. The ratio at which the wall effect is negligible has been inconsistently defined and there are opposing views on its exact effect on pressure drop [9]. For example, Meier et al. [10] claim that the wall-effect can be neglected when d/D_p is greater than 40, whereas Torab and Beasely [11] as well as Cohen and Metzner [12] indicate a value of $d/D_p > 30$. Eisfeld and Schnitzlein [9] conducted an extensive analysis of more than 2300 data points from pressure drop experiments, mostly found in the literature, in order to investigate the wall-effect on pressure losses. They concluded that the influence of the wall-effect is dependent on the Reynolds number, i.e. an increasing pressure drop due to the wall-effect appears in streamline or transitional flow and a decreasing or lack of pressure drop due to the wall emerges during turbulent flow. Moreover, Eisfeld and Schnitzlein [9] determined that for streamline flow, the wall-effect is significant when the d/D_p ratio is less than 10. They present a pressure drop correlation that was developed in a study by Reichelt [13] in the form of the dimensionless friction factor and found that it is valid for particle Reynolds numbers of $0.01 \leq Re_p \leq 17635$, and tank-to-particle diameter ratios of $1.624 \leq d/D_p \leq 250$:

$$f = \frac{K_1 A_w^2}{Re_p} \frac{(1-\varepsilon)^2}{\varepsilon^3} + \frac{A_w}{B_w} \frac{1-\varepsilon}{\varepsilon^3} = \frac{\Delta P D_p}{\rho_f L U^2} \tag{5}$$

with the wall correction terms defined as

$$A_w = 1 + \frac{2}{3(\frac{d}{D_p})(1-\varepsilon)} \tag{6}$$

$$B_w = \left[k_1 (\frac{D_p}{d})^2 + k_2 \right]^2 \tag{7}$$

and k coefficients defined in Table 1 for different particle shapes.

Table 1. Coefficients for Reichelt's equation, Eqn (5) [9].

Particle shape	Coefficients		
	K_1	k_1	k_2
Spheres	154	1.15	0.87
Cylinders	190	2.00	0.77
All particles	155	1.42	0.83

In Eqns. (1) and (5), the only packing parameters that are considered in the correlation are the particle diameter and void fraction of the bed. Since the equivalent particle diameter, D_p, may be the same for different shape particles that exhibit dissimilar flow characteristics, Singh et al. [5] developed a new friction factor correlation that incorporates particle sphericity. Experiments were conducted at particle Reynolds numbers ranging between $1047 \leq Re_p \leq 2674$ and on particles of varying diameter, void fraction, and sphericity in order to develop the following correlation [5]:

$$f = 4.466(\mathrm{Re}_p)^{-0.2}(\varepsilon)^{-2.945}[\exp\{11.85(\log\psi)^2\}] = \frac{\Delta P \rho_f D_p}{L G^2} . \tag{8}$$

The above pressure drop correlations based on Ergun, Singh et al., and Eisfeld and Schnitzlein, are dependent on the particle diameter, D_p, which is equal to the diameter of the particle if it is a sphere. If the particle is non-spherical, an equivalent particle diameter must be used. In Ergun's correlation and Eisfeld and Schnitzlein's correlation this is defined as the Sauter-diameter, which is the diameter of a sphere with the same volume-to-surface area ratio as a non-spherical particle:

$$D_{sd} = \frac{6V_p}{A_p} \tag{9}$$

In [1], Ergun states that use of the volume-to-specific surface of solid ratio was tested and is valid for many shapes including spheres, cylinders, and crushed materials such as coke and coal, but does not extend to solids with holes or other special shapes. Singh et al.'s equation incorporates an equivalent particle diameter by volume, D_v, defined as the diameter of a sphere having the same volume as the given particle:

$$D_v = (\frac{6}{\pi}V_p)^{1/3} = \frac{6V_p}{A_{sp}} = \frac{6V_p}{\psi A_p} = \frac{D_{sd}}{\psi} . \tag{10}$$

Thus Ergun's equation can be written as

$$\frac{\Delta P}{L} = 150\frac{(1-\varepsilon)^2}{\varepsilon^3}\frac{\mu U}{(\psi D_v)^2} + 1.75\frac{1-\varepsilon}{\varepsilon^3}\frac{\rho U^2}{\psi D_v} \tag{11}$$

Li and Ma [4] however, conducted experiments and found that use of the Sauter diameter under-predicts the pressure drop when Ergun's equation is used for non-spherical particles. They state that this is due to the fact that the Sauter diameter only considers the specific surface area and does not explicitly account for the sphericity, or shape of the particle. Li and Ma replaced the Sauter diameter with one that considers the sphericity and found that the flow resistance could be predicted with the Ergun equation. They defined this particle diameter, D_{eq}, as:

$$D_{eq} = \psi D_{sd} = \psi^2 D_v = \frac{\psi^2 6V_p}{A_{sp}} = \frac{\psi 6V_p}{A_p} . \tag{12}$$

From the above discussion, it is apparent that there are numerous ways to calculate pressure losses in a packed bed. In the present study, an attempt is made to find a correlation that is best suited for a packed bed of highly irregular shaped solid pellets with moderately low tank-to-particle diameter ratio. The pellets can be considered as crushed rock having flat, jagged surfaces.

2. Experimental test set-up

A schematic of the packed bed system is provided in Fig 1. The final bed height is 0.889 meters and the bed diameter is 0.445 m. The total tank height including the upper and lower plenums is 1.83 m and the tank diameter is 0.711 m. The

difference between the tank diameter and the bed diameter is due to a 0.152 m thick flexible ceramic wool insulation that lines the tank and which will be used for future heat transfer experiments. To estimate the diameter of the bed, an average value was taken by measuring the diameter at various positions. The uncertainty of this measurement was included in the analysis of the air mass flux. A 9.53 mm thick perforated carbon steel plate with 12.7 mm holes and 48% open area is used to support the pellets. The upper and lower elbows are 0.161 m in diameter. A diffuser plate was installed at the exit of the lower elbow to reduce the effects of flow separation in the elbow, which creates non-uniform flow conditions. Room temperature (23°C) air entered the bottom of the system through one of 3 blowers that were used to achieve the desired flow rates. Velocity was measured with a hot wire anemometer at a position that had at least 10 pipe diameters upstream and 4 pipe diameters downstream from the point of measurement. Pressure was measured with a digital manometer. Specifications and accuracy values for the instruments are provided in Table 2.

Table 2. System components.

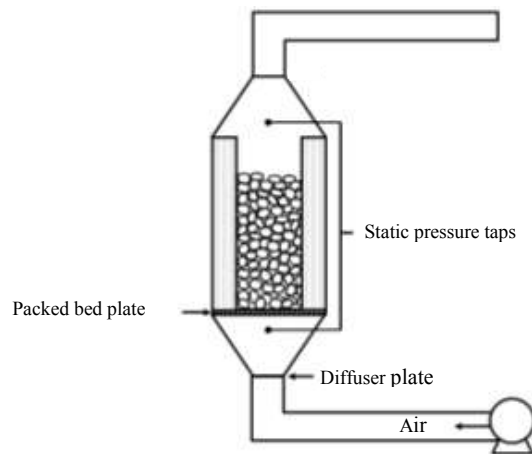| Instrument	Description	Accuracy
Hot wire anemometer	0.15 – 30 m/s range	± 3% reading
Digital manometer	0 -1 in. w.c.(0 – 249.1 Pa) range	± 0.5% F.S./ ± 0.1% F.S. hysteresis
Digital manometer	0 -20 in. w.c.(0 – 4.982 Pa) range	± 0.5% F.S./ ± 0.1% F.S. hysteresis
Dayton Blower	¼ hp	-
Dayton Blower	1/30 hp	-
Heavy Duty Blower	7.5 hp / 5.5 kW, 3 phase induction motor	-

Fig. 1. System schematic.

To calculate bed porosity, the mass of the pellets and bed volume were used to estimate the bulk density which was then used in the following equation to calculate porosity:

$$\varepsilon = 1 - \frac{\text{Bulk density}}{\text{True density}}. \qquad (13)$$

Porosity was also measured in a separate container with a diameter that was similar to the final bed diameter to verify the voidage.

The equivalent diameter of the pellets was obtained by measuring the mass of 35 random samples. The volume of each of the samples was then calculated using the true density, and the average volume of the 35 samples was used to obtain the equivalent particle diameter by volume as defined in Eqn. (10) with a standard deviation of 0.006426 m. The bed parameters are listed in Table 2.

Table 2. Bed parameters

Description	Value
Bed voidage	0.51
D_v, particle diameter by volume	0.04259 m
Solid pellet density	3127 kg/m3
Tank-to-particle ratio	10.4

The packed bed support plate and the restriction created by the insulation at the exit and entrance of the bed introduce an additional pressure drop. To estimate this value, the pressure drop was measured with an empty bed for different velocities. A third-order polynomial best-fit equation was then used to calculate the additional pressure loss and subtract it from the total pressure drop. In order to test whether there was flow maldistribution due to the inlet geometry, velocity was measured at various points in the cross section of the first five to ten centimeters of the bed with a vane anemometer and found to be acceptable.

3. Results

Tests were conducted for particle Reynolds numbers between $353 \leq Re_p \leq 5206$. The particle Reynolds number was calculated by using the equivalent particle diameter by volume, D_v. The results of the pressure measurements as a function of air mass flux are provided in Fig. 2. An error propagation uncertainty analysis was conducted and the pressure gradient ($\Delta P/L$) uncertainties are included in the figure. The air mass flux (kg/m^2-s) uncertainty ranged from 3.1% to 3.8% and was not included in order to maintain clarity within the plot.
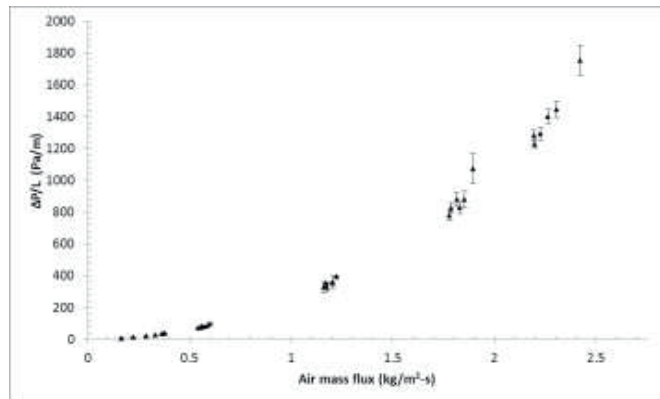

Fig. 2. Measured pressure gradient versus air mass flux.

Since the particle sphericity is difficult to calculate with irregular shaped solids, it was deduced from Ergun's correlation, Eqn. (11), by calculating the root mean square deviation between measured and predicted pressure drop for different sphericity values. This was repeated with Singh et al.'s equation using data that fell in the particle Reynolds number range that was used to develop the correlation, i.e. between $1047 \leq Re_p \leq 2674$. The same sphericity value of 0.495 was obtained using both correlations.

Figure 3 demonstrates the measured pressure drop of the packed bed versus particle Reynolds number. Measured values are plotted with Ergun's original equation, Eqn. (1), and the equation presented in Eisfeld and Schnitzlein, Eqn. (5), which accounts for the wall-effect. The Sauter diameter was used in both equations. In calculating Eqn. (5), the coefficients for "all shapes" were used, as defined in Table 1. Both correlations are plotted with Singh et al.'s equation in Fig. 3, which uses the equivalent particle diameter by volume. Singh's equation shows a relatively good fit in the Reynolds numbers

range of 1166 to 4069, where the difference is between 0 and 18%. The difference between measured and predicted values increases at higher Reynolds numbers (neglecting the 2 outliers), lying between 14 and 20%. At lower Reynolds numbers, i.e. $Re_p < 808$, the difference is between 14 and 33%. This trend in lower predictability at lower and higher Re_p is due to the fact that Singh et al.'s equation has a 1.8^{th} power dependence on bed velocity, whereas the measurements show a 2nd power dependence ($r^2 = 0.9986$).

The difference between Ergun's equation and the measured values is between 1% to 14% over the range of $1166 \leq Re_p \leq 5206$ and 5% to 16% in the lower range of $353 \leq Re_p \leq 808$, which is shown in Fig. 4. Eisfeld and Schnitzlein's equation predicts a lower pressure drop than Ergun's equation for all Reynolds numbers, even in streamline flow. Since the sphericity effectively reduces the particle diameter, the tank-to-particle ratio increases from 10.4 to 20.9 when the Sauter diameter is used, which, per Eisfeld and Schnitzlein, is high enough so that the wall does not have an effect on pressure losses. The measured values are approximately 2% to 14% greater than Eisfeld and Schnitzlein's prediction over the entire Reynolds number range. The percent overall average relative absolute error, defined as

$$\frac{1}{n} \sum_{i=1}^{n} \frac{\left| x_{i,predicted} - x_{i,measured} \right|}{x_{i,measured}}, \tag{14}$$

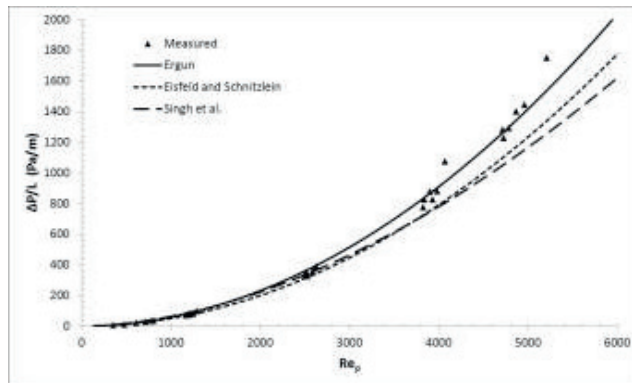for the three correlations are provided in Table 3.

Fig. 3. Pressure gradient as a function of particle Reynolds number. Shape factor of 0.495 is used in the pressure correlations.
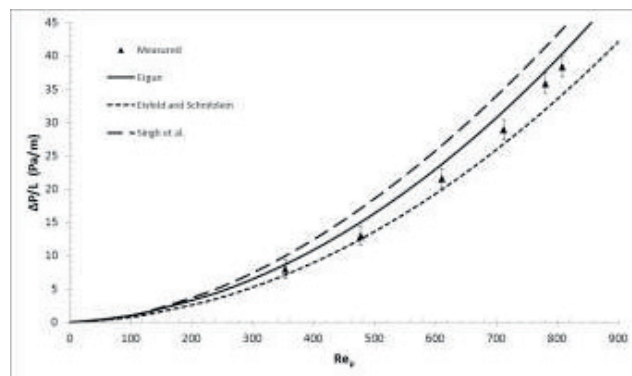

Fig. 4. Correlation between pressure gradient and particle Reynolds number for lower Reynolds number range. Particle shape factor of 0.495 is considered.

Table 3. Percent average relative absolute error (ARAE) for the 3 pressure drop correlations in which sphericity was considered.

Correlation	% ARAE
Ergun	6.9%
Eisfeld and Schnitzlein	10.1%
Singh et al.	14.2%

An attempt was also made to calculate the sphericity using the equivalent particle diameter defined by Li and Ma [4], however the sphericity that was found with this method did not produce reasonable results in Singh et al.'s equation. Figure 5 shows the results of the predictions of the three correlations if the shape factor is determined by the use of Li and Ma's equivalent particle diameter, D_{eq}. This results in a shape factor value of 0.70. As can be seen in the figure, Singh et al.'s equation severely underpredicts the pressure drop when the higher shape factor is introduced. The percent average relative absolute error for Ergun's equation in this instance is 7.6% and for Eisfeld and Schnitzlein is 7.5%. For Singh et al.'s correlation it is 43.9%.
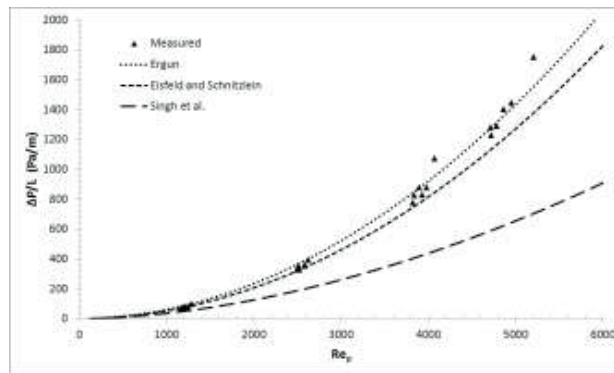

Fig. 5. Pressure gradient as a function of particle Reynolds number. Shape factor of 0.70 is used in the pressure drop correlations.

4. Conclusions

The purpose of this study was to find a pressure drop correlation that can reasonably predict pressure losses in a bed of highly irregular shaped solid pellets. Many correlations are available in the literature and there is little agreement on any universal method that can be used to accurately predict the pressure gradient for non-spherical shapes. There has even been some dispute in the accuracy of Ergun's correlation for the prediction of spherical particles [14]. With the many factors that can come into play, it is of no surprise that there is considerable discrepancy between correlations. Key parameters such as inlet and outlet conditions, tank-to-particle ratio, packing method, bed porosity, and particle shape can affect the studies. In [15] it was found that by simply washing aggregates of crushed stone and rock, pressure losses decreased by a factor of approximately 2. Correlations are also highly sensitive to slight changes in porosity and particle diameter, which can often be difficult to measure accurately.

The equivalent particle diameter by volume, D_v, is the simplest diameter to determine yet it does not necessarily provide a complete picture of the hydraulic behaviour of the bed. The Sauter diameter, which is the product of the sphericity and D_v, is known to be a more appropriate property, however the sphericity is very difficult to measure for

irregular shapes. Therefore it was back-calculated using Ergun's equation which shows a similar dependence on the bed velocity as the measured values. The calculated sphericity also produced reasonable predictions from Singh et al.'s equation, which was based on experiments that utilized well-defined shapes so that the sphericity could easily be calculated. The equivalent diameter, D_{eq}, defined by Li and Ma was also used to determine sphericity but the obtained value did not produce good results with Singh et al.'s equation. Either method alone provided a sphericity that could be used in Ergun's or Eisfeld and Schnitzlein's correlations, however, the physical significance of the smaller shape factor of 0.495 seems to be more reasonable. Since the particles exhibit a shape that is closer to a parallelepiped than a sphere, one would expect the sphericity to be very low. Particle roughness and roundness may also play a role in reducing the shape factor. Their effect is similar to the effect of sphericity in that they change the surface characteristics of the particles, essentially producing smaller channels, which increases inertial resistance within the system [15].

It is difficult to definitively say that the wall does not have an effect on the pressure drop in this instance. Since the tank-to-particle ratio is on the cusp of the limit that was suggested by Eisfeld and Schnitzlein, and there is flexible insulation lining the wall which may reduce the effects of wall channelling, it is likely that the wall effects are negligible. It is also possible that since the solid particles are randomly placed in the bed and there is little structure both within the bed and at the walls, the effect of wall channelling may not be as severe as compared to a bed packing that is highly structured. Additionally, the tests were conducted in the turbulent regime where, as suggested by Eisfeld and Schnitzlein, the wall plays a minor role in pressure losses within the bed. Nevertheless, both Ergun's equation and Eisfeld and Schnitzlein's equation give comparable results in predicting the measured values during turbulent flow when the calculated shape factor is employed.

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