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Mode I fracture toughness of fibre-reinforced concrete by means of a modified version of the two-parameter model

Sabrina Vantadori*, Andrea Carpinteri, Giovanni Fortese, Camilla Ronchei, Daniela Scorza

Dept. of Civil-Environmental Engineering and Architecture (DICATeA) – University of Parma
Parco Area delle Scienze 181/A, 43124 Parma, Italy

Abstract

The present paper proposes a method to calculate Mode I plane-strain fracture toughness of concrete, by taking into account the possible crack deflection (kinked crack), even in the case of a far-field Mode I loading. As a matter of fact, during fracture extension, cracks may deflect as a result of microstructural inhomogeneities inside the material. Concrete is an inhomogeneous mixture due to aggregates embedded in the cementitious matrix, but additional inhomogeneities may be represented by fibres. Firstly, a two-parameter fracture model based on Mode I analytical expressions of the linear elastic fracture mechanics is employed. Then, in order to take into account the possible crack deflection as a result of the above inhomogeneities, a modified version of such a model is here discussed. Three-point bending tests on both plain concrete specimens and concrete specimens reinforced with micro-synthetic polypropylene fibrillated fibres are experimentally performed, and the modified model is applied.

Keywords: Fibre-reinforced concrete; fracture toughness; micro-synthetic polypropylene fibrillated fibres reinforced concrete; two-parameter fracture model

1. Introduction

The Two-Parameter Model (TPM) originally proposed to determine the value of Mode I plane-strain fracture toughness of plain concrete (Jenq and Shah (1985), RILEM (1990), Karihaloo and Nallathambi (1991)) is herein modified in order to take into account the possible crack deflection (kinked crack).

According to the TPM, the value of the fracture toughness is obtained from three-point bending tests on single edge-notched specimens. Firstly, the registration of the applied load against the crack mouth opening displacement

* Corresponding author. Tel.: +39 0521-905962; fax: +39 0521-905924.
E-mail address: sabrina.vantadori@unipr.it

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(CMOD) is needed. Then, the critical stress-intensity factor is computed through the expressions related to a crack loaded in Mode I (Tada et al. (2000)).

However, a crack in concrete may deflect during fracture extension, even in the case of a far-field Mode I loading, as a result of inhomogeneities embedded in the cementitious matrix. Inhomogeneities can be represented by aggregates for a plain concrete, and by aggregates and fibres for a fibre-reinforced concrete. Since the crack is loaded under both Mode I and Mode II in such a case, the TPM cannot be applied in its original formulation being proposed for crack under Mode I loading only. Therefore, to determine the critical stress-intensity factor (or Mode I plane-strain fracture toughness), a modified version of the TPM is here proposed by employing both the Castigliano theorem and the analytical solutions for the SIFs of a bent crack (Kitagawa et al. (1975), Cotterell and Rice (1980)).

Three-point bending tests on concrete specimens are performed in order to assess the proposed model, by considering the inhomogeneities represented by both only aggregates (for plain specimens) and aggregates and randomly-distributed micro-synthetic polypropylene fibrillated fibres with a fibre volume content equal to 2.5% (for fibre-reinforced specimens).

Nomenclature

$\underline{a}$	effective critical crack length
a_0	notch length
A	cracked area
B	specimen thickness
C_i	initial compliance
C_u	unloading compliance
E	elastic modulus
F	virtual load
G	total energy rate
K_{IC}^S	Mode I critical stress-intensity factor
$K_{(I+II)C}^S$	Mixed Mode critical stress-intensity factor
$P_{\max}$	peak load
S	specimen loading span
U_T	total energy
W	specimen depth
α_0	relative notch length
α	relative crack length
Δ_F	displacement along F - direction

2. Two-Parameter Model

According to the Two-Parameter Model (TPM) (Jenq and Shah (1985), RILEM (1990), Karihaloo and Nallathambi (1991)), the specimens have a prismatic shape and present a notch in the lower part of the middle cross section (Fig. 1(a)). The tests are performed under three-point bending loading and crack mouth opening displacement (CMOD) control.

Each specimen is monotonically loaded up to the peak load. When such a load is achieved, the post-peak stage follows and, when the force is equal to about 95% of the peak load, the specimen is fully unloaded. Then, the specimen is reloaded up to failure.

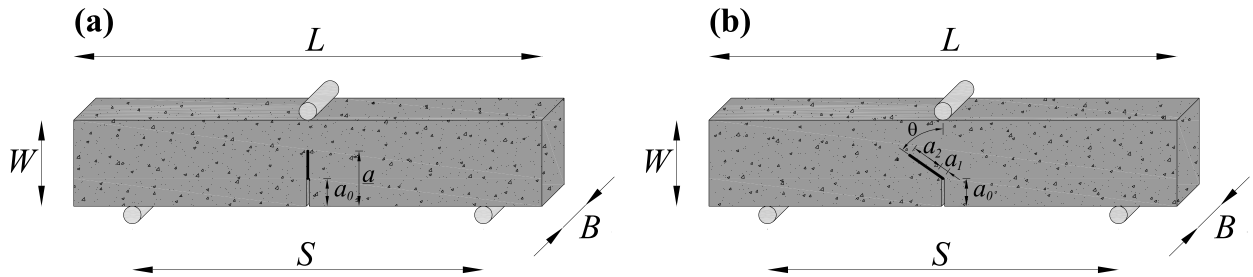

Fig. 1. Crack propagates under: (a) pure Mode I; (b) Mixed Mode.

The initial compliance, C_i, is used to calculate the elastic modulus, E (Tada et al. (2000)):

$$E = \frac{6 S\, a_0\, V(\alpha_0)}{C_i W^2 B} \tag{1}$$

where S, W and B areloading span, depth and thickness of the specimen, respectively, a_0 is the notch length (Fig. 1(a)), C_i is the linear elastic compliance. Further, the parameter $V(\alpha_0)$ is expressed as follows (Tada et al. (2000)):

$$V(\alpha_0) = 0.76 - 2.28\alpha_0 + 3.87\alpha_0{}^2 - 2.04\alpha_0{}^3 + \frac{0.66}{1-\alpha_0{}^2} \qquad \text{with} \qquad \alpha_0 = \frac{a_0}{W} \tag{2}$$

Therefore, if the crack propagates under pure Mode I loading, the effective critical crack length, $\underline{a}$, is determined through the following equation, by employing an iterative procedure (Tada et al. (2000)):

$$E = \frac{6 S\, \underline{a}\, V(\alpha)}{C_u W^2 B} \tag{3}$$

where C_u is the unloading compliance, and $V(\alpha)$ is obtained from Eq.(2) by replacing a_0 with $\underline{a}$. Since a stable three-point bend test cannot be performed in some cases, the value C_u can approximately be computed by assuming that the unloading path will return to the origin.

Finally, the Mode I critical stress-intensity factor, K_{IC}^S, is computed by employing the measured value of the peak load, $P_{\max}$, as follows (Tada et al. (2000)):

$$K_{IC}^S = \frac{3 P_{\max} S}{2 W^2 B} \sqrt{\pi \underline{a}}\; f(\alpha) \tag{4}$$

where:

$$f(\alpha) = \frac{1}{\sqrt{\pi}} \frac{1.99 - \alpha(1-\alpha)(2.15 - 3.93\alpha + 2.70\alpha^2)}{(1+2\alpha)(1-\alpha)^{3/2}} \qquad \text{with} \qquad \alpha = \frac{a}{W} \tag{5}$$

Note that the value of K_{IC}^{S}, computed by assuming an unloading path to the origin, is about 10 to 25% higher than the corresponding one computed using the actual unloading compliance.

3. Modified Two-Parameter Model

Now a modified procedure is proposed when crack propagates under Mixed Mode loading (Mode I and Mode II). Specimens geometry and experimental test procedure are analogous to those discussed in the previous Section. Firstly, the elastic modulus is determined according to Eq.(1).

Under Mixed Mode loading, the effective critical crack length, $\underline{a} = a_0 + a_1 + a_2$ (Fig. 1(b)), is obtained from the following equation by employing an iterative procedure:

$$E = \frac{6S}{C_u W^2 B} a_0 \left\{ V\left(\frac{a_0}{W}\right) + \right.$$
$$+\left[\cos^6\frac{\theta}{2} + \sin^2\frac{\theta}{2}\cos^4\frac{\theta}{2}\right]\left[(a_0 + a_1\cos\theta)V\left(\frac{a_0 + a_1\cos\theta}{W}\right) - a_0 V\left(\frac{a_0}{W}\right)\right] + \tag{6}$$
$$\left. +\left[\cos^3\theta + \sin^2\theta\cos\theta\right]\left[(a_0 + a_1\cos\theta + a_2\cos\theta)V\left(\frac{a_0 + a_1\cos\theta + a_2\cos\theta}{W}\right) - (a_0 + a_1\cos\theta)V\left(\frac{a_0 + a_1\cos\theta}{W}\right)\right]\right\}$$

Equation (6) is deduced by employing the Castigliano theorem in the manner suggested by Paris (1957), being θ the crack kinking angle (Fig. 1(b)) and $a_1 = 0.3\,a_0$. More precisely, the Castigliano theorem states that the displacement, Δ_F, of any load F (in its own direction) may be computed as follows:

$$\Delta_F = \frac{\partial U_T}{\partial F} \tag{7}$$

where U_T is the total energy expressed by:

$$U_T = U_{No\ Crack} + \int_0^A \frac{\partial U_T}{\partial A}\, dA \tag{8}$$

being dA an increase in the cracked area. By assuming constant loading forces, the total energy rate G is equivalent to the rate of increase of the total strain energy U_T, that is:

$$G = \frac{\partial U_T}{\partial A} \tag{9}$$

the displacement, Δ_F, of a virtual load F can be computed by replacing Eqs (8) and (9) in Eq.(7):

$$\Delta_F = \left(\frac{\partial U_T}{\partial F}\right)_{F=0} = \left(\frac{\partial U_{No\ Crack}}{\partial F}\right)_{F=0} + \int_0^A \left(\frac{\partial G}{\partial F}\right)_{F=0} dA \tag{10}$$

For the plane-stress problem herein examined, that is, a prismatic specimen tested under three-point bending (Fig. 2), the first term on the right-hand side of Eq.(10) is equal to zero, because it corresponds to the displacement produced by F (in their own direction) in an uncracked beam, whereas the second term is a function of the SIF values for each Mode of loading (Kitagawa et al. (1975), Cotterell and Rice (1980), Tada et al. (2000)) due to both the loading force, P, and the virtual force, F.

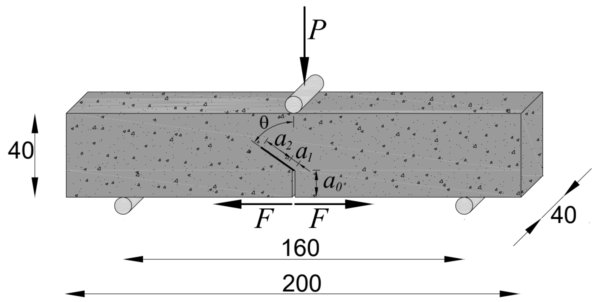

Fig. 2. Specimen geometry and the actual and the virtual forces in Castigliano theorem. Lengths are in mm.

Note that, as is shown in Figure 2, the kinked crack path consists of two segments, named a_1 and a_2. If the value of a_2 obtained from Eq.(6) is negative, it means that the effective crack length is $\underline{a} = a_0 + a_1$ with $a_1 < 0.3\, a_0$. Such a length a_1 is obtained from the following equation by employing an iterative procedure:

$$E = \frac{6S}{C_u W^2 B}\left\{a_0\, V\left(\frac{a_0}{W}\right) + \left[\cos^6\frac{\theta}{2} + \sin^2\frac{\theta}{2}\cos^4\frac{\theta}{2}\right]\left[(a_0 + a_1\cos\theta)\, V\left(\frac{a_0 + a_1\cos\theta}{W}\right) - a_0\, V\left(\frac{a_0}{W}\right)\right]\right\} \tag{11}$$

Finally, the critical stress-intensity factor, $K^S_{(I+II)C}$, is computed through Eqs (4) and (5) by considering a straight crack having length equal to the projected length of the effective kinked crack (Kitagawa et al. (1975); Cotterell and Rice (1980)):

$$K^S_{(I+II)C} = \frac{3P_{\max}}{2W^2 B}\sqrt{\pi\left[a_0 + (a_1 + a_2)\cos\theta\right]}\; f(\alpha) \quad \text{with:} \quad \alpha = \frac{a_0 + (a_1 + a_2)\cos\theta}{W} \quad \text{when } a_1 = 0.3 a_0 \tag{12}$$

or

$$K^S_{(I+II)C} = \frac{3P_{\max}}{2W^2 B}\sqrt{\pi\left[a_0 + a_1\cos\theta\right]}\; f(\alpha) \quad \text{with:} \quad \alpha = \frac{a_0 + a_1\cos\theta}{W} \quad \text{when } a_1 < 0.3 a_0 \tag{13}$$

4. Experimental and theoretical results

Specimens are tested under three-point bending (Figure 2). Testing is performed by means of an Instron 8862 testing machine under crack mouth opening displacement (CMOD) control, employing a clip gauge at an average speed equal to 0.1 mm h^{-1}.

The specimen matrix is a cementitious matrix characterised by the following proportions: cement: water: aggregates (by weight) = 1: 0.7 : 3.6. This mixture presents a compressive strength of 30MPa at 28 days.

Two types of specimens are tested: plain concrete specimens (from P-1 to P-3 in Table 1) and concrete specimens reinforced by randomly-distributed micro-synthetic polypropylene fibrillated fibres (from R25-1 to R25-3 in Table 1). Such fibres are generally used for concrete secondary reinforcement and to control the plastic shrinkage of concrete. The fibre aspect ratio and content are equal to 0.003 (fibre length equal to 18mm) and 2.5% by volume, respectively. The maximum aggregate size is 4mm.

Values of fracture toughness $K^S_{(I+II)C}$ are computed according to Eqs (12) e (13). Values of $K^S_{(I)C}$ are also computed according to Eq. (4) and listed in the last column of Table (1).

Table 1. Elastic modulus, *E,* and critical SIF under Mixed, $K^S_{(I+II)C}$, and Mode I, K^S_{IC} , loading.

Specimen No.	E (MPa)	$K^S_{(I+II)C}$ (MPa m$^{1/2}$)	K^S_{IC} (MPa m$^{1/2}$)
P-1	16028.85	0.460	0.547
P-2	16294.31	0.423	0.473
P-3	16242.12	0.488	0.503
R25-1	17068.80	0.622	0.639
R25-2	16862.08	0.664	0.696
R25-3	16838.15	0.600	0.660

For each specimen, crack growth under Mixed Mode (Figs 3 and 4) is observed. Higher values of crack kinked angle are generally found for plain concrete specimens.

In Table 1, it can be noted that fracture toughness values, $K^S_{(I+II)C}$, for reinforced specimens (averaged value equal to 0.629 MPa m$^{1/2}$) are significantly different from those related to plain specimens (averaged value equal to 0.457 MPa m$^{1/2}$).

Therefore, it can be concluded that randomly-distributed micro-synthetic polypropylene fibrillated fibres are able both to reduce the value of the kinked angle and to increase the concrete resistance to fracture. This is probably due to the fact that such fibres, tending to slip with respect to the matrix, reduce shear stress field with respect to that related to the case where only aggregates are embedded in the matrix. Since lower shear stresses produce a reduction of Mode II loading, both a decrease in kinked angle values and an increase in fracture toughness, $K^S_{(I+II)C}$, are expected to occur.

As is listed in Table 1, the fracture toughness value determined by employing Eq.(4) instead of Eqs (12) and (13) is overestimated up to 19% for the plain concrete specimens, and up to 10% for the fibre-reinforced concrete specimens here examined.

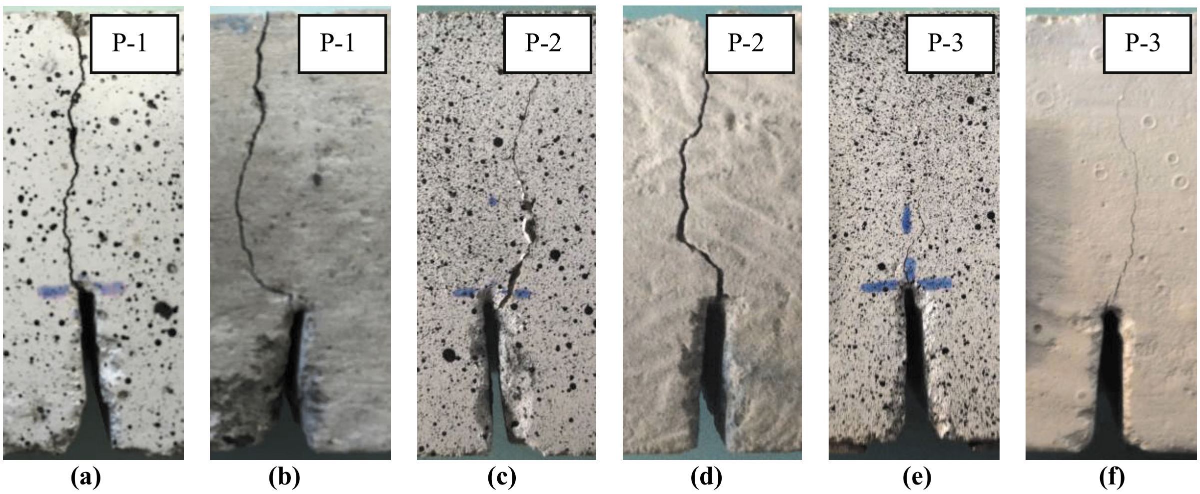

Fig. 3. Front and back side of fracture region of plain concrete specimens: (a)-(b) P-1; (c)-(d) P-2; (e)-(f) P-3.

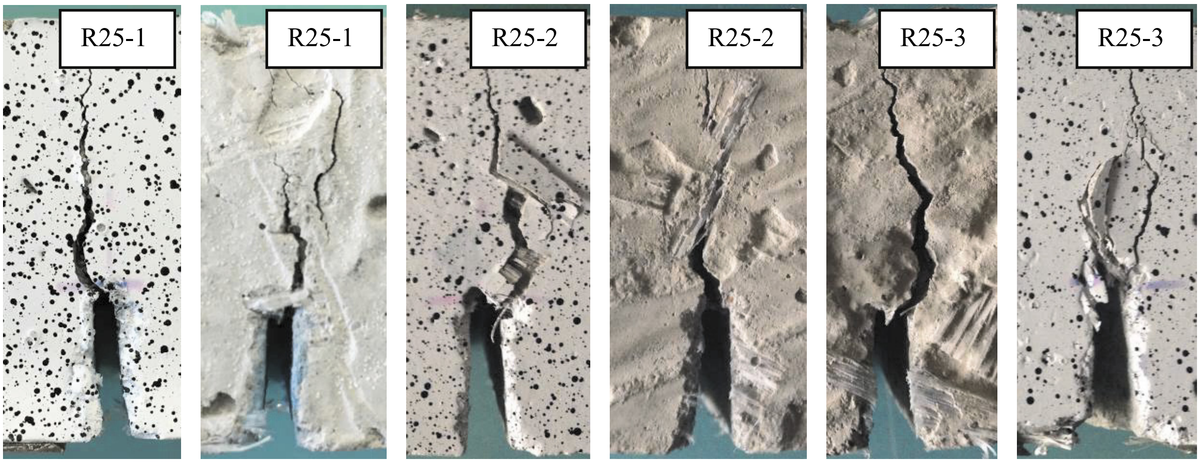

Fig. 4. Front and back side of fracture region of fibre-reinforced concrete specimens: (a)-(b) R25-1; (c)-(d) R25-2; (e)-(f) R25-3.

5. Conclusions

In the present paper, a method to calculate Mode I plane-strain fracture toughness of concrete, by taking into account the possible crack deflection (kinked crack), has been proposed. Concrete and fibre-reinforced concrete are inhomogeneous mixtures due to aggregates and fibres embedded in the cementitious matrix. Due to such microstructural inhomogeneities, cracks may deflect during fracture extension. Therefore, to take into account such a possibility, a modified version of the TPM model has been proposed. Three-point bending tests on both plain concrete specimens and concrete specimens reinforced with micro-synthetic polypropylene fibrillated fibres have experimentally been performed in order to assess the proposed model. It can be concluded that, by applying the TPM according to its original formulation, Mode I plane-strain fracture toughness is overestimated up to 19% for plain concrete and up to 10% for fibre-reinforced concrete.

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