# Quarkonium dissociation in strongly coupled far-from-equilibrium matter: holographic description 

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#### Abstract

The heavy quarkonium real-time dissociation in a strongly coupled non-Abelian matter relaxing to equilibrium is described in a holographic approach. Boundary sourcing, impulsive distortions of the boundary metric, are used to mimic effects driving the matter far-from-equilibrium. Quarkonium is represented by a string with endpoints kept close to the boundary, and its evolution in the time-dependent geometry is studied.


Keywords: Holography and quark-gluon plasma, Quarkonium properties in a thermal medium

## 1. Introduction

The discovery that the QCD matter produced in heavy ion collisions at ultrarelativistic energies behaves as a strongly coupled hot fluid raises many questions, in particular how to describe the real-time relaxation of the fluid from the initial far-from-equilibrium state [1]. Quarkonium dissociation is among the various phenomena to be described in such conditions. Using gauge/gravity duality ("holographic") methods, the relaxation of an out-of-equilibrium strongly coupled system can be mapped into a gravity problem [2]. Although the existence of a gravitational dual of QCD has not been established, insights can be gained by the study of theories sharing various properties with high-T QCD, namely $\mathcal{N}=4$ large $N_{c}$ SYM, dual to $A d S_{5} \times S^{5}$ with a black-hole (BH). In this framework, quarks are represented by open strings in the higher dimensional space [3], and the string evolution is governed by the equations of motion from the NambuGoto action [4]. Among other properties, the screening length for the quarkonium moving in the plasma has been evaluated in this approach [5]. Here, we consider the evolution of a string stretched between two endpoints (representing the heavy quark and antiquark) kept close to the boundary: the string falls down under gravity and can reach the BH horizon, condition interpreted as quarkonium dissociation [6].

A holographic description of out-of-equilibrium matter can be achieved considering a time-dependent dual geometry resulting from a quench. This is a distortion of the boundary metric mimicking an impulsive effect which drives the boundary system far from equilibrium [7]. Denoting the $4 D$ boundary coordinates as $x^{\mu}=\left(x^{0}, x^{1}, x^{2}, x^{3}\right)$, with $x^{3}=x_{\|}$the axis in the collision direction, and considering the case where boost invariance along this axis is imposed, together with rotational and translation invariance in the $x_{\perp}=\left\{x^{1}, x^{2}\right\}$
plane, the $4 D$ line element can be written in terms of the proper time $t$ and of the spatial rapidity $y: d s_{4}^{2}=$ $-d t^{2}+d x_{\perp}^{2}+t^{2} d y^{2}$. A quench is introduced as an anisotropic distortion of the boundary metric:

$$
\begin{equation*}
d s_{4}^{2}=-d t^{2}+e^{\gamma(t)} d x_{\perp}^{2}+t^{2} e^{-2 \gamma(t)} d y^{2} \tag{1}
\end{equation*}
$$

with profile $\gamma(t)$ which is time-dependent. Using Eddington-Finkelstein $5 D$ coordinates, the bulk metric can be written as

$$
\begin{equation*}
d s_{5}^{2}=2 d r d t-A d t^{2}+\Sigma^{2} e^{B} d x_{\perp}^{2}+\Sigma^{2} e^{-2 B} d y^{2} \tag{2}
\end{equation*}
$$

with the boundary reached as the radial coordinate $r \rightarrow \infty$. The metric functions $A(r, t), \Sigma(r, t)$ and $B(r, t)$ are determined solving $5 D$ Einstein vacuum equations with negative cosmological constant, imposing the condition that the geometry (2) reproduces Eq.(1) for $r \rightarrow \infty$, and that the metric functions correspond to the $\mathrm{AdS}_{5}$ ones before the quench is switched on. Several quench profiles have been investigated in [8]. We report the results of two cases, model $\mathcal{A}(2)$ consisting of two overlapping short pulses, and model $\mathcal{B}$, a smooth distortion with a superimposed pulse. Their profiles $\gamma(t)$ are drawn in the upper panels of Fig. 2. The numerical solutions of the Einstein equations determine $A, \Sigma$ and $B$ [8], yielding the time dependence of boundary energy density, parallel and longitudinal pressures and several non-local observables [9]. In all the models, at early-times the geometry abruptly responds to the external distortion, and a black-hole is formed with a rapidly changing horizon position [7, 8]. After the pulses are switched off, a black-brane geometry is recovered, with horizon position given in terms of a $\Lambda$ parameter typical of each model:

$$
\begin{equation*}
r_{H}(t)=\frac{\pi \Lambda}{(\Lambda t)^{1 / 3}}\left[1-\frac{1}{6 \pi(\Lambda t)^{2 / 3}}+\frac{-1+\log 2}{36 \pi^{2}(\Lambda t)^{4 / 3}}+\frac{-21+2 \pi^{2}+51 \log 2-24(\log 2)^{2}}{1944 \pi^{3}(\Lambda t)^{2}}+O\left(\frac{1}{(\Lambda t)^{8 / 3}}\right)\right] \tag{3}
\end{equation*}
$$

and related metric functions dual to viscous hydrodynamics [8].

## 2. Quarkonium evolution and dissociation

We describe a heavy quark-antiquark pair in the far-from-equilibrium medium by a classical string in the space with metric (2) and endpoints kept close to the boundary. The Nambu-Goto action

$$
\begin{equation*}
S_{N G}=-T_{f} \int d \tau d \sigma \sqrt{-g}=\int d \tau d \sigma \mathcal{L}_{N G} \tag{4}
\end{equation*}
$$

with $T_{f}=\frac{1}{2 \pi \alpha^{\prime}}, \alpha^{\prime}=\frac{L^{2}}{\sqrt{\lambda}}, L$ the $\operatorname{AdS}_{5}$ radius, $\lambda$ the 't Hooft coupling, $g$ the determinant of the induced world-sheet metric, and $(\tau, \sigma)$ the world-sheet coordinates, allows to obtain the equation of motion governing the string evolution. The string embedding functions are $X^{M}(\tau, \sigma)$. For strings living in a threedimensional slice of the dual space described by the coordinates $(t, w, r)$, we consider the cases $w=x$, with $x=x_{1}$ or $x=x_{2}$ one of the two transverse coordinates and the string endpoints kept fixed at distance $2 L$ close to the boundary, and $w=y$ along the rapidity axis, representing a quark and an antiquark mutual moving away from each other in the longitudinal direction $x_{\|}$with rapidity $y_{L}$.

In the gauge $\tau=t$ and $\sigma=w$ the string profile is described by the function $r(t, w)$. The Nambu-Goto action $S_{N G}$ is expressed in terms of the functions in Eq. (2):

$$
\begin{equation*}
S_{N G}=-T_{f} \int d t d w \sqrt{\Sigma_{w}(t, r)\left(A(t, r)-2 \partial_{t} r\right)+\left(\partial_{w} r\right)^{2}} \tag{5}
\end{equation*}
$$

where $\Sigma_{w}=\bar{\Sigma}=\Sigma^{2} e^{-2 B}$ if $w=y$, and $\Sigma_{w}=\tilde{\Sigma}=\Sigma^{2} e^{B}$ if $w=x$, then the equation of motion can be worked out. In three simple cases with Eddington-Finkelstein coordinates considered in [10], $\mathrm{AdS}_{5}, \mathrm{AdS}_{5}$ with a BH , and $\mathrm{AdS}_{5}$ with a black-brane (a BH with time-dependent horizon position), the string profile has been determined analytically. The dissociation time is finite in this coordinate system.

The equations for $r(t, w)$, the string profile, have been solved for $w=y$ or $w=x$ imposing suitable initial and boundary conditions. At the initial time $t_{i}$ the string is completely stretched close to the boundary. The


Fig. 1. Quench model $\mathcal{B}$ : string profile $r(t, w)$ corresponding to $\left\{t_{i}, v, L\right\}=\{3,-1,0.1\}$ for the transverse $w=x$ configuration. Left: profile as a function of $x$ at different $t$. Right: solution $r(t, x)$ (sea-green surface). The blue surface corresponds to the horizon.
string endpoints are kept fixed at $w_{Q}=-L$ and $w_{\bar{Q}}=L$ for the transverse configuration, and $w_{Q}=-y_{L}$ and $w_{\bar{Q}}=y_{L}$ for the $w=y$ configuration. We study the dependence on the spatial and rapidity separation varying $L$ and $y_{L}$, as well as the velocity $\dot{r}\left(t_{i}, w\right)=v$ [10].

In Fig. 1 we depict the string profile for model $\mathcal{B}$ in the case $w=x$ and separation $L=0.1$. Analogous results are obtained for the $w=y$ configuration and for model $\mathcal{A}(2)$. The dissociation time $t_{D}$ is the one at which the horizon is reached. The dependence of $t_{D}$ on the starting time $t_{i}$ is shown in Fig. 2 for the two models. In each column we draw the quench profile (upper panel), $t_{D}$ for the longitudinal $w=y$ configuration with $y_{L}=10$ (middle panel), and $t_{D}$ for the transverse $w=x$ configuration with $L=10$


Fig. 2. Quarkonium dissociation time $t_{D}$ versus $t_{i}$ for quench model $\mathcal{A}(2)$ (left) and $\mathcal{B}$ (right), with profile $\gamma(t)$ drawn in the top panels. The middle plots refer to the $w=y$ string configuration (with $y_{L}=10$ ), the bottom plots to $w=x$ (with $L=10$ ), choosing three values of $v$.


Fig. 3. Quarkonium dissociation time $t_{D}$ for model $\mathcal{A}(2)$ (upper plots) and $\mathcal{B}$ (lower plots) and initial velocity $v=-1$. Left panels: $t_{D}$ vs $t_{i}$ and rapidity separation $y_{L}$ in the $w=y$ string configuration. Right panels: $t_{D}$ vs $t_{i}$ and separation $L$ for the $w=x$ transverse configuration.
(bottom panel). The dissociation time exhibits abrupt fluctuations during the quenches. Soon after the end of the quenches, $t_{D}$ changes smoothly, approaching the same value in both models. During the quenches the behavior of $t_{D}$ is different for the longitudinal and transverse string configurations. Dissociation is faster for the string placed in a plane transverse to the collision axis: this agrees with other studies showing that the orientation of the string and the position of its endpoints are relevant for the description of the quarkonium properties using the dual framework [5,11]. The results obtained varying the separation between the string endpoints are illustrated in Fig.3.

Dissociation is a fast phenomenon, regardless of the quenches. Similarly to other observables, such as the boundary energy density, as soon as the last pulse is switched off the behavior of $t_{D}$ in a geometry dual to viscous hydrodynamics is recovered.

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## References

[1] P. Braun-Munzinger, V. Koch, T. Schäfer, J. Stachel, Phys. Rept. 621 (2016) 76. P. Romatschke, Eur. Phys. J. C77 (1) (2017) 21.
[2] J. M. Maldacena, Int.J.Theor.Phys. 38 (1999) 1113. E. Witten, Adv.Theor.Math.Phys. 2 (1998) 253. S. Gubser, I. R. Klebanov, A. M. Polyakov, Phys.Lett. B428 (1998) 105.
[3] A. Karch, E. Katz, JHEP 06 (2002) 043.
[4] S. S. Gubser, Phys. Rev. D74 (2006) 126005. C. P. Herzog, A. Karch, P. Kovtun, C. Kozcaz, L. G. Yaffe, JHEP 07 (2006) 013. J. Casalderrey-Solana, D. Teaney, Phys. Rev. D74 (2006) 085012. M. Chernicoff, J. A. Garcia, A. Guijosa, JHEP 09 (2006) 068. P. M. Chesler, K. Rajagopal, Phys. Rev. D90 (2) (2014) 025033. P. M. Chesler, K. Jensen, A. Karch, Phys. Rev. D79 (2009) 025021. P. M. Chesler, K. Jensen, A. Karch, L. G. Yaffe, Phys. Rev. D79 (2009) 125015. S. S. Gubser, D. R. Gulotta, S. S. Pufu, F. D. Rocha, JHEP 10 (2008) 052.
[5] H. Liu, K. Rajagopal, U. A. Wiedemann, Phys. Rev. Lett. 98 (2007) 182301; JHEP 03 (2007) 066. S. I. Finazzo, J. Noronha, JHEP 01 (2015) 051. M. Ali-Akbari, F. Charmchi, A. Davody, H. Ebrahim, L. Shahkarami, Phys. Rev. D93 (8) (2016) 086005.
[6] S. Lin, E. Shuryak, Phys. Rev. D77 (2008) 085013. I. Iatrakis, D. E. Kharzeev, Phys. Rev. D93 (2016) 086009.
[7] P. M. Chesler, L. G. Yaffe, Phys. Rev. Lett. 102 (2009) 211601; Phys. Rev. D82 (2010) 026006.
[8] L. Bellantuono, P. Colangelo, F. De Fazio, F. Giannuzzi, JHEP 07 (2015) 053.
[9] L. Bellantuono, P. Colangelo, F. De Fazio, F. Giannuzzi, S. Nicotri, Phys. Rev. D94 (2) (2016) 025005.
[10] L. Bellantuono, P. Colangelo, F. De Fazio, F. Giannuzzi, S. Nicotri, Phys. Rev. D96 (3) (2017) 034031.
[11] M. Natsuume, T. Okamura, JHEP 09 (2007) 039. K. Bitaghsir Fadafan, S. K. Tabatabaei, Phys. Rev. D94 (2) (2016) 026007.

